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ABSTRACT

The description of the flow of two-phase fluids is important in many engineering devices. Unexpected transient conditions which occur in these devices cannot, in general, be treated with single-component momentum equations. Instead, the use of momentum equations for each phase is necessary in order to describe the varied transient situations which can occur. These transient conditions can include phases moving in the opposite directions, such as steam moving upward and liquid moving downward, as well as phases moving in the same direction. The derivation of continuity and momentum equations for each phase and an overall energy equation for the mixture are presented. Terms describing interphase forces are described. A seriated (series of) continuum is distinguished from an interpenetrating medium by the representation of interphase friction with velocity differences in the former and velocity gradients in the latter. The seriated continuum also considers imbedded stationary solid surfaces such as occur in nuclear reactor cores. These stationary surfaces are taken into account with source terms. Sufficient constitutive equations are presented to form a complete set of equations. Methods are presented to show that all these coefficients are determinable from microscopic models and well known experimental results. Comparison of the present derivation with previous work is also given. The equations derived here may also be employed in certain multiphase, multicomponent flow applications.

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GOVERNING EQUATIONS FOR A SERIATED CONTINUUM: AN UNEQUAL VELOCITY MODEL FOR TWO-PHASE FLOW

I. INTRODUCTION

Theoretical investigation of the continuum mechanics of mixtures of fluids, both reacting and nonreacting, has been of interest for some time. In these investigations, mass, momentum, and energy field balance equations and constitutive equations are written for each of the constituents of the mixture. Several approaches have been taken depending on the primary interest of each researcher.

Basic differences exist between a seriated continuum and an interpenetrating continuum. A two-phase mixture is an example of a seriated continuum, whereas a mixture of gases is an example of an interpenetrating continuum. Due to the definition of a seriated continuum, all interpenetrating continuums are seriated continuums but not conversely. Therefore, the basic equations of change are to be expected to contain more terms for a seriated continuum than for an interpenetrating continuum.

Comprehensive, formal mathematical theories of interpenetrating mixtures have been given by Truesdell and Toupin (1960), Truesdell and Noll (1965), Kelly (1964), Green and Naghdi (1965, 1967, 1971), Eringen and Ingram (1965), Ingram and Eringen (1967), and Bowen (1967). Differences have been evidenced in the results among the workers in this area due in part to the originality and difficulty of the subject matter. Truesdell (1969) has summarized existing mixture continuum theories and indicated that the results of Müller (1967, 1968) and Dunwoody and Müller (1968) are rigorously consistent with classical continuum mechanics theory. These generalized continuum mechanics studies are, for the most part, formal mathematical investigations that often result in equations with undetermined coefficients. In particular, the constitutive equations associated with these theories are quite complex and numerical values of most of the coefficients are not available. Application of the results of these theories is limited to special cases such as the mixture of linearly viscous, incompressible fluids considered by Craine (1971).

Parts of the results of the above mentioned studies have been employed in the more practical areas of engineering fluid flow such as solid-fluid suspensions; combustion; dispersed single-component, two-phase flow; chemically reacting gas flows; the flow of blood in circulatory systems; fluidization; and petroleum industry processes. Many of these areas have been discussed in the books by Soo (1967) and Wallis (1969).

A large number of derivations of the balance equations are available for multiphase, multicomponent systems for engineering applications. The fact that each researcher is primarily interested in a particular system or flow regime has led to conflicts and differences in the resulting equations. A few derivations in the primary area of interest are listed here: Murray (1965), fluidized beds; Panton (1968), gas-particle mixtures; Anderson and Jackson (1967), fluidized beds; Kalinin (1970), liquid-phase dispersed, two-phase, single-component flows with interphase mass transfer; Bouré et al (1971) and Delhaye (1969), gas-liquid flows; and Mecredy and Hamilton (1972), who have adapted gas-solid equations given by Soo (1967) to investigate the isentropic sound speed of liquid-vapor (single-component) mixtures.

In the work reported herein, the simultaneous flow of the liquid and vapor phases of a single-component fluid is considered. In addition, the phase change processes of evaporation of the liquid phase and condensation of the vapor phase are considered. These flows present an extremely complex situation for analytical and experimental study due to the presence of both phases of the fluid, the phase change processes that occur, and the phase distribution patterns that arise. The existence of continuous and discrete regions of liquid and vapor phases is important in both theoretical and experimental studies of these flows. In particular, the use of pseudothermophysical coefficients (for example, component coefficients weighted by the void fraction) in existing classical constitutive equations cannot be considered valid for analysis of these flows. Instead, rigorous theoretical analyses of two-phase, single-component flows should be conducted by solving the Navier-Stokes equations (if each phase obeys a linear stress-rate-of-strain relation) for each continuous vapor and liquid region in the flow field and coupling these with the associated interphase mass, momentum, and energy transfer conditions. The difficulties associated with forced convection evaporation and condensation analyses have led to most of the knowledge of these flows having been gained through experimental data. Little of these data have been analyzed with the use of differential models that apply to each phase. Instead, in many cases, correlations and concepts are extrapolated from single-phase flow analyses and are employed in attempts to correlate the two-phase flow data. Although some success has been attained by this method, correlation of two-phase flow phenomena, in general, has been less accurate and dependable than correlation of single-phase turbulent flow phenomena. For example, the pressure gradient in two-phase, single-component flow is of great practical importance and, consequently, has been one of the most studied areas associated with these flows. However, continuous evaluation of experimental data and comparisons of correlation predictions with these data have shown that prediction methods of the two-phase pressure gradient have yet to be formulated in a general manner. Brodkey (1967) has discussed two-phase pressure gradient correlations and their associated accuracies.

Some theoretical analyses are available for special flow regimes (that is, for phase distribution patterns) of steady state flow evaporation and condensation. The liquid (or vapor) dispersed regime and the annular flow regime have been studied more than others primarily due to their relative simplicity and similarity to other engineering flow situations. During transients that may be associated with two-phase flow equipment, several flow regimes may be encountered and, thus, if transient momentum equations are to be written and solved, to allow for the occurrence of all flow regimes seems best.

This report is concerned with momentum equations and mechanical constitutive equations that are required to describe transient, two-phase, single-component evaporating and condensing flows. Momentum field balance equations are derived for each phase on the basis of a seriated-continuum approach. The present formulation introduces those concepts required to extend generalized continuum analyses and formulations to a system of equations that can be used to calculate the velocity of each phase and the pressure gradient for two-phase, single-component flows. In Section II, the limiting process used to define the model of the two-phase media being considered is described. The resulting definition is applied in Sections III and IV to obtain continuity and momentum equations. The energy equation for the mixture is derived in Section V in terms of component velocities. The remaining equations which are required to close the set of equations, such as the equations of state, are described in Section VI. The correlations and analyses required to represent the mechanical constitutive equations are discussed in Section VII.

II. LIMITING PROCESS WHICH DEFINES A SERIATED CONTINUUM

Several methods have been employed to obtain field balance equations for flowing mixtures. Generalized continuum studies have for the most part employed a general property balance integral equation written for each constituent of the mixture along with discontinuity conditions for the constituent interfaces. Time-and-area-averaging techniques have been employed in several derivations associated with engineering applications. For example, Panton (1968) employed area averages of time-averaged flow field properties, Delhaye (1969) used space-averaging, time-averaging, and space-and-time-averaging procedures, and Birkhoff (1964) has indicated that the use of averages with mixture balance equations must be corrected for the effect of using the product of the averages of factors instead of the average of a product of factors. Statistical methods have been employed by Tam (1969), Buyevich et al (1969), and Buyevich (1971) in derivations of multiphase flow equations.

Two different Eulerian methods may be used to obtain the space-averaged, two-phase equations. The first method involves the derivation of balances on a control volume of finite size in analogy to the method employed in Bird, Stewart, and Lightfoot (1960). The second method begins with the well known single-phase balance equations. These equations are then integrated over a control volume. The resulting integrals are approximated in terms of averaged properties. This method is illustrated by Panton (1968) and Anderson and Jackson (1967). Both methods should yield equivalent equations. Although only the first method is presented in this derivation, the authors and their associates have used the second method to provide a check on this derivation. The first method is better for determining explicit forms for constitutive relations. It also indicates the spacial resolution which can be expected from the results of such an equation set. The second method is better for determining the assumptions required in deriving averaged quantities.

Figure 1 shows a portion of a vapor-liquid mixture with three possible control volumes superimposed. The limitations associated with the control volume-limiting process procedure can be illustrated by considering the density of the liquid phase as determined by the control volume. The partial densities of the liquid and vapor phases could but will not be defined as

$$\tilde{\rho}^{\ell} \equiv \lim_{\Delta V \to 0} \bar{\rho}^{\ell} \tag{1}$$

and

$$\tilde{\rho}^{g} \equiv \lim_{\Delta V \to 0} \bar{\rho}^{g} \tag{2}$$

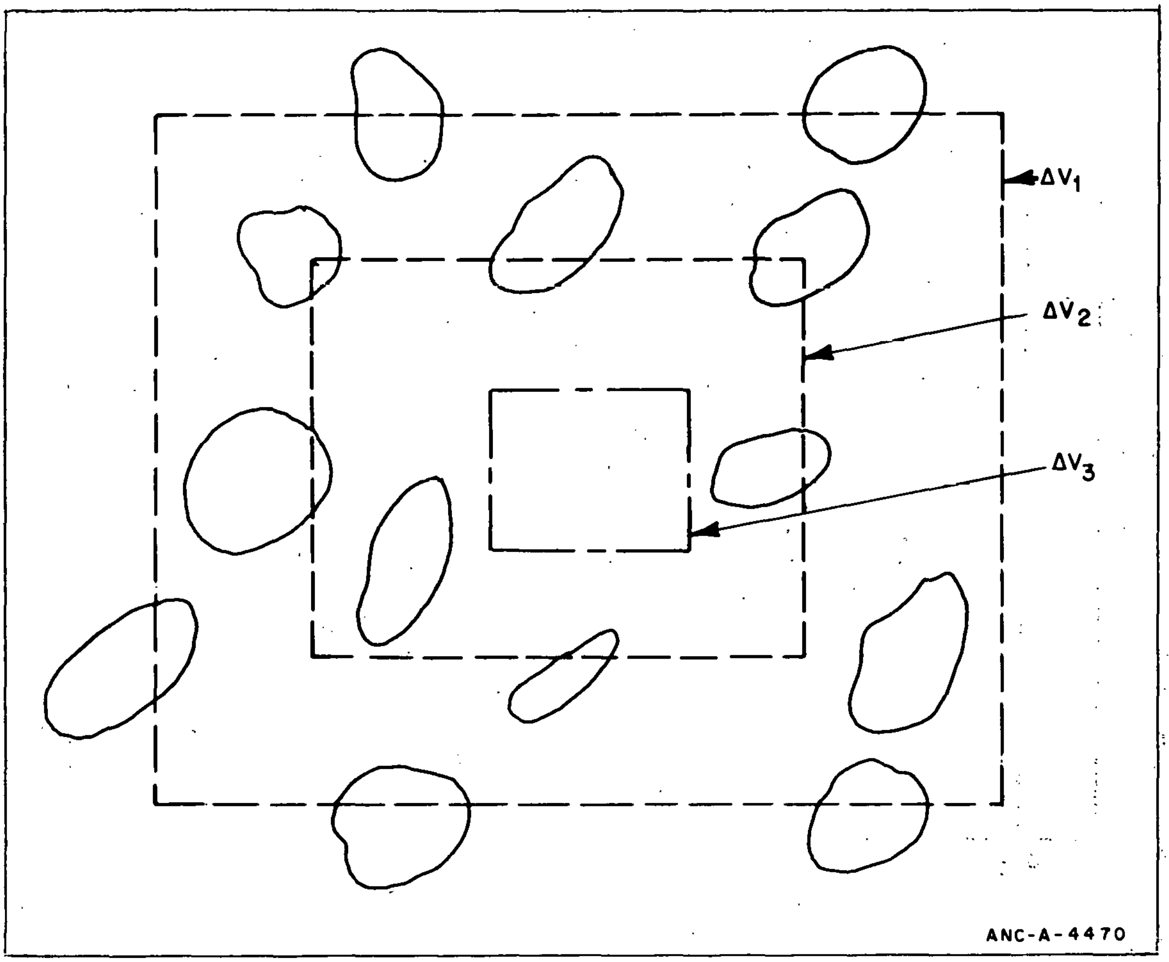

Fig. 1 A two-phase mixture with superimposed control volumes.

where $\bar{\rho}^{\ell}$ = (mass of liquid in the volume)/(total volume)

and $\bar{\rho}^{g}$ = (mass of vapor in the volume)/(total volume).

These definitions are unsatisfactory for the following reasons. A hypothetical distribution of $\bar{\rho}^{\ell}$ for different size control volumes is shown in Figure 2 where ρ_{ℓ} is the thermodynamic density of the liquid. If the control volume ΔV_3 had been located in a region occupied entirely by vapor, the value of $\bar{\rho}^{\ell}$ would have been zero rather than ρ_{ℓ}. Thus, this definition of the partial density of the liquid may exhibit a discontinuous distribution. The same discontinuity occurs in the mechanics of a single-phase continuum. The mass-to-volume ratio at a point in a single-phase continuum would differ considerably depending upon whether the point in question was located in a molecule (which would yield a large mass-to-volume ratio) or in the space between (which would yield a zero ratio). The difficulty could be further compounded by considering the spaces within the molecule. This difficulty in continuum mechanics is resolved by defining the density as

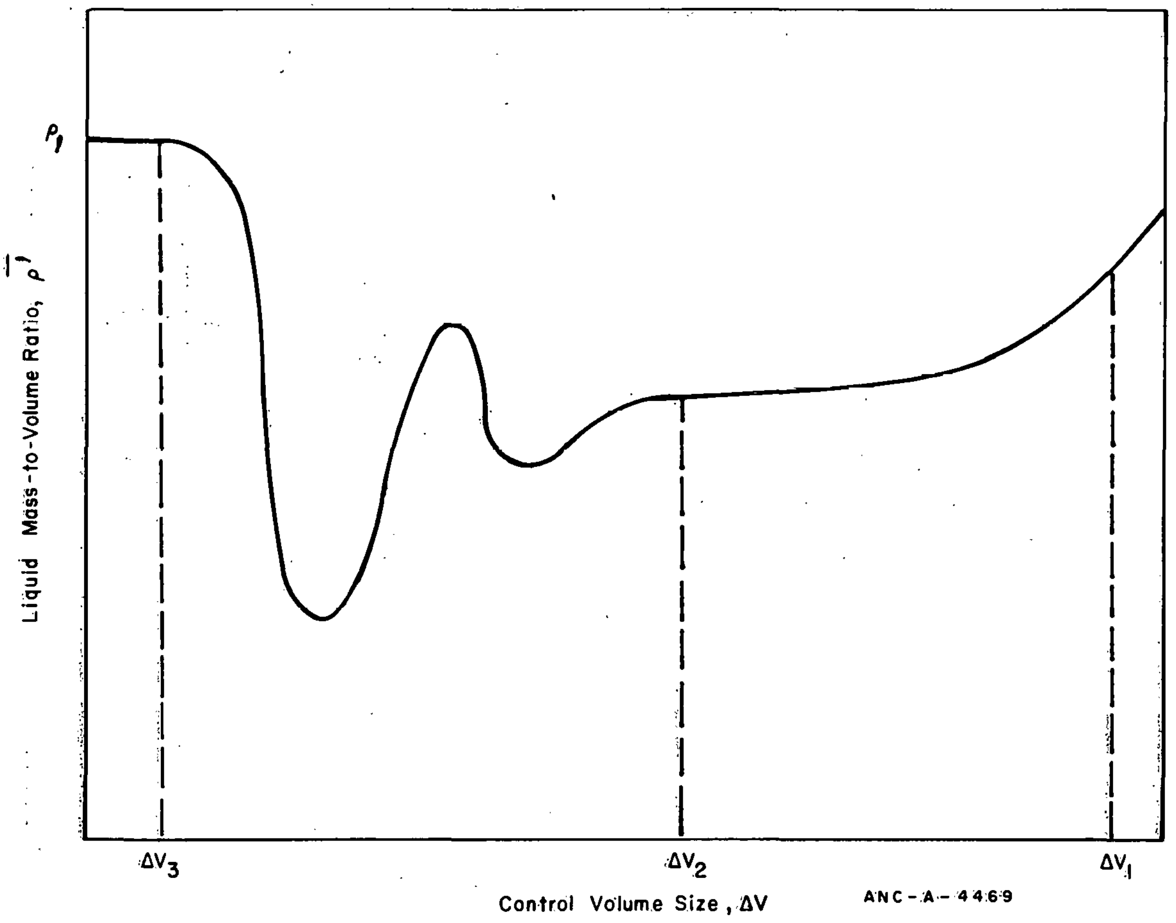

Fig. 2 Variation of liquid mass-to-volume ratio in different size volumes.

$$\rho \equiv \lim_{\Delta V \to \varepsilon} \frac{m}{\Delta V}$$

where ϵ is a small volume size. This volume size must be small relative to the perturbations of interest in the continuum but large enough to contain a sufficient number of molecules so that ρ is a continuous function. Thus, this definition of density further implies a definition of a continuum.

In a similar manner, a seriated continuum is defined by property definitions such as the partial densities of the liquid and the gas phases. These quantities are defined as

$$\rho^{\ell} \equiv \lim_{\Delta V \to \varepsilon_s} \bar{\rho}^{\ell} \tag{3}$$

$$\rho^{g} \equiv \lim_{\Delta V \to \varepsilon_s} \bar{\rho}^{g} \tag{4}$$

where ϵ_s is a volume size which is small relative to the perturbations of interest in the seriated continuum but is large enough to contain a sufficient number of bubbles or droplets or both so that ρ^ℓ and ρ^g are continuous functions. A seriated continuum consists of a series of phases of components which may exhibit definite interfaces. Various portions of one phase or component may be separated, as bubbles are separated from one another in a two-phase mixture. However, the definitions of ρ^ℓ and ρ^g imply that each phase or component is continuous. At the same time, the rather large size of ϵ_s allows each phase to be considered discrete when the constitutive equations are derived.

The assumptions involved in deriving equations for a seriated continuum are observed to be more severe than the assumptions required in continuum theory because $\epsilon_s >> \epsilon$. The volume ϵ is on the order of a mean free path, whereas the volume ϵ_s is on the order of the size of a bubble. Solutions obtained from seriated-continuum theory do not yield information concerning local fluid and flow conditions. For example, seriated-continuum theory cannot describe the flow or density field around a bubble.

The curve in Figure 3 represents the computational results obtained from seriated-continuum solutions. If the density is evaluated at a point in a droplet, the value of ρ^ℓ will be obtained instead of the thermodynamic density ρ_ℓ. A similar statement can be made about all of the other properties of the flow field. The numerical solutions of the seriated-continuum equations will involve finite difference techniques. The increments may in some cases be smaller than the size of a bubble or droplet. A stable convergent numerical scheme will yield the value of ρ^ℓ as a solution no matter how small the increment size. In some instances, results may have to be calculated for increment sizes smaller than a bubble or droplet in order to obtain an accurate solution from the finite difference scheme.

Although the value of ϵ_s is on the order of the size of bubbles and droplets, other considerations can determine its minimum value. For example, if a two-phase flow in a pipe is to be considered as a one-dimensional problem, ϵ_s would be on the order of the cross-sectional area of the pipe times the average distance between bubbles.

In the following derivations, the smoothed functions, as shown in Figure 3, will be used. The quantities ρ^ℓ and ρ^g are defined in a manner equivalent to Equations (3) and (4) as

$$\rho^\ell \equiv \operatorname*{lims}_{\Delta V \to 0} \bar{\rho}^\ell \tag{5}$$

$$\rho^g \equiv \operatorname*{lims}_{\Delta V \to 0} \bar{\rho}^g \tag{6}$$

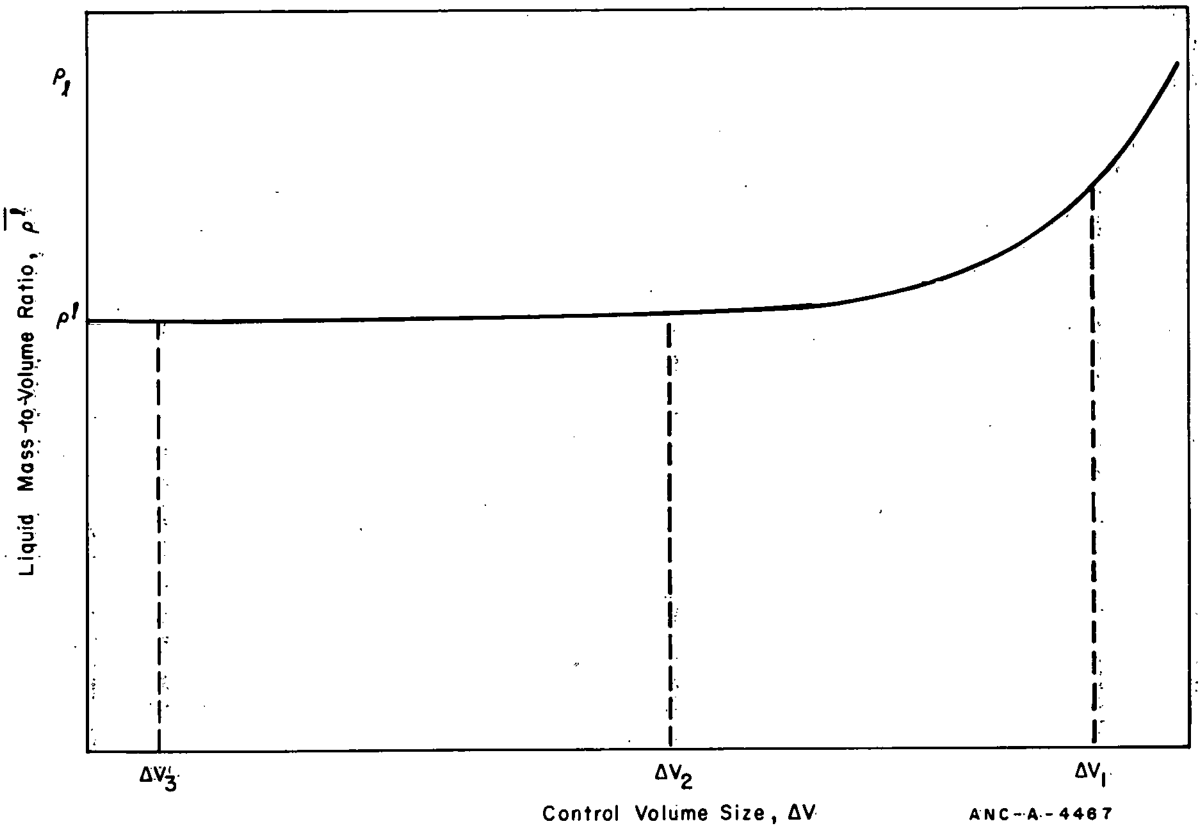

Fig. 3 Idealized limiting process for a seriated continuum.

where the $\underset{\Delta V \to 0}{\text{lims}}$ operator indicates the limit of the smoothed function. This procedure is required to use the definition of a derivative in the derivation of the balance equations. For example, the law of the mean guarantees that a value of $\frac{\partial \rho^\alpha v^\alpha_x}{\partial x}$ exists between x and x + Δx such that

$$\rho^\alpha v^\alpha_x \Big|_{x + \frac{\Delta x}{2}} - \rho^\alpha v^\alpha_x \Big|_{x - \frac{\Delta x}{2}} = \frac{\partial \rho^\alpha v^\alpha_x}{\partial x} \Big|_\xi \Delta x$$

where $x - \frac{\Delta x}{2} \leq \xi \leq x + \frac{\Delta x}{2}$.

After the term on the right side is divided by Δx, the operator $\underset{\Delta x \to 0}{\text{lims}}$ may be used to obtain

$$\frac{\partial \rho^\alpha v^\alpha_x}{\partial x} \equiv \underset{\Delta x \to 0}{\text{lims}} \frac{\partial \rho^\alpha v^\alpha_x}{\partial x} \Big|_\xi \quad . \tag{7}$$

The use of the smoothed functions alleviates the difficulty of attempting to define the derivative in terms of the operator $\lim_{\Delta V \to \epsilon_s}$.

1. GEOMETRY OF BOUNDING SURFACES

The geometries which are of interest in this development range from the one-dimensional representation of flow through a constant area pipe to the three-dimensional flow of a fluid in a nuclear reactor core. The reactor core of a present day pressurized water reactor generally consists of a vertical right cylinder 12 feet in diameter. The core region is filled with evenly spaced fuel rods which supply heat to the fluid. The fuel rods are vertical and, in general, about 14 feet long, about 0.422 inch in diameter, and number about 30,000.

The minimum size of the control volume, ϵ_s, for a nuclear reactor core includes several fuel rods within its boundaries. The volume of the control volume includes the volume of the fuel rods and the volume encompassed by the fluid. The total volume of the control volume can be written $V_T = V_\ell + V_g + V_s$ where V_ℓ, V_g, and V_s are the volumes occupied by the liquid, gas, and solid phases, respectively. The volume fractions must sum to unity so that

$$\alpha^\ell + \alpha^g + \alpha^s = 1 \tag{8}$$

where

α^ℓ ≡ the ratio of the volume of the liquid phase to the total volume

α^g ≡ the ratio of the volume of the gas phase to the total volume

α^s ≡ the ratio of the volume of the solid phase to the total volume.

If the quality θ is defined as $1 - \alpha^s$, that is, the ratio of the fluid volume to the total volume, Equation (8) can be written:

$$\alpha_\ell + \alpha_g = 1 \tag{9}$$

where now

α_ℓ ≡ the ratio of the volume of the liquid phase to the fluid volume = α^ℓ/θ

α_g ≡ the ratio of the volume of the gas phase to the fluid volume = α^g/θ.

The equations which will be developed will yield values of properties of the fluid at all points even at points within the fuel rods. The fuel rods are essentially treated in the equations as generation terms of friction and energy. The property values of points within the fuel rods are meaningless. The velocity values calculated are absolute velocities and, therefore, are not reduced by the ratio of the volume of the fuel rods to the total volume. The flow rate w^a past a plane would be calculated by $w^a = \rho^a v^a A$ where A is the average area available for flow. The average flow area is assumed to be constant in a given direction with distance, time, and direction. This assumption implies that any obstructions in the flow must be uniform and evenly spaced. Coupling or boundary conditions can be used to couple two different regions with different density of obstructions. Consideration of a variable obstruction density in space, time, and direction is beyond the scope of this report.

2. LOCAL DISTRIBUTION CORRECTIONS FOR PRODUCTS

A question as to whether the limit of the product is equal to the product of the limit of the factors remains with regard to the use of the smoothed limit operator. That is, does

$$\lim_{\Delta V \to \varepsilon_s} \overline{v_x \cdot v_x} = \left(\lim_{\Delta V \to \varepsilon_s} \bar{v}_x \right)^2 \ ? \tag{10}$$

This question arises in the more elementary problem of single-phase flow through a pipe and is not particular to two-phase flow problems. In general, Equation (10) is not an equality. A well known example for which Equation (10) is not true is in fully developed, steady state flow of a Stokesian fluid in a pipe (parabolic velocity distribution). Since the limiting process must include the entire area of the pipe for a one-dimensional representation, Equation (10) does not hold for this situation. As far as the authors know, this question cannot be resolved except by solving for these velocity distributions in detail (at a tremendous increase in effort to solve this problem). One possibility, suggested by many authors (for example, Panton, 1968) is to use a factor to correct each of these products. For example, a K could be defined such that

$$\lim_{\Delta V \to \varepsilon_s} \overline{v_x \cdot v_x} = K \left(\lim_{\Delta V \to \varepsilon_s} \bar{v}_x \right)^2 = K v_x^{\ 2} \ . \tag{11}$$

This definition suffers from K being dependent on the profile, for example:

$$K = \frac{\int_{A_c} v_x \cdot v_x dA}{\left(\int_{A_c} v_x dA \right)^2}$$

where A_c is the cross-sectional area and v_x is the local velocity. Evaluation of K requires as much information as a more complete solution does. It is a function of position, time, and direction but is often approximated from assumed profiles or steady state data (Hancox and Nicoll, 1972). A more detailed solution is beyond the intent of this discussion. Analysis for a nuclear reactor with approximately 30,000 flow channels which may have dissimilar flow patterns and heat addition would be beyond the scope of present day computers. In this discussion, the authors have choosen to assume that Equation (10) is an equality. Anytime a product appears in the equations derived, the reader may wish to append constant K factors to attempt to account for this effect. Due consideration of an anisotropic medium would have to be given if K were assumed to depend on direction.

III. DERIVATION OF THE MASS BALANCE EQUATIONS

The continuity equation, or mass balance equation, for each phase of the two-phase, single-component mixture is obtained as follows. The center of a control volume of volume ΔV containing both phases is considered to be situated at location (x, y, z) in a rectangular Cartesian coordinate system. The lengths of the sides of the volume in the x, y, and z directions are Δx, Δy, and Δz, respectively. The phase change processes of evaporation and condensation may be occurring due to both energy transfer and pressure changes in the flow field. The mass balance equations are obtained by equating the accumulation and net transport into the volume of each phase to the generation or growth of mass of each phase. This balance can be expressed as

$$\begin{Bmatrix}\text{Rate of Accumulation}\\ \text{of Phase 'a'}\end{Bmatrix} - \begin{Bmatrix}\text{Rate of Influx}\\ \text{of Phase 'a'}\end{Bmatrix} + \begin{Bmatrix}\text{Rate of Efflux}\\ \text{of Phase 'a'}\end{Bmatrix}$$

$$= \begin{Bmatrix}\text{Rate of Generation}\\ \text{of Phase 'a'}\end{Bmatrix} . \qquad (12)$$

The rate of accumulation of phase 'a' is represented by

$$\iiint_{\Delta V} \frac{\partial \rho^a \phi}{\partial t} \, dxdydz \qquad (13)$$

where Φ is the ratio of unobstructed volume to the total volume in the control volume and is referred to as the volume porosity. The mean value theorem of integral calculus may be used to represent this integral as

$$\iiint_{\Delta V} \frac{\partial \rho^a \phi}{\partial t} \, dxdydz = \left.\frac{\partial \rho^a \phi}{\partial t}\right|_{\xi_1} \Delta x \Delta y \Delta z \qquad (14)$$

where ξ_1 represents the coordinates of a point in the volume ΔV.

The efflux minus the influx of mass in the x direction is given by

$$\left(\rho^a v_x^a \phi \Big|_{x + \frac{\Delta x}{2}, y, z} - \rho^a v_x^a \phi \Big|_{x - \frac{\Delta x}{2}, y, z} \right) \Delta y \Delta z \qquad (15)$$

where Φ is also assumed to be the ratio of the area of the fluid in the x direction to the total area in the x direction.

The porous-media literature, for example, Scheidegger (1957), defines a velocity which is based on the total area rather than the unobstructed flow area. The velocity used here is based on the latter. The two velocities are related by

$$v_{xp}^a = v_x^a \phi$$

where v_{xp}^a is the velocity used in the porous-media literature. Either of these velocities may be used to determine the total flow rate past a plane:

$$w_x^a = \rho^a v_{xp}^a A_T = \rho^a v_x^a \phi A_T$$

where A_T is the unobstructed total flow area. Although the velocity v_{xp}^a does not represent an absolute velocity, no difficulty arises in the porous-media literature because momentum effects are usually neglected. Momentum effects are included in this report and, consequently, use of the velocity v_x^a is more meaningful here. This velocity represents the absolute velocity when the value of Φ represents the ratio of unobstructed area to the total area in all directions. If this ratio is direction dependent, v_x^a is an approximation to the absolute velocity. The assumption is made in this report that Φ may be considered to be independent of direction.

The quantity $\rho^a v_x^a \Phi \Big|_{x+\frac{\Delta x}{2},y,z}$ is an appropriately averaged quantity of the area $\Delta y \Delta z$ centered at the location $x + \frac{\Delta x}{2},y,z$. The law of the mean may be applied to Equation (15) to obtain

$$\frac{\partial \rho^a v_x^a \phi}{\partial x} \Big|_{\xi_2} \Delta x \Delta y \Delta z \tag{16}$$

where ξ_2 represents the coordinates of a point in ΔV. The efflux minus the influx in the y and z directions can be represented similarly as

$$\left(\frac{\partial \rho^a v_y^a \phi}{\partial y} \Big|_{\xi_3} + \frac{\partial \rho^a v_z^a \phi}{\partial z} \Big|_{\xi_4} \right) \Delta x \Delta y \Delta z \ . \tag{17}$$

Fluid mass is not generated within the control volume; however, mass is exchanged between phases. This mass exchange occurs across interfaces within the control volume as, for example, across the surface of droplets or bubbles. Mass appears to be generated or consumed with respect to each phase even though the total mass remains constant. The total mass exchange of phase 'a' is represented by

$$\iiint_{\Delta V} \dot{m}^a \phi \, dx dy dz$$

where $\dot{m}^a$ represents the mass generation of phase 'a' per unit volume. The mean value theorem of integral calculus may be used to show that

$$\iiint_{\Delta V} \dot{m}^a \phi \, dx dy dz = \dot{m}^a \phi \Big|_{\xi_5} \Delta x \Delta y \Delta z \ . \tag{18}$$

The Expressions (14) and (16) through (18) may be substituted into Expression (12), the resulting expression divided by ΔxΔyΔz, and the operator $\lim_{\Delta V \to 0}$ applied to the equation to obtain the mass balance equation for phase 'a':

$$\frac{\partial \rho^a \phi}{\partial t} + \left(\rho^a v_i^a \phi\right)_{,i} = \dot{m}^a \phi \,. \tag{19}$$

In Equation (19), the superscript 'a' refers to the liquid or the gas phase. Cartesian tensor notation is used. The subscript refers to any one of the coordinate directions x, y, or z; the comma indicates differentiation; and repeated subscripts imply summation over the three coordinate directions.

Since mass is not being generated but only exchanged between phases, the total mass must be conserved. That is,

$$\dot{m}^\ell + \dot{m}^g = 0 \,. \tag{20}$$

Equations (19) and (20) are three independent relations that contain 4 + 2n unknown dependent quantities where n is the number of dimensions. Equation (19) may be summed over the phases and combined with Equation (20) to obtain the expression

$$\frac{\partial \rho \phi}{\partial t} + (\rho v_i \phi)_{,i} = 0 \tag{21}$$

where the density of the mixture, ρ, is defined as

$$\rho \equiv \sum_{a=\ell,g} \rho^a \tag{22}$$

and the baricentric velocity v_i of the mixture is defined as

$$\rho v_i \equiv \sum_{a=\ell,g} \rho^a v_i^a \,. \tag{23}$$

Equation (21) is not an independent relation because it is the sum of Equations (19) and (20).

IV. DERIVATION OF THE MOMENTUM BALANCE EQUATIONS

Momentum balance equations are derived in this section for each phase of a liquid-vapor mixture of a single-component fluid. The areas wherein the occurrence of phase change processes enter in the terms appearing in the balance equations are indicated. The mechanical constitutive equations accounting for the stresses acting on each phase are also given. These expressions are given as tensor quantities in Cartesian coordinates. The forms for other coordinates can be obtained by suitable transformations. In Section IV-1, a general balance expression is stated and the accumulation and transport of momentum terms are given. In Section IV-2, the momentum growth term is discussed. In Section IV-3, the shearing forces acting on each phase are formulated and the pressure, transient flow forces, and body forces are discussed in Sections IV-4, IV-5, and IV-6, respectively. The final form of the momentum equation is given in Section IV-7.

1. TRANSPORT OF MOMENTUM

In this derivation, the hypothesis is adopted that each phase may be considered as continuous for some purposes but may be considered as discrete for other purposes. This hypothesis is analogous to continuum theory in which the basic equations of change are derived from continuum considerations, but constitutive equations for transport properties may be derived from discrete considerations. This hypothesis has been employed in all derivations of engineering two-phase flow equations, either explicitly or implicitly. To a large extent, phase distribution patterns (flow regimes) in boiling and condensing flow fields are physically characterized by one phase being continuous and the other phase being discrete.

For the system under consideration, the momentum balance in a given direction for phase ‘a’ may be written as

$$\begin{aligned}
&\begin{bmatrix}\text{Rate of Accumulation of}\\ \text{Momentum of Phase 'a'}\end{bmatrix} + \begin{bmatrix}\text{Rate of Momentum}\\ \text{Efflux of Phase 'a'}\end{bmatrix} - \begin{bmatrix}\text{Rate of Momentum}\\ \text{Influx of Phase 'a'}\end{bmatrix}\\
&= \begin{bmatrix}\text{Rate of Momentum}\\ \text{Growth of Phase 'a'}\end{bmatrix} + \begin{bmatrix}\text{Sum of the Shearing}\\ \text{Forces Acting on}\\ \text{Phase 'a'}\end{bmatrix}\\
&\quad + \begin{bmatrix}\text{Sum of the Pressure}\\ \text{Forces Acting on}\\ \text{Phase 'a'}\end{bmatrix}\\
&\quad + \begin{bmatrix}\text{Transient Flow}\\ \text{Forces Acting on}\\ \text{Phase 'a'}\end{bmatrix}\\
&\quad + \begin{bmatrix}\text{Sum of the Body Forces}\\ \text{on Phase 'a' Due to}\\ \text{Field Sources}\end{bmatrix} .
\end{aligned} \tag{21}$$

The momentum equation for a phase is derived for the x direction. Cartesian tensor notation is used to generalize to equations to account for the y and z directions.

The rate of accumulation of momentum of phase ‘a’ in the x direction is given by

$$\iiint_{\Delta V} \frac{\partial \phi \rho^{a} v_{x}^{a}}{\partial t} \, dxdydz = \left. \frac{\partial \phi \rho^{a} v_{x}^{a}}{\partial t} \right|_{\xi_1} \Delta x \Delta y \Delta z \tag{22}$$

where ξ_1 represents the coordinate of an appropriate point within the control volumc ΔV. The momentum which is being convected in and out of the sides of the volume normal to the x direction is $\Phi\rho^{a}v_{x}^{a}$. The net flux of momentum may be expressed as

$$\begin{aligned} &\left. \frac{\partial \phi \rho^{a} v_{x}^{a} v_{x}^{a}}{\partial x} \right|_{\xi_2} \Delta x \Delta y \Delta z + \left. \frac{\partial \phi \rho^{a} v_{x}^{a} v_{y}^{a}}{\partial y} \right|_{\xi_3} \Delta x \Delta y \Delta z \\ &+ \left. \frac{\partial \phi \rho^{a} v_{x}^{a} v_{z}^{a}}{\partial z} \right|_{\xi_4} \Delta x \Delta y \Delta z \end{aligned} \tag{23}$$

where ξ_2, ξ_3, and ξ_4 represent the coordinates of three points within the control volume ΔV. The next term considered in Equation (21) is the contribution of the mass exchange processes to the momentum balance of each phase.

2. MOMENTUM GROWTH

The phase change processes of evaporation of the liquid phase and condensation of the vapor phase can occur due to energy transfer from stationary surfaces within or bounding the flow field, energy transfer between the phases, and the existence of a pressure change. The effect of these mass exchange processes on the momentum of a phase is represented by the first term on the right side of Equation (21). These processes occur at the interfaces between the phases within the control volume. In general, the interface moves with its own intrinsic velocity $\hat{v}_{I}^{a}$ and not with the velocity of either phase.

For unequal phase temperature fluids, condensation and evaporation can occur due to several processes. The fluid considered in this report is an equal temperature fluid and, consequently, the only mass exchange process considered is the thermal equilibrium process. The net mass exchange rate $\dot{m}^{a}$ may be calculated from thermodynamic considerations. The individual contributions of evaporation and condensation from kinetic theory are considered equal in this report. Since the mass exchange rate for an equal temperature fluid is a single process, only one velocity may be used to represent the velocity of mass generation of phase ‘a’.

The momentum growth in the control volume may be expressed as

$$\iiint_{\Delta V} \phi \, \dot{m}^a \, \hat{v}_i^a \, dxdydz = \phi \dot{m}^a \hat{v}_i^a \Big|_{\xi_5} \Delta x \Delta y \Delta z \tag{24}$$

where ξ_5 represents the coordinates of a point within ΔV. The intrinsic velocity $\hat{v}_i^a$ is another unknown for each phase. It should be related to the velocity v_i^a and other quantities. Some controversy exists in the literature as to whether these relations should both be constitutive equations or whether one should be replaced with a momentum balance. The latter view is accepted in this report for the following reason.

A particle of mass m is assumed to leave phase 'a' and enter phase 'b'. This particle does not change its center of mass when changing phase. On a continuum basis, no time or force is required to change phase. Change of phase of a mass simply means that a different equation is used to account for the momentum of that mass. The momentum of the mass must be conserved in the phase transition. That is,

$$m \, \hat{v}_i^a = m \hat{v}_i^b$$

or

$$\hat{v}_i^a = \hat{v}_i^b \equiv \hat{v}_i \tag{25}$$

which states that the intrinsic velocities are equal. This relation would conflict with the assumption of setting the intrinsic velocity of a phase equal to the velocity of the phase. That is, the relations

$$\hat{v}_i^a = v_i^a \tag{26}$$

cannot be valid except in the simplified case in which the velocities of both phases are always equal. Thus, Equation (26) is dismissed as being, in general, physically impossible for a seriated continum.

An additional equation is necessary to relate $\hat{v}_i$ to other variables. This relation is a constitutive equation which can depend on the flow regime and is represented by the following:

$$\hat{v}_i = \hat{v}_i \left(v_i^g , v_i^\ell \right) \tag{27}$$

where

g refers to the gas phase

ℓ refers to the liquid phase.

Other variables might be included in the argument list of Equation (27) for some flow regimes. Some possibilities for the function $\hat{v}_i$ are given in but not restricted to the following:

(1) The first possibility is

$$\hat{v}_i = v_i^a \tag{28}$$

where $\dot{m}^a < 0$. This relation implies that the intrinsic velocity is equal to the velocity of the phase that is losing mass.

(2) The second possibility is

$$\hat{v}_i = v_i^a \tag{29}$$

where 'a' is the dispersed phase. This model may be derived by considering the flow from a droplet uniform in all directions. The vapor leaves the surface with a velocity relative to the surface of

$$v_n = \frac{\dot{m}}{\rho_g A}$$

where v_n is the velocity normal to the surface. The average value of this velocity in any direction is zero. Therefore, the vapor leaves the droplet at the velocity of the droplet.

(3) The third possibility is

$$\hat{v}_i = \beta\, v_i^g + (1 - \beta)\, v_i^\ell \tag{30}$$

where β is a weighting factor ($0 \leqslant \beta \leqslant 1$). This relation implies that a continuous velocity distribution exists between the gas and liquid phases. When β is between zero and unity, the interface velocity is bounded by these two velocities.

A brief literature review is included in the following to summarize what other authors have used. Panton (1968) who analyzes a gas-particle mixture assumes that the velocity of the dispersed phase, Equation (29), should always be used. Green and Naghdi (1969) and Craine, Green, and Naghdi (1970) analyze a multicomponent mixture and hypothesize that $\hat{v}_i^a$ is a mean velocity which will depend on the velocities of the remaining components. The simplest expression which satisfies this hypothesis for a two-phase mixture is

$$\hat{v}_i^a = v_i^b \quad . \tag{31}$$

This expression would conflict with Equation (25) except when the velocities are equal. Naghdi, in a private communication, indicated that v_i^a should also be included. This modification would be consistent with Equation (30). Müller (1968) indicates that the momentum growth consists of two parts: (1) thermomechanical interactions and (2) phase changes (chemical production). The latter portion is equivalent to the momentum growth considered in this section. He states that $\hat{v}_i^a$ is the velocity of the newly created mass. The first part is presumably related to diffusive forces which are considered nonexistent in a two-phase mixture. Marble (1969) states that as a result of condensation or vaporization at the droplet surface, mass is transferred between the velocities of the bulk liquid and bulk vapor. This velocity is consistent with Equation (30). The particular problem he considers is liquid droplets dispersed in a steam environment. He states that with the assumption of no shear induced circulatory motion, the mass exchanged at the interface has the velocity of the liquid phase. This velocity is consistent with Equation (29).

Mecredy and Hamilton (1972) subdivide the mass exchange process into an evaporation rate Γ_e and a condensation rate Γ_c. These rates are related to the mass exchange in the present work by the relation

$$\dot{m} = \Gamma_e - \Gamma_c \quad . \tag{32}$$

Expressions from kinetic theory are used to compute Γ_e and Γ_c. The momentum growth of the gas phase is defined by Mecredy and Hamilton as

$$\Gamma_e v_i^{\ell} - \Gamma_c v_i^{g} \tag{33}$$

where ℓ and g refer to the liquid and gas phases, respectively. These expressions were not used in the present work because they predict that $\dot{m} = 0$ for equal phase temperatures. This model for $\dot{m}$ precludes a net evaporation or condensation during a pressure change and appears to be physically unrealistic for the present work.

The conclusion which might be drawn from the literature is that no agreement exists as to what this term should be; however, we believe that the conclusion which should be drawn is that no universal method exists for computing the intrinsic velocity $\hat{v}_i$. The method of calculation will depend upon the particular physical situation or flow regime. Consequently, the intrinsic velocity calculation should be done with a correlation which is dependent upon the two-phase flow regime. As noted by Panton (1968) for the case in which averages are used to eliminate solving for three-dimensional velocity distributions around single particles, an assumption or model, regarding the distribution of the mass exchange process around a droplet or bubble, is required. For example, if evaporation is occurring uniformly around a droplet, no net effect should be produced on the momentum of the droplet. If condensation is occurring on one half of the droplet and evaporation on the other half, an effect would be produced. However, with only averaged information available, directional effects associated with the mass exchange processes cannot be accounted for unless these effects are included through correlations for $\hat{v}_i$.

3. SHEARING FORCES ACTING ON PHASE A

The shearing forces considered in this section are due entirely to velocity gradients. The continuum mechanics literature usually includes the pressure forces and shear forces due to velocity gradients in a single term herein called the total shear stress (for example, Craine et al, 1970). The forces due to pressure are considered in Section IV-4. The shearing forces are considered to be made up of three components which are given in the following equation:

$$\begin{bmatrix}\text{Sum of the Shearing}\\ \text{Forces Acting on}\\ \text{Phase 'a'}\end{bmatrix} = \begin{bmatrix}\text{Sum of the Shearing Forces}\\ \text{Exerted on Phase 'a' by}\\ \text{Stationary Surfaces Inside}\\ \text{the Control Volume}\end{bmatrix} + \begin{bmatrix}\text{Sum of the Shearing Forces}\\ \text{Exerted on Phase 'a' by the}\\ \text{Other Species Inside the}\\ \text{Control Volume}\end{bmatrix} + \begin{bmatrix}\text{Sum of the Shearing Forces}\\ \text{Exerted by Phase 'a' Outside}\\ \text{the Control Volume on the}\\ \text{Outside Surfaces of the}\\ \text{Control Volume}\end{bmatrix} . \tag{34}$$

These three components of the shearing forces may be visualized by considering the model shown in Figure 4. This figure illustrates a typical control volume which contains a two-phase fluid as well as some stationary surfaces such as fuel rods. The two-phase fluid is assumed to be made up of a continuous phase and a discrete phase. Due to the limiting process used to define a seriated continuum, stationary surfaces must be included as well as both phases in the control volume. The shear forces on the continuous phase are seen to be due to (a) velocity gradients at the interfaces between the stationary surface and the continuous phase, (b) velocity gradients at the interfaces between the continuous phase and the discrete phase, and (c) velocity gradients in the continuous phase on the surface of the control volume.

The same forces can be applied to the discrete phase. Stationary surface friction can be exerted on the bubbles or droplets on the walls. Interphase friction is exerted by the continuous phase on the discrete phase, and intraphase friction is exerted on the droplets or bubbles imbedded in the surface of the control volume. Thus, Equation (33) is valid for both the continuous phase and the discrete phase. The assumption which is inferred for the discrete phase is that the forces which are applied to the discrete particles are effectively applied to the discrete phase as an aggregate. In this sense, the discrete phase is treated as a continuum.

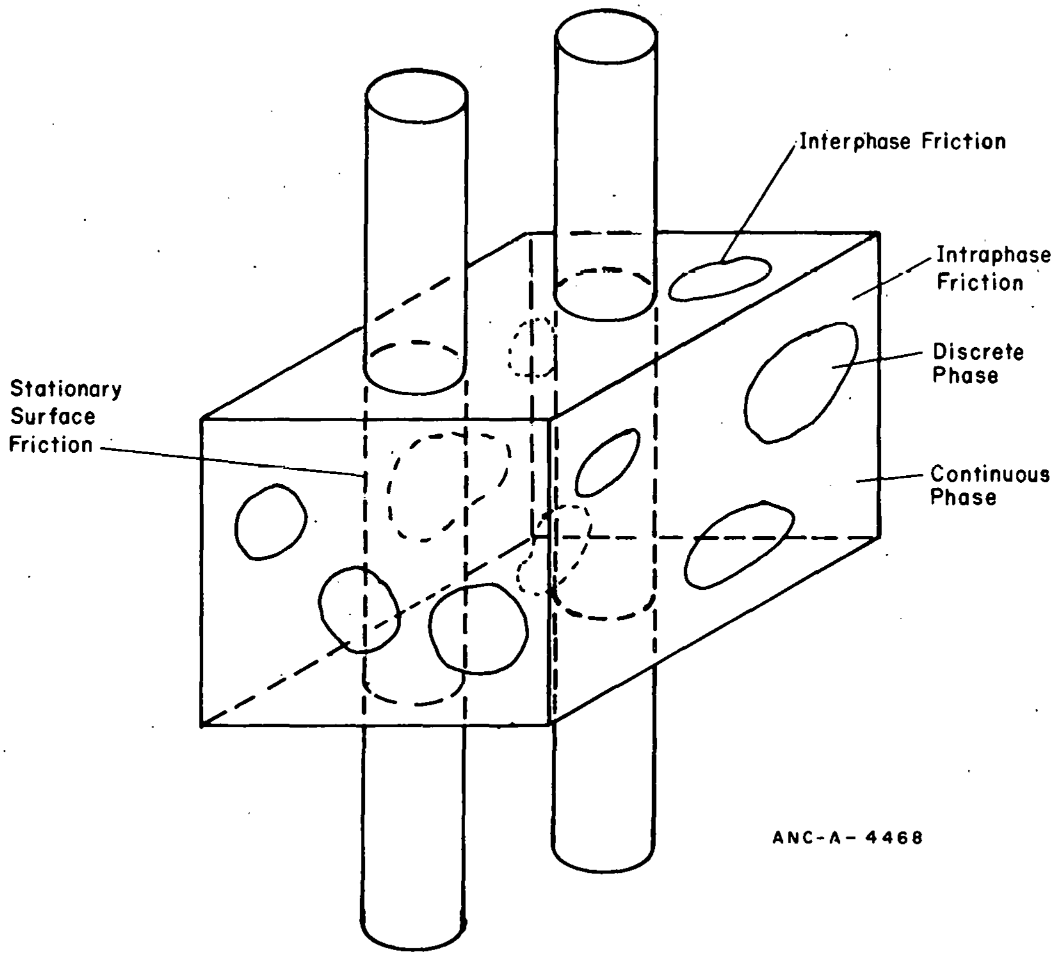

Fig. 4 Shearing force model.

Some two-phase flow regimes are not characterized by a continuous phase and a discrete phase. For example, annular flow is two continuous phases, one inside the other. Equation (34) is also valid for this and other flow regimes. The phenomena-based origin of these forces is discussed more fully in the following sections.

3.1 Shearing Forces Due to Stationary Surfaces and Other Species

The shearing forces due to other species and stationary surfaces both result in forces which appear to be field forces similar to gravitational forces. The shearing force exerted by the wall on the fluid is looked at first by considering laminar or turbulent flow of a single-phase fluid between parallel plates. The velocity profile for this situation is sketched in Figure 5. The shear stress between the fluid and the wall is directly related to the velocity

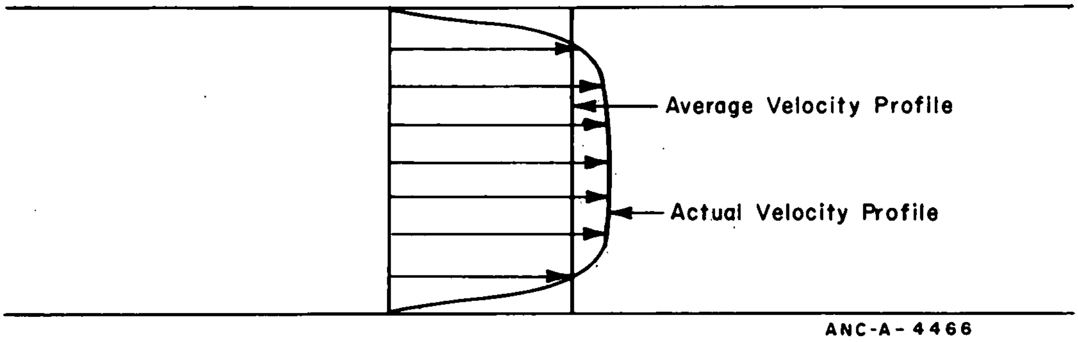

Fig. 5 Velocity profile between parallel plates.

gradient at the wall for both laminar and turbulent flow. The model being developed is based on the assumption that many surfaces exist within the control volume but that only average velocities will be obtained as solutions. Thus, the velocity gradient at the wall is available from the solution. This gradient is approximated by the difference between the average velocity of the fluid and the velocity of the wall (which is zero) multiplied by some suitable coefficient. This force acts in the opposite direction of the velocity. This model is considerably different for shear than the usual viscous shear model. In this model, a velocity profile may be spatially invariant yet significant shear may occur. Viscous shear, on the other hand, would be zero in the instance for which the true velocity gradient is zero. For the physical situation for which flow is present in the axial direction of a single-phase fluid in a constant area flow section (invariant with axial direction), this type of shearing force can be associated with pressure drops calculated by the use of Fanning friction factors. This concept may also be extended to flow in the direction of a repeated geometry such as in the case of flow normal to tube banks.

The presence of a second phase can change this shear force on the fluid by distorting the velocity gradient on the wall. A bubble attached to the wall or mass exchange for a bubble will distort the velocity gradient in the region near the bubble. This effect may be accounted for by adjustment of the coefficient of the shear term.

The second shear force in a mixture consisting of two fluids exists because the second phase may be moving at a different speed and direction than the first. This shearing force is related to velocity gradients along the interfaces between the two phases. This force is represented as the difference between the average velocities of both phases in the control volume multiplied by a suitable coefficient. This effect may be related to the measurement of drag forces on various geometric shapes with and without boundary layer injection.

An approximate analytical expression for these shear forces can be obtained by considering the surfaces (both stationary and phase interface surfaces) within the control volume. The total force within the control volume may be expressed as

$$F_i^a = \sum_s \int_{A_s^a} \sigma_{is}^a \, dA \tag{35}$$

where

σ_{is}^a = the shearing force per unit surface area in the i^{th} direction exerted on phase 'a' by the surface 's'

A_s^a = the area of surface 's' exposed to phase 'a'

and $\sum_s$ indicates summation over all the surfaces in the control volume. This expression may be written in terms of average shear forces:

$$F_i^a = \sum_s \overline{\sigma_{is}^a} \; A_s^a \tag{36}$$

where

$$\overline{\sigma_{is}^a} \equiv \frac{1}{A_s^a} \int_{A_s^a} \sigma_{is}^a \, dA \; .$$

For convenience, Equation (36) may be written in the form

$$F_i^a = \sum_s \overline{\sigma_{is}^a} \; \frac{A_s^a}{\Delta V_f} \; \phi \Delta x \Delta y \Delta z \tag{37}$$

where

ΔV_f = $\Phi \Delta x \Delta y \Delta z$ and is the total volume of the fluid in the control volume.

Finally, this expression is further modified to introduce the area per unit volume $\frac{A_s^a}{\Delta V_f}$

$$F_i^a = \sum_s \overline{\overline{\sigma_{is}^a}} \; \overline{A_s^a} \; \phi \Delta x \Delta y \Delta z \tag{38}$$

where

$$\overline{A_s^a} \equiv \lim_{\Delta V_f \to 0} \frac{A_s^a}{\Delta V_f}$$

and

$$\sigma_{is}^a \equiv \frac{1}{\overline{A_s^a}} \; \overline{\sigma_{is}^a} \; \frac{A_s^a}{\Delta V_f} \; .$$

This is the form of the shear stress, $\overline{\overline{\sigma_{is}^{a}}}$, for which an approximate analytical expression will be obtained. Both transient and steady state effects contribute to this stress. Only the portion of this stress which is related to friction factor forces will be considered in this section. The remainder of this stress is due to transient effects and curved streamlines which are considered in Section IV-5.

Note should be taken that the area per unit volume $\overline{A_s^a}$ is related simply to the inverse of the hydraulic diameter. In the instance of steady state fully developed flow between parallel plates, as shown in Figure 6, the area per unit volume for the upper plate is

$$\overline{A_1}\,\sigma = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \int \sigma \cdot dA$$

or, since σ is a constant,

$$\overline{A_1} = \lim_{\Delta V \to 0} \frac{A_1}{\Delta V} = \frac{ab}{abL} = \frac{1}{L} .$$

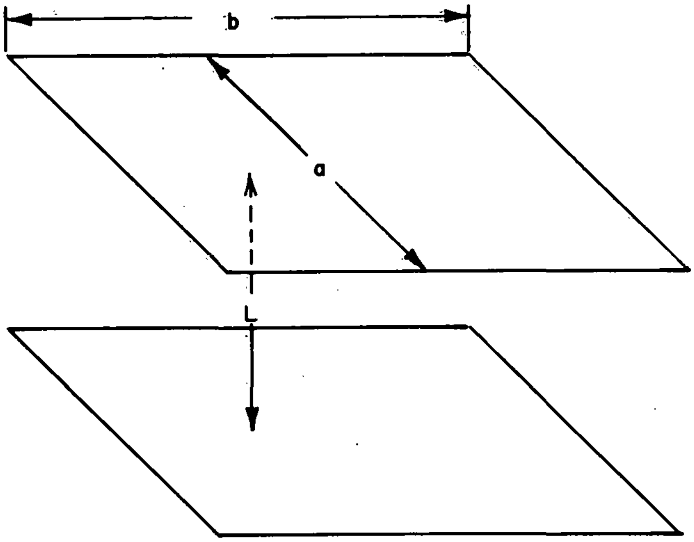

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Fig. 6 Parallel plate geometry.

Physically, the limiting process may not be carried out for volumes which do not include both plates. Conceptually, however, this process offers no difficulty and the limiting process may be performed. Note should be taken that the total area per unit volume for the parallel plates will be the same as if each were considered separately and then both summed. That is,

$$\overline{A_T} = \lim_{\Delta V \to 0} \frac{A_1 + A_2}{\Delta V} = \lim_{\Delta V \to 0} \frac{A_1}{\Delta V} + \lim_{\Delta V \to 0} \frac{A_2}{\Delta V} = \overline{A_1} + \overline{A_2} .$$

The shearing force per unit area between two fluids or a fluid and a surface is assumed to be linearly related to the velocity gradient:

$$\sigma^a_{is} = -K_1 \frac{\partial v^a_i}{\partial n} \tag{39}$$

where n is the normal to the surface, K_1 is an appropriate coefficient, and v^a_i is the local velocity profile near the interface or surface. Since the local profile will not be calculated, the following approximation is made for the derivative

$$\sigma^a_{is} \approx -K_1K_2 \frac{v^a_i - v^s_i}{\ell} \tag{40}$$

where ℓ is a characteristic length which depends on the physical configuration, v^a_i is the velocity of phase 'a', v^s_i is the velocity of the other phase or the surface, and K_2 is a coefficient which attempts to account for the difference between $\frac{\partial v}{\partial n}$ and $\frac{v^a_i - v^s_i}{\ell}$. Substitution of this expression into Equation (26) yields

$$\overline{\sigma^a_{is}} \approx -\frac{1}{A^a_s} \int_{A^a_s} \frac{K_1K_2}{\ell}\left(v^a_i - v^s_i\right) dA^a_s$$

$$\overline{\sigma^a_{is}} \approx -\frac{K_1K_2}{\ell}\left(v^a_i - v^s_i\right)\Big|_{\xi_s} \tag{41}$$

where ξ_s represents the coordinate of an appropriate point on the surface 's'. The desired shear stress $\overline{\overline{\sigma^a_{is}}}$ is obtained from the expression

$$\overline{\overline{\sigma^a_{is}}} = -B^a_s\left(v^a_i - v^s_i\right)\Big|_{\xi_5} \tag{42}$$

where

$$B^a_s = \frac{K_1K_2 \; A^a_s}{\ell \; \overline{A^a_s} \; \Delta V_f} .$$

Substitution yields the expression

$$F^a_i = -\sum_s B^a_s \; \overline{A^a_s}\left(v^a_i - v^s_i\right) \phi\Delta x\Delta y\Delta z \quad . \tag{43}$$

In extracting the quantity B^a_s from experimental data, what to assign to $\overline{A^a_s}$ and what to assign to B^a_s is somewhat flexible. In many cases, for convenience, an expression is assigned to the product $\overline{A^a_s}\, B^a_s$.

Tensor considerations do not allow B_s^a to be a function of the velocity component $v_i^a - v_i^s$. If this coefficient were allowed to be a function of this velocity component, the resultant force would depend upon the orientation of the coordinate system. B_s^a can be a function of

$$\sqrt{\left(v_i^a - v_i^s\right)\left(v_i^a - v_i^s\right)}$$

which is a scalar and hence acceptable.

For solid surfaces, the coefficient B_s^a is related to friction factors reported for single-phase, single-component flows. However, as previously noted, these coefficients may require modification to account for the effects of the presence of two phases on the velocity gradient at the surface. The coefficient B_s^a and the area per unit volume are discussed in Section VII.

These forces can, for a two-component fluid, be segregated into a sum of force terms which represent the stationary surfaces ($v_i^s = 0$) and forces which represent the other phase ($v_i^s = v_i^b$). Thus, the expression which represents the first two terms on the right-hand side of Equation (33) is

$$F_i^a = \left[-\sum_s B_s^a \overline{A_s^a}\, v_i^a - B_{ab}\, \overline{A_{ab}} \left(v_i^a - v_i^b\right)\right] \phi \Delta x \Delta y \Delta z \tag{44}$$

where $B_s^a \overline{A_s^a}\, v_i^a$ are the appropriate coefficients for the stationary surfaces, and $B_{ab}\overline{A_{ab}}$ are the appropriate coefficients for the interphase forces.

3.2 Intraphase Shearing Forces

The third shearing force which is due to the same species acting on the surface of the control volume is related to velocity gradients in the fluid at these surfaces. These velocity gradients will be approximated by gradients in the average fluid velocity. Thus, although changes exist in the velocity gradients along these surfaces due to the presence of another phase at the surface, the assumption is made that these forces are included in the shearing force term due to other species. The shearing forces on the surface of the control volume can be visualized by considering a single-phase fluid moving in a tube bundle as is shown in Figure 7. This figure illustrates the top view and two cross sections of a fluid moving in the axial direction in a control volume in a fuel rod bundle. This figure shows that although the shearing forces of the friction factor type account for the shearing forces on the rods, the possibility of shear forces between the fluid in the various channels still exists. These forces can be represented in the usual manner. The shear stresses in the x direction are shown in Figure 8 in two dimensions for component ‘a’.

The force on the control volume is seen to be

$$F_i^a = \left(\phi \alpha^a \sigma_{ji}^a\right)_{,j} \Delta x \Delta y \Delta z \quad . \tag{45}$$

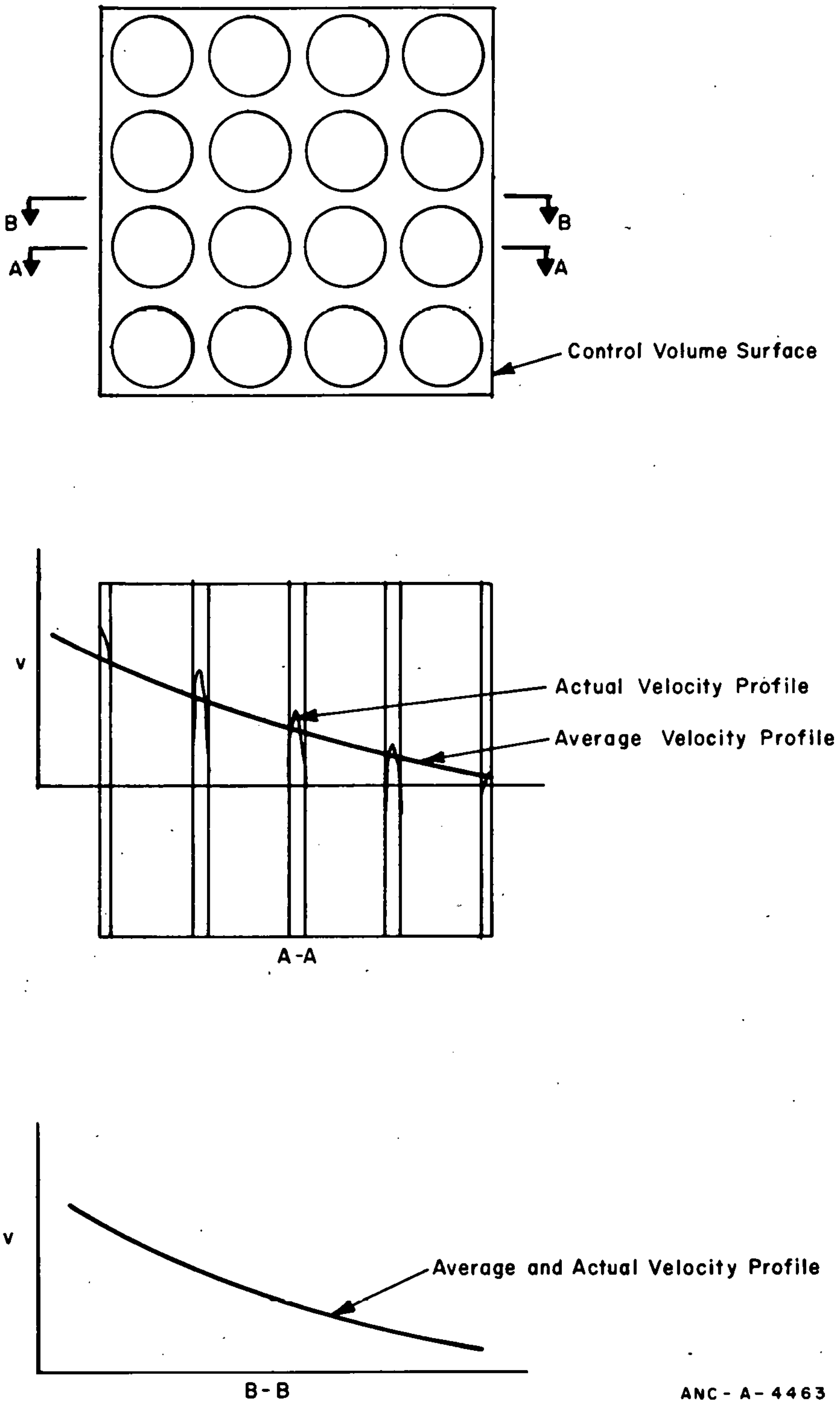


Fig. 7 Illustration of the Intraphase Shear Term.

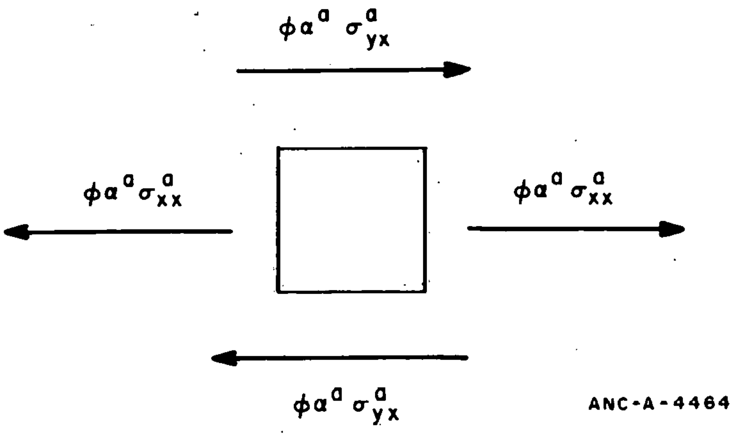


Fig. 8 Intraphase shear diagram.

Since this force should be zero if all parts of the fluid are moving at the same velocity, the stresses σ^a_{ji} are related to velocity gradients rather than to velocities. The assumption is that

$$\sigma^a_{ji} = F^a\left(v^a_{i,j}\ ,\ v^a_{j,i}\right) \quad . \qquad (46)$$

That is, the stresses are related to the velocity gradients of a given component. If σ^a_{ji} is assumed to be a linear symmetric tensor, the stress may be written as

$$\sigma^a_{ji} = -2\mu^a\left(\varepsilon^a_{ij} - \frac{1}{3}\varepsilon^a_{kk}\delta_{ij}\right) - \mu^a_v\varepsilon^a_{kk}\delta_{ij} \qquad (47)$$

where

$$\varepsilon^a_{ij} \equiv \frac{1}{2}\left(v^a_{i,j} + v^a_{j,i}\right)$$

and

μ^a is the kinematic viscosity

μ^a_v is the bulk viscosity.

The viscosity coefficients should be considered as dependent on position if turbulent components are to be part of the viscous stresses.

Thus, the total force due to shear is

$$F^a_i = F^a_{i\ \mathrm{walls}} + F^a_{i\ \mathrm{phase\ b}} + F^a_{i,\ \mathrm{viscous}}$$

or

$$F_i^a = \left[- \phi \overline{A_s^a}\; B_s^a\,(\theta_s,\; \theta_a)\; v_i^a - \phi \overline{A_{ab}}\; B_{ab} \left(v_i^a - v_i^b \right) + \left(\phi \alpha^a \sigma_{ji}^a \right)_{,j} \right] \Delta x \Delta y \Delta z \; . \tag{48}$$

The coefficient $B_s^a(\theta_s, \theta_a)$ is assumed to be a function of the angle of approach of the fluid, θ_a, to the angle of the solid surface, θ_s.

4. PRESSURE FORCES ACTING ON PHASE A

The pressure forces acting on phase 'a' are considered to be made up of

$$\left[\begin{array}{l}\text{Sum of the Pressure}\\ \text{Forces Acting on}\\ \text{Phase 'a'}\end{array}\right] = \left[\begin{array}{l}\text{Sum of the Pressure Forces}\\ \text{Exerted on Phase 'a' by}\\ \text{Stationary Surfaces Inside}\\ \text{the Control Volume}\end{array}\right] + \left[\begin{array}{l}\text{Sum of the Pressure Forces}\\ \text{Exerted on Phase 'a' by}\\ \text{the Other Species Inside the}\\ \text{Control Volume}\end{array}\right] + \left[\begin{array}{l}\text{Sum of the Pressure Forces}\\ \text{Exerted by Phase 'a' Outside}\\ \text{the Control Volume on the}\\ \text{Outside Surfaces of the}\\ \text{Control Volume}\end{array}\right] . \tag{49}$$

Expressions for these forces are obtained in the following manner. A control volume with a discrete or continuous phase 'b' inside a continuous phase 'a', as shown in Figure 9, is considered.

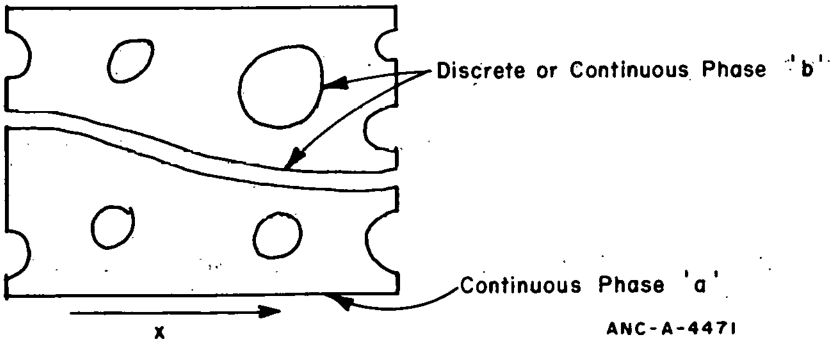


Fig. 9 Pressure force model.

The pressure forces exerted by phase 'b' on phase 'a' in the x direction are

$$\sum_S \int P_S \, n_x dA$$

where the $\sum_S$ indicates a summation over all the internal surfaces, P_S is the pressure on the surface, and n_X is the unit normal to the surface. Two forces are generated internal to the control volume from this expression for both the stationary surfaces and other species. One is due to both phases being assumed continuous. The second is due to phases being considered discrete.

These forces may be derived by considering a somewhat simpler picture than that presented. The model in Figure 10 is considered. Since the volume fraction is treated as a continuous quantity, the continuous analogy of this model might be interpreted as shown in Figure 11.

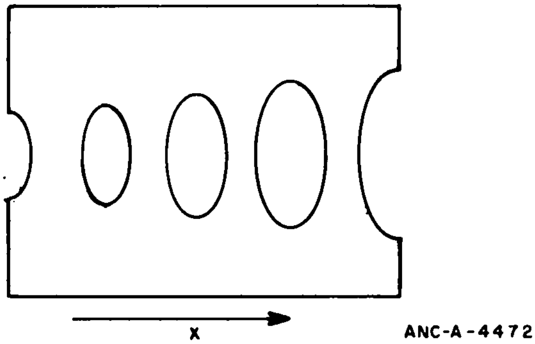


Fig. 10 Pressure forces due to continuous and discrete contributions.

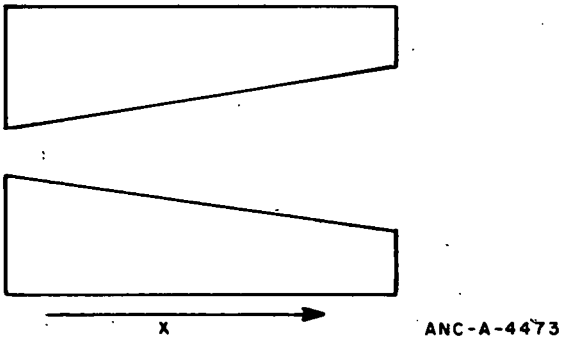


Fig. 11 Continuous portion of the discrete phase distribution.

The force in the x direction due to the pressure distribution is

$$\int_x^{x+\Delta x} P_S \, n_x dA.$$

The value of $n_x dA$ is

$$n_x dA = \lim_{\Delta x \to 0} A^\ell_{x + \Delta x} - A^\ell_x$$

therefore

$$\int_x^{x+\Delta x} P_s n_x dA = \int_x^{x+\Delta x} P_s \lim_{\Delta x \to 0} \frac{A^\ell_{x+\Delta x} - A^\ell_x}{\Delta x} dx$$

$$= \int_x^{x+\Delta x} P_s \frac{\partial A^\ell}{\partial x} dx = \int_x^{x+\Delta x} P_s \frac{\partial \alpha^\ell \phi}{\partial x} AVdx.$$

so that

$$\int_x^{x+\Delta x} P_s n_x dA = P_s \left.\frac{\partial \alpha^\ell \phi}{\partial x}\right|_{\xi_6} \phi \Delta x \Delta y \Delta z \cdot \tag{50}$$

This force is due to a pressure distribution within a volume which has a change in the volume fraction. Another pressure force is the form loss force. The existence of this force can be observed in the case for which no change occurs in the volume fraction. A force would still be exerted on the bubbles or droplets and fuel rods by the other phase as long as the velocities of the two phases are different. Such forces are well known and have been studied in such cases as flow around a sphere. These forces exist even when the fluid is inviscid. These pressure distribution forces are expressed in the same manner as the interphase drag. That is,

$$F_i = \left[- \sum_s C^a_s \overline{A^a_s} v^a_i - C_{ab} \overline{A_{ab}} \left(v^a_i - v^b_i \right) \right] \phi \Delta x \Delta y \Delta z \tag{51}$$

where

C^a_s is the appropriate coefficient for the stationary surfaces

and

C_{ab} is the appropriate coefficient for the interphase forces.

In certain flow regimes such as smooth annular flow, this force may be zero. This coefficient may depend upon pressure drop because this force should be zero when no pressure gradient is present. The transient contributions from these forces will be discussed in Section IV-5.

The pressure forces exerted on the liquid on the surface is given by

$$\Sigma F_p^a = F_p^a\Big|_{x+\frac{\Delta x}{2}} + F_p^a\Big|_{x-\frac{\Delta x}{2}}$$

where

$$F_p\Big|_{x+\frac{\Delta x}{2}} = -\alpha^a P\Big|_{x+\frac{\Delta x}{2},\,y,z} \phi\Delta y\Delta z$$

$$F_p\Big|_{x-\frac{\Delta x}{2}} = \alpha^a P\Big|_{x-\frac{\Delta x}{2},\,y,z} \phi\Delta y\Delta z \ .$$

The factor α^a is used to account for only a portion of the surface of the unit volume exposing the liquid to the pressure P. [Bouré et al (1971) have discussed using different pressures for each phase.] The quantity α^a is defined as the $\alpha^a \equiv$ volume of the component 'a'/total volume. Thus, on a surface, this quantity is the area of component 'a' divided by the total area. The resulting expression is given as

$$\Sigma F_p^a = -\left(\phi\alpha^a P\Big|_{x+\frac{\Delta x}{2},y,z} - \phi\alpha^a P\Big|_{x-\frac{\Delta x}{2},y,z}\right) \Delta y\Delta z \tag{52}$$

$$\Sigma F_p^a = -\frac{\partial\phi\alpha^a P}{\partial x}\Big|_{\xi_7} \Delta x\Delta y\Delta z$$

where ξ_7 represents the coordinates of a point in ΔV. Thus, the total force due to shear is

$$F_i^a = \left[P_s \frac{\partial\alpha^a\phi}{\partial x}\Big|_{\xi_6} - \frac{\partial\phi\alpha^a P}{\partial x}\Big|_{\xi_7} - \phi C_s^a \overline{A_s^a}\, v_i^a - \phi C_{ab}\overline{A_{ab}}\left(v_i^a - v_i^b\right)\right] \Delta x\Delta y\Delta z \ . \tag{53}$$

5. TRANSIENT FLOW FORCE

The forces acting on phase 'a' as discussed in Sections IV-3 and IV-4 were limited to the steady state shear and pressure forces, respectively. However, the shear forces of Equation (35) and the pressure forces of Equation (49) are averages over the interfaces in the flow field and, therefore, may contain time dependent quantities. Of course, even in transient one-dimensional, single-phase flow, common practice is to employ a steady state friction factor correlation evaluated at the local instantaneous channel average velocity. Slattery (1972, pp 182-183) has compared the exact and the area-averaged, one-dimensional solutions for the case of a single-phase fluid starting from rest in a circular conduit of constant cross section. For this case, the approximate solution overpredicts the flow through the conduit.

The flow situation that is usually used to illustrate the effect of transient pressure and velocity distributions is the Basset-Boussinesq-Oseen solution for unsteady Stokes' motion of a single solid spherical particle in an incompressible constant property fluid at rest (Tchen, 1947). The equation of motion, for the particle starting from rest, is given by

$$\frac{4}{3}\pi a^3 \rho_p \frac{dV_p}{dt} = -\frac{1}{2}\left(\frac{4}{3}\pi a^3\right)\rho_f \frac{dV_p}{dt} - 6\pi\mu_f a V_p$$

$$-6\left(\pi\rho_f\mu_f\right)^{1/2} a^2 \int_0^t \frac{dV_p}{dt}\frac{1}{\sqrt{t-T}}\,dT + \frac{4}{3}\pi a^3\left(\rho_f-\rho_p\right) g \qquad (54)$$

where

a = radius of spherical particle

V_p = velocity of particle

ρ_p,ρ_f = density of particle, density of fluid

μ_f = viscosity of fluid.

The terms on the right-hand side of Equation (54) are as follows. (a) The first term represents the added mass due to the difference between the transient and steady state pressure distribution around the particle; (b) the second term represents the steady state Stokes' drag due to shear and pressure, evaluated at the instantaneous particle velocity V_p; (c) the third term represents the Basset force (Tchen, 1947) which accounts for the transient viscous drag; and (d) the fourth term represents the gravitational force on the particle. Thus, for this flow situation, the transient pressure and velocity distribution effects are additive to the complete steady state effects evaluated at the instantaneous transient velocity.

Figures 12 and 13 are presented to show the relative importance of the transient force terms in the slow flow regime. Solutions of Equation (54) are presented in these figures as a solid line for a steam bubble rising in water. The effect of pressure is manifested in Equation (54) only in terms of density ratio. The effect of the transient flow forces was determined by re-solving Equation (54) with the Basset term neglected and then with both the Basset and added mass term neglected. The results are presented as the ratio of the instantaneous velocity V_p to the terminal velocity V_∞ as a function of a dimensionless time. The effects of these terms are observed to be significant in this situation. The effects of these terms would be much less for a droplet in a vapor and may be less in high speed flow.

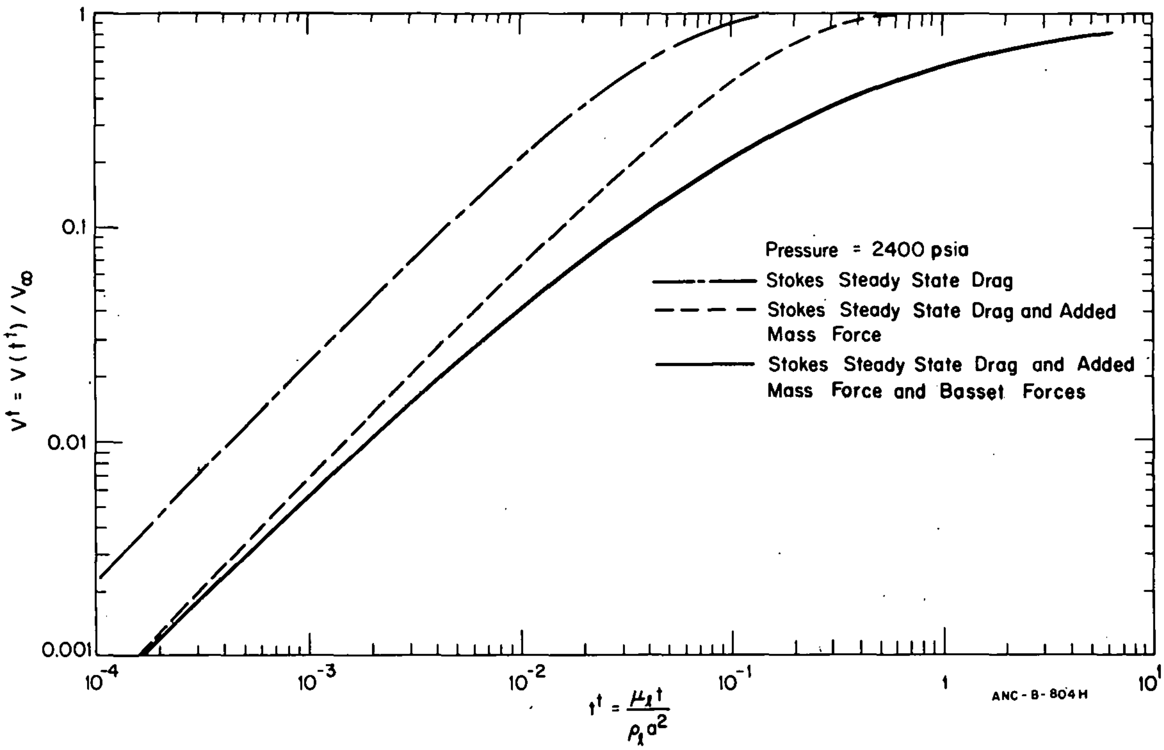


Fig. 12 Effect of transient fluid flow forces on velocity of a vapor bubble at 2400 psia.

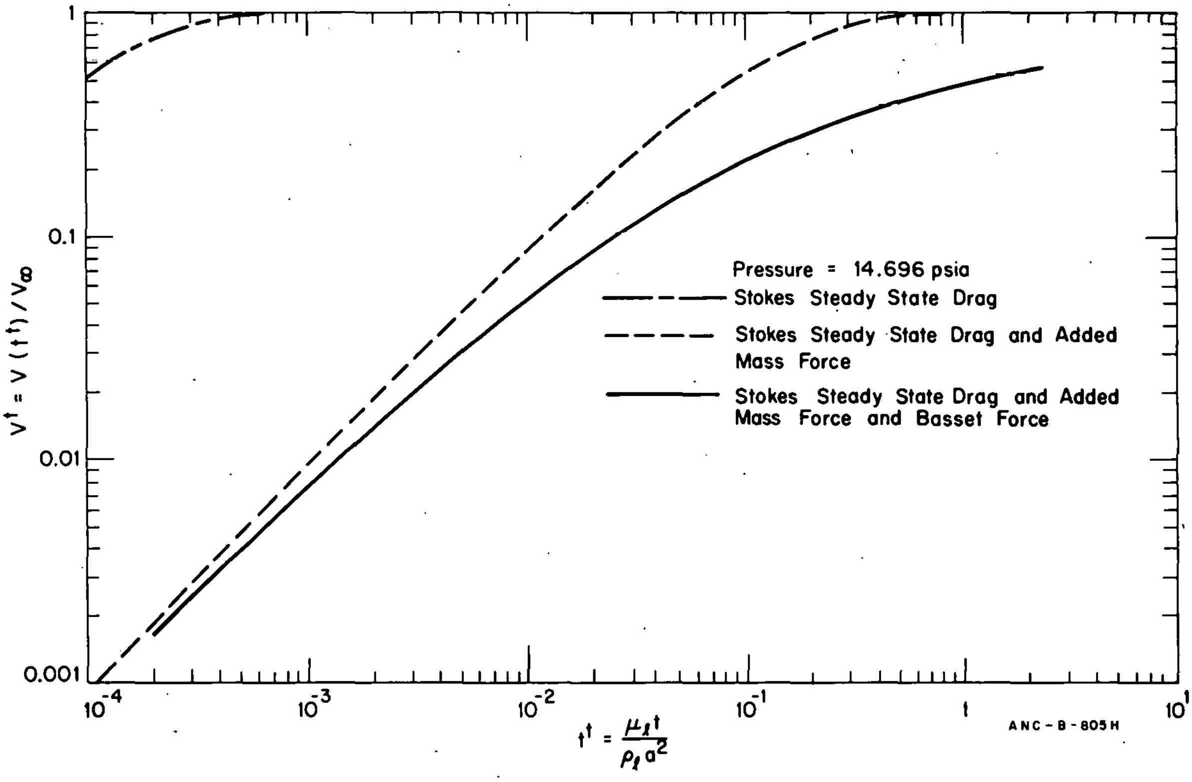


Fig. 13 Effect of transient fluid flow forces on velocity of a vapor bubble at 14.696 psia.

The most restrictive assumption associated with Equation (54) is the slow flow assumption. However, the equation is employed in various forms in the inertial, or high speed, flow regime essentially by mere extrapolation. Thus, as shown by Tchen (1947) for the case of a single particle, Equation (54) can be written

$$\begin{aligned} 4/3\ \pi a^3 \rho_p \frac{dV_p}{dt} &= -C_{AM}(Re,A_c)\ \rho_f\ 4/3\ \pi a^3 \frac{dV_p}{dt} - C_{DA}(Re,A_c) 1/2 \rho_f \pi a^2 V_p \\ &- C_B(Re,A_c)(\pi \rho_f \mu_f)^{1/2}\ a^2 \int_0^t \frac{dV_p}{dt} \cdot \frac{1}{\sqrt{t-T}}\, dT \\ &+ 4/3\ \pi a^3 \left(\rho_f - \rho_p\right) g \end{aligned} \tag{55}$$

where

Re = particle Reynolds number = $\dfrac{2V_p a \rho_f}{\mu_f}$

A_c = particle acceleration number = $\dfrac{V_p^2}{2a\dfrac{dV_p}{dt}}$

$C_{AM}(Re,A_c)$ = added mass force coefficient

$C_{DA}(Re,A_c)$ = velocity drag force coefficient

$C_B(Re,A_c)$ = Basset force coefficient

and the assumption that V_p and $\frac{dV_p}{dt}$ do not change sign has been employed. For the case of an ensemble of particles, Equation (55) is further extrapolated by assuming that the coefficients C_{AM}, C_{DA}, and C_B are functions of the volume fractions of the phases. Application of various forms of Equation (55) to momentum balance equations for two-phase flow has been discussed by Murray (1965), Anderson and Jackson (1967), Buyevich (1971), and Mecredy and Hamilton (1972). Forms of Equation (55) have been compared with experimental data by Odar and Hamilton (1964), Odar (1968a, 1968b), Tunstall and Houghton (1968), Torobin and Gauvin (1959) and Clift and Gauvin (1971). The results of these studies indicate that the extrapolations of Equation (54) have not yet been justified and that the coefficients C_{AM}, C_{DA}, and C_B have not been determined for general conditions.

Both the added mass and Basset force effects relative to the Stokes' drag are functions of the particle-fluid density ratio. For the case of liquid-vapor flow of a single-component fluid, the density ratio will depend on the flow regime. That is, the particles will be analogous to bubbles in one flow regime and to liquid droplets in another. Thus, in order to be general enough so that the possibility that a significant flow force is not omitted, some form of the added mass and Basset forces needs to be included in the momentum balance equations. However, in addition to the extrapolations and assumptions noted, additional assumptions are required with regard to (a) the form of the relative acceleration required due to the fact that both phases may be accelerating, (b) inclusion of the Basset force effect into the coefficients C_{AM} and C_{DA} in order to lessen numerical solution problems associated with evaluation of the integral of the Basset force, and (c) the equation of motion obtained for the one-dimensional flow case being considered to be a single component composed of vector forces. Further, additional analyses are required to apply the preceding solutions to fluid particles (Levich, 1962).

Adoption of the assumptions and extrapolations noted enables additive transient force effects to be included for both the phase-to-phase and phase-to-stationary-surfaces forces. In analogue with the pressure and shear force per unit volume derivations, the transient forces are given by

$$F^{ab}_{AM_i} = -\bar{A}_{ab} \, C_{AM_{ab}} \left(\frac{\partial v_i^{\,a}}{\partial t} - \frac{\partial v_i^{\,b}}{\partial t} \right) \phi \Delta x \Delta y \Delta z \tag{56}$$

for the interphase transient forces and by

$$F^{a}_{AM_{wi}} = - \sum_{s} \bar{A}^{a}_{s} \, C^{a}_{AM_s} \left(\frac{\partial v_i^{\,a}}{\partial t} \right) \phi \Delta x \Delta y \Delta z \tag{57}$$

for the phase-to-stationary-surface forces. An explicit representation of the Basset force is omitted. Other forms of the relative acceleration are possible and, indeed, the version given in Equation (56) is not invariant.

The example discussed is a highly idealized representation of a two-phase flow. In addition to the factors already discussed, several other differences between the example and reality exist. Among these differences for the phase interface are (a) nonspherical particle shapes, (b) compressibility of the particles and continuous fluid, (c) rotation of the particles, (d) phase changes at the liquid-vapor interfaces, and (e) acceleration of both the particle and the fluid. Nonetheless, forces due to transient effects do exist and must be accounted for in analyses. In general, these forces are due to the presence of curved streamlines in the flow field. All flow regimes of interest in two-phase flow will have associated with them curved streamlines. In some of these regimes, such as annular wavy flow, the transient effects may be small.

6. BODY FORCES

The only field force source of interest here is that due to gravity. The force on phase 'a' is given by

$$\iiint_{\Delta V} \phi\rho^a g_i \, dxdydz = \left(\phi\rho^a g_i\right)\Big|_{\xi_8} \Delta x \Delta y \Delta z \tag{58}$$

where ξ_8 represents the coordinates of a point in ΔV, and g_i is the component of the gravity field.

7. FINAL FORM OF THE MOMENTUM BALANCE EQUATION

The expressions for the components of the momentum equation, Equations (22), (23), (24), (48), (53), and (58), may be substituted into Equation (21), the resulting expression divided by $\Delta x \Delta y \Delta z$, and the operator $\lim_{\Delta V \to 0}$s applied to the equation to obtain the momentum equation for phase 'a':

$$\begin{aligned}
\frac{\partial \phi\rho^a v_i^a}{\partial t} + \left(\phi\rho^a v_j^a v_i^a\right)_{,j} = {} & \phi \dot{m}^a \hat{v}_i^a \\
& - \phi \sum_s \overline{A}_s^a \, B_s^a \left(\theta_s, \theta_a\right) v_i^a - \phi \overline{A}_{ab} B_{ab} \left(v_i^a - v_i^b\right) + \left(\phi\alpha^a \sigma_{ji}^a\right)_{,j} \\
& - \phi\alpha^a P_{,i} - \phi \sum_s C_s^a \, \overline{A}_s^a v_i^a - \phi C_{ab} \overline{A}_{ab} \left(v_i^a - v_i^b\right) \\
& - \phi \overline{A}_{ab} C_{AM_{ab}} \left(\frac{\partial v_i^a}{\partial t} - \frac{\partial v_i^b}{\partial t}\right) - \phi \sum_s \overline{A}_s^a \, C_{AM_s}^a \left(\frac{\partial v_i^a}{\partial t}\right) + \phi\rho^a g_i
\end{aligned} \tag{59}$$

where $\hat{v}_i^a = \hat{v}_i^b \equiv \hat{v}_i$

and σ_{ji}^a is approximated by Equation (47).

Additional interphase force terms can be included in this equation in a manner similar to the method used to include the second phase. For the two phases considered here, all interphase forces are assumed equal. That is,

$$\overline{A}_{ab}B_{ab} = \overline{A}_{ba}B_{ba}$$

$$\overline{A}_{ab}C_{ab} = \overline{A}_{ba}C_{ba}$$

$$\overline{A}_{ab}C_{AM_{ab}} = \overline{A}_{ba}C_{AM_{ba}} \quad .$$

The assumption also made is that $P_s = P$ because the pressures of both phases are assumed equal. Note should be taken that this momentum equation does not include diffusive forces. If multiple species are to be considered, this force should be added. Truesdell (1962) points out that the normal equations which are used to account for diffusion are actually approximations to momentum equations for each species for which momentum terms are neglected. Diffusive forces are beyond the scope of this present work.

The mixture momentum equation may be obtained by summation of the component momentum equations to obtain

$$\frac{\partial \phi \rho v_i}{\partial t} + \left(\phi \sum_{\ell,g} \rho^a v_j^a v_i^a\right)_{,j} = \phi \sum_{\ell,g} \sum_s \overline{A}_s^a B_s^a v_i^a + \left(\phi \sigma_{ji}\right)_{,j}$$
$$- \phi P_{,i} - \phi \sum_{\ell,g} \sum_s C_s^a A_s^a v_i^a - \phi \sum_{\ell,g} \sum_s \overline{A_s^a} \; C_{AM_s}^a \left(\frac{\partial v_i^a}{\partial t}\right) + \phi \rho g_i \qquad (60)$$

where

$\sum_{\ell,g}$ indicates summation over the components.

Note should be taken that the momentum flux terms do not yield a term which can be written completely in terms of the average velocity. The momentum flux may be written in terms of the average velocity and a velocity difference $u_i^a \equiv v_i - v_i^a$. Solution for v_i^a yields

$$v_i^a = v_i - u_i^a$$

so that

$$\Sigma \rho^a v_j^a v_i^a = \Sigma \rho^a \left(v_j - u_j^a\right) \left(v_i - u_i^a\right)$$

$$= \Sigma\rho^a v_j v_i - \Sigma\rho^a v_j u_i^a - \Sigma\rho^a u_j^a v_i + \Sigma\rho^a u_j^a u_i^a$$

therefore

$$\Sigma\rho^a v_j^a v_i^a = \rho v_j v_i = \Sigma\rho^a u_j^a u_i^a$$

because

$$\Sigma\rho^a v_j u_i^a = v_j \left(\Sigma\rho^a v_i - \Sigma\rho^a v_i^a\right) = 0 \quad .$$

Several authors, for example Green and Naghdi (1969), prefer to write the momentum flux in terms of $\rho v_j v_i$ and include the $\Sigma\rho^a u_j^a u_i^a$ as part of the total shear stress tensor.

V. DERIVATION OF THE ENERGY EQUATION

The overall energy balance for the two-phase seriated continuum is derived in this section. Whereas a continuity and a momentum are required for each phase, only one overall energy equation is necessary because the phase temperatures are assumed equal.

The first law of thermodynamics states that

$$\left\{\begin{matrix}\text{Rate of Change}\\ \text{of Energy}\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate of Heat}\\ \text{Added to the}\\ \text{System}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of Work}\\ \text{Done by the System}\end{matrix}\right\} . \tag{61}$$

This equation applies to a constant mass (Lagrangian) system.

For the Eulerian system under consideration, this equation may be written as

$$\begin{aligned}&\left\{\begin{matrix}\text{Rate of Accumulation}\\ \text{of Total Energy}\end{matrix}\right\} + \left\{\begin{matrix}\text{Rate of Energy}\\ \text{Efflux}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of Energy}\\ \text{Influx}\end{matrix}\right\}\\ &= \left\{\begin{matrix}\text{Rate of Heat}\\ \text{Influx through}\\ \text{the Walls}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of Heat}\\ \text{Efflux through}\\ \text{the Walls}\end{matrix}\right\} + \left\{\begin{matrix}\text{Rate of Heat}\\ \text{Generated in the}\\ \text{Control Volume}\end{matrix}\right\}\\ &- \left\{\begin{matrix}\text{Rate of Work}\\ \text{Done by the}\\ \text{System on the}\\ \text{Surroundings}\end{matrix}\right\} .\end{aligned} \tag{62}$$

Expressions for these terms are derived in the following sections.

1. TRANSPORT OF ENERGY

The total energy E^a of phase 'a' is defined as the sum of the internal energy U^a plus the kinetic energy, $\frac{1}{2} v_i^a v_i^a$. This total energy is expressed as

$$E^a = U^a + \frac{1}{2} v_i^a v_i^a \quad .$$

The rate of accumulation of energy in a control volume is given by

$$\iiint\limits_{\Delta V} \frac{\partial\phi}{\partial t} \left(\rho^g E^g + \rho^\ell E^\ell\right) dxdydz = \frac{\partial\phi}{\partial t} \left(\rho^g E^g + \rho^\ell E^\ell\right)\Big|_{\xi_1} \Delta x \Delta y \Delta z \tag{63}$$

where ξ_i is an appropriate point in the control volume. The energy which is being convected through each side of the control volume is $\Phi(\rho^g E^g + \rho^\ell E^\ell)$. The net flux (efflux minus influx) of energy may be expressed as

$$\begin{aligned} &\frac{\partial}{\partial x}\left[\phi\left(\rho^g E^g v_x^g + \rho^\ell E^\ell v_x^\ell\right)\right]\Big|_{\xi_2} \Delta x \Delta y \Delta z \\ &+ \frac{\partial}{\partial y}\left[\phi\left(\rho^g E^g v_y^g + \rho^\ell E^\ell v_y^\ell\right)\right]\Big|_{\xi_3} \Delta x \Delta y \Delta z \\ &+ \frac{\partial}{\partial z}\left[\phi\left(\rho^g E^g v_z^g + \rho^\ell E^\ell v_z^\ell\right)\right]\Big|_{\xi_4} \Delta x \Delta y \Delta z \quad . \end{aligned} \tag{64}$$

2. ADDITION OF HEAT TO THE SYSTEM

The heat flux in the direction "i" due to phase 'a' will be denoted by q_i^a. The net influx minus efflux of heat through the walls is given by

$$\begin{aligned} &\left\{ - \frac{\partial}{\partial x}\left[\phi\left(q_x^g + q_x^\ell\right)\right]\Big|_{\xi_5} - \frac{\partial}{\partial y}\left[\phi\left(q_y^g + q_y^\ell\right)\right]\Big|_{\xi_6} \right. \\ &\left. - \frac{\partial}{\partial z}\left[\phi\left(q_z^g + q_z^\ell\right)\right]\Big|_{\xi_7} \right\} \Delta x \Delta y \Delta z \quad . \end{aligned} \tag{65}$$

The conduction term will be considered to be the major portion of q_i^a. A representative form for this term is

$$q_i^a = - \alpha^a K^a T_{,i}$$

where k^a is the thermal conductivity of phase 'a' and T is the temperature.

The volumetric heat generation rate may be expressed as

$$\iiint_{\Delta V} \phi\left(q^g + q^\ell\right) \, dxdydz = \phi\left(q^g + q^\ell\right)\Big|_{\xi_8} \Delta x \Delta y \Delta z \quad .$$

The quantity q^a is considered to be made up of two components: (1) heat transfer from the stationary surfaces within the control volume, q^a_{conv}, and (2) heat generation from volume sources such as gamma heating, q^a_v, and thermal radiation, q^a_r. The term q^a_{conv} is related to the heat transfer per unit wall area, Q^a_w, by the relation

$$q^a_{conv} \quad \Delta V = A^a_w \; Q^a_w$$

or

$$q^a_{conv} = \overline{A}^a_w \; Q^a_w \quad . \tag{67}$$

3. RATE OF WORK DONE BY THE SYSTEM

The possible contributions to the work terms are:

(1) Shearing forces due to stationary surfaces

(2) Intraphase shearing forces

(3) Pressure forces

(4) Body forces.

The remaining two forces described in Section IV do not contribute to the work terms. The first of these, the shearing forces between phases, does work on each phase but produces no work on the surroundings. The second of these, the transient flow forces, results from accelerative effects and is, thus, not actually a force term.

3.1 Work Due to Stationary Surface Shearing Forces

The work due to shearing forces from stationary surfaces can be obtained from multiplying the surface shear force in Equation (35) by the velocity of the surface to obtain

$$W^a = \Sigma \int_{A^a_s} \sigma^a_{is} v^s_i \, dA = 0 \quad . \tag{68}$$

Since this velocity is zero, this work term is zero. This expression is equivalent to multiplying the stationary shear force term in Equation (44) by v^s_i.

3.2 Work Due to Intraphase Shearing Forces

The work done on the surface of the control volume by both phases is obtained by multiplying the forces shown in Figure 8 by the appropriate velocities and then summing over the x, y, and z directions as well as over both phases. The total work due to these forces is given by

$$W^a = \sum_{\ell,g} \left(\phi\alpha^a\sigma_{ji}^a v_i^a\right)_{,j}\Big|_{\xi_9} \Delta x\Delta y\Delta z \quad . \tag{69}$$

3.3 Work Due to Pressure Forces

The work due to pressure forces will be calculated as the negative of the work done by the surroundings on the system.

The pressure forces are exerted by phase 'a' on the control volume on the faces in the direction of an inward normal. These forces in the x direction are

$$F^a_{x-\frac{\Delta x}{2},y,z} = \phi\ \alpha^a P\Big|_{x-\frac{\Delta x}{2},y,z} \quad \Delta y\Delta z$$

and

$$F^a_{x+\frac{\Delta x}{2},y,z} = -\phi\ \alpha^a P\Big|_{x+\frac{\Delta x}{2},y,z} \quad \Delta y\Delta z \quad .$$

These forces are exerted with the velocity of phase 'a'. Thus, the work terms for both phases and on all sides of the control volume may be written as

$$\begin{aligned}&\Bigg\{-\frac{\partial}{\partial x}\left[\phi P\left(\alpha^g v_x^g + \alpha^\ell v_z^\ell\right)\right]\Big|_{\xi_{10}} - \frac{\partial}{\partial y}\left[\phi P\left(\alpha^g v_y^g + \alpha^\ell v_y^\ell\right)\right]\Big|_{\xi_{11}} \\ &- \frac{\partial}{\partial z}\left[\phi P\left(\alpha^g v_z^g + \alpha^\ell v_z^\ell\right)\right]\Big|_{\xi_{12}}\Bigg\} \Delta x\Delta y\Delta z \quad .\end{aligned} \tag{70}$$

The work done by the system on the surroundings is the negative of this expression.

3.4 Work Due to Body Forces

The work due to body forces will be derived as the work done by the system on the surrounding force field. Only body forces will be considered. The force exerted on phase 'a' by the body force field is

$$\iiint_{\Delta V} \phi\rho^a g_i \quad dxdydz$$

where g_i is the gravitational constant in the x_i direction. The force exerted by the fluid opposing this force is the negative of this expression. This force is being moved with the velocity of phase 'a'. Consequently, the work term associated with body forces on both phases is

$$\begin{aligned} & -\Bigg[\phi\Big(\rho^g g_z v_x^g + \rho^g g_y v_y^g + \rho^g g_z v_z^g \\ & + \rho^\ell g_x v_x^\ell + \rho^\ell g_y \; v_y^\ell + \rho^\ell g_z v_z^\ell\Big)\Bigg] \Big|_{\xi_{13}} \; \Delta x \Delta y \Delta z \quad . \end{aligned} \tag{71}$$

4. RESULTANT ENERGY EQUATION

The resultant energy equation is obtained by substituting Expressions (63) to (66) and (68) to (71) into Equation (62), dividing by ΔxΔyΔz, and then applying $\lim_{\Delta V \to 0}$ to obtain

$$\begin{aligned} \frac{\partial \phi \rho E}{\partial t} + \phi \sum_{\ell,g} \rho^a E^a v_i^a = & \left(\phi \sum_{\ell,g} \alpha^a k^a T,_i\right),_i \\ & + \phi q + \left(\phi \sum_{\ell,g} \alpha^a \sigma_{ji}^a v_i^a\right),_i \\ & - \left(\phi P \sum_{\ell,g} \alpha^g v_i^g\right),_i \\ & + \phi \sum_{\ell,g} \rho^a g_i v_i^a \end{aligned} \tag{72}$$

where

$$\rho E \equiv \rho^g E^g + \rho^\ell E^\ell$$

$$q = q^\ell + q^g \quad .$$

VI. ADDITIONAL RELATIONS REQUIRED TO COMPLETE THE EQUATION SET

Thus far, 5 equations and 12 unknowns have been introduced. The equations are mass balance equations for each phase, momentum equations for each phase, and the energy equation. The unknowns are

$\rho^{\ell} \quad \rho^{g}$

$\rho_{\ell} \quad \rho_{g}$

$v^{\ell} \quad v^{g}$

$a^{\ell} \quad a^{g}$

$U^{\ell} \quad U^{g}$

P

and $\dot{m} \equiv \dot{m}^{g} = - \dot{m}^{\ell}$.

The seven additional relations which are needed to complete the equation set are given in the following:

(1) Relation between the partial densities and the thermodynamic densities:

$$\rho^{g} = a^{g}\rho_{g}$$

$$\rho^{\ell} = a^{\ell}\rho_{\ell}.$$

(2) Equations-of-state for the thermodynamic densities along the phase boundary [steam-water properties are given in Meyer et al (1967)]:

$$\rho_{g} = \rho_{g}(P)$$

$$\rho_{\ell} = \rho_{\ell}(P).$$

(3) Equations-of-state for the thermodynamic energies along the phase boundary [Meyer et al (1967)]:

$$U^{g} = U^{g}(P)$$

$$U^{\ell} = U^{\ell}(P).$$

(4) Summation of the volume fractions:

$$a^g + a^\ell = 1.$$

The equations-of-state are assumed known.

These equations form the closure of the equation set. The momentum equations require empirical information for evaluation of some of the coefficients in the equation. The following section indicates how and where estimates for these correlations may be obtained.

VII. CORRELATIONS AND ANALYSES REQUIRED TO COMPLETE THE MOMENTUM BALANCE EQUATIONS

The momentum balance for each phase as given by Equation (59) contains several coefficients that must be estimated in order to obtain numerical results. These coefficients represent the effects on each phase of pressure and shear force distributions around the discrete regions of each phase and around stationary surfaces within the flow field. Analogous to transient, area-averaged, one-dimensional, single-phase flow, these coefficients are in fact representations of local solutions of the appropriate form of the Navier-Stokes equations for the geometry and flow conditions of interest. However, in most cases of engineering interest, experimental data must be used because analytical solutions are impractical. In the case of two-phase flow, even fewer analytical solutions are available, and reliance on experimental data is much more necessary. Again analogous to single-phase flow, the empirical correlations will be more reliable if simple physical models are employed as obtained from analytical solutions of simpler problems. Table I gives a summary of the required coefficients.

TABLE I

COEFFICIENTS APPEARING IN MOMENTUM BALANCE EQUATIONS

	Interphase Coefficients	Stationary Surfaces Coefficients
Shear Forces	B_{ab}	B_s^a
Pressure Forces	C_{ab}	C_s^a
Transient Effects of Shear and Pressure Forces	$C_{AM_{ab}}$	$C^a_{AM_s}$
Area per Unit Volume	$\bar{A}_{ab}$	$\bar{A}_s^a$

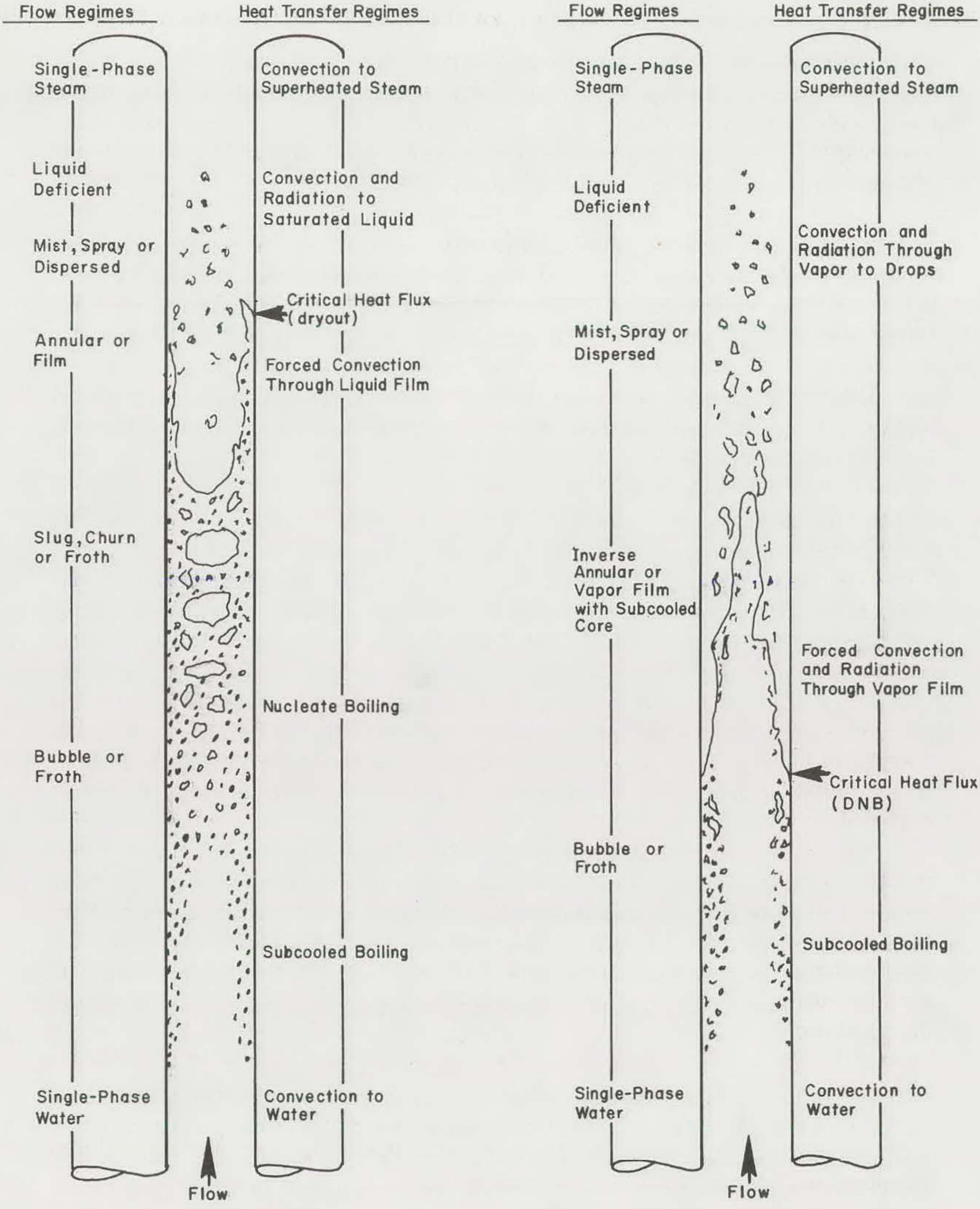
Proper evaluation of these eight terms is important if the balance equations are to describe accurately two-phase flows with mass exchange processes. These coefficients are discussed in this section with emphasis on the special requirements of two-phase flows. That is, the phase distribution patterns that arise in two-phase flows are presented along with methods of describing the effects of these patterns on the momentum of each phase. A heuristic discussion is presented here because a complete discussion of the determination of these constitutive relations is beyond the scope of this report.

The seriated (discrete-continuous) nature of two-phase, single-component flows has been noted earlier. The results of various experimental studies have indicated that several phase distribution patterns exist in these flows and, in fact, accurate determination of the conditions for a particular pattern would lead to greater understanding of the basic nature of two-phase flow. That is, if a given flow regime could be predicted to always be present for a fixed flow field and boundary conditions, improved accuracy could be attained with two-phase flow empirical correlations. One reason the ranges of conditions under which these correlations are derived should always be adhered to, if accuracy is to be considered, is that the phase distibution pattern has an effect on the quantities of interest, that is, on the pressure gradient, heat transfer coefficients, and liquid-vapor velocities, among others.

The various phase distribution patterns depend on many flow field properties and bounding surface thermal-hydraulic conditions. Brodkey (1967) and Hewitt and Hall-Taylor (1970) have given discussions of the flow regimes for vertical and horizontal flow channels. Figure 14 illustrates the possible flow regimes for the case of subcooled liquid entering the bottom of a heated array of rods and changing phase to become superheated steam at the exit of the array. The flow regimes are indicated for both a low and a high surface heat flux. Clearly, all or only a few of the regimes may be present depending primarily on the extent of net vapor phase generation possible for the flow into the channel and the heat flux at the bounding surfaces. Further discussion of flow regimes can be found in the books of Brodkey and Hewitt and Hall-Taylor.

An accounting of all occurrences in each flow regime is a complicated task. For example, in the early stages of bubble or froth flow, as shown in Figure 14(a), vapor regions are not distributed totally across the flow channel. Thus, if one-dimensional calculations are employed, an average vapor volume fraction across the whole channel would be required which clearly involves an approximation. Additionally, in this flow regime as the flow progresses up the channel, the vapor regions (bubbles) are of a wide range of sizes. In the annular, or film, flow regime, vapor generation at the heated surface may be suppressed under some conditions. The flow of the vapor phase past the liquid film surface will generate surface waves that may grow into roll waves on the bounding surface. Breakup of the tips of these waves adds liquid droplets to the vapor core, some of which may deposit on the liquid surface. In the mist and liquid deficient regimes, the vapor forms the continuous phase and discrete regions of liquid droplets of various sizes exist across the flow channel. As in the bubble regime, the discrete regions may not be distributed totally across the flow channel and may not attain contact with the heated surface under some conditions. The preceding is only a brief summary of a very large number of occurrences associated with two-phase, single-component flows with phase change.

The problem then is to account for the physical situation shown in Figure 14 with the coefficients of Table I. As a specific example, the force coefficients for interphase momentum exchange B_{ab} and C_{ab} are considered. For the case of a single solid particle at



(a) Low Heat Flux

(b) High Heat Flux

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Fig. 14 Flow and heat transfer regimes in rod array with vertical upflow.

rest in an infinite flow field at steady state conditions, these coefficients are well defined. These well known coefficients could be used as a first approximation for interphase momentum transfer in the flow regimes that consist of a discrete phase contained in a continuous phase. However, improvement of the description of this process can be obtained by incorporating the following information into the coefficients.

Firstly, liquid and vapor, unlike solid particles, will undergo deformation in response to the forces acting on them. The shape of the region determined by these forces in turn will influence the forces accounted for by the coefficients B_{ab} and C_{ab}. Secondly, the presence of stationary surfaces, adjacent particles, and trailing and following particles is known to also affect the forces acting on the particles. Lastly, the occurrence of mass exchange processes at the liquid-vapor interfaces will affect the velocity gradient at surfaces and thus affect the coefficients accounting for interphase forces. Boundary layer studies with blowing or suction have included an analogous effect.

In the annular flow regime shown in Figure 14(a), the interphase momentum exchange occurs at the vapor core-liquid film interface and between the liquid regions in the vapor core and the vapor. The coefficients B_{ab} and C_{ab} in this regime must account for the unique nature of the vapor flow past a wavy liquid surface. That is, as discussed by Cohen and Hanratty (1968) and Hewitt and Hall-Taylor (1970), the velocity distribution in the vapor does not correspond to that expected by analogy to roughened surfaces. In addition, the nonrigid liquid surface and the breakup of the tips of the waves is expected to also influence the velocity gradient at the interface. Again, as in the bubble flow regime, with one-dimensional calculations, the vapor volume is not uniformly distributed across the flow channel, and thus approximations will be associated with these calculations in this regime.

The surface areas per unit volume, $\overline{A}_{ab}$ and $\overline{A}_s^a$, will also be required to account for flow regime effects. For example, if no vapor covers part of a stationary surface, $\overline{A}_s^g$ is zero; and if the vapor is the only phase adjacent to the surfaces, $\overline{A}_s^g$ is the total area per unit volume available. Since the interphase area is composed of phase regions of various sizes, an average value will be determined or the effect of a distribution of sizes may be included in this term. Of course, if only one phase is present, the forms of $\overline{A}_{ab}$ and $\overline{A}_s^a$ must reflect this fact also.

The preceding brief discussion evidences that ultimately, to employ the model developed in this work, the conditions that determine which phase distribution pattern will exist must be understood. Determination of the phase distribution pattern is clearly an area that requires study and research of two-phase, single-component flows at a basic level. Additionally, interpretation of experimental data with a model that accounts for the effects of each phase will enable direct incorporation of experimental results into theoretical models.

We have previously indicated that the interphase and phase-to-stationary-surface coefficients of Table I can be estimated from existing data of single-phase and particle-fluid flows. Modifications that account for the uniqueness of two-phase, single-component flows with phase change have been noted as being necessary. These modifications and other additional information required for the coefficients can be obtained from both experimental data and theoretical analyses. In particular, theoretical analyses need be conducted only on a simpler problem that simulates the required situation and the results formulated in terms of the information contained in the field balance equation in order to improve the balance equations.

VIII . CONCLUSIONS

A set of equations has been derived for an unequal phase velocity and equal phase temperature seriated continuum. These equations have been reduced to a solvable form. That is, a complete set of equations has been formed, 12 equations and 12 unknowns, and how the correlations needed in this model may be estimated has been discussed. One point should be noted concerning the transient flow forces included in these equations. If these terms are neglected, Jarvis (1965) points out that the equation set is elliptic in nature and is not a properly posed initial value problem. Hence, the conclusion is that these terms are important for a complete solution of these equations.

We have noted in Section I that several methods have been used to derive momentum balance equations for two-phase or multiphase, multicomponent flows. The wide range of different flow situations of interest in the multiphase, multicomponent area has led to several sets of momentum equations in the literature. In general, these equations have resulted from different interpretations of the makeup of the flow field and the properties of the field. We have assumed that the material within the flow field may be considered continuous for some purposes and considered discrete for others. And whereas the primary area of interest is the simultaneous flow of the vapor and liquid phases of a single-component fluid, the resulting momentum balance, Equation (59), is applicable to other multiphase flow situations.

The momentum balance of Equation (59) applies to each phase in the flow field and also to each coordinate direction. The velocity employed in the derivation and resulting equations is a time- and volume-averaged value of each coordinate direction component. The constitutive relations accounting for the forces acting on each phase are also expressed as vector quantities and, in addition, by virtue of the averaging process employed in the derivation, the effect of stationary surfaces that bound the flow field is included in the momentum balance equation. The accuracy and completeness of the constitutive equations necessary to account for the forces acting on each phase will ultimately determine the accuracy of momentum balance equations of multiphase, multicomponent flows. In Section VII, brief discussion was given indicating methods of determining the information required to complete our forms of the constitutive equations for the case of flow of the liquid and vapor phases of a single-component fluid with evaporation and condensation occurring.

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