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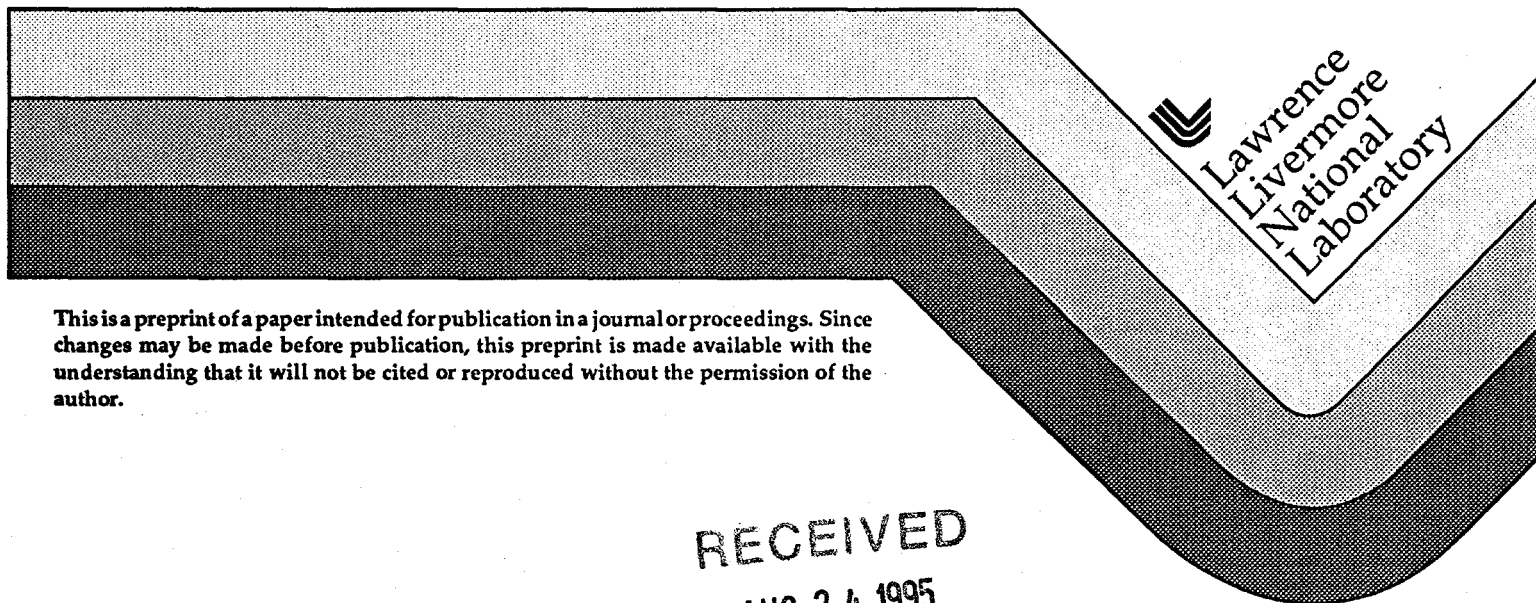
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MODEL-BASED INTERNAL WAVE PROCESSING

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A model-based approach is proposed to solve the oceanic internal wave signal processing problem that is based on state-space representations of the normal-mode vertical velocity and plane wave horizontal velocity propagation models. It is shown that these representations can be utilized to spatially propagate the modal (depth) vertical velocity functions given the basic parameters (wave numbers, Brunt-Vaisala frequency profile etc.) developed from the solution of the associated boundary value problem as well as the horizontal velocity components. Based on this framework, investigations are made of model-based solutions to the signal enhancement problem for internal waves.

I. INTRODUCTION

When operating in a stratified environment like the upper ocean with relatively sharp density gradients, then any excitation that disturbs the pycnocline (density profile) will generate *internal waves* that propagate away from this region (Apel,[1]). Internal waves are volume gravity waves having maximum vertical displacement typically at a plane where the density gradient are largest and are detectable far above and below this interface (Clay,[2]). They can be generated from tidal flow against islands, sea mounts and continental shelf edges or surface/internal wave interactions created displacements in the pycnocline. For instance, a ship traveling along the surface of a stratified ocean creates various visible wakes: the turbulent or centerline wake, the Kelvin wake and, of most interest in this work, the surface generated internal waves. Internal waves have been measured experimentally both in controlled environments as well as the open ocean (Garrett,[3]) and observed using synthetic aperture radar processing techniques from satellite imagery (Alpers,[4]; Thompson,[5]). From the scientific viewpoint, it is of high interest to understand the effect of internal waves on acoustic propagation in the ocean [2] as well as the ability to measure their effect directly using current sensor technology. Military applications are obvious, since a submerged body moving through the ocean environment disturbing the pycnocline generates internal wave signatures.

The inclusion of a propagation model in any oceanic signal processing scheme provides a means of introducing environmental information in a self-consistent manner. Recent work in ocean acoustics (Candy and Sullivan,[6]) has shown that a propagation model can be imbedded into a signal processing scheme to solve various enhancement, localization and detection problems. In this paper, we propose a model-based approach to the internal wave signal processing problem founded on a state-space representation of the normal mode and plane wave models of propagation. Specifically, using the normal mode model of the wave velocity field, the vertical velocity modal functions and the horizontal velocity can be estimated from noisy sensor array measurements in the following way. First, the propagation model is cast into state-space form. It is shown that this representation can be used to propagate the modal functions, given the basic parameters (wave numbers etc.) developed from the solution of the associated boundary value problem. There are basically two sets of equations in this representation: the state equation and the measurement

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equation. The state equation describes the evolution in space of the vertical velocity modal and horizontal velocity functions, whereas the measurement equation relates the states to the actual array measurements. In the stochastic case, an approximate Gauss-Markov representation evolves. Once this framework is established, as will be shown in what follows, we investigate model-based solutions to the signal enhancement. We characterize a realizable, recursive processor as shown in Figure 1 (Candy,[7]; Jazwinski,[8]).

In the next section the state space representation of the internal wave propagation model is developed for both vertical and horizontal propagation. In the vertical, the so-called "forward propagator" is defined, while in the horizontal, plane wave propagators adequately characterize the propagation. Next the measurement equations are developed for both a vertical and then horizontal sensor array. The state-space model is then explicitly formulated for the case of all model parameters known where estimates of the velocity field and the modal functions are made and various noise models can be exercised in order to simulate different sources of modal and measurement noise. Next a dispersive internal wave model-based processor is designed which uses a horizontal array of sensors modeling the configuration employed in the Loch Linnhe experiments performed in Scotland during the summer of 1994. It is shown that the internal wave can be successfully estimated from noisy measurements. The final section summarizes the results.

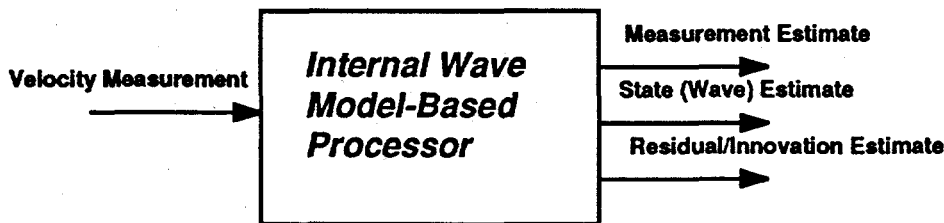


Figure 1. Basic Internal Wave Model-Based Processor.

II. INTERNAL WAVE STATE-SPACE PROPAGATION MODELS

In this section we investigate the feasibility of developing state-space propagation models from the corresponding internal wave dynamics. The development of the wave equation associated with internal wave propagation evolves from the small perturbation momentum equations under the assumptions of linearity, incompressibility, zero mean shear, a Boussinesq fluid, flat or slowly varying ocean bottom, and a horizontally homogeneous or slowly varying density field. Under these assumptions, the small perturbation component momentum equations governing the propagation of the *vector* velocity field defined are by $\chi(x, y, z, t) := [u(x, y, z, t)v(x, y, z, t)w(x, y, z, t)]'$ where u , v , w represent the respective *on-track velocity*, *cross-track velocity*, and *vertical velocity* components of the vector velocity field.

The solution to the homogeneous wave equation is accomplished by using the separation of variables approach (see Apel,[1]) with velocity function given by $w(x, y, z, t) = \mu(x)\nu(y)\phi(z)\tau(t)$.

where plane wave propagation in the respective horizontal dimensions, μ and ν are identical, ϕ is the vertical distribution and τ is the corresponding temporal function. Substituting, collecting terms, etc., it is possible to show that the function associated with each independent variable can be separated yielding a set of coupled ordinary differential equations with the corresponding separation constants shown as

$$\begin{aligned}
 \frac{d^2}{dx^2}\mu(x) + \kappa_x^2\mu(x) &= 0 \\
 \frac{d^2}{dy^2}\nu(y) + \kappa_y^2\nu(y) &= 0 \\
 \frac{d^2}{dz^2}\phi(z) + \kappa_h^2\left(\frac{N^2(z)}{\omega^2} - 1\right)\phi(z) &= 0 \\
 \frac{d^2}{dt^2}\tau(t) + \omega^2\tau(t) &= 0
 \end{aligned} \tag{1}$$

where κ_x and κ_y are the horizontal wave numbers with $\kappa_h^2 = \kappa_x^2 + \kappa_y^2$, and ω^2 the temporal frequency (a function of κ_h). These relations describe the dynamics of the vertical velocity component of an internal wave in a constant depth ocean. It is clear that the first two and last equations above admit harmonic solutions, that is, $\mu(x) = e^{i\kappa_x x}$ $\nu(y) = e^{i\kappa_y y}$ $\tau(t) = e^{-i\omega t}$ while the depth relation is an eigen-equation in z with

$$\frac{d^2}{dz^2}\phi_m(z) + \kappa_h^2\left(\frac{N^2(z)}{\omega_m^2} - 1\right)\phi_m(z) = 0, \quad m = 1, \dots, M \tag{2}$$

whose eigen-solutions $\{\phi_m(z)\}$ are the so-called *modal functions* associated with the vertical velocity component, κ_h is the *horizontal wavenumber*, $N(z)$ is the *Brunt-Vaisala frequency profile* (BVP) and ω_m is the corresponding *eigenvalue* associated with the m^{th} -modal function. State-space models will eventually be employed as "forward" propagators in model-based signal processing schemes (Candy,[7]). Note that this approach does *not* offer a new solution to the resulting boundary value problem, but, in fact, it requires that solution be available *a-priori* in order to propagate the normal-modes recursively in an initial value scheme.

Since Eq. 2 is a linear, space-varying coefficient (for each layer) differential equation, it can easily be placed in state-space form where the state-vector for each mode is defined as $\mathbf{x}_m := [\phi_m(z) \quad \frac{d}{dz}\phi_m(z)]'$. Examining mode propagation in more detail, we see that each mode is characterized by a set of ordinary differential equations which can be written as

$$\frac{d}{dz}\mathbf{x}_m(z) = A_m(z)\mathbf{x}_m(z) + B_m(z)\mathbf{u}_m(z) = \begin{bmatrix} 0 & 1 \\ -\kappa_h^2 \left(\frac{N^2(z)}{\omega_m^2} - 1 \right) & 0 \end{bmatrix} \mathbf{x}_m(z) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u}_m(z) \quad (3)$$

where $\mathbf{u}(z) = \frac{ds(z)}{dz}$ is the source driving function (a scalar in our problem). With the parameters $\{[\kappa_h], N(z), \{\omega_m\}\}$ known, we see that we have constructed a *forward propagator* for this problem driven by the source and initial condition vector, $\mathbf{x}_m(z_0)$ for each mode. From linear systems theory (see Chen,[9]), it is well-known that the general solution to this equation is governed by the state-transition matrix, $\Phi(z, z_0)$, where the state equation is solved by

$$\mathbf{x}_m(z) = \Phi_m(z, z_0)\mathbf{x}_m(z_0) + \int_{z_0}^z \Phi_m(z, \alpha)B_m(\alpha)\mathbf{u}_m(\alpha)d\alpha, \quad m = 1, \dots, M \quad (4)$$

and the state transition matrix satisfies

$$\frac{d}{dz}\Phi_m(z, z_0) = A_m(z)\Phi_m(z, z_0), \quad m = 1, \dots, M \quad (5)$$

with $\Phi_m(z_0, z_0) = I$.

If we include all of the M -modes in the model, then we obtain

$$\frac{d}{dz}\mathbf{x}_w(z) = \begin{bmatrix} A_1(z) & \dots & O \\ \vdots & & \vdots \\ O & \dots & A_M(z) \end{bmatrix} \mathbf{x}_w(z) + \begin{bmatrix} B_1(z) \\ \vdots \\ B_M(z) \end{bmatrix} \mathbf{u}(z) \quad (6)$$

or simply

$$\frac{d}{dz}\mathbf{x}_w(z) = A(z)\mathbf{x}_w(z) + B(z)\mathbf{u}(z) \quad (7)$$

The general solution to these modal state-space vertical velocity propagation equations are given by Eq. 10 over all the modes as

$$\mathbf{x}(z) = \Phi(z, z_0)\mathbf{x}(z_0) + \int_{z_0}^z \Phi(z, \alpha)B(\alpha)\mathbf{u}(\alpha)d\alpha \quad (8)$$

and as before the state transition matrix satisfies

$$\frac{d}{dz}\Phi(z, z_0) = A(z)\Phi(z, z_0), \quad \Phi(z_0, z_0) = I \quad (9)$$

with $\Phi(z, z_0) = \text{diag}[\Phi_1(z, z_0), \dots, \Phi_M(z, z_0)]$.

The solutions in the other horizontal dimensions, x, y are also possible in state-space form, if desired, and take on the same form for each relation with the separation constants interchanged. For instance, the component $\nu(y)$ satisfies the following state equations with the state vector defined as $\mathbf{x}(y) := [\nu(y) \quad \frac{d}{dy}\nu(y)]'$

$$\frac{d}{dy}\mathbf{x}(y) = A_y\mathbf{x}(y) \quad (10)$$

where

$$A_y := \begin{bmatrix} 0 & 1 \\ -\kappa_y^2 & 0 \end{bmatrix} \quad (11)$$

which gives the plane wave solution of Eq. 7 above. We note that in this formulation the *strain rate*, which is an important quantity for radar images, appears as a component of the state vector. Thus, we see that using the state-space formalism enables us to characterize the propagation (forward) of the vertical internal wave velocity component which will prove useful for processing. Next we consider characterizing the corresponding measurement system.

Sensor technology enables us to measure components of internal wave cross-track velocity at depth z , $v(x, y, z, t)$ as well as on-track velocity $u(x, y, z, t)$. Let us investigate the deployment of a horizontal array of current meter sensors positioned to measure the cross-track velocity. Actual sensors sample the cross-track velocity in time at given spatial positions. To obtain the sensor output we must specify the functions $\mu(x)$, $\nu(y)$, and $\tau(t)$ as well as $\phi_v(z)$. The measurement at the n^{th} horizontal sensor at the location (x_n, y_n, z_ℓ) is given by

$$v(x_n, y_n, z_\ell, t) = \mu(x_n)\nu(y_n)\phi_v(z_\ell)\tau(t) = A \left(\frac{\kappa_y}{\kappa_h^2} \frac{d\phi(z_\ell)}{dz} e^{i(\kappa_x x_n + \kappa_y y_n - \omega(\kappa_h)t + \pi/2)} \right) \quad (12)$$

where A is a complex amplitude, and $\omega(\kappa_h)$ is the angular frequency which is a function of the wave

number magnitude given by, $\kappa_h = \sqrt{\kappa_x^2 + \kappa_y^2}$. The sensor measures the real part of the velocity field so we can write (using the magnitude phase form of A)

$$v(x_n, y_n, z_\ell, t) = -|A| \frac{\kappa_y}{\kappa_h^2} \frac{d\phi(z_\ell)}{dz} \sin(\kappa_x x_n + \kappa_y y_n - \omega(\kappa_h)t + \theta) \quad (13)$$

Since we have M -modal velocities, then (as before)

$$v(x_n, y_n, z_\ell, t) = \mathbf{C}_y^T \mathbf{x}(z_\ell) \quad (14)$$

with $\mathbf{C}_y^T = \frac{\kappa_y}{\kappa_h^2} [0 \ \sigma_1(x, y, t) \ | \ \dots \ | \ 0 \ \sigma_M(x, y, t)]$, and $\sigma_m(x_m, y_m, t) = |A_m| \sin(\kappa_x x_n + \kappa_y y_n - \omega_m(\kappa_h)t + \theta_m)$.

Incorporating *both* the vertical and horizontal state-space representations simultaneously leads to the set of coupled equations:

$$\begin{aligned} \frac{d}{dz} \mathbf{x}_w(z) &= A(z) \mathbf{x}_w(z) + B(z) \mathbf{u}(z) \\ \phi(z_\ell) &= \mathbf{C}_\phi^T \mathbf{x}_w(z_\ell) \end{aligned} \quad (15)$$

where $\mathbf{C}_\phi^T = \frac{\kappa_y}{\kappa_h^2} e^{i\pi/2} [0 \ 1 \ | \ \dots \ | \ 0 \ 1]$

$$\begin{aligned} \frac{d}{dy} \mathbf{x}(y) &= A_y \mathbf{x}(y) \\ \nu(y_n) &= \mathbf{C}_\nu^T \mathbf{x}(y_n) \end{aligned} \quad (16)$$

with $\mathbf{C}_\nu^T = [1 \ 0 \ | \ \dots \ | \ 1 \ 0]$ and the overall velocity measurement is

$$\begin{aligned} v(x, y_n, z_\ell, t) &= \nu^T(y_n) \mathbf{C}_v(x, t) \phi_v(z_\ell) \\ \mathbf{C}_v(x, t) &= \text{diag}[A_1 \mu_1(x) \tau_1(t) \ \dots \ A_M \mu_M(x) \tau_M(t)] \end{aligned} \quad (17)$$

which is the equivalent of a two-dimensional measurement taking into account both vertical and horizontal velocity measurements. So we see that the state-space representations enables alot of flexibility in modeling both the phenomenology (internal wave dynamics) as well as the accompanying measurement systems.

This constitutes a complete deterministic representation of the normal-mode and plane wave models in state-space form. However, since propagation in the ocean is effected by inhomogeneities in the water, slow variations in the BVP and motion of the surface, the model must be modified in order to include these effects. This can be done in a natural way by placing the model into a Gauss-Markov representation which includes the second order statistics of the measurement as well as the velocity noise. The measurement noise can represent the near-field velocity noise field, flow noise on the current meter and electronic noise. The modal noise can represent BVP errors, distant shipping noise, errors in the boundary conditions, sea state effects and ocean inhomogeneities. Besides the ability to lump the various noise terms into the model, the Gauss-Markov representation provides a framework in which the various statistics associated with the model such as the means and their associated covariances can be computed. This completes the state-space representations of internal wave dynamics. Next we investigate horizontal solutions using dispersive plane wave representations in the next section, develop the model-based processor, perform various simulations and apply the results to actual measurement data.

III. CROSS-TRACK VELOCITY ESTIMATION

In this section we use a model developed for the nondispersive case [12] and extend it to the dispersive by including more sophisticated dispersion models based on some empirical results [13] and approximating the appropriate temporal derivative. To extend these results (approximately) to the dispersive case, that is, the case where the temporal frequency is no longer constant (narrowband) but varies temporally, we return to the original development of the plane wave processor of the previous section. Recall that the states were defined as, $\mathbf{x}(t) := [v(t) \frac{d}{dt} v(t)]^T$ and now from Eq. 45 with both frequency and wave number temporal functions, we have

$$v(y, t) = \cos(\kappa_y(t)y - \omega(t)t) = \cos \kappa_y(t)y \cos \omega(t)t + \sin \kappa_y(t)y \sin \omega(t)t \quad (18)$$

From our definition of the temporal state vector, we have that $\tau(t) := \cos \omega(t)t$ and approximate its corresponding derivative using

$$\frac{d}{dt} \tau(t) = -\omega(t) \left[1 + \frac{\dot{\omega}(t)}{\omega(t)} t \right] \sin \omega(t)t \approx -\omega(t) \sin \omega(t)t \quad (19)$$

Defining the notation for the dispersive case, we have for the ℓ^{th} -sensor

$$\alpha_\ell(t) := \cos \kappa_y(t)y_\ell, \quad \beta_\ell(t) := \frac{\sin \kappa_y(t)y_\ell}{\omega(t)} \quad (20)$$

therefore, the measurement becomes

$$v(y_\ell, t) = \alpha_\ell(t) \cos \omega(t)t - \beta_\ell(t) \sin \omega(t)t = \alpha_\ell(t)\tau_1(t) - \beta_\ell(t)\omega(t)\tau_2(t) \quad (21)$$

Next we must develop the relations for the dispersive case. For a plane wave propagating in the y-direction, the value of wave number at time t is a function of *group speed* $c_g(\kappa_y)$, that is,

$$y_\ell = c_g(\kappa_y(t)) t \quad (22)$$

with corresponding temporal frequency $\omega(\kappa_y(t), t)$. Using the Barber approximation [13] for internal wave dispersion and group speed, we have

$$\omega(\kappa_y(t), t) = \frac{\kappa_y(t)C_o}{1 + \kappa_y(t)C_o/N_o} \quad (23)$$

and

$$c_g(\kappa_y(t)) = \frac{C_o}{(1 + \kappa_y(t)C_o/N_o)^2} \quad (24)$$

Substituting Eq. 24 for c_g in Eq. 22 and solving for the wave number at the ℓ^{th} -sensor, we obtain

$$\kappa_y(t) = \frac{N_o}{C_o} \left(\sqrt{\frac{C_o t}{y_\ell}} - 1 \right) \quad (25)$$

Substituting this result into Eq. 23, the corresponding temporal frequency is given by

$$\omega_\ell(t) = N_o \left(1 - \sqrt{\frac{y_\ell}{C_o t}} \right) \quad t > \frac{y_\ell}{C_o} \quad (26)$$

Further assuming a bandlimited pulse with minimum frequency ω_o and observation time at the ℓ^{th} -sensor given by

$$t_{o\ell} = \frac{y_\ell}{C_o(1 - \omega_o/N_o)^2} \quad (27)$$

With this approximate dynamic dispersive model, we developed a Gauss-Markov simulation with the corresponding frequency and wave numbers varying temporally according to the dispersion relation. This presents no particular problem in the Gauss-Markov formulation, since it can support nonlinear dynamics as well as nonstationary statistics. We performed the simulation with the same basic set of parameters and the results for a $-13dB$ SNR were quite successful indicating an optimal design.

Next we consider the application of the dispersive model-based processor to an internal wave field experiment performed in Loch Linnhe, Scotland in September of 1994 (Mantrom,[14]; Robey,[15]). The general objective of this experiment is to examine the relationship between modulations observed in radar images and ship generated, internal waves by fusing the data with that measured by an array of current meter sensors. Loch Linnhe is a narrow, salt water estuary located on the west coast of Scotland which possesses favorable subsurface environmental conditions (stratification) which are conducive for the generation and propagation of internal waves. The internal waves are generated by surface ships with concurrent oceanographic measurements (temperature, salinity, BV frequency, meteorology, etc.) performed by other vessels in the Loch. Internal waves are typically imaged by airborne radar systems employing synthetic aperture radar techniques; however, this experiment concentrated on real aperture radar images of the waves from low-grazing angle marine radars positioned on the surrounding hillsides and an array of current meter sensors which provide measurements of hydrodynamic currents associated with the internal waves. The Lawrence Livermore National Laboratory (LLNL) current meter array (CMA) measurement platform consists of ten (10) standard S4 current meters uniformly spaced at $3.75m$, longitudinally, creating a $34m$ aperture, positioned at a depth of $2m$ and sampled at a $10sec$ rate. The CMA is positioned to measure both cross-track and on-track velocities. A typical measurement is shown in Figure 3a, where we see the output of each sensor channel typifying the phase front of an internal wave and subsequent ambient noise.

We processed the Loch Linnhe experimental data using the dispersive rather than plane wave processor and the results are shown in Figure 2. Here we see the differences in using the dispersive over the nondispersive approach. First, the processing doesn't really begin until the "onset" has begun and then the MBP utilizes the dispersive model, until that time a simple random walk model is employed (constant + gaussian noise). Next we note that each internal wave component temporal signal is clearly visible especially those which meet the onset criterion established by the dispersion relation. Those non-propagating wavefronts are attenuated quite heavily. We see the enhanced internal wave propagating across the array based on the onset and all other events attenuated significantly. We note the overall selectivity of the MBP to events that "match" both its dispersive signal model as well as onset. This completes the development of the dispersive MBP for internal wave enhancement and its application to experimental data, we summarize our results and place them in perspective.

IV. DISCUSSION

We have developed the model-based approach to processing internal waves. It was shown

how to develop the underlying state-space representations of the internal wave propagation model. In the vertical, a normal-mode propagation model resulted, while in the corresponding horizontal plane wave propagation models were developed. Since our primary motivation was to process measurements from a horizontal array of current meter sensors, we developed the horizontal (plane wave) processors for both the nondispersive and dispersive cases. After using the model to develop Gauss-Markov stochastic simulations, the model-based processors were designed using the extended Kalman filter (EKF) algorithm [7] for signal enhancement. After designing the MBP, the minimum variance estimates were shown capable of extracting the desired wavefronts from simulated data quite effectively.

We then applied the various MBP to sets of data gathered from experiments performed in Loch Linnhe, Scotland during the summer of 1994 [14,15] and the results are quite promising. It was shown that the nondispersive plane wave MBP was capable of selecting all wave fronts that matched the correct temporal frequency and prescribed onset arrival times while attenuating all others. The dispersive MBP design approximated the true dispersive solution for slow changes in temporal frequency and appears to be a more effective approach, since it depends heavily on the pre-specified wavefront onset arrivals at the array and therefore is more discriminating than the nondispersive design.

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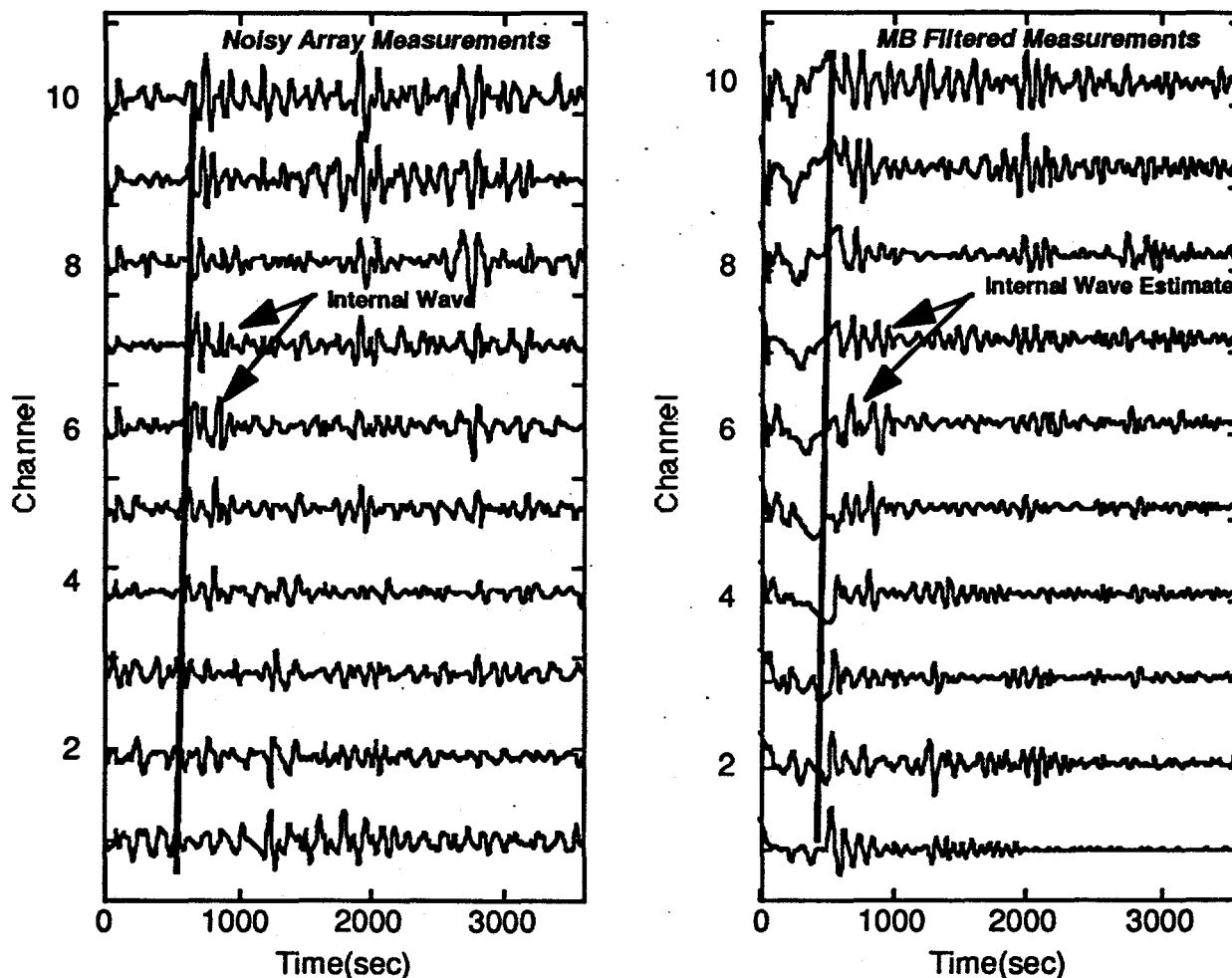


Figure 2. Dispersive Wave MBP Design for Loch Linnhe Experimental Array (10-elements): (a) Preprocessed Sensor Signals (s7r1). (b) Enhanced Sensor Estimates.

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