MASTER

ORNL-4406, Part I
UC-34 – Physics
UC-20 – Controlled Thermonuclear Processes

REVISED FORMULATION OF THE MACROSCOPIC MAXWELL THEORY. I. FUNDAMENTALS OF THE PROPOSED FORMULATION

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OAK RIDGE NATIONAL LABORATORY
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

Printed in the United States of America. Available from Clearinghouse for Federal
Scientific and Technical Information, National Bureau of Standards,
U.S. Department of Commerce, Springfield, Virginia 22151
Price: Printed Copy $3.00; Microfiche $0.65

ORNL-4406, Part I

Contract No. W-7405-eng-26

HEALTH PHYSICS DIVISION

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NOVEMBER 1969

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ABSTRACT

A modified version of the macroscopic Maxwell theory is developed with the purpose of removing some of the current contradictions and inconsistencies. One of the crucial problems of the current electromagnetic theory concerns the concept of energy. It appears that energy which has a clearly defined physical significance in all fields of physics has not been properly incorporated into the Maxwell-Lorentz theory. Therefore, the electromagnetic energy in a dissipative system as interpreted in the current version of the theory is not "energy" in the same sense as one usually interprets this concept in classical mechanics. The main object of this investigation is, therefore, to clarify the meaning of energy and to suggest a revised version in which the concept of energy is incorporated in accordance with the logical requirements of the theory. The proposed version is based on an assumption that the "electrical energy" both in nondissipative and dissipative media has a meaning only to the extent to which the electrical-field theory can be developed as a theory of a dynamical system. This assumption leads to a formulation of a principle which states that the energy density U_e of an electrical field in an absorbing medium is determined solely by the electrical intensity $\mathbf{E}$ and is, therefore, independent of any parameter which represents absorption. The proposed version differs from the traditional theory in the interpretation of the dielectric constant and of conductivity. The dielectric constant is considered to be independent of conductivity in the sense that a change in conductivity does not produce a corresponding alteration of the dielectric constant. Part I of this investigation deals with the fundamentals of the proposed reformulated version. These fundamentals are then applied in Part II to the extension of Brillouin's theory of wave propagation and group velocity to absorbing media.

I. INTRODUCTION

Substance and Scope of the Investigation

There are inconsistencies and contradictions in the macroscopic Maxwell theory. The achievements of the electromagnetic theory during the last several decades were nothing short of marvelous, but in spite of the spectacular accomplishments, one is still confronted with several fundamental problems which have neither been solved nor clarified. Difficulties are still encountered in the analysis of the energetic aspects of the theory, and several basic questions concerning the mechanism of propagation of electromagnetic energy from one region of space to another cannot be explained in the context of the present theory. Although the literature is immense and the level of sophistication is very high, several simple and ordinary questions such as "What is the energy of an electromagnetic disturbance in a dissipative medium?" and "What is the velocity of the disturbance?" have not been answered to date. The failure to clarify the most fundamental aspects of the theory is embarrassing. One is tempted to conjecture that the assumptions on which the current formulation is based are not entirely adequate and that perhaps there is a need for a critical evaluation of the entire theoretical structure.

Maxwell's theory is a phenomenological theory in the sense that it describes the electric and magnetic properties of matter by means of certain macroscopic parameters without seeking to make any further analysis of the origin of these quantities. The first insight into the microscopic structure of matter was made by Lorentz.[1] The intention of Lorentz was to account for the entire electromagnetic theory on the basis of microscopic structure of individual molecules and atoms. According to Lorentz, every atom or molecule contains a certain number of electrons more or less tightly bound to the necessary positive charge to make it neutral. When a monochromatic light wave impinges upon such an assembly of molecules, its main effect is to induce the electrons into an oscillatory motion at the same frequency as that of the incident wave. All these oscillating electrons emit, in turn, secondary waves which interfere with each other and with the incident wave and eventually give rise to the propagation through the medium of a single resultant wave of the same frequency but of a modified phase velocity. The variation of the phase velocity with frequency constitutes the phenomenon dispersion.

A portion of the energy of the waves is absorbed by the molecular medium, and the crucial problem in this analysis is to account for both dispersion and absorption. It appears that the mechanism of absorption has not been adequately described by Lorentz. Lorentz assumed that the absorption per se does not attenuate the wave, and expressed attenuation by means of a frictional force applied to each electron and directed against its motion. According to the point of view presented in this investigation, the absorption attenuates the wave, and, therefore, in the proposed formulation the frictional forces do not explicitly appear.

Using the concepts introduced by Lorentz, one arrives at a formulation in which the dielectric constant for an absorbing medium is expressed as $\epsilon = \epsilon(\omega,\gamma)$; that is, it depends not only on frequency, ω, but also on a frictional parameter, γ, which defines energy absorption. The validity of $\epsilon = \epsilon(\omega,\gamma)$ is, however, open to some questions. I have recently shown[2] that a physically meaningful dielectric constant should be independent of γ even if there is absorption. Thus if one takes into account all the physical factors involved, one obtains $\epsilon = \epsilon(\omega)$ for both absorbing and nonabsorbing media.

Both conductivity and absorption are discussed in this investigation from a more general point of view than the one in the previous paper.[2] The conceptual structure of the entire theory is analyzed, with particular emphasis on the idea of energy and on its significance in the study of electromagnetic interactions. Energy has a unique status in the description of nature. Because of its universal character it is effective in combining all physical systems such as electrical, mechanical, or chemical within a framework of a single superstructure. The knowledge that various types of energy are transformable, one into another, and that exact numerical correspondence is found in such transformations suggests that an interaction between any two physical systems may be considered as a transfer of energy from one system to another.

It is shown here that the interpretation of energy in the macroscopic Maxwell theory is misleading. In a study of the assumptions on which the current theory is based, it was concluded that energy which has a clearly defined significance in all fields of physics has not been properly incorporated into the current electromagnetic theory. Therefore, "electromagnetic energy" as interpreted in the current version of the theory is not "energy" in the same sense as one usually interprets this concept in classical mechanics. This conclusion is of some significance, since it concerns the entire electromagnetic theory and its place in the integrated structure of all physical sciences.

The main objective is, therefore, to clarify the meaning of energy in the macroscopic version of the electromagnetic-field theory. The concept of energy has its roots in mechanics. Because of an analogy between the expression for energy in classical mechanics and similar expressions in other dynamical systems, it has been possible to generalize this concept and extend its meaning. The extension into the electromagnetic-field theory can be best accomplished by means of the Lagrangian formulation, since the Lagrange equations of motion expressed in terms of generalized coordinates and generalized momenta apply equally well to dynamics of mechanical and electrical systems. In such a manner one would expect to be able to establish an analogy between the formulation of electrical energy in the context of the Lagrange theory and the corresponding formulation obtained from the macroscopic Maxwell theory, and also show that this analogy applies both to nondissipative and dissipative systems. This, however, is not the case. There is a curious discrepancy in the interpretation of energy in a dissipative dynamical system as applied in classical mechanics and the corresponding interpretation in the Maxwell-Lorentz theory. In classical mechanics the energy is expressed in terms of the state of the system and is not explicitly dependent on any parameter which defines dissipation. However, this is not the case in the current version of the electromagnetic theory.

The two conflicting situations can be illustrated by taking as an example a material particle moving with velocity $v(t)$ in a viscous medium and assuming that γ represents viscosity. The kinetic energy is then expressed as

$$U_{\text{kinetic}} = \frac{1}{2} m[v(t)]^2 \;, \tag{1-1}$$

and it is, therefore, not explicitly dependent on γ. The expression (1-1), which is obvious in elementary mechanics, is, however, not consistent with the interpretation of energy from the point of view of the current electromagnetic theory. If we assume that the dynamical laws of mechanics are developed along the same lines as the laws of electrodynamics within the context of the Lorentz theory, then the kinetic energy of a particle in a viscous medium must depend not only on the instantaneous velocity but also on γ. We would, therefore, have

$$U_{\text{kinetic}} = U[v(t), \gamma] \;. \tag{1-2}$$

The formulations (1-1) and (1-2) are incompatible.

Using as reference the empirical evidence produced by daily observations on a rather simple level, it would be difficult to reject (1-1) in favor of (1-2). We assume, therefore, that energy has a meaning in a dissipative electrical system only to the extent to which electrical-field theory can be developed as a theory of a dynamical system. Using the analogy between the laws of generalized dynamics and the electrical-field theory, we arrive at a formulation in which the energy of an electrical field is dependent solely on field intensity, even if there is dissipation. Such a formulation satisfies our intuitive feeling that the laws of dynamics are the same, both in mechanics and in the electrical-field theory. It provides, however, a point of view which is different from the one currently assumed to be valid.

Other aspects of electromagnetic theory are investigated. Following the point of view expressed by Planck, "energy" is regarded as a fundamental concept; therefore, the field intensity is a derived concept defined operationally in terms of energy. Expressions are obtained which represent energy density and energy flow in an absorbing medium, and the concept of velocity of propagation of an electromagnetic disturbance is analyzed both from the kinematic and the energetic point of view. The current theory is based on the work of Brillouin, and it applies specifically to nonabsorbing media. Therefore, by taking into account absorption, we remove the present restrictions which limit the validity of the Brillouin results.

It is currently assumed that a disturbance in an absorbing medium can be represented analytically in the form of a single nonhomogeneous plane wave, and then the energy attenuation is expressed by a quantity known as the extinction index which characterizes such a wave. A plane wave, whether homogeneous or not, cannot, however, be considered as an appropriate vehicle for transmitting energy through space; therefore, the extinction index is physically misleading. In the revised version the disturbance is expressed by a group of plane waves which are homogeneous when there is no absorption but become nonhomogeneous if absorption occurs. We describe, therefore, energy propagation in accordance with that which is actually observed: a wave packet rather than a single wave. Then a stationary wave packet in a nonabsorbing medium decays exponentially when the medium is absorbing (or grows exponentially when the medium is emitting). The decay of energy is represented by the attenuation of a wave packet and not by the extinction index.

One can classify kinematically or energetically electromagnetic disturbances into convective and nonconvective. It is shown that the attenuation of a convective disturbance can be expressed either by the relaxation length or by the relaxation time, whereas a nonconvective disturbance can only be represented by the relaxation time. The previous work[2] concerning the energetic formulation of Ohm's law for nonconvective disturbances has been generalized so as to include all disturbances, both convective and nonconvective. We define Ohm's law in a convective disturbance as representing a mechanism in which the rate of energy conversion is proportional to both the energy density and to the energy flow.

This investigation is divided into two parts. Part I deals with the fundamental aspects of the proposed formulation. Part II, to be published later, will deal with an analysis of the propagation of an electromagnetic disturbance on the basis of the assumption made in Part I.

Symbolism and Notation

Conventional notation is used in which **E** is the electric intensity, **B** is the magnetic induction, **D** is the electric displacement, **H** is the magnetic intensity, U_e is the electrical energy density, U_m is the magnetic energy density, $U = U_e + U_m$ is the energy density of the electromagnetic field, **S** is the energy flow, and Q is the rate of energy conversion. When $Q > 0$ (absorbing medium) the electromagnetic energy is transformed into nonelectrical energy such as chemical, mechanical, or heat, and the process is reversed when $Q < 0$ (emitting medium). When $Q = 0$ the medium is nonabsorbing, which also means that it is nonemitting.

In comparing the two versions of the electromagnetic theory a different notation is used to designate quantities which appear to be conceptually similar but are interpreted differently. The quantities used in the current version of the theory such as the electric intensity $\mathbf{E}_s$, the magnetic induction $\mathbf{B}_s$, the dielectric constant ϵ_s, the conductivity σ_s, etc., are provided with a subscript s, whereas the corresponding quantities in the revised version have no subscripts (viz., the electric intensity **E**, magnetic induction **B**, dielectric constant ϵ, conductivity σ, etc.).

The field quantities **E**, **B**, etc., vary with the position vector **r** and time t, and, therefore, $\mathbf{E} \equiv \mathbf{E}(\mathbf{r},t)$; $\mathbf{B} \equiv \mathbf{B}(\mathbf{r},t)$, etc. The Fourier components of **E**, **B**, etc., are expressed as $\mathbf{E}_{\mathbf{k},\omega} \equiv \mathbf{E}_{\mathbf{k},\omega}(\mathbf{k},\omega)$, $\mathbf{B}_{\mathbf{k},\omega} \equiv \mathbf{B}_{\mathbf{k},\omega}(\mathbf{k},\omega)$, etc., where **k** is the wave vector. Each of these field quantities on the macroscopic scale is an average of the corresponding microscopic quantities taken over space regions and time intervals which are large compared with atomic dimensions and periods. Such space-time regions, although containing many atoms and extending in time over many atomic periods, may still be sufficiently small with respect to the macroscopic scale to be regarded as "macroscopically infinitesimal." The quantity **B** and not **H** is the average macroscopic magnetic density.[3]

The medium is assumed to be molecular, nonmagnetic ($\mathbf{B} = \mathbf{H}$), and isotropic. Therefore, the magnetic permeability is 1, and the dielectric constant is a scalar. Although the main emphasis is on molecular and absorbing media, this analysis is basically applicable to plasmas, both absorbing and emitting. In the latter case the absorption is negative; therefore, the term "absorbing" should be interpreted as "absorbing or emitting."

There is an implied understanding that the dielectric constant is independent of the temperature of the medium. This is justified, since the heat generated during the process of polarization or depolarization is negligible.[4]

II. UNRESOLVED PROBLEMS AND UNANSWERED QUESTIONS IN THE CURRENT ELECTROMAGNETIC THEORY

Weaknesses and failures of the electromagnetic theory are emphasized; this subject has not received much attention and has been mentioned only occasionally. The successes in several specialized areas were so outstanding and the achievements so brilliant that one is tempted to assume that the electromagnetic theory as a branch of physics and of technology is complete — that there is nothing fundamental to explore. I strongly feel that this is not the case. Although relatively little is said about the current pitfalls and failures, they do nevertheless exist, and the frustrations in trying to understand what cannot be explained are very real.

There is a well-defined area within the scope of electromagnetic theory in which there is almost as much confusion as understanding. Many fundamental concepts, such as the dielectric constant, conductivity, energy density, group of waves, group velocity, and several others, are generally taken for granted with the understanding that their meaning is clear and unambiguous. However, this is not true.

Before we undertake a critical scrutiny of the entire theory, a few comments will be made concerning that part of the theory which is based on the concepts of "group of waves" and "group velocity" and which deals with the propagation of electromagnetic disturbances through dispersive media. Although the concepts of group of waves and group velocity originally formulated by Stokes[5] and subsequently by Rayleigh[6] are relatively old, the first consistent effort to incorporate them into the electromagnetic theory was made by Brillouin.[7,8] Brillouin proceeded from Maxwell's differential equations and described within the context of Maxwell's theory the details of energy propagation in an integral form. The main achievement of the Brillouin theory is in establishing the well-known relationships between energy density, energy flow, velocity of energy transport, and group velocity.

Brillouin did not make, however, any provision for energy absorption, and his results, therefore, are valid only to the extent to which they can be applied to nonabsorbing media. In spite of recent efforts to remove the present restrictions and generalize the Brillouin theory, relatively little has been accomplished along these lines to date. The results of these efforts were not, however, entirely fruitless.[9,10] In the course of the investigation on how to generalize the Brillouin theory, it became clear that certain assumptions previously taken for granted need to be reconsidered. Particular credit is due to Ginzburg in pointing out the following:[10] "Despite the fact that the problem of the conservation law and the expression for the energy density in electrodynamics is a fundamental one, there are certain aspects of it which have not been yet elucidated, and in particular for the case of an absorbing dispersive medium. For example, the familiar expression,

$$W_E = \frac{1}{4\pi} \int_{-\infty}^{0} \mathbf{E} \cdot (\partial \mathbf{D}/\partial t)\, dt \, ,$$

even when reduced to the form constant $\times\, E^2$, cannot be regarded as the total energy density when absorption is present, if only because the value of W_E may be negative." He also stated that[11] ". . . one needs to underline that in presence of absorption it does not appear to be possible in a general case to introduce phenomenologically the concept of the mean electromagnetic energy."

In order to justify more fully the efforts to reformulate the theory, it will be useful to outline its present status with emphasis on weakness rather than strength. I hope to accomplish this objective by pointing out several problems of very fundamental nature which, surprisingly, cannot be solved by the current theory. These problems are as follows.

Calculation of Energy Stored by an Electrical Disturbance in an Absorbing Medium. — There is a great deal of misunderstanding in the current theory concerning the meaning of the electrical energy in an absorbing medium. We think of energy of a system as dependent on its physical state at the time considered. If we apply, however, the current procedure to calculate energy, we obtain an expression which depends not only on the time considered but also on previous times. Therefore, it would appear that energy can have different values in accordance with the manner in which a particular state was reached.

Principle of Conservation of Energy. — The entire structure of all physical sciences is based on the principle that energy is conserved, a principle which has never been seriously questioned or challenged. Surprisingly, however, the validity of this principle cannot be established in the context of the current theory. The difficulty is due to the failure of the theory to explain the meaning of energy, and if we attempt to follow the standard procedure to prove that energy is conserved, we arrive at an equality which has either an obscure physical significance or is misleading.

Velocity of a Disturbance. — Both the kinematic velocity (group velocity) and the energetic velocity (velocity of the energy transport) can be said to have a physical meaning if we assume that there is no absorption. However, absorption distorts the shape of the disturbance, and if absorption occurs it is not clear how one can define the kinematic velocity. Furthermore, since energy in an absorbing medium is not adequately defined, one cannot formulate an expression for the velocity of energy transport.

Attenuation of an Electromagnetic Disturbance. — The current calculation of the effects of attenuation leads to a puzzling result which implies that absorption is a necessary condition, sine qua non, for the transmission of energy. In other words, if a medium is nonabsorbing it cannot transmit electromagnetic energy! Apparently a physically meaningful description of the absorption process has not been formulated to date.

III. ENERGY IN CONSERVATIVE AND DISSIPATIVE SYSTEMS

The definition of energy is a crucial matter in this investigation. In order to fully comprehend the complications which have arisen from different interpretations of this concept, one needs to follow the logical evolution from the original definition of energy in mechanics to its very abstract involvement in electromagnetic-field theory. The concept of energy is so thoroughly a part of our existence and so completely taken for granted that a brief critical evaluation of some of its elementary aspects in mechanics appears to be essential in order to clarify the main points of this analysis.

Energy in a Conservative System

According to Newton's law, the behavior of a system of N particles is described by a set of N equations of the type

$$\mathbf{F}_u^0 = \frac{d}{dt}(m_u \dot{\mathbf{r}}_u^0) \qquad (u = 1, 2, \ldots, N) , \tag{3-1}$$

where $\mathbf{F}_u^0$ is the force applied to each particle, m_u is the mass, and $\mathbf{r}_u^0$ is the position of the particle. In seeking a measure of the total effect of a force, it seems natural to consider its line integral taken over the path of the particle. From (3-1) we have

$$\int_{(a)}^{(b)} \mathbf{F}_u^0 \cdot d\mathbf{r}_i = \int_{(a)}^{(b)} \frac{d}{dt} m_u \mathbf{r}_u^0 \cdot d\mathbf{r}^0 = \frac{1}{2}[m_u(\dot{\mathbf{r}}_u^0)^2]_{(a)}^{(b)} , \tag{3-2}$$

where the integration extends from the initial time a to the final time b. This line integral is defined as the work done by the force. It is assumed that this work is stored in the particle as the energy of motion, and the quantity $T_u^0 = \frac{1}{2} m(\dot{\mathbf{r}}_u^0)^2$ is, therefore, referred to as the kinetic energy. Summing over all particles of the system we have

$$\int_{(a)}^{(b)} \sum_{i=1}^{N} \mathbf{F}_u^0 \cdot d\mathbf{r}_s^0 = \sum_{u=1}^{N} (T_u^0)_{(a)}^{(b)} = (T^0)_{(a)}^{(b)} , \tag{3-3}$$

where $(T^0)^{(a)}$ and $(T^0)^{(b)}$ are the final and initial values of the total kinetic energy of the system. The relationship (3-3) can be expressed in a differential form as

$$\frac{dT^0}{dT} = \sum_{u=1}^{N} \mathbf{F}_u^0 \cdot \dot{\mathbf{r}}_u^0 . \tag{3-4}$$

In certain cases $\sum_{u=1}^{N} \mathbf{F}_u^0 \cdot \dot{\mathbf{r}}_u$ is the complete differential of a uniform, one-valued function $-V^0$ of $\mathbf{r}_1^0, \mathbf{r}_2^0, \ldots, \mathbf{r}_N^0$, and then

$$\sum_{u=1}^{N} \mathbf{F}_u^0 \cdot d\mathbf{r}_u^0 = -dV^0 , \tag{3-5}$$

or

$$\frac{dV^0}{dT} = \sum_{S=1}^{N} \frac{\partial V^0}{\partial \mathbf{r}_u^0} \cdot \dot{\mathbf{r}}_u^0 = \sum_{S=1}^{N} \mathbf{F}_u^0 \cdot \dot{\mathbf{r}}_u^0 . \tag{3-6}$$

Such a system is known as conservative, and in such case we obtain

$$\frac{d}{dt}(T^0 + V^0) = 0 , \tag{3-7}$$

or

$$(V^0)^{(a)} + (T^0)^{(a)} = (V^0)^{(b)} + (T^0)^{(b)} . \tag{3-8}$$

One refers to (3-7) or (3-8) as representing the principle of energy conservation, and

$$U^0 = T^0 + V^0 \tag{3-9}$$

is known as the energy of the system. The concept of energy is, therefore, associated with motion of material particles, and energy is defined as "that which is produced by work, or which can be transformed into work."

Consider now the idea of energy as interpreted in electromagnetic-field theory. The most significant achievement of electromagnetic theory is that a continuum which was previously considered as an empty space became a locus of dynamical processes controlled by dynamical laws and that these laws were found to be in some respects similar to the laws of classical mechanics. Assumption was therefore made that the same variational concepts associated with the Hamilton principle and with the Lagrange equations of motion, which proved to be so fruitful in the study of mechanical systems, can also be applied to the analysis of electromagnetic fields. The fundamental merit of the Hamilton-Lagrange approach is not only its emphasis on energy and work rather than on force, but also that the generalized coordinates (q^0) and the generalized velocities ($\dot{q}^0$) do not have to refer directly to the positions and velocities of actual particles. By a suitable identification of coordinates, one can describe a Lagrangian density in electromagnetic-field theory; therefore, one can derive Maxwell's equations from the Lagrangian formulation in electrodynamics in the same manner as Newton's equations are derived from the Lagrangian formulation in mechanics.[12] In that sense, electrodynamics in the dynamics of an electrical field and certain conceptual aspects of the Lagrangian formulation, particularly those which concern energy, are equally applicable in both cases.

It was therefore recognized that the expression

$$\delta \int_{t_1}^{t_2} L \, dt \tag{3-10}$$

for Hamilton's principle and the appropriate formulation for the Lagrangian function $L = L(q_i^0, \dot{q}_i^0)$ $(i = 1, 2, \ldots, n)$ can be used in describing the laws of mechanics as well as those of the electrical-field theory, and that the Lagrangian equations of motion

$$\frac{d}{dt} \frac{\partial L(q_i^0, \dot{q}_i^0)}{\partial \dot{q}_i^0} - \frac{\partial L(q_i^0, \dot{q}_i^0)}{\partial q_i^0} = 0 \tag{3-11}$$

apply equally well in both cases. [The Lagrangian L was assumed not to depend explicitly on time, and the variation in (3-10) was taken over an arbitrary path $q_i^0(t)$ such that δq_i vanishes at t_1 and t_2.]

One of the important aspects of the Lagrangian formulation is that it leads to a relationship

$$\frac{d}{dt} \left[\sum_{i=1}^{n} \dot{q}_i^0 \frac{\partial L(q_i^0, \dot{q}_i^0)}{\partial \dot{q}^0} - L(q_i^0, \dot{q}_i^0) \right] = 0 \tag{3-12}$$

which can be expressed as

$$\sum_{i=1}^{n} \dot{q}_i^0 \frac{\partial L(q_i^0, \dot{q}_i^0)}{\partial \dot{q}^0} - L(q_i^0, \dot{q}_i^0) = U^0 \ , \tag{3-13}$$

where U^0 is a constant. The expression (3-13) is known as the Jacobi integral. It is also called energy, because for a Lagrangian which describes the motion of material particles the expression (3-13) reduces to the form (3-9). We find it convenient to think of the coordinates $q_i(t)$ as typifying "the state of the system" in the sense that when $q_i(t)$ and $\dot{q}_i(t)$ are known at a given time t, the state of the system is said to be known at that time. The energy is therefore a scalar which depends on the physical state but remains constant at all times.

Energy in a Dissipative System

It may happen that the given forms are such that an equality of the type (3-6) does not occur, and then the above procedure for defining energy cannot be used. In such case the system is known to be nonconservative. Consider a nonconservative system of N particles in which $\mathbf{F}_u$ is the force applied to a particle u, defined by its position $\mathbf{r}_u$. Experience tells us that we can describe the observed motion reasonably well by distinguishing two types of forces, $\mathbf{F}_{u1}$ and $\mathbf{F}_{u2}$, and recognizing that in some instances $\sum_{u=1}^{N} \mathbf{F}_{u1} \cdot d\mathbf{r}_u$ can be expressed as a perfect differential of an appropriate function $-V$ of $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N$, and then

$$\frac{dV}{dt} = \sum_{u=1}^{N} \frac{\partial V}{\partial \mathbf{r}_u} \cdot \dot{\mathbf{r}}_u = \sum_{u=1}^{N} \mathbf{F}_{u1} \cdot \dot{\mathbf{r}}_u . \tag{3-14}$$

The forces $\mathbf{F}_{u1}$ are said to be conservative forces and are thought of as constituting a single physical entity to which the remaining forces $\mathbf{F}_{u2}$ are extraneous. In such case we have

$$\frac{dT}{dt} = \sum_{u=1}^{N} \mathbf{F}_{u1} \cdot \dot{\mathbf{r}} + \sum_{u=1}^{N} \mathbf{F}_{u2} \cdot \dot{\mathbf{r}} = -\frac{dV}{dt} + \sum_{u=1}^{N} \mathbf{F}_{u2} \cdot \dot{\mathbf{r}}_u , \tag{3-15}$$

from which we obtain

$$\frac{d}{dt}(T + V) = \sum_{S=1}^{N} \mathbf{F}_{u2} \cdot \dot{\mathbf{r}}_u . \tag{3-16}$$

There is a formal similarity between (3-6), which deals with all the forces in a conservative system, and (3-14), which accounts for the conservative forces in a nonconservative system. Partly because of this similarity and partly because of intuition guided by experience, we assume that (3-16) represents conservation of energy and that

$$U = T + V \tag{3-17}$$

is the energy of a nonconservative system. Therefore, we consider (3-16) as representing a process in which a change in energy is compensated by the work of extraneous forces. If the extraneous forces are due to friction, the energy is converted into heat. Then (3-17) is said to represent energy of a dissipative system.

The assumption that (3-17) represents energy gives a new outlook on the entire problem. One should note that the definition of energy in a conservative system was based on the existence of a function V^0 defined by (3-5) and that such a function does not exist in a dissipative system. We are attaching, therefore, a physical significance to a concept which cannot be logically formulated in the same manner as before. The hypothetical nature of (3-17) was strongly emphasized in several discussions on the conceptual problems in classical mechanics. According to Lindsay and Margenau,[13] "The very definition of mechanical energy rests upon the discovery of something which remains constant during motion and now we wish to apply the concept to the case where this constancy does not exist." Lindsay[14] in his analysis of a damped harmonic oscillator observed that "Strictly speaking there is no longer an energy because there no longer exists a potential energy function. However, in many practical problems the rate of decrease in amplitude of motion is small and it becomes convenient to introduce a fiction of energy which decreases with time at the rate indicated." Although the physical meaning of (3-17) was not accepted without reservation, it is now generally agreed that this expression represents energy,[15] and, as pointed out by Lindsay and Margenau,[13] "The concept of energy has been extended and precise meaning given to its variation with time."

The key problem in this investigation deals with the extension of (3-17) into the electromagnetic-field theory. Therefore, (3-17) will be rephrased in terms of generalized coordinates in the same manner that (3-13) was obtained by rephrasing (3-9), and then we assume that electrical energy has a meaning only to the extent to which the electrical-field theory can be developed as a theory of a dynamical system.

There is, however, a difficulty. The dynamical energy (3-17) is transformed into heat, and heat as pointed out by Whittaker[16] is a form of energy not recognized in dynamics. In other words, the Hamilton-Lagrange formalism is not adapted to the study of thermal processes.[17] One can, however, describe the effects of dissipation by relatively simple mathematical means through the introduction of a function of the velocity in the Lagrange equations of motion. In this way, suggested by Rayleigh,[18] it becomes a simple matter to describe the evolution of a dissipative system in terms of generalized coordinates and to consider the problem in its broad aspects, both in the dynamics of a mechanical system and in the electrical-field theory.

We find it convenient in this investigation to treat the Rayleigh dissipation function as a small perturbation. A conservative (unperturbed) system which was previously expressed in terms of the unperturbed coordinates q_i^0 is now described by the perturbed coordinates q_i. Therefore, by including the Rayleigh dissipation function

$$R = \frac{1}{2} \sum_{i=1}^{n} \gamma_i \dot{q}_i \tag{3-18}$$

into the Lagrangian equations of motion, we obtain

$$\frac{d}{dt} \frac{\partial L(q_i,\dot{q}_i)}{\partial \dot{q}_i} - \frac{\partial L(q_i,\dot{q}_i)}{\partial q_i} = Q \ , \tag{3-19}$$

where

$$Q = -\partial R/\partial \dot{q}_i \tag{3-20}$$

are the extraneous forces which depend on the frictional parameters γ_i. Since the perturbation is small, we have

$$|Q_i| << |\partial L(q_i,\dot{q}_i)/\partial q_i| \ . \tag{3-21}$$

The perturbed coordinates $q_i(t)$ constitute a single physical set to which the quantities involving γ_i are extraneous. Therefore, one can think of a physical state in a dissipative system as expressed by q_i in analogy to the physical state of a conservative system expressed in terms of q_i^0.

It remains now to define the energy of a perturbed and therefore dissipative system. There is, however, no unique way of arriving at an appropriate formulation, and any choice of what should be regarded as energy is based to a large extent on intuition. We adopt a heuristic approach and assume that the dependence between the energy and the physical state of the system is not altered in any manner by the perturbation. In other words, the relationship between the perturbed energy U and the perturbed coordinates q_i is assumed to be the same as the one between the unperturbed energy U^0 and the unperturbed coordinates q_i^0. Therefore,

$$U = \sum_{i=1}^{n} \dot{q}_i \frac{\partial L(q_i, \dot{q}_i)}{\partial \dot{q}_i} - L(q_i,\dot{q}_i) \ . \tag{3-22}$$

We cannot, however, accept (3-22) on its face value unless we can show that the sum of all the energies is conserved. The conservation of energy occupies such a central position in our thought that when we encounter a dynamical problem in which the energy decreases, we must find another form of energy such as heat which is simultaneously produced. Therefore, we return to Eq. (3-19) and derive the following equality:

$$\frac{d}{dt}\left(\sum_{i=1}^{n} \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L \right) - \sum_{i=1}^{n} \gamma_i \dot{q}_i^2 = 0 \ , \tag{3-23}$$

which represents energy conservation, provided we interpret

$$Q = \sum_{i=1}^{n} \gamma_i \dot{q}_i^2 \tag{3-24}$$

as the rate of generation of heat.

Our justification for interpreting (3-22) as a generalized form of energy is based on empirical evidence. The following two examples will show that in case of a simple mechanical system, (3-22) can be reduced to what is usually considered as energy.

Energy Dissipation in Simple Mechanical Systems

Consider a material particle moving along a straight line in the absence of a potential field. It can be shown that if there is no friction ($\gamma = 0$), Eq. (3-13) can be reduced to the form

$$U^0 = \tfrac{1}{2}m(\dot{q}^0)^2 \ . \tag{3-25}$$

On the other hand, if there is friction ($\gamma \neq 0$), we obtain, according to (3-22),

$$U = \tfrac{1}{2}m\dot{q}^2 \ . \tag{3-26}$$

It is evident that (3-26) represents an extension of (3-25), and both represent energy.

The most important empirical justification for (3-22) is provided by a harmonic oscillator. If there is no dissipation the Lagrangian of a harmonic oscillator can be expressed as

$$L = \tfrac{1}{2}[(\dot{q}^0)^2 - \omega_0^2(q^0)^2] \ , \tag{3-27}$$

where ω_0^2 is a constant. Substituting (3-27) in (3-11), one obtains the equality $\ddot{q}^0 + \omega_0^2 q^0 = 0$, which yields $q^0 = a \cos(\omega_0 t + \alpha)$. Consequently, in accordance with (3-13) we have

$$U^0 = \tfrac{1}{2}[(\dot{q}^0)^2 + \omega_0^2(q^0)^2] \ . \tag{3-28}$$

However, if the oscillator is perturbed by dissipation, then using the Rayleigh term $R = \tfrac{1}{2}\gamma(\dot{q})^2$, we obtain from (3-19) the equality $\ddot{q} + \gamma\dot{q} + \omega_0^2 q = 0$, which yields

$$q = ae^{-\lambda t}\cos(\omega t + \alpha) \ , \tag{3-29}$$

where $\lambda = \gamma/2$, and $\omega = (\omega_0^2 - \lambda^2)^{1/2}$. Since the perturbation is small, the difference between ω and ω_0 is of second degree of smallness. Consequently, we assume that

$$q = ae^{-\lambda t}\cos(\omega_0 t + \alpha) \tag{3-30}$$

and express the perturbed energy in accordance with (3-22) as

$$U = \tfrac{1}{2}(\dot{q}^2 + \omega_0^2 q^2) = U^0 e^{-2\lambda t} \ . \tag{3-31}$$

The expression (3-31) represents the familiar attenuating process in mechanical vibrations. In this context (3-31) is generally referred to as the "energy" of the vibration, and in that sense (3-31) is an extension of (3-28).

Dependence Between the Energy and the State of the System

It would seem at first sight that the result which represents in a generalized form the energy of a dissipative system is intuitively obvious, and one may question the need for emphasizing the importance of a procedure which leads to the derivation of (3-22). From a methodological point of view, such judgment is shortsighted. What is "intuitively obvious" is a question of semantics, since otherwise two conflicting definitions of the same physical concept would not have occurred. The purpose of the formulation (3-22) is to establish a criterion for determining which one of the two conflicting definitions of energy represents a physically meaningful concept. In simple mechanical systems we all meet on common ground when we share a vivid intuition of the motion of a particle. But when we pass to more complicated dynamical systems, the intuition is harder to follow, and in such case the mathematical structure in the form of expression (3-22) may lead more effectively to practical results.

By mere inspection of (3-22) and by comparing (3-22) with (3-13), we can now state two requirements which have to be satisfied in any formulation of energy in a dissipative dynamical system. These requirements are as follows:

(*a*) Energy must be expressed by the coordinates of the system and be, therefore, independent of the dissipational parameters. Consequently,

$$U = U(q_i) \; , \tag{3-32}$$

not

$$U = U(q_i, \gamma_i) \; . \tag{3-33}$$

(*b*) The relationship between the energy and the coordinates must be the same whether there is dissipation or not. Consequently,

$$U(q_i) = U^0(q_i^0)_{q_i^0 = q_i} \; . \tag{3-34}$$

Electromagnetic Energy in a Dissipative Medium

An electromagnetic field in a nonabsorbing medium can be defined by the electrical intensity $\mathbf{E}^0$ and the magnetic induction $\mathbf{B}^0$. The significance of these quantities as representing the physical state of a system is, therefore, analogous to q_i^0 and $\dot{q}_i^0$ in a dynamical particle system. Therefore, it is possible to associate with an electromagnetic field a Lagrangian density as $\mathcal{L} = \mathcal{L}(\mathbf{E}^0, \mathbf{B}^0)$ and to derive the laws of electrodynamics, using a variational approach. The equations of motion will then appear in the form of Maxwell's equation, and we can think of the energy density in analogy to $U = U^0(q_i, \dot{q}_i)$ as having the form

$$U^0 = U^0(\mathbf{E}^0, \mathbf{B}^0) \; . \tag{3-35}$$

The analogy between classical mechanics and electrodynamics can also be extended to dissipative systems. Therefore, one can modify the Lagrangian formulation of an electromagnetic field by incorporating therein a dissipation function similar to (3-18), which for our present purposes needs not to be specified. The electromagnetic field will then be defined by perturbed coordinates **E**, **B** such that $\mathbf{E} \neq \mathbf{E}^0$ and $\mathbf{B} \neq \mathbf{B}^0$. We will then assume that the perturbed energy U depends on the perturbed field intensities **E**, **B** in the same manner that the unperturbed energy U^0 is dependent on the unperturbed field intensities $\mathbf{E}^0$, $\mathbf{B}^0$. Consequently,

$$U = U(\mathbf{E},\mathbf{B}) \ , \tag{3-36}$$

$$U(\mathbf{E},\mathbf{B}) = [U^0(\mathbf{E}^0,\mathbf{B}^0)]_{\mathbf{E}^0=\mathbf{E};\,\mathbf{B}^0=\mathbf{B}} \ . \tag{3-37}$$

One can separate the electric field from the magnetic field, and relationships similar to (3-36) and (3-37) can be shown to apply to the electric field. It is well known that if there is no dissipation the energy density can be expressed as $U^0 = U_e^0 + U_m^0$, where $U_e^0 = U_e^0(\mathbf{E})$ depends solely on the electric intensity, and $U_m^0 = U_m(\mathbf{B})$ depends solely on the magnetic induction. One can then assume that in a dissipative system, $U = U_e + U_m$, where

$$U_e = U_e(\mathbf{E}) \ , \tag{3-38}$$

$$U_e(\mathbf{E}) = [U^0(\mathbf{E}^0)]_{\mathbf{E}^0=\mathbf{E}} \ . \tag{3-39}$$

(A similar relationship holds for the energy stored in the magnetic field.) It is therefore seen that the relationship between the field energy and the field intensity is the same in absorbing and non-absorbing media.

IV. VALIDITY OF THE CURRENT MACROSCOPIC THEORY

Fundamental Structure of the Electromagnetic Theory

The most distinctive feature of the electromagnetic theory deals with a new form of energy which can be stored in, removed from, and travel freely through a medium which appears to our senses to be mere empty space. Essentially there are two postulates: the postulate on conservation of energy and the postulate on contiguous action. The principle on conservation of energy states that the new form of energy satisfies the same requirements as any other energy form. Therefore,

$$\operatorname{div} \mathbf{S} + \frac{\partial U}{\partial t} + Q = 0 \ . \tag{4-1}$$

The principle of contiguous action deals with the mechanism of energy transmission and states that this mechanism operates in accordance with the following two relationships:

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} , \tag{4-2}$$

$$\operatorname{curl} \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\sigma \mathbf{E} \ , \tag{4-3}$$

where

$$\mathbf{D}_{\mathbf{k},\omega} = \epsilon \, \mathbf{E}_{\mathbf{k},\omega} \ . \tag{4-4}$$

Although the expressions for the new form of energy do not explicitly appear, these equations describe nothing more than the transmission of energy, and the field intensities **E**, **B**, and **D** are considered to be merely auxiliary quantities. The term "contiguous" indicates that causality is expressed by a relationship in which **E**, **B**, and **D** at each point are connected only with the values at adjacent, that is, contiguous, points and only indirectly with the conditions at finitely distant points. Consequently, whatever happens at a certain place at a certain time is completely and uniquely defined by the events which have occurred immediately preceding it in the immediate neighborhood of the place. This is essentially the distinction between the Maxwellian action-through-contact theory and the pre-Maxwellian action-at-a-distance theory.

The general structure of the theory is based on a dual representation of an electromagnetic field which can be described either in terms of its energy or in terms of its intensity. The postulate on the conservation of energy deals solely with the quantities which define the energy of the field ($U = U_e + U_m$, **S**, and Q), whereas the postulate on the contiguous action is expressed in terms of the intensity of the field (**E** and **B**). In order to combine the two parts into a single system, the theory provides two supplementary equations which give a relationship between the field energy and the field intensity. These are

$$\frac{\partial U_e}{\partial t} = \frac{1}{4\pi} \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t}, \tag{4-5}$$

where **D** is related to **E** by (4-4), and

$$\frac{\partial U_m}{\partial t} = \frac{1}{4\pi} \mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t}. \tag{4-6}$$

The crucial problem in this investigation concerns the formulation of the dielectric constant ϵ [Eq. (4-4)] and of the expression for the rate of change of the electrical energy [Eq. (4-5)]. It will now be shown that in the present version of the theory neither (4-4) nor (4-5) is correctly formulated. Consequently, the current formulation of (4-5) does not provide a valid link between the expression representing energy conservation and the one representing the principle of contiguous action.

Macroscopic Description of a Dispersive Medium According to the Lorentz Theory

Electrical and "Extraneous" Fields. – It is intended to show in this section that the Lorentz theory does not take into account the distinction between "electrical" fields and "extraneous" electrical fields in dissipative media. An electrical field is considered to have its origin in the Maxwellian process of energy transmission and is directly related to the Maxwellian form of energy. An extraneous field is associated with the non-Maxwellian form of energy and manifests itself by the presence of heat, which is dissipated in the medium as a result of the absorption process. It is shown that an accurate physical representation can only be obtained by taking into account both electrical and extraneous effects.

Although the Lorentz theory was published in the early part of this century, some of the assumptions made by Lorentz can be traced back to 1875. At that time a paper by von Helmholtz[19] appeared in which the problem of dispersion of light in a dissipative medium was treated in some respects in a manner similar to the one of Lorentz. At the time of von Helmholtz the prevalent point of view was based on existence of ether; therefore, in his theory, ether was considered to be the appropriate medium for transmitting energy through space. Von Helmholtz assumed that an assembly of vibrating charged particles (a molecular medium) was coupled to another assembly of vibrating particles which represented ether, and then he analyzed their mutual interaction in an oscillating electrical field. To a modern reader this physical picture is very archaic, but nevertheless the significant point is that von Helmholtz suggested, apparently for the first time, that the effects of energy dissipation are analogous to those produced by a frictional drag in a viscous medium. He postulated a mechanism in which each electron is acted upon by a frictional force proportional to the velocity of the electron and directed oppositely to the motion.

The idea of von Helmholtz was endorsed by Lorentz. According to Lorentz, the motion of an oscillating electron having mass m, charge e, and exposed to an electrical field $\mathbf{E}_0 \cos \omega t$ having frequency ω can be expressed as

$$m(\ddot{\mathbf{r}}_s + \gamma \dot{\mathbf{r}}_s + \omega_0^2 \mathbf{r}_s) = e\,\mathbf{E}_0 \cos \omega t \,, \tag{4-7}$$

where $\mathbf{r}_s$ is the displacement of the electron from its position of equilibrium, $m\omega_0^2\mathbf{r}_s$ is the elastic binding, and $m\gamma\mathbf{r}_s$ is the von Helmholtz term which represents energy absorption. Obviously, if there is no absorption, then $\gamma = 0$, and instead of (4-7) one has

$$m(\ddot{\mathbf{r}}_s + \omega_0^2 \mathbf{r}_s) = e\,\mathbf{E}_0 \cos \omega t \,. \tag{4-8}$$

There is an implied understanding in the Lorentz theory that the effects of absorption are expressed by the von Helmholtz term. Therefore, it would appear that the absorption by itself has no effect on the intensity of the electrical field, since the same expression $\mathbf{E}_0 \cos \omega t$ is used in (4-7) and in (4-8).

According to the point of view expressed in this analysis, (4-7) does not properly describe the physical process. The physical effects should be described by using an entirely different approach. One needs to follow the assumption made in the recent publication[2] that the driving force applied to an oscillating electron in an atomic dissipative medium has two components. One component derived from an electrical field accounts for the Maxwellian displacement, whereas the other, associated with an extraneous field, accounts for dissipation. Consequently, the oscillatory motion in an absorbing medium should be described by an equality

$$m(\ddot{\mathbf{r}} + \gamma \dot{\mathbf{r}} + \omega_0^2 \mathbf{r}) = e\,\mathbf{E}_{\text{electrical}} + e\,\mathbf{E}_{\text{extraneous}} \,, \tag{4-8a}$$

which represents two distinct processes: an electrical and an extraneous process. The above two processes can be formally separated and expressed as

$$m(\ddot{\mathbf{r}} + \omega_0^2 \mathbf{r}) = e\,\mathbf{E}_{\text{electrical}} \,, \tag{4-8b}$$

$$m\gamma\dot{\mathbf{r}} = e\,\mathbf{E}_{\text{extraneous}} \,. \tag{4-8c}$$

Extraneous fields, generally known in the literature as "impressed electrical fields," have been widely used in describing electrical phenomena in the presence of energy sources. An impressed (extraneous) field occurs whenever there is a conversion of nonelectrical (chemical or thermal) energy into the Maxwellian form of energy, and it represents a source of an electrical field. Such a source is in many cases continuously distributed in space. A particularly significant application of extraneous forces is in the formulation of Ohm's law, which has been differently defined in the absence and in the presence of a continuously distributed source. In the former case, Ohm's law is expressed by an equality

$$\mathbf{j} = \sigma \mathbf{E}_{\text{electrical}} \tag{4-8d}$$

(j is the electrical current and σ is the conductivity), whereas in the latter case the corresponding formulation is

$$\mathbf{j} = \sigma(\mathbf{E}_{\text{electrical}} + \mathbf{E}_{\text{extraneous}}) \ . \tag{4-8e}$$

An extraneous field can occur, for instance, in a medium in which there is a variation of concentration within a dilute aqueous solution of a strong electrolyte such as HCl. In such a medium an electric current sets in which is due to the difference of mobility and, therefore, to the difference of diffusion velocity of H^+ and Cl^- ions. Another significant case in which an extraneous force is known to appear is in a nonhomogeneous medium having a nonuniform temperature distribution.

It should be noted that the underlying basis of (4-8*a*) is the same as that of (4-8*e*). In both cases an extraneous field is known to occur whenever there is a transformation of electrical (Maxwellian) energy into nonelectrical energy or vice versa. Such a transformation results in the occurrence of a source or of a sink of electrical energy. In the case described by (4-8*e*) the extraneous field represents the energy source, whereas in (4-8*a*) it represents the sink of electrical energy.

Let us now examine the formulation (4-8*a*). The occurrence of dissipative effects tends to decrease the magnitude of the electrical field. It is, therefore, reasonable to assume that the dissipation of electrical energy represented by $\mathbf{E}_{\text{extraneous}}$ results in an exponential attenuation of the electrical field. Consequently, one has

$$\mathbf{E}_{\text{electrical}} = \mathbf{E}_0 e^{-\beta t} \cos \omega t \tag{4-8f}$$

where β is the attenuative parameter. Thus, Eq. (4-8*b*) can be expressed as

$$m(\ddot{\mathbf{r}} + \omega_0^2 \mathbf{r}) = e\,\mathbf{E}_0 e^{-\beta t} \cos \omega t \ . \tag{4-9}$$

In the proposed formulation the absorption is thus manifested directly by the attenuation of the electrical field. The significance of exp $(-\beta t)$ is in this version similar to that of the von Helmholtz term $m\gamma\dot{\mathbf{r}}$ in the analysis of Lorentz.

Dielectric Constant and Conductivity Based on the Lorentz Theory. — Clearly, the physical content of (4-7) is not the same as the one of (4-9); therefore, further theoretical developments based on (4-7) will differ substantially from those based on (4-9). We will now discuss some of these differences, particularly those which concern the formulation of the Maxwellian displacement and of the dielectric constant.

Essentially, the displacement represents the response of a molecular medium to an electrical field. The field is effective in displacing bound electrons from atomic nuclei and therefore accounts for polarization. If $\mathbf{r}$ is the displacement of an electron and N is the number of electronic dipoles per unit volume, the polarization density is $\mathbf{P} = Ne\mathbf{r}$, and the electric displacement is

$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$. Consequently, in order to define $\mathbf{D}$ in terms of $\mathbf{E}$, one needs to establish a relationship between $\mathbf{r}$ and $\mathbf{E}$. Using the relationship (4-7), the Maxwellian displacement is currently defined as

$$(\mathbf{D}_{\mathbf{k},\omega})_s = \epsilon_s(\mathbf{E}_{\mathbf{k},\omega})_s \, , \tag{4-10}$$

where

$$\epsilon_s = \epsilon_s^{(r)} + i\epsilon_s^{(i)} = 1 + \frac{\omega_e^2}{\omega_0^2 - \omega^2 - i\gamma\omega} \tag{4-11}$$

is the dielectric constant, and $\omega_e^2 = 4\pi Ne/m$. Both (4-10) and (4-11) are complex quantities which depend on the von Helmholtz parameter γ.

It seems that complex numbers are not always convenient in practical use. A preference was therefore shown for a dielectric constant expressed by a real number, and the most "obvious" assumption was to identify the dielectric constant with the real component of (4-11). An "effective" dielectric constant was introduced and defined as

$$\epsilon_s^{\mathrm{eff}} = \epsilon_s^{(r)} = 1 + \frac{\omega_e^2(\omega_0^2 - \omega^2)}{[(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2]} \, , \tag{4-12}$$

and the corresponding expression for the "effective" displacement was assumed to be

$$(\mathbf{D}_{\mathbf{k},\omega}^{\mathrm{eff}})_s = \epsilon_s^{\mathrm{eff}}(\mathbf{E}_{\mathbf{k},\omega})_s \, . \tag{4-13}$$

It was also found desirable to associate the imaginary component of the dielectric constant with conductivity, and conductivity was therefore defined as

$$\sigma_s = \frac{\omega\epsilon_s^{(i)}}{4\pi} = \frac{\omega_e^2\omega\gamma}{4\pi[(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2]} \, . \tag{4-14}$$

At first sight the change from (4-11) to (4-12) would seem innocuous, but a closer inspection reveals some drastic modifications in the structure of the theory. In order to incorporate the "effective" dielectric constant, it would seem to be necessary to provide an explicit formulation for "work" in terms of field intensity. The work would pertain to the electric and magnetic forces, and it should be defined in terms of field intensity in the same manner as one usually defines energy of the field. Such a modification will not, however, eliminate the difficulties and will tend to confuse rather than clarify the issues. We will return to this point later.

A close scrutiny of the relationship between the conductivity (4-14) and the effective dielectric constant (4-12) reveals a very intriguing situation. Since γ enters in both expressions, the dielectric constant and the conductivity are intimately related. Any alteration of conductivity would unavoidably result in a change in the dielectric constant.

Electrical Energy in the Current Macroscopic Theory

Having defined the Maxwellian displacement, we will now proceed to the formulation of energy within the framework of the Maxwell-Lorentz theory. Suppose that an absorbing medium interacts with a slowly growing, almost monochromatic field which was zero at $t = -\infty$. Then the electrical intensity can be expressed as

$$\mathbf{E}_s = 0.5[\mathcal{E}_s(t)e^{-i\omega t} + \text{c.c.}] \ , \tag{4-15}$$

which represents a rapidly oscillating function modulated by a slowly growing amplitude $\mathcal{E}_s(t)$ which was zero at $t = -\infty$ (c.c. denotes a complex conjugate). The corresponding expression for the displacement can be obtained from (4-10) and is

$$\mathbf{D}_s = 0.5[\mathcal{D}_s(t)e^{-i\omega t} + \text{c.c.}] \ , \tag{4-16}$$

where $\mathcal{D}_s(t)$ is an appropriate modulating function.

The rate of change of the energy density can then be expressed as

$$\frac{\partial (U_e)_s}{\partial t} = \frac{1}{4\pi} \mathbf{E}_s \cdot \frac{\partial \mathbf{D}_s}{\partial t} \ . \tag{4-17}$$

Since the processes are oscillatory, we will consider, instead of $\partial(U_e)_s/\partial t$, its mean value $\partial(\overline{U}_e)_s/\partial t$ (the bar above a symbol is used to indicate a time average and/or space average value). Substituting in (4-17) $\mathbf{E}_s$ and $\mathbf{D}_s$ as defined by (4-15) and (4-16), respectively, and applying well-known operational techniques, we obtain after some manipulations[20]

$$\frac{\partial (U_e)_s}{\partial t} = \frac{1}{16\pi}\frac{d}{d\omega}[\omega\epsilon_s^{(r)}(\omega,\gamma)]\frac{d}{dt}(\mathcal{E}_s \cdot \mathcal{E}_s^*) + \frac{1}{8\pi}\omega\epsilon^{(i)}(\omega,\gamma)\mathcal{E}_s \cdot \mathcal{E}_s^* \ . \tag{4-18}$$

Therefore, the energy density of an electrical field can be expressed as

$$(U_e)_s = \frac{1}{16\pi}\frac{d}{d\omega}\epsilon_s^{(r)}(\omega,\gamma)\mathcal{E}_s \cdot \mathcal{E}_s^* + \frac{1}{8\pi}\omega\epsilon^{(i)}(\omega,\gamma)\int_{-\infty}^{t}\mathcal{E}_s \cdot \mathcal{E}_s^* \ . \tag{4-19}$$

It is evident that (4-19) depends on the parameter γ, which defines dissipation. It does not, therefore, satisfy the requirements (3-38) and (3-39), and it does not represent energy in a sense in which one interprets this concept in generalized dynamics.

There is an additional point which militates against the acceptance of (4-19) as representing energy. The second term in (4-19) depends on $\mathcal{E}_s$ at all times from $t = -\infty$ until the instant considered. Therefore, it would seem that the energy of a system depends on the history as well as on the state of the system. This implies that energy is not a unique function of the state of a system but has different values, depending on the manner in which the system arrived at this state.

Principle of Conservation of Energy

We will now examine one of the most fundamental and universal principles of physics, the principle of conservation of energy, and show that one cannot establish the validity of this principle within the framework of the current macroscopic theory.

Suppose that an almost monochromatic wave interacts with an absorbing wave. The wave will be expressed by

$$\mathbf{E}_s = 0.5[\mathcal{E}_s(x,t)e^{i(kx-\omega t)} + \text{c.c.}] \ , \tag{4-20}$$

$$\mathbf{B}_s = 0.5[\mathcal{B}_s(x,t)e^{i(kx-\omega t)} + \text{c.c.}] \ , \tag{4-21}$$

where $\exp i(kx - \omega t)$ is a carrier wave, and $\mathcal{E}_s(x,t)$ and $\mathcal{B}_s(x,t)$ are modulating functions which vary slowly with x and t. We apply the customary procedure to obtain conservation of energy by forming the scalar products of (4-2) and (4-3) with $\mathbf{B}_s$ and $\mathbf{E}_s$, respectively, determining the mean values of these products, and subtracting one product from the other.[21] In the process of averaging with respect to t and x, we apply operational techniques similar to those used in obtaining (4-19). The derivations are lengthy, and only the results apparent from the nature of the problem are quoted here. The expression which purportedly represents energy conservation can then be expressed as follows:

$$\frac{\partial(\bar{U}_e)_s}{\partial t} + \frac{1}{16\pi}\frac{d}{dt}(\mathcal{B}_s \cdot \mathcal{B}_s^*) + \text{div}\,\bar{\mathbf{S}}_s + \bar{Q}_s = 0 \ , \tag{4-22}$$

where $\partial(\bar{U}_e)_s/\partial t$ has a form similar to (4-18),

$$\bar{Q}_s = 0.5\sigma(\mathcal{E}_s \cdot \mathcal{E}_s^*) \ , \tag{4-23}$$

$$\bar{\mathbf{S}}_s = \left(\frac{1}{16\pi}\right)[(\mathcal{E}_s \times \mathcal{B}_s^*) + \text{c.c.}] \ . \tag{4-24}$$

The interpretation of (4-22) is extremely difficult. The term $\partial(\bar{U}_e)_s/\partial t$ cannot be considered as a valid expression for the change of energy for the reasons outlined above. Furthermore, if we accept the assumption that Q_s represents the rate of production of Joule's heat, it remains to be shown that $\mathbf{S}_s$ is a physically meaningful quantity, which we cannot do.

An Alternative Approach

Replacement of "Energy" by "Work." – We will now return to (4-12) and (4-14) and will consider a formulation expressed in terms of "work" rather than "energy." Although not specifically stated in the usual theory, the rate of work $\partial W/\partial t$ pertains to forces which one has to overcome in order to store energy in the medium; the rate can be expressed as

$$\frac{\partial W}{\partial t} = \frac{\partial W_e}{\partial t} + \frac{\partial W_m}{\partial t} , \tag{4-25}$$

where $\partial W_e/\partial t$ and $\partial W_m/\partial t$ are associated with electric and magnetic forces respectively. We assume that

$$\frac{\partial W_e}{\partial t} = \frac{1}{4\pi} \mathbf{E}_s \cdot \frac{\partial \mathbf{D}_s}{\partial t} , \tag{4-26}$$

$$\frac{\partial W_m}{\partial t} = \frac{1}{4\pi} \mathbf{B}_s \cdot \frac{\partial \mathbf{B}_s}{\partial t} , \tag{4-27}$$

where

$$(\mathbf{D}_{\mathbf{k},\omega})_s = \epsilon_s (\mathbf{E}_{\mathbf{k},\omega})_s , \tag{4-28}$$

and ϵ_s is a complex quantity defined by (4-11). The general intent of this approach is to consider $\partial W_e/\partial t$ as a sum of two terms which represent energy storage and energy dissipation, and to associate these two terms with the real and imaginary components of ϵ_s. Since the relations (4-26) and (4-27) are not the same as the supplementary equations (4-5) and (4-6), the entire structure of the theory has changed.

Let us proceed on the assumption that (4-26) and (4-27) are valid formulations of a physical process. Substituting in (4-26) $\mathbf{E}_s$ and $\mathbf{D}_s$ as expressed in (4-15) and (4-16), respectively, we obtain[20]

$$\frac{\partial \overline{W}_e}{\partial t} = \frac{1}{16\pi} \frac{d}{d\omega} [\omega \epsilon_s^{(r)}(\omega,\gamma)] \frac{d}{dt} (\mathcal{E}_s \cdot \mathcal{E}_s^*) + \frac{1}{8\pi} \omega \epsilon_s^{(i)}(\omega,\gamma) \mathcal{E}_s \cdot \mathcal{E}_s^* . \tag{4-29}$$

Since the work performed by electric forces is effective not only in storing energy but also in accounting for the generation of heat, we would be safe in assuming that

$$\frac{\partial W_e}{\partial t} = \left[\frac{\partial U_e^{(h)}}{\partial t} \right] + Q^{(h)} , \tag{4-30}$$

where $\partial U_e^{(h)}/\partial t$ is an expression for the change of energy, and $Q^{(h)}$ is an expression for the rate of energy dissipation. However, the quantities $\partial U_e^{(h)}/\partial t$ and $Q^{(h)}$ are not known, since there is nothing in the theory which would tell us how to divide the sum (4-29) in two parts in order to obtain separate expressions for the energy stored and the energy dissipated.

In order to find a convenient way to solve the problem, a conjecture is made that

$$\frac{\partial U_e^{(h)}}{\partial t} = \frac{1}{16\pi} \frac{d}{d\omega} [\omega \epsilon^{(r)}(\omega,\gamma)] \frac{d}{dt} (\mathcal{E}_s \cdot \mathcal{E}_s^*) \tag{4-31}$$

and that

$$Q^{(h)} = \left(\frac{1}{8\pi}\right)\omega\epsilon_s^{(i)}(\omega,\gamma)\mathcal{E}_s \cdot \mathcal{E}_s^* . \tag{4-32}$$

It appears, however, that both (4-31) and (4-32) cannot be justified on logical grounds. The most that can be said about (4-31) is that it "looks like" an expression

$$\frac{\partial U_e}{\partial t} = \frac{1}{16\pi}\frac{d}{d\omega}[\omega\epsilon(\omega)]\frac{d}{dt}(\mathcal{E} \cdot \mathcal{E}^*) , \tag{4-33}$$

which is a valid formulation when $\gamma = 0$, and that (4-32) "looks like" the expression for Ohm's law, provided (4-14) represents conductivity. Therefore, it would appear that in order to calculate the change of energy in an absorbing medium, one needs merely to substitute the dielectric constant for a nonabsorbing medium by the effective dielectric constant defined by (4-12). It is obvious that (4-31) is not a physically meaningful formulation, since it leads to an expression for energy,

$$U_e^{(h)} = \frac{1}{16\pi}\int_{-\infty}^{t}\frac{d}{d\omega}[\omega\epsilon_s^{(r)}(\omega,\gamma)]\frac{d}{dt}(\mathcal{E}_s \cdot \mathcal{E}_s^*)\, dt , \tag{4-34}$$

which depends on γ, and is, therefore, not consistent with the meaning of this term in generalized dynamics.

A simple argument shows that (4-34) leads to a paradox. Consider the accumulative period from $t = -\infty$ during which the energy was gradually stored in the medium, and assume that during the period γ varied with time as $\gamma(t)$. It would then appear that the energy of a system has different values depending on $\gamma(t)$, that is, on the manner according to which the particular physical state was reached.

Conservation of Energy. – Let us see whether the modified formulation based on "work" will lead to a valid expression for the conservation of energy.

It is evident that the assumptions (4-25) to (4-28) require a change in the equations of contiguity and that in place of (4-2) and (4-3) we have now

$$\text{curl } \mathbf{E}_s = -\frac{1}{c}\frac{\partial \mathbf{B}_s}{\partial t} , \tag{4-35}$$

$$\text{curl } \mathbf{B}_s = \frac{1}{c}\frac{\partial \mathbf{D}_s}{\partial t} . \tag{4-36}$$

It should be noted that the conductivity current $(4\pi/c)\sigma\mathbf{E}$ does not appear on the right-hand side of (4-36), since the formulation (4-26) includes the effects of absorption.

Forming the scalar products of (4-35) and (4-36) with $\mathbf{B}_s$ and $\mathbf{E}_s$, respectively, determining the mean values of the products, and subtracting one from the other, we obtain

$$\left(\frac{\partial \overline{W}_e}{\partial t}\right) + \text{div } \mathbf{S}_s = 0 . \tag{4-37}$$

The intent of the above expression is to represent energy conservation. Since we have not been able to separate $\partial\overline{W}_e/\partial t$ into two terms expressing energy storage and energy dissipation, we cannot assume that (4-37) represents a physically meaningful relationship. The best that can be said for the expression (4-37) and for the previously derived expression (4-22) is that they represent sums of certain quantities which depend on the intensity of the electromagnetic field. These quantities are not clearly defined, and by adding them together we obtain zero.

V. ENERGY, DIELECTRIC CONSTANT, AND CONDUCTIVITY

Fundamental Assumptions in the Proposed Formulation

The revised version of the theory is characterized by three main features as follows: (*a*) to reformulate the concept of energy by assuming that the relationship between the field energy and the field intensity is independent of energy dissipation; (*b*) to suggest that energy be considered as a fundamental quantity in the sense that field intensity can only be defined operationally in terms of energy; (*c*) to treat the effects of energy dissipation as a small perturbation.

Our main emphasis is, therefore, on energy, and in order to clarify the meaning of energy in the electromagnetic-field theory, we must also consider this concept from the historical point of view.[22] The idea of energy envisaged by Galileo, as that which is gained in power is lost in speed, and subsequently developed by Bernoulli, Leibnitz, Lagrange, and others was originally a concept associated with motion of masses. With the development of electromagnetic theory, energy ceased to be dependent on matter as a vehicle for its transport and attained the status of an independent physical existence. An electromagnetic field can, therefore, be considered as nothing more than a spatial distribution of energy which varies with time.

Planck[23] strongly emphasized the meaning of energy which came with the electromagnetic theory. In his interpretation an electromagnetic field can only be defined by energy, and he suggested that the field intensity be expressed in terms of energy. "It is not easy to define absolute value of the electric intensity. To arrive at, one starts from the concept of the energy of the field." Consequently, electric- and magnetic-field intensities are secondary concepts used insofar as necessary to explain the Maxwellian mechanism of energy propagation in terms of the principle of contiguity. This point of view has influenced the formal presentation of the revised version of the theory.

The assumption that dissipation is small is consistent with the basic principles of the electromagnetic theory. The intent of the Maxwellian hypothesis is to explain energy transmission and, incidentally, to take into account the effects of dissipation. Therefore, the dissipation expressed by the term Q is a secondary process, and Q can be regarded as a parameter which represents a small perturbation. The analysis of an electromagnetic field is based, therefore, on a perturbation procedure in which a formal distinction is made between the unperturbed field quantities in a nonabsorbing medium ($Q = 0$) and the corresponding perturbed quantities ($Q \neq 0$) in an absorbing medium. When $Q = 0$, the field quantities (unperturbed) are provided with a superscript "0" such as in $\mathbf{E}^0$, $\mathbf{B}^0$, $\mathbf{D}^0$, U^0, U_e^0, U_m^0, and $\mathbf{S}^0$, whereas when $Q \neq 0$ the corresponding perturbed quantities are $\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$, U, U_e, U_m, and $\mathbf{S}$ with no superscript attached. A similar notation is applied to the dielectric constant expressed by ϵ^0 when the medium is nonabsorbing and by ϵ when there is absorption. The relationship between ϵ and ϵ^0 will be considered later.

The distinction between an unperturbed field and a perturbed field should carefully be noted. It is clear that the effect of the perturbation is to change the spatial distribution of energy and also its dependence on time. Therefore,

$$U_e \neq U_e^0 , \quad U_m \neq U_m^0, \quad \text{and} \quad \mathbf{S} \neq \mathbf{S}^0 . \tag{5-1}$$

Consequently

$$\mathbf{E} \neq \mathbf{E}^0 \quad \text{and} \quad \mathbf{B} \neq \mathbf{B}^0 . \tag{5-2}$$

The apparently obvious inequalities (5-1) and (5-2) are emphasized because of a special reason which concerns the Lorentz postulate (4-7). Lorentz assumed that one can take into account the effects of dissipation by means of the von Helmholtz frictional term and that the electric field is not influenced by absorption. There is a clear distinction in the physical content of the assumption (5-2) on which this analysis is based and the assumption that $\mathbf{E} = \mathbf{E}^0$ made by Lorentz.

Description of an Electromagnetic Field

Nonabsorbing Medium. – Planck observed that "among the laws of physics none is so universal and so easy to grasp as the principle of conservation of energy." Since transformation of energy from one form into another is of frequent occurrence in nature, and since the sum of the energies of all kinds in the universe is constant, energy became a common coin of exchange in any interaction between two physical systems, and energy conservation can be considered to be a connecting link between the "mechanical processes and motions of material points" and the "electrical and magnetic or electrodynamic phenomena which form an equally uniform whole." Planck assumed, therefore, that the principle of energy conservation should have a "priority of position" over the principle of contiguous action. A similar sequence is followed in this analysis.

It is evident that when the field is unperturbed, the only form of energy to be considered is the electromagnetic energy which is conserved. Therefore,

$$\left(\frac{\partial U^0}{\partial t}\right) + \operatorname{div} \mathbf{S}^0 = 0 . \tag{5-3}$$

Following Planck, $\mathbf{E}^0$ is defined in terms of U_e^0 by means of

$$\frac{\partial U_e}{\partial t} = \frac{1}{4\pi} \mathbf{E}^0 \cdot \frac{\partial \mathbf{D}^0}{\partial t} , \tag{5-4}$$

where

$$\mathbf{D}^0_{\mathbf{k},\omega} = \epsilon^0 \mathbf{E}^0_{\mathbf{k},\omega} , \tag{5-5}$$

and ϵ^0 is the dielectric constant. Similarly, we define $\mathbf{B}^0$ in terms of U^0_m by means of

$$\frac{\partial U^0_m}{\partial t} = \frac{1}{4\pi}\mathbf{B}^0 \cdot \frac{\partial \mathbf{B}^0}{\partial t}, \tag{5-6}$$

and write the equations of contiguity as

$$\text{curl } \mathbf{E}^0 = -\frac{1}{c}\frac{\partial \mathbf{B}^0}{\partial t}, \tag{5-7}$$

$$\text{curl } \mathbf{B}^0 = \frac{1}{c}\frac{\partial \mathbf{D}^0}{\partial t}. \tag{5-8}$$

We apply the customary requirement that Eqs. (5-7) and (5-8) should be compatible with the principle of conservation of energy. This is ascertained in the usual manner by assuming that

$$\mathbf{S}^0 = \frac{c}{4\pi}(\mathbf{E}^0 \times \mathbf{B}^0) \tag{5-9}$$

is the energy flow.

In the above formulation we did not follow precisely the pattern established by Planck and attempted in some respects to combine the traditional point of view with the one proposed by Planck. Planck introduced his subject with the expression for the principle of energy conservation [Eq. (5-3)]. Then using (5-4) to (5-6) and (5-9) as definitions, he transformed (5-3) into an expression which holds for every volume element and represents an identity. Then equating the coefficients of **E** and **B** separately he obtained the formulation of the principle of contiguous action [Eqs. (5-7) and (5-8)]. In his derivation the principle of contiguity appeared as a direct consequence of conservation of energy and of certain auxiliary relationships representing the dependence between the field energy and the field intensity. Our derivation represents a compromise between the approach of Planck and the one in the current theory. Instead of deriving equations of contiguity from the principle of energy conservation, as Planck did, we postulated the existence of the differential relationships (5-7) and (5-8) and then verified their validity by showing that they are compatible with the principle of energy conservation, provided energy flow is defined by (5-9). Both points of view are equivalent.

Absorbing (or Emitting) Media. – The medium is assumed to be energetically active, which means that there is an energy transfer from the electromagnetic form into a "nonelectric" form, or vice versa. The electromagnetic energy per se is not conserved, and the equality (5-3) is now replaced by

$$\left(\frac{\partial U}{\partial t}\right) + \text{div } \mathbf{S} + Q = 0\ , \tag{5-10}$$

which contains the perturbing parameter Q.

As previously, **E** is defined in terms of U_e by means of

$$\frac{\partial U_e}{\partial t} = \frac{1}{4\pi} \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t}, \tag{5-11}$$

where

$$\mathbf{D}_{\mathbf{k},\omega} = \epsilon \mathbf{E}_{\mathbf{k},\omega}, \tag{5-12}$$

and ϵ is the dielectric constant. We define **B** in terms of U_m by means of

$$\frac{\partial U_m}{\partial t} = \frac{1}{4\pi} \mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t}. \tag{5-13}$$

It is assumed that the mechanism of energy conversion is expressed by Ohm's law, and, therefore,

$$Q = \sigma \mathbf{E}^2. \tag{5-14}$$

Since Q and **E** are already known, the expression (5-14) can be considered to be a definition of conductivity. Henceforth σ will be used in place of Q as the parameter which represents perturbation. If Q and consequently σ are positive, the medium is absorbing, and conversely when Q and σ are negative the medium is emitting.

The principle of contiguity is expressed in its usual form as

$$\text{curl } \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{5-15}$$

$$\text{curl } \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi\sigma}{c}\mathbf{E}. \tag{5-16}$$

We comply here with the customary requirement that (5-15) and (5-16) should be consistent with the principle of energy conservation, and, therefore, the energy flow is defined as

$$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{B}). \tag{5-17}$$

Dielectric Constant

The dielectric constant for nonabsorbing and absorbing media has been expressed by ϵ^0 in (5-4) and (5-5) and by ϵ in (5-11) and (5-12). Therefore, it can be defined in both cases as "a measure of the ability of a substance to store electrical energy under the influence of an electrical field."[24]

There is some amount of confusion in the current literature concerning the dielectric constant and what is considered to be its meaning. For a number of years this concept has been used in various contexts to define molecular media[25] and more recently misused in the definition of plasmas.[26] Of particular significance to our analysis is a curious situation that the definition of the dielectric constant as a "measure of the ability to store energy" is entirely meaningless as far as the current theory is concerned. The current theory has failed to provide a meaningful concept of energy and cannot, therefore, tell us how to measure energy.

In this revised version the problem of defining electrical energy was reduced to the corresponding problem in generalized dynamics, and in such case there is no ambiguity as to the meaning of the dielectric constant as a measure of the ability to store energy. Furthermore, in this formulation the relationship between U or $\partial U/\partial t$ and $\mathbf{E}$ is the same as the one between U^0 or $\partial U^0/\partial t$ and $\mathbf{E}^0$. Consequently,

$$\epsilon^0 = \epsilon \, , \tag{5-18}$$

which means that the dielectric constant for a nonabsorbing medium is the same as the one for an absorbing medium.

The equality (5-18) has been derived on general theoretical considerations, and in establishing this equality no specific assumptions were made concerning the structure of the medium in which the energy is both stored and dissipated. However, it is evident that the dielectric constant depends on the structural properties of the medium and would be useful for practical purposes to obtain an expression in which such dependence is explicitly shown.

Consider a molecular model similar to the one postulated by Lorentz. We assume at first that there is no absorption and represent the motion of the elastically bound electron in the form

$$m(\ddot{\mathbf{r}}^0 + \omega_0^2 \mathbf{r}) = e\mathbf{E}^0 \, , \tag{5-19}$$

which is identical to the form

$$m(\ddot{\mathbf{r}}_s^0 + \omega_0^2 \mathbf{r}_s) = e\mathbf{E}_s^0 \tag{5-20}$$

in the Lorentz theory. If, however, there is absorption, then our description becomes significantly different from the one of Lorentz. According to Lorentz, (5-20) should be replaced by

$$m(\ddot{\mathbf{r}}_s + \gamma\dot{\mathbf{r}}_s + \omega_0^2 \mathbf{r}_s) = e\mathbf{E}_s \, , \tag{5-21}$$

where

$$\mathbf{E}_s = \mathbf{E}_s^0 \, ; \tag{5-22}$$

whereas in our version (5-19) should be replaced by

$$m(\ddot{\mathbf{r}} + \omega_0^2 \mathbf{r}) = e\mathbf{E} \, , \tag{5-23}$$

where

$$\mathbf{E} \neq \mathbf{E}^0 \, . \tag{5-24}$$

Note that in our formulation the relationship between $\mathbf{r}^0$ and $\mathbf{E}^0$ as expressed in (5-19) is the same as the one between $\mathbf{r}$ and $\mathbf{E}$ expressed in (5-23). If the medium is nonabsorbing, one has

$$\mathbf{r}^0_{\mathbf{k},\omega} = \chi(\omega)\mathbf{E}^0_{\mathbf{k},\omega} , \tag{5-25}$$

whereas the corresponding relationship for an absorbing medium is

$$\mathbf{r}_{\mathbf{k},\omega} = \chi(\omega)\mathbf{E}_{\mathbf{k},\omega} , \tag{5-26}$$

where

$$\chi(\omega) = \frac{e}{m(\omega_0^2 - \omega^2)} \tag{5-27}$$

is the same in (5-25) and (5-26).

Assuming that N is the number of oscillating electrons, the polarization density in a nonabsorbing medium can be expressed as $\mathbf{P}^0 = Ne\mathbf{r}^0$, and if the medium is absorbing it is $\mathbf{P} = Ne\mathbf{r}$. Consequently, the dielectric constant in the former case is $\mathbf{D}_{\mathbf{k},\omega} = \epsilon^0\mathbf{E}^0_{\mathbf{k},\omega} = \mathbf{E}^0_{\mathbf{k},\omega} + 4\pi\mathbf{P}^0_{\mathbf{k},\omega}$, and in the latter case it is $\mathbf{D}_{\mathbf{k},\omega} = \epsilon\mathbf{E}_{\mathbf{k},\omega} = \mathbf{E}_{\mathbf{k},\omega} + 4\pi\mathbf{P}_{\mathbf{k},\omega}$. The dielectric constant in our version of the theory can, therefore, be expressed as

$$\epsilon = \epsilon^0 = 1 - \frac{\omega_e^2}{\omega^2 - \omega_0^2} , \tag{5-28}$$

and it is, therefore, the same for absorbing and nonabsorbing media. The formulation (5-28) represents one of the most distinctive features of the proposed theory. It should be recalled that in the traditional formulation the dielectric constant depends on absorption. In the proposed formulation such a dependence does not occur. One can therefore increase or decrease the absorbing properties of a medium without altering the dielectric constant.

There are also other distinctions which characterize the relationship between the dielectric constant and the conductivity. In the proposed version the definition of the dielectric constant precedes the one of conductivity. At first, one defines ϵ by (5-11) and (5-12), and then one obtains an expression for σ from Ohm's law. On the other hand, in the current version both ϵ_s and σ_s are derived from the complex dielectric constant, and there is no requirement that the formulation of one of these parameters should precede the other. Furthermore, in the proposed formulation ϵ and σ are always real quantities. However, this is not the case in the current theory, where a substance can be specified by a complex dielectric constant

$$\epsilon_{\text{complex}} = \epsilon_s + (4\pi i\sigma_s/\omega) , \tag{5-29}$$

even when $\epsilon_s = 0$, or by a complex conductivity

$$\sigma_{\text{complex}} = \sigma_s - (i\omega\epsilon_s/4\pi) , \tag{5-30}$$

even when $\sigma_s = 0$. These designations are physically confusing. In this formulation we avoid a terminology in which complex dielectric constant can define conductivity, or complex conductivity can define dielectric constant.

Perturbation Operator

Consider now the perturbation treatment in which σ is the perturbation parameter. Assume that there exists an operator $\hat{K}$ which depends on σ and which can be defined as follows: (*a*) $\hat{K}$ is effective in transforming an unperturbed field quantity $\mathbf{E}^0$ or $\mathbf{B}^0$ (in a nonabsorbing medium) into a corresponding perturbed quantity $\mathbf{E}$ or $\mathbf{B}$ (in an absorbing medium). Therefore,

$$\mathbf{E} = \hat{K}\mathbf{E}^0 \quad \text{and} \quad \mathbf{B} = \hat{K}\mathbf{B}^0 \ . \tag{5-31}$$

(*b*) If $\mathbf{E}^0$ and $\mathbf{B}^0$ are plane waves, the operation performed by $\hat{K}$ is equivalent to that of multiplying $\mathbf{E}^0$ or $\mathbf{B}^0$ by a number K,

$$K = e^{\alpha x - \beta t} \ , \tag{5-32}$$

where α and β are appropriate constants. Therefore, if $\mathbf{E}^0$ and $\mathbf{B}^0$ are expressed as

$$\begin{aligned} \mathbf{E}^0 &= 0.5[\mathbf{E}_0^0 e^{i(kx-\omega t)} + \text{c.c.}] \ , \\ \mathbf{B}^0 &= 0.5[\mathbf{B}_0^0 e^{i(kx-\omega t)} + \text{c.c.}] \ , \end{aligned} \tag{5-33}$$

respectively, then using (5-32) one obtains

$$\begin{aligned} \mathbf{E} &= 0.5\{\mathbf{E}_0^0 e^{i[(k-i\alpha)x-(\omega-i\beta)t]} + \text{c.c.}\} \ , \\ \mathbf{B} &= 0.5\{\mathbf{B}_0^0 e^{i[(k-i\alpha)x-(\omega-i\beta)t]} + \text{c.c.}\} \ , \end{aligned} \tag{5-34}$$

respectively ($\mathbf{E}_0$ and $\mathbf{B}_0$ are constant amplitudes). Consequently, a homogeneous plane wave in an unperturbed medium becomes an inhomogeneous plane wave in a perturbed medium. The wave number, which for an unperturbed wave was expressed by a real k, now has a complex value $k - i\alpha$. Similarly, the frequency, which for an unperturbed wave was expressed by a real ω, now has a complex value $\omega - i\beta$.

Substituting (5-33) in the contiguity equations (5-7) and (5-8), one obtains

$$F = F(k,\omega) = (c^2k^2/\omega) - \omega\epsilon = 0 \ . \tag{5-35}$$

Similarly, substituting (5-34) in the contiguity equations (5-15) and (5-16), we have

$$F(k - i\alpha, \omega - i\beta) = 4\pi i\sigma \ , \tag{5-36}$$

where $F(k - i\alpha, \omega - i\beta)$ is obtained by replacing in $F(k,\omega)$ the term k by $k - i\alpha$ and the term ω by $\omega - i\beta$.

It has been assumed that the effects of the perturbation are small. Therefore, there is an implied understanding that the energy decrease in an absorbing medium (or increase in an emitting medium) during a "significant" time interval is small when compared with the energy stored in the medium. (What constitute a "significant" time interval and the smallness of the perturbation will be discussed in Part II of this investigation.)

Since the perturbation is small, one can assume that

$$|\alpha| << |k| \quad \text{and} \quad |\beta| << |\omega| \ . \tag{5-37}$$

Therefore, expanding $F(k - i\alpha, \omega - i\beta)$ in the form of a Taylor series in which the derivatives of order higher than 1 are neglected, we transform (5-36) into a relationship

$$F - i\alpha \frac{\partial F}{\partial k} - i\beta \frac{\partial F}{\partial \omega} - 4\pi i\sigma = 0 \ . \tag{5-38}$$

Then considering (5-35), we obtain

$$\alpha \frac{\partial F}{\partial k} + \beta \frac{\partial F}{\partial \omega} + 4\pi\sigma = 0 \ . \tag{5-39}$$

The two equalities (5-35) and (5-39) will be designated as the "characteristic" equations. They define the response of a dispersive medium to an electromagnetic disturbance and take into account both transmission and absorption (or emission) effects. The role of the characteristic equations in the revised version of the theory is similar to the role of the dispersion equation in the traditional version. The characteristic equations will be used in Part II in the analysis of transmission and absorption (or emission) effects for convective and nonconvective disturbances in dispersive media.

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