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REVISED FORMULATION OF THE KINETIC PLASMA THEORY

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ABSTRACT

Using the Maxwellian macroscopic approach and analyzing the formulation of the dielectric constant, I have shown that the concept of energy has not been properly incorporated into the current kinetic plasma theory. The difficulties are due to the Boltzmann collisional term $(\partial F/\partial t)_{\text{coll}}$, which accounts for a change in the velocity distribution due to collisions alone. If one attempts to rephrase the Boltzmann-Vlasov theory in terms of the Maxwellian macroscopic formulation, one obtains an expression for energy which is not consistent with the meaning of this concept in generalized dynamics. In a revised version developed in this analysis the Boltzmann collisional term has been eliminated, and an appropriate collisional operator is introduced which is believed to describe collisional processes in a plasma more adequately. It is assumed that the collisional operator can be applied directly to the electrical intensity of the field interacting with the plasma and is effective in transforming the intensity in a collisionless plasma into a corresponding intensity in a collisional plasma. At the same time the relationship between the electron velocity distribution function and the field intensity is considered to be the same, whether there are collisions or not. In other words, not only a collisionless but also a collisional plasma are assumed to be controlled by a Vlasov-type mechanism which does not explicitly take into account the Boltzmann term $(\partial F/\partial t)_{\text{coll}}$.

1. INTRODUCTION

There are inconsistencies in the current formulation of the plasma theory. Many fundamental concepts, such as the dielectric constant, conductivity, energy density, group of waves, and group velocity, are generally taken for granted with the understanding that their meaning is clear and unambiguous. However, this is not always true. Although the concepts of group of waves and group velocity are relatively old, the first consistent effort to incorporate them into the electromagnetic theory was made by Brillouin (1932, 1960). Brillouin proceeded from Maxwell's differential equations and described, within the context of Maxwell's theory, the details of energy propagation in an integral form. The main achievement of the Brillouin theory is in establishing the well-known expressions for the energy density, energy flow, and group velocity in dispersive media. These expressions are currently applied both in molecular media and in plasmas.

However, Brillouin did not make any provision for energy absorption, and therefore his results are valid only to the extent to which they can be applied to nonabsorbing media. In spite of recent efforts to remove the present restrictions and generalize the Brillouin theory, relatively little has been accomplished along these lines to date. In the course of the investigation on how to generalize the Brillouin theory, it became clear that certain assumptions which were previously taken for granted need to be reconsidered. Particular credit is due to Ginzburg (1964*b*) for pointing out that: "Despite the fact that the problem of the conservation law and the expression for the energy density in electrodynamics is a fundamental one, there are certain aspects of it which have not been yet elucidated, and in particular for the case of an absorbing dispersive medium. For example, the familiar expression

$$W_E = \frac{1}{4\pi} \int_{-\infty}^{0} \mathbf{E} \cdot (\partial \mathbf{D}/\partial t) dt ,$$

even when reduced to the form constant $\times\ E^2$, cannot be regarded as the total energy density when absorption is present, if only because the value of W_E may be negative."

Maxwell's theory is a phenomenological one in the sense that it describes the properties of matter by means of certain macroscopic parameters such as the dielectric constant and conductivity. Before we undertake a critical scrutiny of the current formulation, a few comments will be made concerning the significance of the dielectric constant in the macroscopic Maxwell theory. It is currently assumed that if absorption is present the dielectric constant for a dispersive medium is a function not only of frequency ω (and sometimes of the wave vector $\mathbf{k}$), but it also depends on a parameter which defines absorption. The validity of such a formulation is, however, open to question. The author of this investigation has shown that a physically meaningful dielectric constant should be independent of the dissipational parameter, even if there is absorption (Neufeld, 1966). Thus, if one takes into account all the physical factors involved, the analytical expression for the dielectric constant should be the same whether or not absorption occurs.

Although the previous discussion (Neufeld, 1966) on the dielectric constant dealt primarily with molecular media, the fundamental assumptions made therein are also applicable to plasmas. The purpose of this investigation is to analyze the dielectric and energetic properties of a plasma both from the point of view of the macroscopic Maxwellian theory and from the point of view of the Boltzmann-Vlasov formulation. It is intended in this analysis to incorporate the concept of the dielectric constant into the framework of the Boltzmann-Vlasov theory and to show that the inconsistencies previously discussed for molecular media also occur in the current version of the kinetic theory of plasma.

One of the principal absorption mechanisms in a plasma is due to collisions, and the crucial term in the current version of the kinetic plasma theory is the Boltzmann collisional term $(\partial F/\partial t)_{\mathrm{coll}}$, which accounts for a change in the velocity distribution due to collisions alone (Delcroix, 1965). It will be shown in this analysis that if one attempts to rephrase the Boltzmann-Vlasov theory in terms of the phenomenological Maxwellian formulation, one obtains an expression for the dielectric constant which is explicitly dependent on collisions. In other words, any change in $(\partial F/\partial t)_{\mathrm{coll}}$ would produce a corresponding alteration in the dielectric behavior of a plasma. However, according to the results previously obtained (Neufeld, 1966), a meaningful expression for the dielectric constant should be independent of the parameter which defines absorption. Consequently, the dielectric constant should be the same whether $(\partial F/\partial t)_{\text{coll}} = 0$ or not.

In order to obtain physically meaningful results it appears to be necessary to modify the kinetic plasma theory and to provide a different interpretation of the collisional mechanism in a plasma. It is therefore proposed in this analysis to eliminate $(\partial F/\partial t)_{\text{coll}}$ even if collisions occur and to replace $(\partial F/\partial t)_{\text{coll}}$ by an appropriate collisional operator which is believed to describe collisional processes in a plasma more adequately. It is shown that the collisional operator can be applied directly to the electrical intensity of the field interacting with a plasma and is effective in transforming the intensity in a collisionless plasma into the corresponding intensity in a collisional plasma. At the same time the relationship between the electron distribution function and the field intensity is considered to be the same whether there are collisions or not. In other words, the collisional operator does not change the form of such a relationship. This investigation represents, therefore, a fundamental departure from the traditional point of view. It is assumed here that not only a collisionless but also a collisional plasma are controlled by a Vlasov-type mechanism which does not explicitly take into account the Boltzmann collisional term $(\partial F/\partial t)_{\text{coll}}$.

2. FORMULATION OF THE DIELECTRIC CONSTANT AND OF ELECTRICAL ENERGY IN THE BOLTZMANN-VLASOV THEORY

The discussion in this section will deal with the current status of the plasma theory, with emphasis on some of the inconsistencies previously discussed by Ginzburg (1964*a*, *b*; 1967). It is intended to derive from the Boltzmann-Vlasov equation an expression for the dielectric constant in the presence of collisions and then to calculate energy density within the framework of the macroscopic Maxwell theory. It will then appear that because of the inclusion of the Boltzmann term $(\partial F/\partial t)_{\text{coll}}$, one cannot obtain a meaningful expression for energy.

Derivation of the Dielectric Constant

Consider an electron gas in a stationary background of heavy particles which has just enough uniformly distributed positive charge density to make the plasma macroscopically neutral when there is no electrical field. Let

$$E = 0.5[E_1 e^{i(kx-\omega t)} + \text{c.c.}] \tag{1}$$

be the electrical intensity in a small-amplitude longitudinal wave traveling in the x direction (E_1 is a constant and c.c. denotes a complex conjugate). Assume that $F_0 = F_0(v)$ is the electron velocity distribution function when $E = 0$ and that $F(x, v, t)$ is the corresponding distribution function when $E \neq 0$ (v is the electron velocity in the x direction). One has

$$nF = nF_0 + f, \tag{2}$$

where

$$f = 0.5[f_1(v)e^{i(kx - \omega t)} + \text{c.c.}] \ , \tag{3}$$

and n is electron density. The term f represents an alternating component of the distribution function which is produced by (1) and which accounts for polarization. The applicable form of the Boltzmann-Vlasov equation is known to be

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{ne}{m} E \frac{\partial F_0}{\partial v} = \left(\frac{\partial F}{\partial t}\right)_{\text{coll}} \ . \tag{4}$$

From the methodological point of view the most appropriate procedure for incorporating $(\partial F/\partial t)_{\text{coll}}$ into the macroscopic Maxwellian formulation is to assume a simple relaxation model in which

$$(\partial F/\partial t)_{\text{coll}} = -\nu n(F - F_0) \ , \tag{5}$$

where ν is the mean collision frequency between electrons and ions (Bhatnagar, Gross, and Krook, 1954).

Combining (4) with (2) and (5), one obtains

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{ne}{m} E \frac{\partial F_0}{\partial v} = -\nu f \ . \tag{6}$$

Equation (6) is supplemented by

$$\partial E/\partial x = 4\pi\rho \ , \tag{7}$$

where

$$\rho = -e \int_{-\infty}^{\infty} f \, dv \tag{8}$$

is the charge density.

Using polarization variables, one can replace (7) and (8) by

$$\frac{\partial D}{\partial x} = \frac{\partial(E + 4\pi P)}{\partial x} \ ,$$

where

$$D = 0.5[D_1 e^{i(kx - \omega t)} + \text{c.c.}]$$

is the electric displacement, and

$$P = 0.5[P_1 e^{i(kx - \omega t)} + \text{c.c.}]$$

is the polarization density. One has

$$\frac{\partial P}{\partial t} = -e \int_{-\infty}^{\infty} fv \, dv \,. \tag{9}$$

The propagation is longitudinal, and therefore **E**, **P**, and **D** are aligned along the x axis.

Substituting (1) and (3) in (6), one obtains

$$f_1 = \frac{ine}{m} E_1 \frac{\partial F_0/\partial v}{\omega - i\nu - kv} = f_1^{(r)} + if_1^{(i)} \,, \tag{10}$$

where

$$f_1^{(r)} = -\frac{ne}{m} E_1 \frac{\partial F_0}{\partial v} \frac{\nu}{(\omega - kv)^2 + \nu^2} \tag{11}$$

and

$$f_1^{(i)} = \frac{ne}{m} E_1 \frac{\partial F_0}{\partial v} \frac{\omega - kv}{(\omega - kv)^2 + \nu^2} \,.$$

Combining (3) and (10) with (9) and using the relationship $D_1 = E_1 + 4\pi P_1 = \epsilon E_1$, where ϵ is the dielectric constant, one obtains

$$\epsilon = \epsilon^{(r)} + i\epsilon^{(i)} \,,$$

where

$$\epsilon^{(r)} = \epsilon^{(r)}(k, \omega, \nu) = 1 + \frac{\omega_0^2}{k} \int_{-\infty}^{\infty} \frac{(\partial F_0/\partial v)(\omega - kv)}{(\omega - kv)^2 + \nu^2} \, dv \tag{12}$$

and

$$\epsilon^{(i)} = \epsilon^{(i)}(k, \omega, \nu) = -\frac{\omega_0^2 \nu}{k} \int_{-\infty}^{\infty} \frac{\partial F_0/\partial v}{(\omega - kv)^2 + \nu^2} dv ;$$

here $\omega_0^2 = 4\pi ne^2/m$.

It is customary to assume that the dielectric properties of an absorbing medium can be expressed by an "effective" dielectric constant which has the form

$$\epsilon^{\mathrm{eff}} = \epsilon^{(r)}(k, \omega, \nu) = 1 + \frac{\omega_0^2}{k} \int_{-\infty}^{\infty} \frac{(\partial F_0/\partial v)(\omega - kv)}{(\omega - kv)^2 + \nu^2} dv \tag{13}$$

and is therefore dependent on ν. The parameter ν defines dissipation and is analogous to the frictional parameter γ which enters in the expression $\epsilon(\omega, \gamma)$ for the dielectric constant of molecular media. Our objective is to point out that the formulation based on $\epsilon^{\mathrm{eff}}(k, \omega, \nu)$ leads to the same inconsistencies as those previously discussed in the analysis of molecular media (Neufeld, 1966).

It is evident that when $\nu = 0$ the effective dielectric constant has the form

$$\epsilon^{\mathrm{eff}} = \epsilon(k, \omega) = 1 - \frac{\omega_0^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial F_0/\partial v}{[v - (\omega/k)]} dv . \tag{14}$$

The equality (14) was derived by Gertzenshtein (1952) and further analyzed by Neufeld and Ritchie (1955), Neufeld (1961), and others.

Energy Density According to the Boltzmann-Vlasov Theory

Having defined the effective dielectric constant, one can now apply the customary procedure for deriving an expression for energy stored by an electrical field in a collisional plasma. Suppose that

$$E_{\mathrm{am}} = 0.5[\mathcal{E}_{\mathrm{am}}(x, t)e^{i(kx - \omega t)} + \mathrm{c.c.}] \tag{15}$$

is the electrical intensity in an almost monochromatic field. The factor $\exp[i(kx - \omega t)]$ represents a carrier wave, and $\mathcal{E}_{\mathrm{am}}(x, t)$ is a slowly growing modulating function such that $\mathcal{E}_{\mathrm{am}}(x, t) = 0$ at $t = -\infty$.

The usual procedure for determining energy density is straightforward. It is based on the relationship

$$\partial U/\partial t = (1/4\pi) E_{\mathrm{am}}(\partial D_{\mathrm{am}}/\partial t) ,$$

where $\partial U/\partial t$ is the rate of change of the energy density and D_{am} is the electrical displacement defined by E_{am} and ϵ^{eff}. Using the method outlined by Brillouin (1932, 1960) and assuming that (13) is a physically meaningful expression for the dielectric constant, one obtains

$$\overline{U} = \frac{1}{16\pi} \frac{\partial[\omega\, \epsilon^{eff}(k, \omega, \nu)]}{\partial \omega} \mathcal{E}_{am} \mathcal{E}^*_{am} , \tag{16}$$

where the bar above $\overline{U}$ indicates an averaging process and the asterisk denotes a complex conjugate.

Strictly speaking, expression (16) is not the one derived by Brillouin. Brillouin's theory is valid provided there is no absorption, and in such a case the dielectric constant for a plasma is $\epsilon(k, \omega)$ and not $\epsilon^{eff}(k, \omega, \nu)$. Therefore, according to Brillouin, one has

$$\overline{U} = \frac{1}{16\pi} \frac{\partial[\omega\, \epsilon(k, \omega)]}{\partial\omega} \mathcal{E}_{am} \mathcal{E}^*_{am} .$$

Expression (16) is usually interpreted as representing a generalization of the Brillouin theory to absorbing media. There is, however, a question whether such a generalization provides a physically meaningful result. This question was raised by Ginzburg, who expressed serious doubts concerning the validity of (16). According to Ginzburg (1967): ". . . one needs to underline that in presence of absorption it does not appear to be possible in a general case to introduce phenomenologically the concept of the mean electromagnetic energy." In some instances the current formulation leads to paradoxical results; namely, when $\omega \ll \nu$, the energy density appears to be negative (Ginzburg, 1964*b*).

There is an additional point which militates against the acceptance of (16) as representing energy. From the standpoint of generalized dynamics, energy of a system depends solely on the parameter which defines the physical state, whether there is dissipation or not. However, the expression for energy cannot be explicitly dependent on a parameter which defines dissipation (Neufeld, 1969). Therefore, there is an inconsistency since the expression (16) contains ν. In view of the universal character of energy, this conceptual discrepancy is of some significance. The main object of this investigation is therefore to clarify the meaning of energy and to suggest a revised formulation in which the concept of energy is incorporated in accordance with the logical requirements of the theory.

3. FUNDAMENTALS OF THE PROPOSED FORMULATION

Introductory Comments

The definition of energy is a crucial matter in this investigation. The wave-plasma interaction will, therefore, be considered as a transformation of energy from one form to another. The forms of energy which are particularly relevant to this discussion are the "electric" energy, which in the macroscopic Maxwellian sense includes vacuum and polarization energy, and the "nonelectric energy"; the latter is defined by the translational motion of electrons and is dependent on the velocity distribution $F_0(v)$.

From the Maxwellian macroscopic point of view, the parameter which defines energy transformation is the conductivity σ. When $\sigma > 0$ the electric energy is transformed into nonelectric energy, whereas an inverse process occurs when $\sigma < 0$. The conductivity is considered to be a sum of two terms, namely, $\sigma = \sigma_{coll} + \sigma_{res}$, where the subscripts refer to collisional and resonance conductivities. The resonance conductivity is due to electrons traveling at or near the velocity of the wave and having velocities comprised in the range

$$\frac{\omega}{k} - \Delta v \leqq v \leqq \frac{\omega}{k} + \Delta v \tag{17}$$

($\Delta v << v$), known as the resonance range. The collisional conductivity accounts for the conversion of the polarization energy stored in electron oscillations into heat, which manifests itself in the form of random electron motion.

We assume that the conductivity is small and acts as a perturbation. Therefore when $\sigma = 0$ the field is considered to be unperturbed. In such a case there is no energy conversion, and the energy stored by an electrical disturbance neither grows nor decays. However, in the presence of a perturbation the energy grows when $\sigma < 0$ and decays when $\sigma > 0$. The effects of the collisional and the resonance conductivity are such that σ_{coll} is always positive, whereas σ_{res} may be positive or negative.

Unperturbed Electrical Field ($\sigma = 0$)

We will assume that $\sigma = 0$ and subsequently consider the changes which occur when $\sigma \neq 0$. To facilitate the discussion the electrical intensity E^0 and other quantities characterizing an unperturbed field will be symbolized by a superscript zero, whereas the electrical intensity E and the corresponding quantities characterizing a perturbed field will not. Suppose that a plasma interacts with an oscillating field

$$E^0 = 0.5[E_1^0 \, e^{i(k^0 x - \omega^0 t)} + \text{c.c.}] \, . \tag{18}$$

Since $\sigma = 0$, there is no energy conversion either by collision or by resonance. Consequently, electrons having velocities within the range (17) are nonexistent. Such a plasma in which there are no resonance electrons will be denoted as the "main plasma."

Let $F_{m0} = F_{m0}(v)$ be the velocity distribution of the main plasma when $E^0 = 0$ and $F_m^0 = F_m^0(v, t)$ be the corresponding distribution when $E^0 \neq 0$. One has

$$nF_m^0 = nF_{m0} + f_m^0(x, v, t),$$

where

$$f_m^0(x, v, t) = 0.5[f_{m1}^0(v)\, e^{i(k^0 x - \omega^0 t)} + \text{c.c.}] \tag{19}$$

is the oscillating component produced by (18).

The applicable form of the kinetic equation is the one due to Vlasov. Therefore

$$\frac{\partial f_m^0}{\partial t} + v\frac{\partial f_m^0}{\partial x} - \frac{ne}{m} E^0 \frac{\partial F_{m0}}{\partial v} = 0 , \tag{20}$$

and

$$\partial E^0/\partial t = 4\pi\rho^0 , \tag{21}$$

where

$$\rho^0 = -e \int_{-\infty}^{\infty} f_m^0 \, dv . \tag{22}$$

Using polarization variables, one can replace (21) and (22) by

$$\partial E^0/\partial t + 4\pi j_{\text{pol}}^0 = 0 ,$$

where

$$j_{\text{pol}}^0 = -e \int_{-\infty}^{\infty} f_m^0 v \, dv$$

is the polarization current.

Substituting (18) and (19) in (20), one obtains

$$f_{m1}^0 = i\,\frac{ne}{m} E_1^0 \frac{\partial F_{m0}/\partial v}{\omega^0 - k^0 v} .$$

Then, using the relationship $D^0 = E^0 + 4\pi P^0 = \epsilon^0 E^0$ (where ϵ^0 is the dielectric constant), one obtains

$$\epsilon^0 = \epsilon^0(k^0, \omega^0) = 1 - \frac{\omega_0^2}{(k^0)^2} \int_{-\infty}^{\infty} \frac{\partial F_{m0}/\partial v}{v - (\omega^0/k^0)} \, dv \, . \tag{23}$$

Combining the above relationships, one obtains

$$\omega^0 \, \epsilon^0(k^0, \omega^0) = 0 \ , \tag{24}$$

which represents the dispersion equation for the longitudinal waves in a plasma.

Perturbed Electrical Field ($\sigma \neq 0$)

Since $\sigma \neq 0$ and particularly since $\sigma_{res} \neq 0$, the electrons within the resonance range (17) will now be taken into account. Therefore the velocity distribution has two components: one represents the resonance electrons within the range (17), and the other represents the main plasma, in which the electron velocities are outside of the range (17).

Suppose that

$$E = 0.5[E_1 \, e^{i(kx - \omega t)} + \text{c.c.}] \tag{25}$$

is the electrical field interacting with the plasma. We assume at first that $E = 0$ and consider the electron velocity distribution $F_{m0}(v)$ of the main plasma and also the velocity distribution $F_{r0}(v)$ of the resonance electrons. It is evident that when $E \neq 0$ the corresponding velocity distributions are altered. In the latter case these distributions will be expressed by $F_m = F_m(x, v, t)$ and $F_r = F_r(x, v, t)$ respectively.

Considering the main plasma, one has a relationship

$$nF_m = nF_{m0}(v) + f_m(x, v, t) \ ,$$

where

$$f_m(x, v, t) = 0.5[f_{m1}(v) \, e^{i(kx - \omega t)} + \text{c.c.}] \tag{26}$$

is the oscillating part which is produced by (25). The corresponding relationship for the resonance electrons is

$$nF_r = nF_{r0}(v) + f_r(x, v, t) \ ,$$

where

$$f_r(x, v, t) = 0.5[f_{r1}(v)\, e^{i(kx - \omega t)} + \text{c.c.}] \tag{27}$$

is the oscillating component produced by (25).

The perturbation produced by σ is considered to be small. It has therefore been assumed that in the absence of the electrical field the velocity distribution which accounts for the translational electron motion is the same when $\sigma = 0$ and when $\sigma \neq 0$ and is $F_{m0}(v)$.

Effects of Perturbation

It will now be useful to compare the unperturbed quantities f_m^0 and E^0 with the corresponding perturbed quantities f_m and E. Obviously, one has $f_m \neq f_m^0$ and $E \neq E^0$ because of the perturbation. The basic novelty of the proposed formulation is in the assumption that the perturbation which accounts for the transition from f_m^0 to f_m and from E^0 to E has no effect on the relationship between the oscillating component of the distribution function and the electrical field intensity. In other words the relationship between f_m and E is assumed to be the same as the one between f_m^0 and E^0. Consequently,

$$\frac{\partial f_m}{\partial t} + v\,\frac{\partial f_m}{\partial x} - \frac{ne}{m}\,E\,\frac{\partial F_{m0}}{\partial v} = 0\ . \tag{28}$$

It should be noted that (28) has the same form as the Vlasov equation (20). A similar assumption is made concerning the velocity distribution of resonance electrons. Therefore

$$\frac{\partial f_r}{\partial t} + v\,\frac{\partial f_r}{\partial x} - \frac{ne}{m}\,E\,\frac{\partial F_{r0}}{\partial v} = 0\ . \tag{29}$$

The formulations (28) and (29) represent a significant departure from the conventional point of view. Although collisions have been assumed to occur, there is no collisional term of the type $(\partial F/\partial t)_{\text{coll}}$ either in (28) or in (29). Our point of view is such that the collisional effects have already been taken into account, since f_m and E, which characterize a collisional plasma, differ from the corresponding quantities f_m^0 and E^0 in a collisionless plasma. Therefore the equality (28) is different from the corresponding equality (20).

Equations (28) and (29) need to be supplemented with additional relationships which describe the electrical field. Therefore

$$\frac{\partial E}{\partial t} + 4\pi(j_{\text{coll}} + j_{\text{res}} + j_{\text{pol}}) = 0\ ,$$

where $j_{coll} = \sigma_{coll} E$ is the conduction current due to collisions,

$$j_{res} = \sigma_{res} E = -e \int_{-\infty}^{\infty} f_r v \, dv \tag{30}$$

is the conduction current due to the resonance electrons, and

$$j_{pol} = \frac{\partial P}{\partial t} = -e \int_{-\infty}^{\infty} f_m v \, dv \tag{31}$$

is the polarization current.

Combining (25) and (26) with (28), one obtains

$$f_{m1} = \frac{ine}{m} E_1 \frac{\partial F_{m0}/\partial v}{\omega - kv} . \tag{32}$$

Then using (26), (31), (32), and the relationship $D = E + 4\pi P = \epsilon E$, one obtains

$$\epsilon(k, \omega) = 1 - \frac{\omega_0^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial F_{m0}/\partial v}{v - (\omega/k)} dv , \tag{33}$$

which represents the dielectric constant of a plasma when $\sigma \neq 0$.

It is evident that the relationship between ϵ^0 and (k^0, ω^0) (Eq. 23) is the same as the one between ϵ and (k, ω) (Eq. 33). We have therefore arrived at a formulation in which the dielectric behavior of a plasma is the same whether $\sigma = 0$ or $\sigma \neq 0$. This result is similar to the one previously obtained for molecular media (Neufeld, 1966).

Assuming that the width Δv of the resonance range tends to zero, one can write

$$\int_{-\infty}^{\infty} \frac{\partial F_{m0}/\partial v}{v - (\omega/k)} dv = ⨍ \frac{\partial F_0/\partial v}{v - (\omega/k)} dv ,$$

where $F_0 = F_{m0} + F_{n0}$ and $⨍$ indicates the principal value. Consequently,

$$\epsilon(k, \omega) = 1 - \frac{\omega_0^2}{k^2} ⨍ \frac{\partial F_0/\partial v}{v - (\omega/k)} dv .$$

It should be noted that there is a complete separation of the dielectric constant from the conductivity in the sense that any alteration of conductivity does not introduce automatically a change in the dielectric constant. In this respect this formulation differs from the one in the conventional theory. In this formulation the dielectric constant depends solely on F_{m0}, whereas σ_{res} depends on F_{r0}. Therefore, in calculating the dielectric constant, it has been assumed that in the distribution $F_0 = F_{m0} + F_{r0}$ the resonance electrons should not be taken into account even if $F_{r0} \neq 0$. In other words, $F_0 = F_{m0}$. The resonance electrons are, however, not neglected, since eventually the distribution F_{r0} will be accounted for in calculating σ_{res}. The point of view in this analysis is the same as the one outlined in the previous investigation (Neufeld, 1966); namely, the dielectric constant of a medium is independent of conductivity and is therefore the same when $\sigma = 0$ and when $\sigma \neq 0$.

Combining the above relationships, one obtains

$$\omega \, \epsilon(k, \omega) + 4\pi i\sigma = 0 \ , \tag{34}$$

which represents a dispersion equation for a plasma when $\sigma \neq 0$.

Characteristic Equations

We will now center our attention on the mechanism which accounts for the transition from E^0 to E. We assume that

$$E = KE^0 \ , \tag{35}$$

where

$$K = e^{\alpha x - \beta t}$$

and α and β are appropriate parameters which define growth or decay. Obviously, α and β depend on σ. Substituting in (35) the expression (18) for E^0, one obtains

$$E = 0.5[E_1^0 \, e^{i[(k^0 - i\alpha)x - (\omega^0 - i\beta)t]} + \text{c.c.}] \ ,$$

and therefore a plane wave which is homogeneous when $\sigma = 0$ becomes nonhomogeneous when $\sigma \neq 0$. The wave number, which for an unperturbed wave was expressed by a real k^0, has now a complex value $k = k^0 - i\alpha$. Similarly, the frequency, which for an unperturbed wave was expressed by a real ω^0, has now a complex value $\omega = \omega^0 - i\beta$.

Substituting $k = k^0 - i\alpha$ and $\omega = \omega^0 - i\beta$ in (34), one obtains

$$(\omega^0 - i\beta) \, \epsilon(k^0 - i\alpha, \omega^0 - i\beta) + 4\pi i\sigma = 0 \ .$$

Since the perturbation is small, one has $|\alpha| << |k^0|$ and $|\beta| << |\omega^0|$. Therefore, using Taylor's expansion in which the derivatives of order higher than 1 are neglected, one has

$$\omega^0\ \epsilon^0(k^0,\ \omega^0) - i\,\frac{\partial[\omega^0\,\epsilon^0(k^0,\omega^0)]}{\partial k^0}\,\alpha - i\,\frac{\partial[\omega^0\,\epsilon^0(k^0,\omega^0)]}{\partial\omega^0}\,\beta + 4\pi i\sigma = 0\ , \tag{36}$$

and considering $\omega^0\ \epsilon^0(k^0,\ \omega^0) = 0$ (see Eq. 24), one obtains

$$\alpha\,\frac{\partial[\omega^0\ \epsilon^0(k^0,\ \omega^0)]}{\partial k^0} + \beta\,\frac{\partial[\omega^0\ \epsilon^0(k^0,\ \omega^0)]}{\partial\omega^0} = 4\pi\sigma\ .$$

To simplify notation we will now replace ϵ^0, k^0, and ω^0 by ϵ, k, and ω respectively. Then

$$\omega\ \epsilon(k,\ \omega) = 0\ , \tag{37}$$

and

$$\alpha\,\frac{\partial[\omega\ \epsilon(k,\ \omega)]}{\partial k} + \beta\,\frac{\partial[\omega\ \epsilon(k,\ \omega)]}{\partial\omega} = 4\pi\sigma\ . \tag{38}$$

Equations (37) and (38) are the "characteristic equations" for the wave plasma system. They provide a relationship between k, ω, α, and β, and their physical content is analogous to that of the dispersion equation in the current plasma theory.

4. PROPAGATION OF AN ELECTRICAL DISTURBANCE IN A PLASMA

Unperturbed and Perturbed Wave Packet

When $\sigma = 0$ an electrical disturbance is often represented by a specified mixture of homogeneous plane waves having a frequency spectrum peaked at an appropriate dominant frequency ω_s (wave packet). The electrical intensity can then be expressed as

$$E^0_{\mathrm{wp}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} d\omega\ E^0_{k,\omega}\ e^{i(kx-\omega t)}\ \delta[k - k(\omega)]\ . \tag{39}$$

The function $k = k(\omega)$ is obtained by solving (37) for ω.

Assuming that $k(\omega)$ slowly varies with ω and neglecting derivatives of $k = k(\omega)$ of order higher than 1, one can express (39) in the form

$$E^0_{\mathrm{wp}} = 0.5[\mathcal{E}^0\ e^{i(kx-\omega t)} + \mathrm{c.c.}]\ , \tag{40}$$

which represents a modulated carrier wave (Jackson, 1962). In the above expression the exponential is the carrier wave and $\mathcal{E}^0$ is a modulating function. One has $\mathcal{E}^0 = \mathcal{E}^0(\psi)$, where $\psi = x - v_g^0 t$ and

$$v_g^0 = \frac{d\omega}{dk} = -\frac{\partial(\omega\epsilon)/\partial k}{\partial(\omega\epsilon)/\partial\omega} \tag{41}$$

is the group velocity. To simplify notation, we have replaced in (40) $\exp\{i[k(\omega_s)x - \omega_s t]\}$ by $\exp[i(kx - \omega t)]$.

Using the expression (40), we will now derive the corresponding expression for a perturbed wave packet ($\sigma \neq 0$). The relationship between the unperturbed electrical intensity E_{wp}^0 and the corresponding perturbed electrical intensity E_{wp} will be expressed symbolically by $E_{\mathrm{wp}} = \hat{O}(E_{\mathrm{wp}}^0)$, where $\hat{O}$ is an appropriate operator.

The procedure to be used consists in applying the operator $\hat{O}$ to each harmonic wave in the integrand of (39). It has been previously assumed that such an operation when applied to a harmonic wave is equivalent to a multiplication by K (Eq. 35). Therefore a wave which was previously homogeneous and was expressed by $\exp[i(kx - \omega t)]$ is now nonhomogeneous and is expressed by $\exp(\alpha x - \beta t)\exp[i(kx - \omega t)]$. The total effect of the perturbation can then be obtained by summing the nonhomogeneous plane waves in the same manner as homogeneous plane waves were summed in E_{wp}^0. We arrive at an expression for the wave packet which has the form

$$E_{\mathrm{wp}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} d\omega\, E_{k\omega}\, e^{i[(k-i\alpha)x-(\omega-i\beta)t]}\,\delta[k - k(\omega)]\ . \tag{42}$$

Now, assuming that α is a slowly varying function of k and that β is a slowly varying function of ω, the perturbed wave packet can be shown to have the form

$$E_{\mathrm{wp}} = 0.5[\mathcal{E} e^{i(kx-\omega t)} + \mathrm{c.c.}]\ , \tag{43}$$

where

$$\mathcal{E} = e^{\alpha x - \beta t}\,\mathcal{E}^0(\psi)\ . \tag{44}$$

Velocity of a Perturbed Wave Packet

Having obtained an expression for the perturbed wave packet, we will now attempt to define the velocity of the disturbance when $\sigma \neq 0$. It is evident that the concept of velocity has a clear physical meaning when $\sigma = 0$, since in such a case there is no distortion and then the velocity is expressed by (41). In order to extend the meaning of propagation velocity to $\sigma \neq 0$, it will be assumed that the disturbance can be localized at its point of maximum intensity, and the velocity of this point will be considered to be the velocity of the disturbance. Since σ acts as a perturbation, the propagation velocity when $\sigma = 0$ will be referred to as the unperturbed velocity, v_g^0, whereas the velocity for $\sigma \neq 0$ will be referred to as the perturbed velocity, v_g.

Differentiating (44) with respect to time, we obtain

$$\frac{\partial \mathcal{E}}{\partial t} = -e^{\alpha x - \beta t}\left(\beta \mathcal{E}^0 + v_g^0 \frac{\partial \mathcal{E}^0}{\partial \psi}\right).$$

Consequently, the disturbance attains its maximum intensity at the points x and at the times t which satisfy the equality

$$\beta \mathcal{E}^0 + v_g^0 \partial \mathcal{E}^0 / \partial \psi = 0 . \tag{45}$$

It should be noted that x and t do not appear explicitly in (45). The independent variable is ψ, and both x and t are contained in ψ by means of the relationship $\psi = x - v_g^0 t$. The root of the equality (45) is ψ_M, which defines the point $M(x_M, t_M)$ at which the disturbance attains its maximum intensity. Thus, x_M and t_M are related to each other by

$$\psi_M = x_M - v_g^0 t_M . \tag{46}$$

Differentiating both sides of (46) with respect to x_M, we obtain $dx_M / dt_M = v_g^0$, and therefore the point M moves with velocity v_g^0. Consequently, the velocity of the disturbance is the same when $\sigma = 0$ and $\sigma \neq 0$. One has

$$v_g = v_g^0 = \frac{d\omega}{dk} = -\frac{\partial(\omega \epsilon)/\partial k}{\partial(\omega \epsilon)/\partial \omega} . \tag{47}$$

Attenuation or Growth of a Wave Packet

Let $\mathcal{E}_M$ be the electrical intensity at M. Consequently,

$$\mathcal{E}_M = e^{\alpha x_M - \beta t_M} \mathcal{E}_M^0 . \tag{48}$$

Since $t_M = (x_M - \psi_M)/v_g$, one can express (48) as

$$\mathcal{E}_M = e^{\beta \psi_M / v_g} e^{\lambda x_M} \mathcal{E}^0(\psi_M) ,$$

where

$$\lambda = \alpha - (\beta / v_g) . \tag{49}$$

The zero point of the ψ scale can be fixed arbitrarily, and therefore, assuming $\psi_M = 0$, one obtains

$$\mathcal{E}_M = e^{\lambda x_M} \mathcal{E}^0(0) . \tag{50}$$

An alternative expression for $\mathcal{E}_M$ can be obtained by substituting $x_M = \psi_M + v_g t_M$ in (48). Then one obtains

$$\mathcal{E}_M = e^{\alpha \psi_M} e^{\delta t_M} \mathcal{E}^0(\psi_M) ,$$

where

$$\delta = \alpha v_g - \beta , \tag{51}$$

and since $\psi = 0$, one has

$$\mathcal{E}_M = e^{\delta t_M} \mathcal{E}^0(0) . \tag{52}$$

Both (50) and (52) describe the same physical event. The variable x_M is the length of the path followed by the point M, and (50) tells how the maximum intensity decays or grows with x_M. On the other hand, t_M is the time at which the disturbance attains its maximum, and consequently (52) indicates how the maximum intensity decays or grows with time.

Calculation of λ

Dividing both sides of (38) by $\partial(\omega\epsilon)/\partial k$, one obtains

$$\alpha + \beta \frac{\partial(\omega\epsilon)/\partial\omega}{\partial(\omega\epsilon)/\partial k} = \frac{4\pi\sigma}{\partial(\omega\epsilon)/\partial k} .$$

Taking into account (47), the above relationship gives

$$\alpha - \frac{\beta}{v_g} = \frac{4\pi\sigma}{\partial(\omega\epsilon)/\partial k} , \tag{53}$$

and combining (49) with (53) gives

$$\lambda = \frac{4\pi\sigma}{\partial(\omega\epsilon)/\partial k}\,. \tag{54}$$

Calculation of δ

Dividing both sides of (38) by $\partial(\omega\epsilon)/\partial\omega$, one obtains

$$\alpha\,\frac{\partial(\omega\epsilon)/\partial k}{\partial(\omega\epsilon)/\partial\omega} + \beta = \frac{4\pi\sigma}{\partial(\omega\epsilon)/\partial\omega}\,.$$

Taking into account (47), the above equality gives

$$-\alpha v_g + \beta = \frac{4\pi\sigma}{\partial(\omega\epsilon)/\partial\omega}\,, \tag{55}$$

and combining (51) with (55), one obtains

$$\delta = -\frac{4\pi\sigma}{\partial(\omega\epsilon)/\partial\omega}\,. \tag{56}$$

Energy Stored by an Electrical Disturbance in a Plasma

It can now be shown that the energy stored by an electrical field in a plasma can be clearly and unambiguously defined even if $\sigma \neq 0$. Since $\epsilon = \epsilon^0$, the relationship between the energy density and the field intensity is known to be the same for $\sigma = 0$ and $\sigma \neq 0$. Therefore the procedure for calculating energy is straightforward. We are not confronted with the situation that is met with in the current theory, in which the same approach when applied to $\sigma = 0$ gives valid results but the results are not valid when $\sigma \neq 0$ (Ginzburg 1964*a*, *b*).

One should recall, however, that the disturbance has been differently defined when $\sigma = 0$ and when $\sigma \neq 0$. In the former case the wave packet is expressed by (40), whereas in the latter case it has the form (43).

Our method for calculating energy when $\sigma \neq 0$ is similar to the one established by Brillouin for $\sigma = 0$. Brillouin used (40) together with the relationship

$$\frac{\partial U^0}{\partial t} = \frac{1}{4\pi} E^0_{\mathrm{wp}}\,\frac{\partial D^0_{\mathrm{wp}}}{\partial t}\,,$$

where $D^0_{\rm wp}$ is the electrical displacement produced by (40), and he derived the well-known expression

$$\frac{\partial \overline{U}^0}{\partial t} = \frac{1}{16\pi}\,\frac{\partial(\omega\epsilon^0)}{\partial\omega}\,\frac{\partial}{\partial t}(\mathcal{E}^0\mathcal{E}^{0*}) ,$$

from which he obtained

$$\overline{U}^0 = \overline{U}^0(\psi) = \frac{1}{16\pi}\,\frac{\partial(\omega\epsilon^0)}{\partial\omega}\,\mathcal{E}^0\mathcal{E}^{0*} .$$

In order to calculate energy when $\sigma \neq 0$, we use (43) and the relationship

$$\frac{\partial U}{\partial t} = \frac{1}{4\pi} E_{\rm wp}\,\frac{\partial D_{\rm wp}}{\partial t} ,$$

where $D_{\rm wp}$ is the electrical displacement produced by (43). One obtains

$$\frac{\partial \overline{U}}{\partial t} = \frac{1}{16\pi}\,\frac{\partial(\omega\epsilon)}{\partial\omega}\,\frac{\partial}{\partial t}(\mathcal{E}\mathcal{E}^*) .$$

Therefore, since $\epsilon = \epsilon^0$, one has

$$\overline{U} = \frac{1}{16\pi}\cdot\frac{\partial(\omega\epsilon)}{\partial\omega}\,\mathcal{E}\mathcal{E}^* = e^{2(\alpha x-\beta t)}\,\overline{U}^0 . \tag{57}$$

Let U_M represent the energy density localized at $M(x_M, t_M)$. Consequently,

$$\overline{U}_M = K_M\overline{U}^0 = e^{2(\alpha x_M-\beta t_M)}\,\overline{U}^0 ,$$

where K_M represents the attenuation or growth. Then, expressing t_M in terms of x_M, one obtains

$$\overline{U}_M = K_M\overline{U}^0(0) = e^{2\lambda x_M}\,\overline{U}^0(0) . \tag{58}$$

Alternatively, expressing x_M in terms of t_M, one has

$$\overline{U}_M = K_M\,\overline{U}^0(0) = e^{2\delta t_M}\,\overline{U}^0(0) . \tag{59}$$

Expressions (58) and (59) describe what actually happens to the energy of the disturbance both in space and in time. The variable x_M gives the successive positions of M, and therefore (58) shows how the energy decays or grows along the path of the disturbance. Similarly, t_M gives the times at which the point M reaches successive positions along its path, and, consequently, (59) shows how the disturbance decays or grows with time.

Let

$$d\bar{U}_M/dt_M = -(K_{\mathrm{coll}} + K_{\mathrm{res}})\bar{U}_M\ , \tag{60}$$

where K_{coll} and K_{res} represent, respectively, the contributions due to the collisional and the resonance effects. Consequently, using (56) and (59), one obtains

$$K_{\mathrm{coll}} = \frac{8\pi\sigma_{\mathrm{coll}}}{\partial(\omega\epsilon)/\partial\omega} \tag{61}$$

and

$$K_{\mathrm{res}} = \frac{8\pi\sigma_{\mathrm{res}}}{\partial(\omega\epsilon)/\partial\omega} \tag{62}$$

respectively. Then combining (60), (61), (62), and (57), one obtains

$$\frac{d\bar{U}_M}{dt_M} = 0.5(\sigma_{\mathrm{res}} + \sigma_{\mathrm{coll}})\mathcal{E}\mathcal{E}^*\ . \tag{63}$$

5. CONCLUDING REMARKS

By modifying the kinetic equations for a plasma, we provided an expression for energy propagation which is free of inconsistencies previously discussed by Ginzburg (1964*a*, *b*). Because of exponential decay due to attenuation, the concept of propagation velocity of an electrical disturbance can now be shown to have a physical meaning even if there is absorption. In the proposed formulation the energetic behavior of the disturbance has been described in terms of the resonance conductivity σ_{res} and collisional conductivity σ_{coll}. In order to complete this discussion it will be useful briefly to outline the procedure for determining σ_{res} and σ_{coll}.

Resonance Conductivity (σ_{res})

Substituting (25) and (27) in (29), one has

$$f_{r1} = -i\frac{ne}{m}E\frac{\partial F_{r0}/\partial v}{v-(\omega/k)}\,. \tag{64}$$

Then combining (27) and (64) with (30), calculating the integral in the neighborhood of the singularity in a manner prescribed by Landau (1946), and assuming that the width Δv of the resonance range tends to zero, one obtains

$$j_{res} = -\frac{\omega_0^2\omega}{4k^2}E\left(\frac{\partial F_{r0}}{\partial v}\right)_{v=\omega/k}\,.$$

Since $j_{res} = \sigma_{res}E$, one has

$$\sigma_{res} = -\frac{\omega_0^2\omega}{4k^2}\left(\frac{\partial F_{r0}}{\partial v}\right)_{v=\omega/k} = -\frac{\omega_0^2\omega}{4k^2}\left(\frac{\partial F_0}{\partial v}\right)_{v=\omega/k}\,. \tag{65}$$

The expression (65) defines the resonance conductivity in terms of ω_0 and $(\partial F_0/\partial v)$ for $v = \omega/k$.

Collisional Conductivity (σ_{coll})

An expression for the collisional conductivity will be obtained from an analysis of the mechanism which accounts for the conversion of the polarization energy due to the oscillatory electron motion into the energy of random motion which manifests itself as heat. To simplify the problem it will be assumed that electrons make many oscillations between collisions and therefore that $\omega >> \nu$. Each collision produces a change in phase of the electron oscillation with respect to the oscillation of the field, and, therefore, at each collision the relative amounts of the oscillatory energy and the energy of the random motion are altered. A succession of collisions results in an increase of the energy due to the random motion of the electron. Thus the nonelectric (random) energy increases at the expense of the ordered (polarization) energy of the oscillating field.

Exact calculations would require a detailed analysis of collisions at all angles at various times during the period of oscillation. For the purpose of this analysis we consider it sufficient to make an estimate based on the calculations of Jancel and Kahan (1963). According to Jancel and Kahan the average energy T which is transferred at each collision into random motion is known to be $T = (e^2/m\omega^2)\,\mathcal{E}\mathcal{E}^*$. Since there are ν collisions per second and n electrons per cubic centimeter, the average amount of energy which is withdrawn from the field per second and per cubic centimeter is

$$-\frac{d\bar{U}_M}{dt_M} = n\nu T = \frac{n\nu e^2}{m\omega^2}\,\mathcal{E}\mathcal{E}^* \,.$$

Therefore, considering (63) and taking $\omega_0^2 = 4\pi ne^2/m$, one obtains

$$\sigma_{\text{coll}} = 0.5\,\omega_0^2\nu/\pi\omega^2 \,. \tag{66}$$

Occurrence of an Instability

In many physical problems one is interested in a condition described as "instability," which occurs when the energy of a disturbance (electric energy) grows at the expense of the nonelectric energy stored in the translational electron motion. An instability occurs when $\sigma = \sigma_{\text{res}} + \sigma_{\text{coll}}$ is negative. Since σ_{coll} is always positive, it is then necessary for σ_{res} to be negative and to exceed σ_{coll} in absolute value. In such a case one has

$$\left(\frac{\partial F_0}{\partial v}\right)_{v=\omega/k} > 0 \tag{67}$$

and

$$\left(\frac{\partial F_0}{\partial v}\right)_{v=\omega/k} > \frac{2k^2\nu}{\pi\omega^3} \,. \tag{68}$$

We will now consider a limiting case when $\sigma \to 0$. In such a case a disturbance which was unstable for $\sigma < 0$ becomes nonevanescent. The condition of nonevanescence when $\sigma = 0$ is expressed by the equality

$$\epsilon\,(k,\,\omega) = 0 \,, \tag{69}$$

which must be satisfied for real k and real ω.

Relationships (67), (68), and (69) represent, therefore, the necessary conditions for an instability.

REFERENCES

Bhatnagar, P. L., E. P. Gross, and M. Krook, 1954, *Phys. Rev.* **94**, 511.

Brillouin, L., 1932, *Congres International d'Electricite* (Paris: Gauthier Villars), vol. 2, pp. 739–88; 1960, *Wave Propagation and Group Velocity* (New York: Academic Press).

Delcroix, J. L., 1965, *Plasma Physics* (New York: John Wiley and Sons), pp. 164–73.

Gertzenshtein, M. E., 1952, *Zh. Eksperim. i Teor. Fiz.* **22**, 303.

Ginzburg, V. L., 1964*a*, *Propagation of Electromagnetic Waves in Plasmas* (London: Pergamon Press), pp. 244–55; 1964*b*, *ibid.*, pp. 477–95; 1967, *Raspostranienie Elektromagnitnykh Voln v Plasme* (Moscow: Izdatelstvo Nauka), p. 364.

Jackson, J. D., 1962, *Classical Electrodynamics* (New York: John Wiley and Sons), p. 210.

Jancel, R., and Th. Kahan, 1963, *Electrodynamics of Plasmas* (New York: John Wiley and Sons), pp. 382–85.

Landau, L., 1946, *J. Phys. (USSR)* **10**, 25.

Neufeld, Jacob, 1961, *Phys. Rev.* **123**, 1; 1966, *Phys. Rev.* **152**, 708; 1969, *Phys. Letters* **29A**, 68; see also Jacob Neufeld, "Revised Formulation of the Macroscopic Maxwell Theory," Parts 1 and 2, to be published in *Il Nuovo Cimento*.

Neufeld, Jacob, and R. H. Ritchie, 1955, *Phys. Rev.* **99**, 1125.

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