NYO-2171-315

RECEIVED BY DTIE JUL 30 1970

Current Commutator Derivation of Mass Difference Relations

by

Ashok Goyal*, Ling Fong Li* and Gino Segre*†
Department of Physics, University of Pennsylvania
Philadelphia, Pennsylvania, 19104

ABSTRACT

We investigate the consequences of the vanishing of certain double commutators to examine the consistency of SU(3) x SU(3) breaking models. We also calculate the contribution to $\Delta I = 2$ mass differences of isospin violating terms other than the electromagnetic interactions, in the Hamiltonian.

†A. P. Sloan Foundation Fellow.

*Supported in part by the U.S. Atomic Energy Commission.

I. Introduction

The principal use of current commutators and the accompanying techniques developed to employ them has been in treating problems involving soft pions.[1] One may also examine the problem of corrections to SU(3) formulae using current commutators as was first done by Furlan et al.[2] With the help of dispersion relations[3], these problems can be treated quite elegantly; we shall not repeat the derivations of Ref. 3, which however are essential to the present paper, but rather refer the interested reader back to it.

Let us begin with a simple example not involving SU(3) The generators of SU(2) are labelled $Q_{I_\pm}$ and Q_{I_3} and the total Hamiltonian by H. We may then consider the following double commutator

$$\left[Q_{I_+}, \left[Q_{I_+}, H\right]\right] = 0 \tag{1}$$

the above relation holding if H contains no I = 2 terms. One may then examine matrix elements of (1) between states belonging to the same isomultiplet, but differing by two charge units such as Σ^+ and Σ^- or π^+ and π^-; to be specific, keeping only the π^o intermediate state we find for the latter case

$$\langle \pi^+ | [Q_{I+}, [Q_{I+}, H]] | \pi^- \rangle =$$

$$\langle \pi^+ | Q_{I+} | \pi^0 \rangle \langle \pi^0 | [Q_{I+}, H] | \pi^- \rangle$$
$$- \langle \pi^+ | [Q_{I+}, H] | \pi^0 \rangle \langle \pi^0 | Q_{I+} | \pi^- \rangle =$$

$$2 (E_{\pi^+} - E_{\pi^0}) \langle \pi^+ | Q_{I+} | \pi^0 \rangle \langle \pi^0 | Q_{I+} | \pi^- \rangle \tag{2}$$

This would imply that $M_{\pi^+} - M_{\pi^0}$ is zero, but of course we have neglected the other contribution of order α, namely the intermediate states with one photon: to take them properly into aocount, we use the assumption of minimal electromagnetic coupling, which allows us to write

$$[Q_{I+}(t), H] = \int \partial_\mu J_{\mu,+}(\vec{x}, t)\, d\vec{x}$$
$$= ie \int J_{\mu,+}(\vec{x}, t)\, A_\mu(\vec{x}, t)\, d\vec{x} \tag{3}$$

where $J_{\mu,+}(x)$ is the isospin current and A_μ the photon field. Using the techniques of Ref. 3 we may also evaluate the matrix element of Q_{I_+} between states involving one and zero photons as

$$\langle a | Q_{I+} | a' \gamma \rangle = \langle a | \int \partial_\mu J_{\mu,+}(x)\, \theta(t - x_0)\, d\vec{x}\, | a' \gamma \rangle \tag{4}$$
$$= \langle a | ie \int J_{\mu,+}(x) A_\mu(x)\, \theta(t - x_0)\, d\vec{x}\, | a' \gamma \rangle$$

and finally combining (2), (3) and (4), we find, to order α

$$E_{\pi^+} - E_{\pi^0} = \Delta E = -e^2 \int d^4q \; e^{iq\cdot x}$$
$$\times \langle 0 | T\{A_\mu(x) A_\nu(0)\} | 0 \rangle$$
$$\times \langle \pi^+ | T\{J_{\mu,+}(x) J_{\nu,+}(0)\} | 0 \rangle$$

$$= -\frac{ie^2}{(2\pi)^4} \int d^4q \frac{e^{iq\cdot x}}{q^2+i\varepsilon} \langle \pi^+ | T\{J_{\mu,+}(x) J_{\mu,+}(0)\} | \pi^- \rangle \, d^4x \tag{5}$$

which is completely equivalent to the standard beginning step of the calculation of the $\pi^+ - \pi^0$ mass difference[4] and so leads to nothing new.

The main purpose of this paper is to use these techniques to explore relations between splitting of different members of multiplets belonging to the same SU(2) or SU(3) representations. Consider for instance the commutator (Q_{K^+} is an SU(3) generator)

$$[Q_{K+}, [Q_{I+}, H]] = 0 \tag{6}$$

whose matrix elements allow one to relate, e.g. $M^2_{\pi^+} - M^2_{\pi^0}$ to $M^2_{K^+} - M^2_{K^0}$ or $M_p - M_n$ to $M_{\Sigma^-} - M_{\Sigma^0}$. We shall show how a difference of two matrix elements of (6) leads to the Coleman-Glashow relation[5]

$$M_{\Xi^-} - M_{\Xi^0} = M_{\Sigma^-} - M_{\Sigma^+} + M_p - M_N \tag{7}$$

Our derivation of this formula is not completely new as Fels[6] has derived the same relation, using the same techniques, but taking instead the difference of matrix elements of

$$[Q_{K^0}, [Q_{K^0}, H]] = 0 \tag{8}$$

We believe our derivation is slightly more transparent in that the electromagnetic splitting is emphasized from the beginning. We will then go on to consider SU(3) mass formulae as obtained from an evaluation of e.g. equation (8) and show one may test SU(3) x SU(3) breaking models using the results.

Finally, we would like to show how relations, which are the consequence of (6), could be used to study if mass differences of particles within a given isomultiplet are entirely electromagnetic in origin, as this assumption has recently come into question.[7,8,9]

II. Electromagnetic Mass Difference Relations

To examine the consequences of (6) it is necessary to know the matrix elements of Q_{K^+} between states which do not belong in the SU(3) limit, to the same irreducible representation. To determine these matrix elements, we calculate

$$\partial_\mu J_{\mu,K_+} = T_{K_+} + ie\, J_{\mu,K^+} A_\mu \tag{9}$$

where J_{μ,K_+} are the SU(3) currents, whose fourth components, integrated over all space, are the generators of SU(3) and where T_{K+} is a measure of the dynamical breaking of the symmetry. For instance, taking the model of Refs. (10), (11), where the Hamiltonian density is

$$H = H_0 - (u_0 + c u_8) + e J_\mu^{e.m.} A_\mu \tag{10}$$

where u_0, u_8 are scalar densities belonging to the $(\bar{3},3)$ + $(3,\bar{3})$ representation of SU(3) x SU(3) and H_0 is invariant, we find that $T_{K+} = - c u_{K+}$.

Utilizing (9) and (3), we find that the matrix element of (6) to order α, between states a and b, receives from intermediate states containing one photon, a contribution

$$\langle b| \left[Q_{K_+}, \left[Q_{I_+}, H\right]\right] |a\rangle \Big|_{\text{int states with one photon}}$$

$$= \frac{ie^2}{(2\pi)^4} \int \frac{d^4q\, e^{iqx}}{q^2+i\epsilon} \langle b| T\{ J_{\mu,K_+}(x)\, J_{\mu,+}(0) \} |a\rangle d^4x \, . \tag{11}$$

exactly as we derived (5). Now let $a = \Sigma^-$, $b = p$ and $a = \Xi^-$, $b = \Sigma^+$ and then subtract the right-hand sides of (11) obtained with these choices. It is easy to see that, in the SU(3) limit, one obtains a vanishing result. This is most readily deduced making use of U-spin since then (p,Σ^+) form one doublet and (Σ^-, Ξ^-) another doublet of the SU(3) supermultiplet. The equality of the two amplitudes is then the U-spin analogue of the equality of

$$K^{*0} + \Xi^0 \rightarrow K^{*-} + P$$

$$K^{*0} + \Xi^- \rightarrow K^{*-} + N \tag{12}$$

which holds in the limit of I spin conservation.[12]

We thus have

$$\langle P | [Q_{K+}, [Q_{I+}, H]] | \Sigma^- \rangle - \langle \Sigma^+ | [Q_{K+}, [Q_{I+}, H]] | \Xi^- \rangle = 0 \tag{13}$$

and that, in the SU(3) limit, intermediate states containing one photon do not contribute to (13) as these states' matrix elements cancel from the difference by use of (11). In the SU(3) limit, the only states contributing to lowest order in the fine structure constant, are the one particle states corresponding to the neutral members of the baryon octet. This then leads to

$$2(M_N - M_P) - 2(M_{\Sigma^-} - M_{\Sigma^0}) - \sqrt{3}\langle \Lambda^0 | [Q_{I+}, H] | \Sigma^- \rangle - 2(M_{\Sigma^0} - M_{\Sigma^+}) + 2(M_{\Xi^-} - M_{\Xi^0}) + \sqrt{3}\langle \Sigma^+ | [Q_{I+}, H] | \Lambda^0 \rangle = 0 \tag{14}$$

or

$$(M_N - M_P) + (M_{\Sigma^+} - M_{\Sigma^-}) + (M_{\Xi^-} - M_{\Xi^0})$$
$$= \frac{\sqrt{3}}{2}\left\{ \langle \Lambda^0 | \dot{Q}_{I_+} | \Sigma^- \rangle - \langle \Sigma^+ | \dot{Q}_{I_+} | \Lambda^0 \rangle \right\} \tag{15}$$

but $\dot{Q}_{I_+}$ is an operator of order α and only its component with isospin equal to one has a non-vanishing matrix element between Λ^0 and Σ^- and this in turn is equal to that between Σ^+ and Λ^0 states. The ensuing relationship between electromagnetic mass differences which follows from the vanishing of the right-hand side of (15), is the so-called Coleman-Glashow[5] formula. Corrections to this formula, arising from SU(3) breaking are, unfortunately, difficult to estimate reliably.

III. Pseudoscalar Meson Mass Formula

From the assumption that the SU(3) breaking part of the Hamiltonian has no twenty-seven components, it follows that

$$[Q_{K_+}, [Q_{K_+}, H]] = 0 \tag{16}$$

Taking the matrix element of (16) between K^+ and K^- states, we find

$$\langle K^+(p_2) | [Q_{K_+}, [Q_{K_+}, H]] | K^-(p_1) \rangle$$
$$= (2\pi)^3 \delta(\vec{p}_1 - \vec{p}_2) E_K (3E_\eta + E_\pi - 4E_K) = 0 \tag{17}$$

where we keep only the one pseudoscalar octet meson intermediate states. This, in turn implies the Gell-Mann-Okubo mass formula

$$3m_\eta^2 + m_\pi^2 - 4m_K^2 = 0 \tag{18}$$

On the other hand we can use the techniques of Fubini, Furlan and Rossetti[3] to write the charges and their time derivatives in terms of four-divergences of currents,

$$\begin{aligned} Q_\alpha(t) &= -\int \partial_\mu J_{\mu,\alpha}(x)\,\theta(t-x_0)\,d^4x \\ &= -c\, f_{\alpha 8 \beta} \int u_\beta(x)\,\theta(t-x_0)\,d^4x \end{aligned} \tag{19}$$

$$\begin{aligned} [Q_\beta, H] &= i\frac{d}{dt}Q_\beta(t) = i\int \partial_\mu J_{\mu,\alpha}(x)\,d\vec{x} \\ &= ic\, f_{\beta 8 \gamma}\int u_\gamma(x)\,d\vec{x} \end{aligned} \tag{20}$$

where we neglect the electromagnetic interaction in calculating the divergence. Then the left-hand side of Eq. (17) can be written as

$$\begin{aligned} &\langle K^+(P_2)|\,[Q_{K_+},[Q_{K_+};H]]\,|K^-(P_1)\rangle \\ &= -ic^2\cdot\tfrac{3}{2}(2\pi)^3\delta(\vec{P}_1-\vec{P}_2)\langle K^+(P_2)|\int d^4y\,\theta(-y_0)\,[u_{K_+}(y),u_{K_+}(0)]\,|K^+(P_1)\rangle \end{aligned} \tag{21}$$

The simplest contributions to the left-hand side of Eq. (21) are those of one-particle intermediate states, in this case π^0 and η. Diagrammatically, these contributions correspond to the following tadpole diagrams:

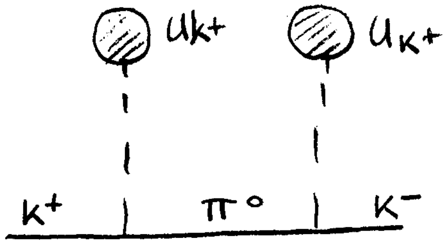

Fig. 1a)

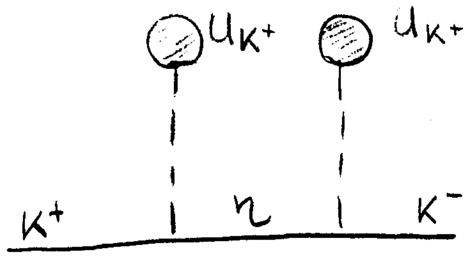

Fig. 1b)

To evaluate these kind of terms, one needs to know the matrix elements of the form $\langle P_i | u_j | P_k \rangle$, where P_i and P_k are the pseudo-scalar meson states, corresponding to the vertex of the tadpole in the diagrams shown in Fig. 1. As pointed out by Gell-Mann, Oakes and Renner,[10,11] these matrix elements, to a good approximation, can be evaluated by soft-meson technique and PCAC

$$\langle P_i | u_j | P_k \rangle = d_{ijk}\,\beta \tag{22}$$

or, using PCAC $\langle 0 | V_i | P_i \rangle = -\beta/2f$ (23)

where $\beta = \Delta m^2/c$ and f is the meson decay constant which appears in the PCAC relation:

$$\partial_\mu J^5_{\mu,i} = \frac{m_i^2}{2f_i} P_i \qquad i = 1 \ldots\ldots 8 \tag{24}$$

Equivalently, one can use PCAC to take the soft-meson limit of the right-hand side of Eq. (21) to get

$$\langle K^+(p_2)|\left[Q_{K^+},\left[Q_{K^+},H\right]\right]|K^-(p_1)\rangle = i\,c^2\,\frac{3}{2}(2\pi)^3(2f)^2$$
$$\times\,\delta(\vec{p}_1-\vec{p}_2)\int d^4y\,\theta(-y_0)\langle 0|\left\{\frac{1}{3}\left[u_4(y)\,u_4(0)+u_5(y)u_5(0)\right]\right.$$
$$\left.+\frac{1}{2}\left[V_3(y)V_3(0)+\frac{1}{3}V_8(y)\,V_8(0)\right]\right\}|0\rangle \tag{25}$$

Then we saturate the right-hand side of (25) with one-particle intermediate states. The first two terms of (25), which contain u_i's will not contribute, because the matrix elements of u_i between vacuum and one particle state is zero, which is another way of saying that there are no low-lying scalar meson states. For the terms containing v_i's only π^o and η contribute, as we have expected. Using the values of the matrix element given in Eq. (22), we obtain from Eq. (24)

$$\langle K^+(p_2)|\left[Q_{K^+},\left[Q_{K^+},H\right]\right]|K^-(p_1)\rangle = (2\pi)^3\,\delta(\vec{p}_2-\vec{p}_1)\frac{(\Delta m^2)^2}{\overline{m}^2} \tag{26}$$

Comparing Eq. (25) with the Gell-Mann Okubo mass formula given in Eq. (18), we see that

$$3m_\eta^2+m_\pi^2-4m_K^2 = -\frac{(\Delta m^2)^2}{2\overline{m}^2} \tag{27}$$

Experimentally, the left-hand side of (26) is

$$\left(3m_\eta^2+m_\pi^2-4m_K^2\right)_{exp} \sim -0.08\ (\mathrm{Bev})^2 \tag{28}$$

If we use the masses of K and η to compute the mass difference Δm^2 and average mass $\bar{m}^2$, we get

$$\frac{(\Delta M^2)^2}{2\bar{M}^2} \simeq 0.16 \, (\mathrm{Bev})^2 \tag{29}$$

which is about twice as large as the experimental value and differs in sign.

It should be understood that the result, eq. (27), does not correspond to an estimate of corrections to the Gell-Mann Okubo mass formula, but rather to model dependent evaluation of the contribution of π^o and η intermediate states to the double commutator in Eq. (17). If this double commutator does in fact vanish, higher intermediate states must cancel the contribution of the π^o and η states. What we have performed is a very rough test of a model of SU(3) x SU(3) breaking in the pseudoscalar mesons, and the result is probably more amusing than enlightening, in that all it shows is that there is no gross discrepancy between an exact and an approximate evaluation of the π^o and η state contributions to the saturation of the double commutator in Eq. (16).

IV. Non-Electromagnetic Isospin Violating Terms

In this section we will try to estimate the contribution of an isospin violating term in the Hamiltonian to $\Delta I = 2$ mass differences. The existence of such a term in the Hamiltonian, proportional to u_3, has been speculated on by various authors.[7,8,9] It is clear that this type of isospin violating term contributes to the mass differences between different members of the same isospin multiplet in addition to the ordinary electromagnetic interactions of hadrons. The reason for looking at $\Delta I = 2$ mass differences is that the usual electromagnetic calculations for π^+-π^o and $\Sigma^+ + \Sigma^- - 2\Sigma^o$ mass differences have been reliably calculated and agree fairly well with the observed mass differences. The contribution coming from the u_3 term would thus furnish at least some rough test of the existence of a non-electromagnetic term.

In the presence of electromagnetic interactions, the Hamiltonian density is assumed to be of the form, e.g.[9]

$$H = H_o - u_o + \sqrt{2}\left(1 - \frac{3}{2}\sin^2\theta\right) u_8 + \frac{\sqrt{6}}{2}\sin^2\theta\, u_3 + e\, J_\mu A_\mu \tag{30}$$

where θ is the Cabibbo[13] angle. Using the above Hamiltonian we would like to estimate the u_3 contribution to π^+-π^o and $\Sigma^+ + \Sigma^- - 2\Sigma^o$ mass differences.

(i) $\pi^+ - \pi^o$ mass difference:

Taking the matrix elements of (1) between π^+, π^- states and proceeding as in section III, we write

$$i(2\pi)^3 \delta(\vec{P}_{\pi^+} - \vec{P}_{\pi^0}) \sum_n d^4y\, \theta(-y_0)$$
$$\times \Big\{ \langle\pi^+| \partial_\mu J_{\mu,+}(y) |n\rangle\langle n| \partial_\nu J_{\nu,+}(0) |\pi^-\rangle$$
$$- \langle\pi^+| \partial_\nu J_{\nu,+}(0) |n\rangle\langle n| \partial_\mu J_{\mu,+}(0) |\pi^-\rangle \Big\} = 0 \tag{31}$$

In (31) we separate the contribution due to π^0 intermediate state which gives

$$(2\pi)^3 \delta(\vec{P}_{\pi^+} - \vec{P}_{\pi^-})\, 2\left(m_{\pi^0}^2 - m_{\pi^+}^2\right) + O(\alpha) + O(c'^2) \tag{32}$$

where c' is the coefficient of u_3 in (30) and we work in the rest frame. The higher order terms in (32) arise from the corrections to the vertex which will be ignored in what follows.

The usual electromagnetic contribution of order α to the mass difference comes from the multiparticle intermediate states which contain a single photon as is shown in section I (Eq. (3) through (5)).

Assuming that the electromagnetic contribution has been calculated we proceed to estimate the contribution of terms of order c'^2 from the one particle intermediate state. This term is given by

$$-i\,c'^2 (2\pi)^3\, \delta(\vec{P}_{\pi^+} - \vec{P}_{\pi^-}) \sum_{\substack{\text{single}\\ \text{particle}}} d^4y\; \theta(-y_0)$$
$$\left[\langle\pi^+|\, u_+(y)\,|n\rangle\langle n|\, u_+(0)\,|\pi^-\rangle - \langle\pi^+|\, u_+(0)\,|n\rangle\langle n|\, u_+(y)\,|\pi^-\rangle \right] \qquad (33)$$

which is obtained from (31) by replacing $\partial_\mu J_{\mu,+}$ by

$$\partial_\mu J_{\mu,+} = -ie\, J_{\mu,+} A_\mu + i c' u_+ \qquad (34)$$

which follows from (30) and then keeping only hadronic one particle intermediate states to estimate the c'^2 contribution.

In the language of Feynman diagrams, Eq. (33) corresponds to the double tadpole irreducible diagram shown in fig. 2(a) where in the intermediate state we have only an η state

(a) (b)

(Fig. 2)

The use of the double commutator (1) to calculate the mass differences is for convenience in taking care of all the numerical factors correctly. What we are actually doing is no more than to calculate the π^+ and π^0 self energies to second order in the isospin violating term $c'u_3$. Diagrammatically, this corresponds to the contribution from the double tadpole diagram shown in Fig. 2(b). These diagrams can then be evaluated by considering the external pions to be soft.

Alternatively, using soft meson techniques and PCAC, Eq. (33) can be written as

$$i c'^2 (2\pi)^3 \delta(\vec{P}_{\pi^+} - \vec{P}_{\pi^+}) \left(2 f_\pi\right)^2 \sum_{\substack{\text{single} \\ \text{particle}}} \int d^4y \; \theta(-y_0)$$
$$\Big\{ -\frac{2}{3} \left[\langle 0| V_8(y) |m\rangle \langle m| V_8(0) |0\rangle - \langle 0| V_8(0) |m\rangle \langle m| V_8(y) |0\rangle \right]$$
$$+ \frac{2}{3} \left[\langle 0| u_-^{+}(y) |m\rangle \langle m| u_+(0) |0\rangle - \langle 0| u_+(0) |m\rangle \langle m| u_-(y) |0\rangle \right] \tag{35}$$

which is evaluated in exactly the same manner as Eq. (25) to give, as the contribution to the double commutator of one particle intermediate states

$$-\frac{1}{3}\left(\frac{\sqrt{6}}{2}\sin^2\theta\right)^2 \frac{\beta^2}{m_\eta^2}$$

Thus the contribution to $\pi^+ - \pi^0$ mass difference in this approximation is given by

$$m_{\pi^+} - m_{\pi^0}\Big|_{u_3 \text{ contrib.}} \simeq \frac{1}{3}\left(\frac{\sqrt{6}}{2}\sin^2\theta\right)^2 \frac{\beta^2}{m_\eta^2}\frac{1}{2m_\pi} = 0.78 \text{ MeV} \tag{36}$$

which is to be compared with[14]

$$(m_{\pi^+} - m_{\pi^0})_{e.m.} = 5.0 \text{ MeV}$$
$$(m_{\pi^+} - m_{\pi^0})_{expt.} = 4.6 \text{ MeV} \tag{37}$$

(ii) $\Sigma^+ + \Sigma^- - 2\Sigma^0$ mass difference:

In this case we need the matrix elements of the scalar density u_i between baryon states. The most general form of the matrix element, in the limit of SU(3) symmetry, is given by

$$-\langle B_i | u_k | B_j \rangle = \bar{u}_i [i F f_{ikj} - D d_{ikj}] u_j \tag{38}$$

where f_{ikj} and d_{ikj} are the usual SU(3) coupling coefficients. F and D are obtained by ascribing the mass splittings between baryons belonging to the same SU(3) multiplet but differing in hypercharge to the symmetry breaking term $-(u_o + cu_8)$ in the Hamiltonian.[10] When this is done we get for F and D

$$F = 174 \text{ MeV}$$
$$D = 60 \text{ MeV} \tag{39}$$

To calculate the contribution to the mass difference, we proceed as in the pion case by taking the matrix elements of (1) between Σ^+ and Σ^- states and write

$$i(2\pi)^3 \delta(\vec{P}_{\Sigma^+} - \vec{P}_{\Sigma^-}) \sum_n \int d^4y\, \theta(-y_0)$$
$$\Big\{ \langle \Sigma^+ | \partial_\mu J^{-}_{\mu,+}(y) | n \rangle \langle n | \partial_\nu J^{+}_{\nu,+}(0) | \Sigma^- \rangle$$
$$- \langle \Sigma^+ | \partial_\nu J^{+}_{\nu,+}(0) | n \rangle \langle n | \partial_\mu J^{-}_{\mu,+}(y) | \Sigma^- \rangle \Big\} \qquad (40)$$

Separating the pole contribution coming from the Σ^0 intermediate state and summing over polarizations, we get, to order α

$$(2\pi)^3 \delta(\vec{P}_{\Sigma^+} - \vec{P}_{\Sigma^-}) \; \frac{M_{\Sigma^+} + M_{\Sigma^-} - 2M_{\Sigma^0}}{2M_{\Sigma^0}}$$
$$\times \; \bar{u}_{\Sigma^+} \left[\gamma_0 (M_{\Sigma^0} + M_{\Sigma^-}) + (M_{\Sigma^0} - M_{\Sigma^-}) \right] u_{\Sigma^-} \qquad (41)$$

As before the usual electromagnetic contribution of order α to the mass difference in (41) comes from the intermediate states containing a single photon. In what follows we will consider this electromagnetic contribution to be calculated separately. When this is done and all terms which are of order higher than α neglected, we have in (41) the contribution to the mass difference from $c'u_3$ terms alone. We now proceed to estimate the contribution, of order c'^2, from the single particle intermediate states. This is given, as in the pion case, by the following expression

$$(2\pi)^3\delta(\vec{P}_{\Sigma^+}-\vec{P}_{\Sigma^-})\Big\{\frac{M_{\Sigma^+}+M_{\Sigma^-}-2M_{\Sigma^0}}{2M_{\Sigma^0}}\,\bar{u}_{\Sigma^+}\Big[\gamma_0(M_{\Sigma^+}+M_{\Sigma^-}) + (M_{\Sigma^0}-M_{\Sigma^-})\Big]u_{\Sigma^-}$$
$$+\, i\,(ic')^2\int d^4y\,\theta(-y_0)\sum_{\text{polarizations}}\Big[\langle\Sigma^+|u_+^\dagger(y)|\Lambda^0\rangle\langle\Lambda^0|u_+^\dagger(0)|\Sigma^-\rangle - \langle\Sigma^+|u_+^\dagger(0)|\Lambda^0\rangle\langle\Lambda^0|u_+(y)|\Sigma^-\rangle\Big]\Big\}=0 \qquad (42)$$

Using (38) to write the matrix elements in the second term, we get

$$M_{\Sigma^+}+M_{\Sigma^-}-2M_{\Sigma^0}\Big|_{\substack{u_3\\ \text{contrib.}}} = \frac{2}{3}c'^2 D^2\frac{1}{M_\Sigma - M_\Lambda} = 0.147\,\text{MeV} \qquad (43)$$

which is to be compared with[15]

$$M_{\Sigma^+}+M_{\Sigma^-}-2M_{\Sigma^0}\Big|_{\text{e.m.}} = 1.54\ \text{MeV}$$
$$M_{\Sigma^+}+M_{\Sigma^-}-2M_{\Sigma^0}\Big|_{\text{exptl.}} = 1.79\ \text{MeV} \qquad (44)$$

The uncertainty in the calculation of the electromagnetic contribution to $\Delta I = 2$ mass differences is sufficiently large however that we do not believe we have provided here a significant test of the presence of a term proportional to u_3 in the Hamiltonian. Admittedly, the contribution to the $\pi^+ - \pi^0$ mass difference is fairly sizeable, circa 20% of the electromagnetic calculated contribution, but one probably should allow for a 20% error in the calculation of the latter.

In addition, we should point out that it may be unrealistic to allow the u_3 term to have such a large coefficient, as this leads to e.g. a large $K^+ - K^o$ mass difference. The electromagnetic contribution to the latter is however hard to calculate reliably so it does not rule out the model of Eq. (30). A coefficient for u_3 appreciably larger than that of Eq. (30) is ruled out, we believe, by our $\pi^+ - \pi^o$ mass difference calculation.

References

1. See, for example, S. Adler and R. Dashen: Current Algebra.
2. G. Furlan, F. Lannoy, C. Rossetti and G. Segré, Nuovo Cimento 38, 1747 (1965); Nuovo Cimento 40, 497 (1965).
3. S. Fubini, G. Furlan and C. Rossetti, Nuovo Cimento 40, 1171 (1965).
4. W. Cottingham, Ann. Phys. 25, 424 (1963).
5. S. Coleman and S. Glashow, Phys. Rev. Letts. 6, 423 (1961).
6. S. Fels, Phys. Rev. 175, 1895 (1968).
7. N. Cabibbo and L. Maiani, Phys. Letts. 28B, 131 (1968).
8. R. Gatto, G. Sartori and M. Tonin, Phys. Letts. 28B, 128 (1968).
9. R. J. Oakes, Phys. Letts. 29B, 683 (1969).
10. M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).
11. S. Glashow and S. Weinberg, Phys. Rev. Letts. 20, 224 (1968).
12. S. Meshkov, C. A. Levinson and H. Lipkin, Phys. Rev. Letts. 10, 361 (1963).
13. N. Cabibbo, Phys. Rev. Letts. 10, 531 (1963).
14. T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low and J. E. Young, Phys. Rev. Letts. 18, 759 (1967).
15. David J. Gross and Heinz Pagels, Phys. Rev. 172, 1381 (1968).