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Carnegie Institute of Technology

Department of Physics

DISPERSION RELATIONS FOR PION SCATTERING

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Donald A. Geffen

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ABSTRACT

The dispersion relations are used to predict the values of the derivatives of the real part of the π^+ and π^- forward scattering amplitudes, $\partial D_\pm(0)/\partial k^2$, at zero kinetic energy. The experimental value of $\partial D_+(0)/\partial k^2$ is fairly well known, and, when compared with the predicted value, yields a determination of the coupling constant, $f^2 = .104 \pm .014$. The predicted value for $\partial D_-(0)/\partial k^2$ disagrees badly with experiment, especially with an f^2 as large as 0.10.

The dispersion relations are modified by introducing an extra energy denominator in such a way as to contain, as the additional constants, the derivatives $\partial D_\pm(0)/\partial k^2$. This enables us to check the values of $\partial D_\pm(0)/\partial k^2$ obtained from the usual dispersion relations as well as the assumption that $\omega^{-2}T_\pm(\omega)$ vanishes at infinity. It is found that as long as the agreement with experiment obtained for the π^+ relation is retained, no appreciable change in the values of $\partial D_\pm(0)/\partial k^2$ is possible and that the high energy behavior of $T_\pm(\omega)$, usually assumed, is correct. The predicted value for $\partial D_-(0)/\partial k^2$ strongly suggests a non-zero effective range for α_1 and a relatively large α_{11}.

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I. INTRODUCTION

The discrepancy with experiment of the π^- dispersion relation, which was first pointed out by Puppi and Stanghellini[1], and subsequently discussed by several authors[2,3,4], is examined in this paper using a slightly different approach. The dispersion relations are used to predict the values of the derivatives of the real part of the π^+ and π^- forward scattering amplitudes, $\partial D_{\pm}(0)/\partial k^2$ at zero kinetic energy. The experimental values of these derivatives depend very strongly on the P wave scattering lengths and the S wave effective ranges. These quantities are fairly well known for T = 3/2 and yield a value for $\partial D_{+}(0)/\partial k^2$ which, when compared with the prediction of the dispersion relations, leads to a determination of the coupling constant, $f^2 = .104 \pm .014$. On the other hand, the dispersion relations predict a value for $\partial D_{-}(0)/\partial k^2$ which disagrees badly with present π^- experiments, especially with an f^2 as large as .10.

To eliminate the unknown high energy contributions to the integrals appearing in the dispersion relations as well as to check the assumption that $\omega^{-2} T_{\pm}(\omega)$ vanishes as ω becomes infinite, the dispersion relations are modified by introducing an extra energy denominator in the integrals. This involves the added subtraction of the real part of the scattering amplitude at an arbitrary energy ω_0. By letting ω_0 approach 1, the relations can be simplified and contain, as added constants, the quantities $\partial D_{\pm}(0)/\partial k^2$. These new relations have the added advantage, then, of enabling us to check the values of $\partial D_{\pm}(0)/\partial k^2$ obtained from the usual dispersion relations. It is found that as long as the agreement with experiment obtained for the π^+ relation is retained, no appreciable change in the latter values of $\partial D_{\pm}(0)/\partial k^2$ is possible. This result indicates the correctness of the assumed high energy behavior of $T_{\pm}(\omega)$ and reinforces the conviction that the values predicted for $\partial D_{\pm}(0)/\partial k^2$ are correct.

The value for $\partial D_{-}(0)/\partial k^2$ predicted by the dispersion relastions is compared with experiment and the discrepancy between these two values is interpreted as being due to the very small $T = \frac{1}{2}$ scattering cross sections that have so far been observed. It will be shown that a resolution of the discrepancy between the theoretical and experimental values of $\partial D_{-}(0)/\partial k^2$ could very well involve changes in present experimental data which would also remove the discrepancy between the predicted and observed values of the real part of the π^- forward scattering amplitude. It would seem reasonable, therefore, to take the failure or success of future experiments to give a value for $\partial D_{-}(0)/\partial k^2$, which agrees with the predicted value, as an indication of the failure or success of the dispersion relations in general.

II. DISPERSION RELATIONS

The dispersion relations obtained by Goldberger[(5)], assuming that $\omega^{-2}\, T_{\pm}(\omega)$ approach zero as ω becomes infinite, can be written for $D_{\pm}(\omega)$, the real part of the $\pi^{\pm}$ forward scattering amplitudes,

$$D_{\pm}(\omega) - \tfrac{1}{2}(1+\omega)D_{\pm}(1) - \tfrac{1}{2}(1-\omega)D_{\mp}(1) =$$

$$= \frac{k^2}{4\pi^2}\, P\int_1^{\infty} \frac{d\omega'}{k'} \left[\frac{\sigma_{\pm}(\omega')}{\omega'-\omega} + \frac{\sigma_{\pm}(\omega')}{\omega'+\omega}\right] \pm \frac{2k^2}{\omega \mp 1/2M}\, f^2 \qquad (2.1)$$

We are using units of $\hbar = c = \mu = 1$. It will be more convenient, in what follows, to write the relations in terms of the more symmetric functions,

$$D^{(1)}(k) = \tfrac{1}{2}\left[D_+(k) + D_-(k)\right]$$

and

$$D^{(2)}(k) = \tfrac{1}{2}\left[D_-(k) - D_+(k)\right] ,$$

the no isotopic spin flip and isotopic spin flip amplitudes, respectively. With $D^{(1)}$ and $D^{(2)}$ written as functions of k, (2.1) takes the form,

$$D^{(1)}(k) - D^{(1)}(0) = \frac{k^2}{4\pi^2} P \int_0^\infty \frac{\left[\sigma_+(k') + \sigma_-(k')\right]}{k'^2 - k^2} dk' + \frac{1}{2M} \frac{2k^2 f^2}{\omega^2 - (1/2M)^2} \qquad (2.2a)$$

$$D^{(2)}(k) - \omega D^{(2)}(0) = \frac{k^2\omega}{4\pi^2} P \int_0^\infty \frac{\left[\sigma_-(k') - \sigma_+(k')\right]}{\omega'(k'^2 - k^2)} - \frac{2\omega k^2 f^2}{\omega^2 - (1/2M)^2} \qquad (2.2b)$$

We assign the integers n = 0, 1, 2, ... to the dispersion relations derived from the assumption that $\omega^{-2n} T_\pm(\omega)$ vanishes at infinity. Relations (2.2a,b), therefore, shall be referred to as the n = 1 relations.

The n = 1 relations can be used to predict the derivatives of the real part of the forward scattering amplitudes at $k^2 = 0$, $\partial D_{\pm}(0)/\partial k^2$. By dividing (2.2a) by k^2 and taking the limit as k^2 approaches zero, it is easily seen that the left hand side becomes $\partial D^{(1)}(0)/\partial k^2$,

$$\frac{\partial D^{(1)}(0)}{\partial k^2} = \frac{1}{4\pi^2} \lim_{k \to 0} P \int_0^{\infty} \frac{\left[\sigma_+(k') + \sigma_-(k')\right]}{k'^2 - k^2} dk'$$

$$+ \frac{1}{M} f^2 \tag{2.3}$$

We neglect the $(1/2M)^2$ in (2.2a,b). To evaluate the limit on the right hand side of (2.3) we must first perform the principal part integration. The principal part can be eliminated, however, by writing,

$$\lim_{k \to 0} P \int_0^{\infty} \frac{\left[\sigma_+(k') + \sigma_-(k')\right]}{k'^2 - k^2} dk'$$

$$= \int_0^{\infty} \lim_{k \to 0} \frac{\left[\sigma_+(k') + \sigma_-(k') - \sigma_+(k) - \sigma_-(k)\right]}{k'^2 - k^2} dk'$$

$$+ \lim_{k \to 0} \left[\sigma_+(k) + \sigma_-(k)\right] P \int_0^{\infty} \frac{dk'}{k'^2 - k^2} \tag{2.4}$$

and noting that the second term on the right hand side of (2.4) vanishes. With the aid of (2.4), (2.3) can be written in the simple form,

$$\frac{\partial D^{(1)}(0)}{\partial k^2} = \frac{1}{4\pi^2} \int_0^{\infty} \frac{\Delta^{(1)}(k')}{k'^2} dk' + \frac{1}{M} f^2 \tag{2.5}$$

with $\Delta^{(1)}(k) = \sigma_+(k) + \sigma_-(k) - \sigma_+(0) - \sigma_-(0)$. The integral in (2.5) is well defined at $k' = 0$ since $\Delta^{(1)}(k')$ vanishes like k'^2. By dividing (2.2b) by k^2 and using the same trick in taking the limit as k^2 vanishes, we also obtain for the derivative of $D^{(2)}$,

$$\frac{\partial D^{(2)}(0)}{\partial k^2} = \frac{1}{2} D^{(2)}(0) - 2f^2 + \frac{1}{4\pi^2} \int_0^\infty \frac{\Delta^{(2)}(k')}{k'^2} \, dk' \qquad (2.6)$$

with $\Delta^{(2)}(k) = \dfrac{\sigma_-(k) - \sigma_+(k)}{\omega} - \left[\sigma_-(0) - \sigma_+(0)\right]$.

Relations for $\partial D_{\pm}(0)/\partial k^2$ can be obtained by adding and subtracting (2.5) and (2.6). By expressing $D_{\pm}(k)$ in terms of phase shifts and differentiating, it is easy to obtain expressions for $\partial D_{\pm}(0)/\partial k^2$ in terms of the S wave scattering lengths and effective ranges, and the P wave scattering lengths. It will be shown in section III that the value of $\partial D_+(0)/\partial k^2$ predicted by (2.6) and (2.5) agrees with experiment with an f^2 slightly greater than 0.10 . The experimental value for $\partial D_-(0)/\partial k^2$, while not as well known, disagrees with the theoretical value, the discrepancy becoming very large for $f^2 \gtrsim 0.10$.

One way to explain this difficulty is to say that the assumption that $\omega^{-2} T_{\pm}(\omega)$ vanishes at infinity is incorrect, so that the $n = 1$ relations must be replaced by $n = 2$ relations. In what follows we shall derive $n = 2$ relations, which contain $\partial D_{\pm}(0)/\partial k^2$, and show that they cannot give any better agreement with experiment than the $n = 1$ relations. Furthermore, the best values of $\partial D_{\pm}(0)/\partial k^2$ are just those values predicted by (2.5) and (2.6), making the $n = 2$ and $n = 1$ relations identical.

The n = 1 relations, (2.1), are dispersion relations for the quantities $(\omega^2 - 1)^{-1}\,T_{\pm}(\omega)$. Analogously, the n = 2 relations we choose shall be dispersion relations for $\left[(\omega^2 - 1)(\omega^2 - \omega_o^2)\right]^{-1} T_{\pm}(\omega)$. ω_o is an arbitrary energy different from 1. The n = 2 relation for $D^{(1)}$ can then be written[6]

$$D^{(1)}(\omega) - \frac{\omega^2 - 1}{\omega_o^2 - 1} D^{(1)}(\omega_o) + \frac{\omega^2 - \omega_o^2}{\omega_o^2 - 1} D^{(1)}(1)$$

$$= \frac{k^2(\omega^2 - \omega_o^2)}{4\pi^2}\, P \int_0^{\infty} \frac{\left[\sigma_+(k') + \sigma_-(k')\right] dk'}{(k'^2 - k^2)(k'^2 - k_o^2)}$$

$$- \frac{1}{2M} \frac{2f^2 k^2 (\omega^2 - \omega_o^2)}{\left[\omega^2 - (1/2M)^2\right]\left[\omega_o^2 - (1/2M)^2\right]} \qquad (2.7)$$

Since we are interested in (2.7) for values of ω, $\omega_o \geqslant 1$, we can neglect the $(1/2M)^2$ in the denominators of the coupling constant term. We now let ω_o approach 1. This simplifies (2.7) somewhat and introduces the $\partial D^{(1)}(0)/\partial k^2$ into the relation. We obtain, after handling the principal part integral in the same way as in Eq. (2.4),

$$D^{(1)}(k) = D^{(1)}(0) + k^2 \frac{\partial D^{(1)}(0)}{\partial k^2} - \frac{1}{M}\frac{k^4}{\omega^2} f^2 + \frac{k^4}{4\pi^2} P \int_0^{\infty} \frac{\Delta^{(1)}(k')\, dk'}{k'^2(k'^2 - k^2)} \qquad (2.8)$$

We can proceed in the same manner for the relation containing $D^{(2)}$. This is slightly more complicated since $D^{(2)}$ is an odd function of ω.

$$\frac{D^{(2)}(k)}{\omega} = D^{(2)}(0) + k^2 \left[\frac{\partial D^{(2)}(0)}{\partial k^2} - \frac{1}{2} D^{(2)}(0) \right]$$

$$+ \frac{k^4}{4\pi^2} P \int_0^{\infty} \frac{\Delta^{(2)}(k')\, dk'}{k'^2(k'^2 - k^2)} + \frac{2k^4}{\omega^2} f^2 \qquad (2.9)$$

Relations (2.8) and (2.9) differ from (2.2a, b) in a number of ways. The unknown high energy contribution to the integrals over the total cross sections is much less important than in (2.2). The price we must pay for this is the presence of the extra constants, $\partial D_{\pm}(0)/\partial k^2$. The n = 2 relations are independent of the n = 1 relations provided the added constants, $\partial D_{\pm}(0)/\partial k^2$, are undetermined by the theory and must be chosen from experiment. On the other hand, if we use the n = 1 relations, namely (2.5) and (2.6), to predict these constants, then it is clear that the n = 2 relations become identical with the n = 1 relations. This can easily be verified by substituting (2.6) into (2.9) and (2.5) into (2.8) to obtain equations (2.2a,b). We know, therefore, that the values of $\partial D_{\pm}(0)/\partial k^2$ predicted by (2.5) and (2.6), when used in (2.8) and (2.9), will predict values of D_+ which agree very well with experiment, and values of D_- which disagree with experiment. What we should like to do is modify the values of $\partial D_{\pm}(0)/\partial k^2$, such that (2.5) and (2.6) no longer hold, and improve the agreement for D_- without destroying the agreement for D_+. We shall show, now, that this cannot be done.

Just as relations (2.1) for $D_{\pm}(\omega)$ each contain a linear combination of $D_+(1)$ and $D_-(1)$, the n = 2 relations for $D_{\pm}(\omega)$ will each depend on a combination of $\partial D_{\pm}(0)/\partial k^2$ as well. Consequently a change in $\partial D_-(0)/\partial k^2$ alone will affect both the relations for D_+ and D_-. In general we must consider changes in both $\partial D_-(0)/\partial k^2$ and $\partial D_+(0)/\partial k^2$. If we call $\delta(\partial D_{\pm}(0)/\partial k^2) = \delta_{\pm}$ the difference between the values used in the n = 2 relation and the values predicted by the n = 1 relations, then it is easy to see that the difference between the values of $D_{\pm}$ predicted by (2.8), (2.9), and (2.11) are given by

$$\delta D_{\pm}(\omega) = \tfrac{1}{2} k^2 \left[(\omega + 1)\delta_{\pm} - (\omega - 1)\delta_{\mp} \right] \qquad (2.10)$$

Since (2.1) for $f^2 = .08$ predicts values of $D_-(\omega)$ which are too low for ω less than 180 Mev and too high for ω larger than 200 Mev, we can improve agreement by taking $\delta D_- \gtrless 0$ for $\omega \lessgtr \omega_0$ with ω_0 of the order of 2.6 (200 Mev). This leads to the result

$$\delta D_{\pm}(\omega) = k^2 \frac{\omega_0 \pm \omega}{\omega_0 - 1} \delta_- \qquad \delta_- > 0 \qquad (2.11)$$

δ_+ has been eliminated by requiring $\delta D_-(\omega_0) = 0$. Equation (2.11) predicts $\delta D_+(\omega) > \left| \delta D_-(\omega) \right|$ so that a large discrepancy in the π^+ relation would result. The reader can easily convince himself that other choices for $\delta_{\pm}$ cannot avoid large corrections for $\delta D_+(\omega)$.

We see, then, that if the n = 2 relations are to retain the good agreement with the π^+ experiments, which is obtained by the n = 1 relations, we must use the values of $\partial D_{\pm}(0)/\partial k^2$ predicted by (2.5) and (2.6), at least to within five per cent. Since this makes (2.8) and (2.9) identical with (2.2a,b), we can conclude that the success of the n = 1 π^+ relation implies that the assumption that $\omega^{-2} T_{\pm}(\omega)$ vanishes at infinity is correct and that (2.5) and (2.6) give the values of $\partial D_{\pm}(0)/\partial k^2$ predicted by the dispersion relations.

III. COMPARISON WITH EXPERIMENT

In this section we compare the predictions of Eqs. (2.5), (2.6), (2.8), and (2.9) with experiment. The total cross sections, used in the integrals, were obtained from all of the latest available data, much of which was either summarized or reported in the 1956 CERN Symposium[7]. The curve used for σ_- is shown in Fig. 1. The only unusual feature of this curve is the height of the peak and the energy at which it occurs. The peak is taken at 175 Mev which is at an energy 5 or 6 Mev lower than usual, and the height is taken to be 70 mb, which is about 6 mb higher than usual. This has the effect of improving, slightly, the agreement of the π^- relation with experiment. The curve for σ_+ is not shown, but it also has its peak at 176 Mev and a peak value of 210 mb, chosen to keep the $T = \frac{1}{2}$ cross section zero. This choice for the σ_+ peak has the effect of making the π^+ relation fit the experiment for an $f^2 = .08$ rather than .10. We shall see, however, that an f^2 of .08 is not consistent with the present experimental values of the π^+ S wave effective range and P wave scattering lengths. This indicates that the σ_+ peak should not be raised so high, and consequently $\sigma_- \gg \frac{1}{3}\sigma_+$ or, equivalently, $\sigma_{1/2} \neq 0$ at these energies. The results obtained from (2.5) and (2.6) will strongly reinforce this statement. Calculations have also been made with the σ_- and σ_+ peaks lowered and placed at 180 Mev.

Two choices for low energy behavior were made. One is Orear's prescription[8] of $\alpha_1 = .165\eta$, $\alpha_3 = -.105\eta$ and with zero effective ranges. To test the effect of non-zero effective ranges, a second choice was made by taking $\alpha_1 = .195\eta - .018\eta^3$ from Anderson's phase shift analysis[9] and taking $\alpha_3 = -.105\eta - .035\eta^3$, which fits the data of Ferrari, et al[7]. Using the Orear S wave phase shifts, an α_{33} of $.235\eta^3$ near zero energies, and taking $\sigma_+ = \sigma_- \cong 30$ mb for energies greater than 1.9 Bev, Eqs. (2.5) and (2.6) lead to values of $\partial D_\pm(0)/\partial k^2$ given by,

$$\frac{\partial D_+(0)}{\partial k^2} = 2(1 + \frac{1}{2M}) f^2 + .104 \pm .008 \qquad (3.1)$$

$$\frac{\partial D_-(0)}{\partial k^2} = -2(1 - \frac{1}{2M}) f^2 + .149 \pm .008 \qquad (3.2)$$

The alternate choice of S wave phase shifts leads to values of $\partial D_\pm(0)/\partial k^2$ which differs from (3.1) and (3.2) by only one to two per cent. The errors quoted here are obtained in part from an estimate of the errors involved in evaluating the integrals appearing in (2.5) and (2.6) and it has been assumed that $\sigma_\pm$ do not become very large at high energies.

As a check on (3.1) and (3.2), these values are used in (2.8) and (2.9) to obtain $D^b_\pm(\omega)$ in the center of mass frame, as a function of energy. Figure 2 shows the predicted curve for D^b_+ compared with experiment. The results, using the Orear S wave phase shifts, would be essentially the same. The good fit for an f^2 of .08 is due to the choice of the σ_+ peak. With the peak in the usual position, results similar to previous authors[1] would be obtained, favoring $f^2 = .10$. Figure 3 compares the predicted values of D_-^b with experiment, using the Orear phase shifts, and Fig. 4 shows D_-^b using the alternate choice of phase shifts. The improved agreement of the latter curves, compared to the former, is due to the choice of a large scattering length for α_1. This leads to an increase in $D_-(0)$ which raises the curve. It is not clear how valid this choice of phase shift is since it leads to a value of $\alpha_1 - \alpha_3 = 0.3\,\eta$, which is rather large. The curves of both Figs. 3 and 4 are raised slightly at 150 Mev by the choice of the high σ_- peak shown in Fig. 1. Even accounting for these differences, this calculation does give a slightly better result than those of previous calculations. While relations (2.8) and (2.9) combined with (2.5) and (2.6) are formally identical with (2.1), the former do present a different way of treating the data. Equations (2.8) and (2.9) are designed to give values of $D_-(\omega)$, for low energies which are consistent with the dispersion relations, and in particular, predicts a slope for $D_-^b(k)$ at $k = 0$ which is zero or slightly negative for $f^2 \geqslant .08$. The form of the relations used require a more careful treatment of the cross sections at energies below 80 Mev.

By varying the cross sections in various energy regions, it is found that the largest changes in D_- occur for changes in the low energy region from 0 to 80 Mev. In particular, agreement is improved by lowering σ_- in this region and agreement can be worsened by increasing σ_-. Despite this improvement, however, the π^- relation is still not satisfactory. Due to the factor of k^4 in the principal part integrals of (2.8) and (2.9), these relations are not particularly good ways of calculating $D_\pm$ at energies near 300 Mev and higher. It is clear, however, from these results, and previous one, that the points of Zinov and Korenchenko[10] at 307 Mev (and 340 Mev not shown in Figs. 3 and 4) is considerably below the curve for D^b_-2with $f^2 = .08$. The error on the experimental point at 307 Mev in Figs. 3 and 4 is drawn asymmetrically in an exaggerated manner to indicate that changes in the data tend to raise the point more easily than lower it.

Nevertheless, it would seem reasonable, from these results and the discussion of Eqs. (2.10) and (2.11) that (3.1) and (3.2) give the values of $\partial D_\pm(0)/\partial k^2$ predicted by the dispersion relations to a good degree of accuracy. Let us compare these predictions with experiment. We use the following notation for the S and P wave phase shifts at low energies.

$$\begin{aligned} \alpha_i &= a_i \eta + b_i \eta^3 + \cdots \qquad i = 1, 3 \\ \alpha_{ij} &= a_{ij} \eta^3 + \cdots \qquad i,j = 1, 3 \end{aligned} \tag{3.3}$$

By writing $D_\pm$ in terms of phase shifts, and using (3.3), we obtain expressions for $\partial D_+(0)/\partial k^2$ in terms of the a_i, b_i, a_{ij}.

$$\left(1 + \frac{1}{M}\right) \frac{\partial D_+(0)}{\partial k^2} = b_3 + 2\,a_{33} + a_{31} + \frac{1}{2M} a_3 - \frac{2}{3} a_3^3 \tag{3.4}$$

$$(1 + \frac{1}{M}) \frac{\partial D_-(0)}{\partial k^2} = \frac{1}{3}(1 + \frac{1}{m}) \frac{\partial D_+}{\partial k^2} + \frac{2}{3}\left[b_1 + 2a_{13} + a_{11}\right]$$

$$+ \frac{1}{3M} a_1 - \frac{4}{9} a_1^3 \qquad (3.5)$$

It is clear that (3.4) and (3.5) are insensitive to the values of a_1 and a_3. Using the Orear values for a_1 and a_3 and substituting in (3.1) and (3.2), we have the following conditions on the T = 3/2, 1/2 S wave effective range and P wave scattering lengths.

$$b_3 + 2a_{33} + a_{31} = 2.469 f^2 + .127 \pm .009 \qquad (3.6)$$

$$b_1 + 2a_{13} + a_{11} = - 4.424 f^2 + .188 \pm .014 \qquad (3.7)$$

To compare (3.6) with experiment, we take $b_3 = -.035 \pm .01$, $a_{31} = -.041 \pm .01$, $a_{33} = + .230 \pm .015$; the errors quoted are only rough estimates. This leads to

$$\left[b_3 + 2a_{33} + a_{31}\right]_{exp} = .384 \pm .033 \qquad (3.8)$$

Comparing this with (3.6) we obtain $f^2 = .104 \pm .014$. It would seem difficult, therefore, to have an $f^2 = .08$ and remain consistent with (3.6) and (3.8). This value of f^2, greater than .10, is inconsistent with the agreement shown in Fig. 1 for D_+^b using an $f^2 = .08$. Since the lower value of f^2 is due to the high peak chosen for σ_{+}, this indicates that the lower peak height usually taken, which favors an $f^2 = .10$, is more consistent with the dispersion relations. Of course the values used in (3.8) for a_{33}, a_{31} and b_3 are rather uncertain, however, since an f^2 greater than .10 would make a fit of (2.1) with experiment very difficult, (3.6) can be taken as evidence that a_{33} cannot be much larger than .230 and, unless a_{33} is as low as .215, $b_3 \neq 0$. Requiring an $f^2 \cong .10$ for the π^- relation would have the advantage of eliminating the discrepancy at high energies. It still would leave a discrepancy, for the π^- relation, at energies near 150 Mev.

The left hand side of (3.7) is not too well known experimentally, though it is generally believed that these quantities are small. If the values given by Anderson in 1956[9] are used, one obtains $b_1 + 2a_{13} + a_{11} \cong -.03$. This value is so small because present experiments observe very small $T = \frac{1}{2}$ scattering above 100 Mev. b_1 has been observed in lower energy π^- experiments partly because the data was analyzed assuming it to be zero. On the other hand, (3.7) predicts, even with an f^2 as low as .08, a value of $-.166 \pm .014$ for $b_1 + 2a_{13} + a_{11}$. This would seem to imply, if the dispersion relations are correct, that one or two of the quantities b_1, a_{13}, or a_{11}, are much larger than has so far been observed. To estimate what values b_1, a_{13}, a_{11} should have to be consistent with the dispersion relations, let us take $f^2 = .10$. Then

$$b_1 + 2a_{13} + a_{11} = -.254 \qquad , f^2 = 0.10 \qquad (3.9)$$

While present estimates of a_{13} are that it is almost zero, let us follow the suggestion of Chew-Low theory [11] and take $a_{13} = a_{31} = -.041$.

Then (3.9) becomes

$$b_1 + a_{11} = -.172, \quad a_{13} = -.041 \tag{3.10}$$

It is interesting that the predictions of the Chew-Low theory that $a_{11} = 4a_{13} = 4a_{31}$ is compatible with (3.9) and (3.10). Reasonable choices for b_1 and a_{11} would be $b_1 = -.04$ or $-.05$ and $a_{11} = -.13$ or $-.12$. This choice has two very nice features. One, the relatively large value of a_{11} would lead us to expect a $T = \frac{1}{2}$ cross section of the order of 7 or 8 mb at energies near 150 Mev. Since present measurements in this region[12] find $\sigma_{1/2} \cong 0$, this means a substantial increase in σ_-, at these energies, of about ten per cent. A correction to σ_- of this order of magnitude would lower the experimental values of D^b_-, in this energy region, by a significant amount. Secondly, a large, negative effective range for α_1 will keep σ_- small, or even decrease it, at low energies, before the P waves become important. We see, then, that this choice for b_1, a_{13}, a_{11} to satisfy (3.9) would predict an energy dependence for σ_- which is qualitatively similar to that assumed by Zaidi and Lomon[2], and which, in effect, raises the theoretical values for D^b_- while lowering the experimental values at the 150 Mev region. It seems to this author, therefore, that the key to the present difficulties with the π^- dispersion relation lies in the large discrepancy that exists between the value of $b_1 + 2a_{13} + a_{11}$ predicted by the dispersion relations and the value obtained from experiment.

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13. The author would like to thank R. Sorensen for communicating these results before publication.

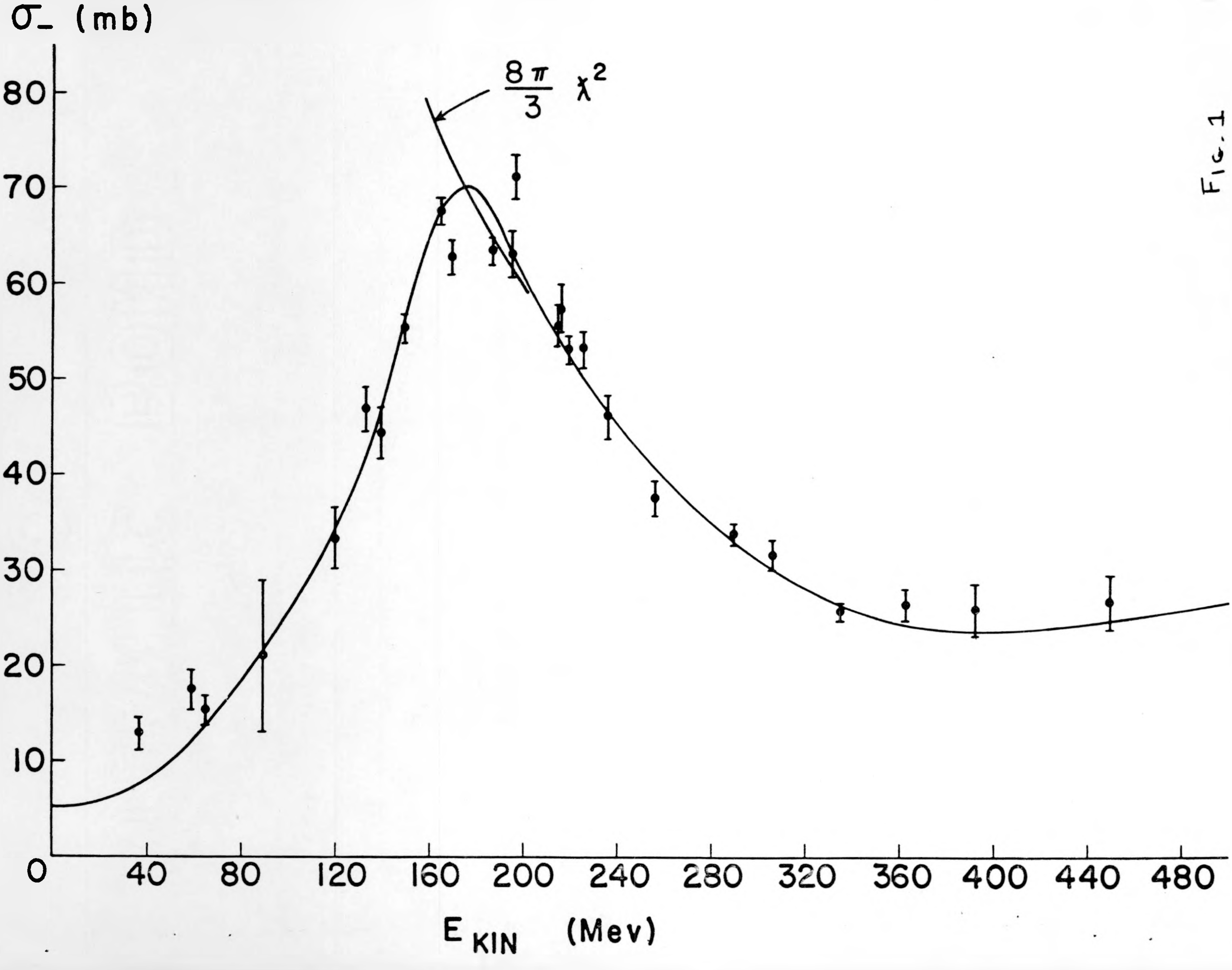
FIGURE CAPTIONS

Fig. 1 Total cross section for negative pions. For simplicity, all of the data are not shown.

Fig. 2 A comparison of the values of $D_+^b(\omega)$ calculated from equations (2.7) and (2.8) using the values of $\partial D_{\pm}(0)/\partial k^2$ given in (3.1) and (3.2) and the values obtained from experiment. $D_+^b(\omega)$ is the real part of the π^+ scattering amplitude in the center of mass system with $\mu = 1$. ω is the meson energy in the laboratory system.

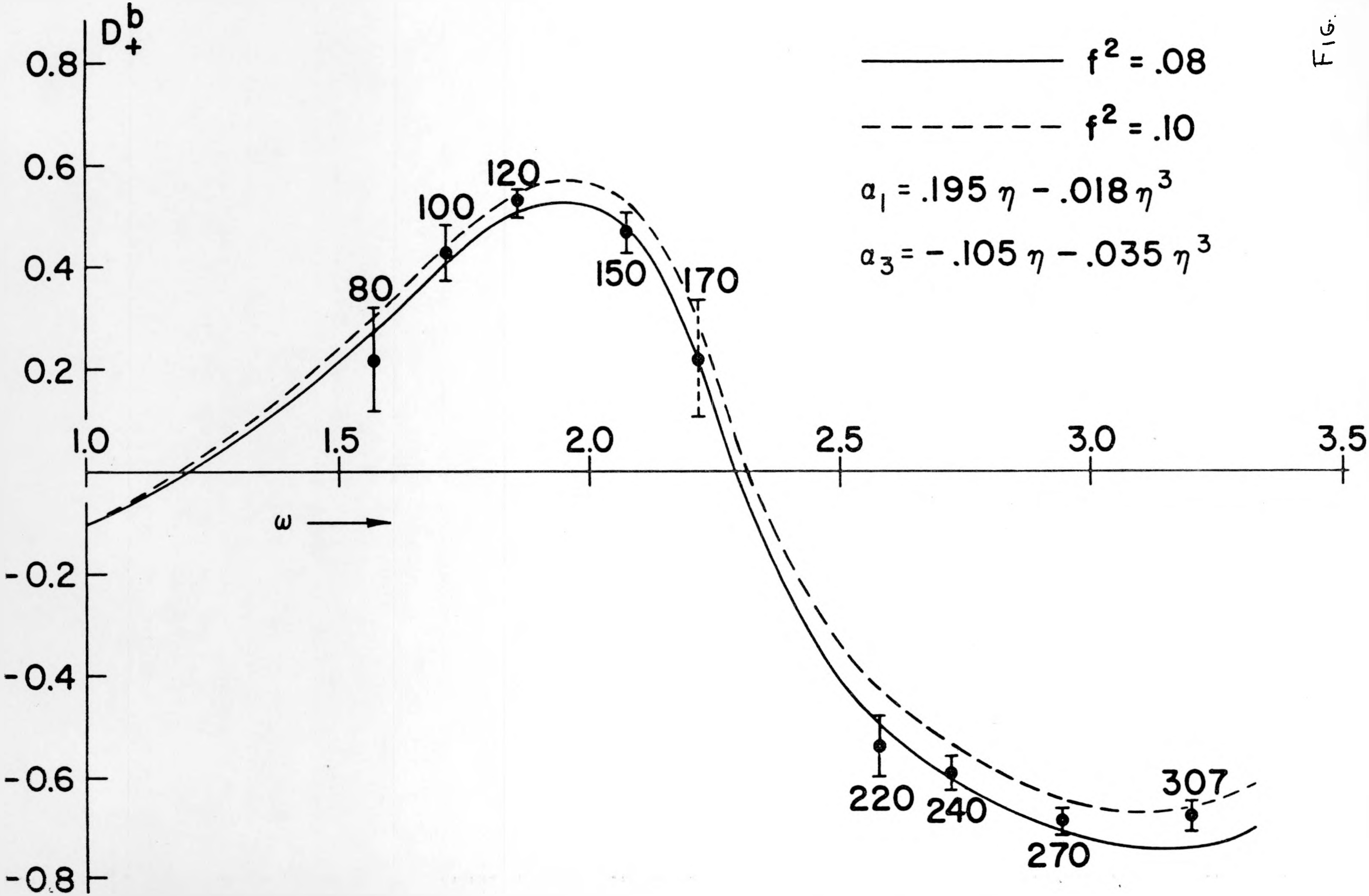
Fig. 3 A comparison of the calculated values of $D_-^b(\omega)$ with experiment using equations (2.7), (2.8), (3.1), and (3.2). The Orear S wave phase shifts are assumed.

Fig. 4 The curve in Fig. 3 is modified by a different choice for the S wave phase shifts, including non-zero effective ranges. This choice leads to a larger value for $D_-(0)$ than is obtained with the Orear S wave phase shifts.



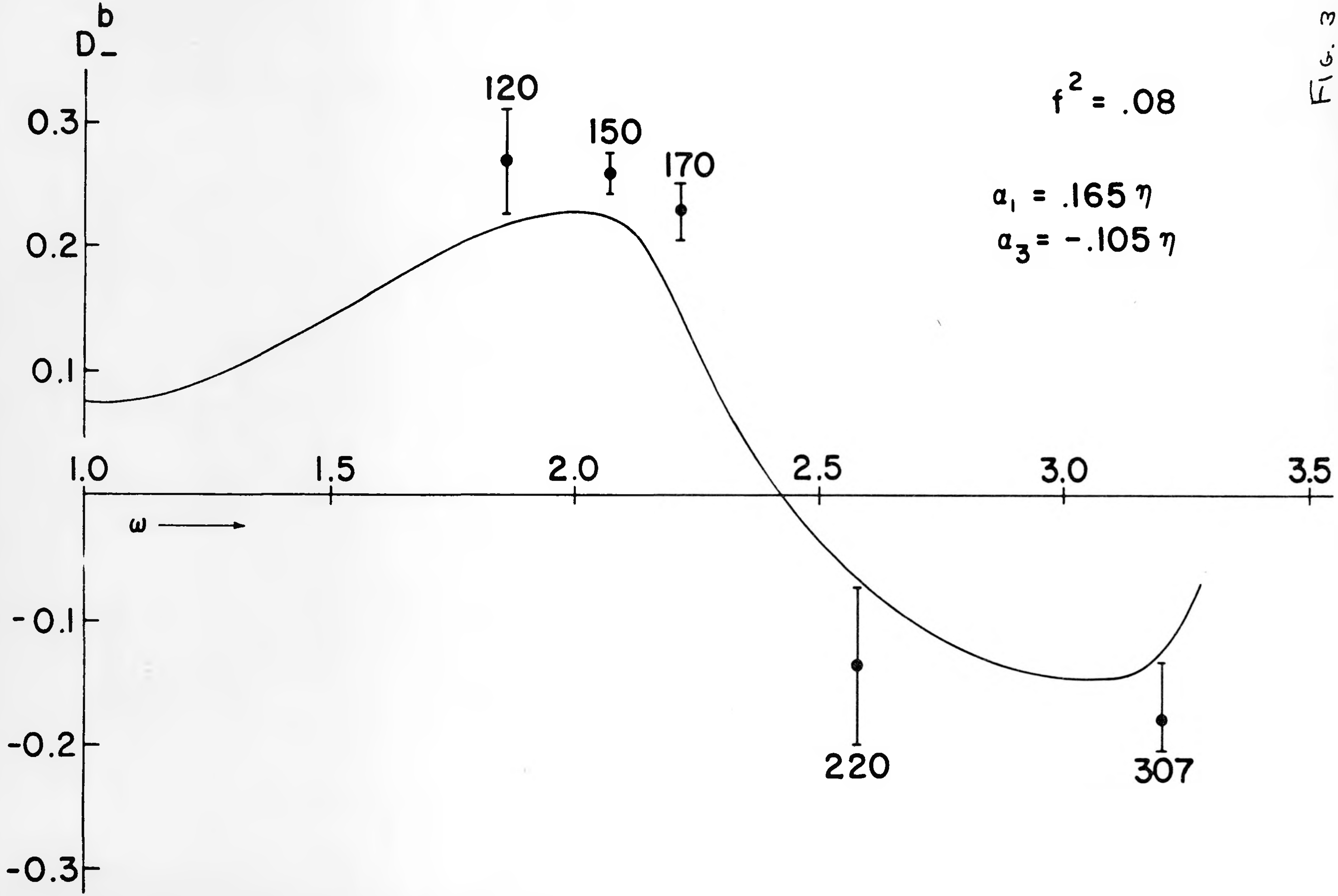
σ₋ (mb)
$\frac{8\pi}{3}\lambda^2$
80
70
60
50
40
30
20
10
0
40
80
120
160
200
240
280
320
360
400
440
480
E KIN (Mev)

Fig. 1



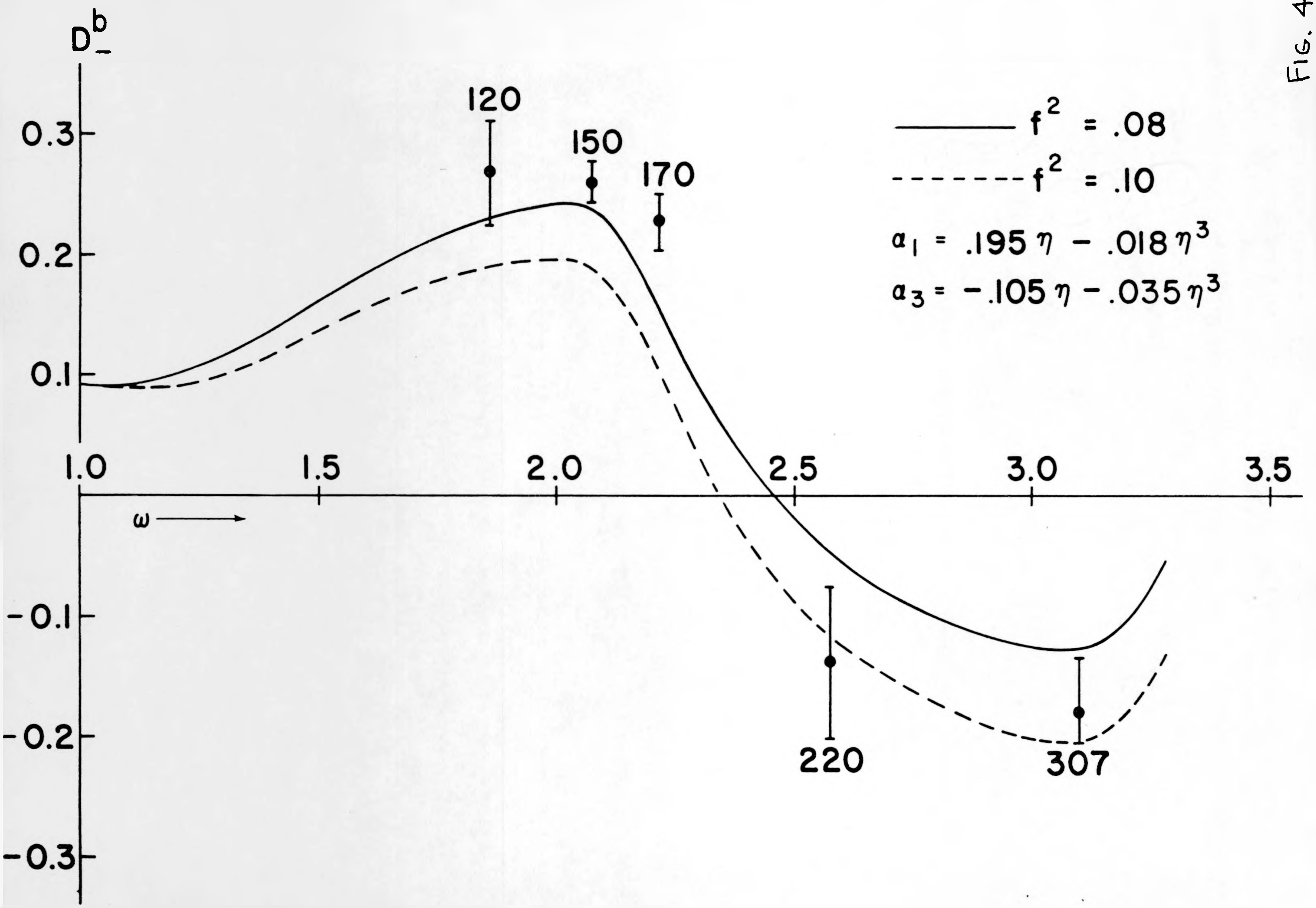
D_+^b
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
-0.8
1.0
1.5
2.0
2.5
3.0
3.5
ω →
$f^2 = .08$
$f^2 = .10$
$a_1 = .195\,\eta - .018\,\eta^3$
$a_3 = -.105\,\eta - .035\,\eta^3$
80
100
120
150
170
220
240
270
307

FIG. 2



D_-^b
0.3
0.2
0.1
-0.1
-0.2
-0.3
1.0
1.5
2.0
2.5
3.0
3.5
ω →
120
150
170
220
307
$f^2 = .08$
$a_1 = .165\,\eta$
$a_3 = -.105\,\eta$

FIG. 3



D_-^b
0.3
0.2
0.1
-0.1
-0.2
-0.3
1.0
1.5
2.0
2.5
3.0
3.5
ω →
120
150
170
220
307
$f^2 = .08$
$f^2 = .10$
$a_1 = .195\eta - .018\eta^3$
$a_3 = -.105\eta - .035\eta^3$

FIG. 4