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TWO LOOP APPROXIMATIONS TO PROPAGATORS IN THE σ MODEL *

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ABSTRACT

A simple and systematic procedure is established for directly generating perturbation theory for the σ model order by order in the number of loops rather than in powers of the bare coupling constant. This self consistent Greens function method is demonstrated by deriving the already known tree and one loop approximations and then is applied to the much more complex problem of the two loop approximations to the propagators. The explicit form is displayed for these and it is demonstrated that in the limit of no symmetry breaking and zero pion bare mass the renormalized pion mass remains at zero as is required by the Goldstone theorem. These are the essential first steps toward calculating a unitary pion-pion scattering amplitude with this model to the two loop order.

I. Introduction

The techniques of current algebra have, for the most part, been used with great success in the description of elementary particle phenomena. However, it is clear that current algebra can at best supply only the basis for the lowest order most model independent approximations to physical processes and that the more detailed understanding of these phenomena and particularly the development of unitary scattering amplitudes requires a more explicit theory. Phenomenological Lagrangian theories have been formulated with the correct chiral symmetry properties[1]. These models automatically result in tree approximations which reproduce the current algebra results with the appropriate definition of the parameters. It is quite reasonable to attempt to extend the predictive power of such models by treating them as true Lagrangian field theories in which, in respect to the symmetry and the first approximation, perturbation order is performed according to the number of closed loops in a diagram. Having made the decision to calculate in this manner it is necessary to suffer the disadvantages as well as the advantages of Lagrangian field theory. In particular, even though the machinery is at hand to systematically calculate approximations to the "true" unitary crossing symmetric scattering amplitudes, we must first contend with the problems of divergences endemic to essentially all perturbation field theories.

In recent papers considerable progress has been made in understanding the renormalization of Lagrangians relevant to generalize current algebraic calculations. B. W. Lee[2] has argued that the broken (intrinsic and or spontaneous) σ model[3] is renormalizable and has explicitly demonstrated a method of calculation and renormalization in the loop approximation. Using more general but formal Wards identity techniques Symanzik[4] has demonstrated that models with symmetry breaking linear in a field are renormalizable if the model with vanishing symmetry breaking is renormalizable. He has also discussed the spontaneous broken symmetry case. It is our intention to study the σ model in the two loop approximation. To do this simply and self consistently we will set up a Greens function procedure in the presence of an external source. Although equivalent to other methods this technique has the advantage that it will avoid any explicit reference to ϕ type lines[2] as they are automatically accounted for. Because of the intrinsic complexity of two loop approximations we will confine this paper to the study of the two field propagators. We shall be able to check the correctness of the forms we derive by demonstrating that in the chiral symmetric zero bare mass but spontaneously broken symmetric limit the Goldstone theorem is explicitly valid and thus the pions physical mass vanishes. We further find that the divergent part of Z_π = divergent part of Z_σ and similarly for the mass corrections so the renormalized Lagrangian has the chiral symmetry broken only by a term linear in σ does the unrenormalized Lagrangian. The mechanism for development of the vertex functions and all important scattering amplitudes is presented but not studied here. In a second paper we shall obtain these functions and use this structure to display the unitary Padé approximations to the partial wave amplitudes and to calculate the width of the σ.

II. The Lagrangian Approximation Scheme

We will perform our calculations using the Lagrangian density.

$$\mathcal{L}(x) = \frac{1}{2}(\partial_\mu \phi_\alpha(x))^2 - \frac{1}{2} m_o{}^2 \phi_\alpha{}^2(x) + \frac{1}{4} g_o \, (\phi_\alpha{}^2(x))^2 \tag{2.1}$$

$$+ J_\alpha(x) \; \phi_\alpha(x)$$

from which follow the field equations

$$(- \partial^2 - m_o{}^2 + g_o \phi^2) \; \phi_\alpha(x) + J_\alpha(x) = 0 \tag{2.2}$$

Here $J_\alpha(x)$ is a c number source. When there are four fields and $J_\alpha(x)$ is constant (2.1) becomes the broken σ model with P.C.A.C. In the case that $J_\alpha(x) = 0$ but

$$\frac{< 0|\phi_\alpha(x)|0 >}{< 0|0 >} = \eta_\alpha \neq 0 \tag{2.3}$$

(2.1) is the Goldstone model for spontaneous symmetry breaking.

This theory has the cannonical commutation relations

$$i[\partial^0 \phi_\alpha(x), \phi_\beta(y)]_{x^o = y^o} = \delta_{\alpha\beta}\, \delta^3(\vec{x} - \vec{y}) \quad .$$

If a local current is defined as $J^\mu{}_{\alpha\beta} \equiv \phi_\alpha(\partial^\mu\phi_\beta)-(\partial^\mu\phi_\alpha)\phi_\beta$, the field equations require that

$$\partial_\mu J^\mu{}_{\alpha\beta} = \phi_\alpha J_\beta - J_\alpha \phi_\beta \quad .$$

In the case of 4 fields this is, of course, the P.C.A.C. condition. Direct calculations leads to the commutation relations

$$i[\int d^3x J^0{}_{\alpha\beta}(x), \phi_\lambda(y)]_{x^o = y^o} = \phi_\alpha(y)\, \delta_{\beta\lambda} - \phi_\beta(y)\delta_{\alpha\lambda} \quad .$$

When $J_\alpha(x) = 0$, the charge $Q_{\alpha\beta}(x^o) \equiv \int d^3x\, J^0{}_{\alpha\beta}(x)$ is independent of time and it is easily seen that if (2.3) holds then the above equation requires that the field ϕ excites a massless particle. We will demonstrate explicitly the appearance of this massless particle in the pion propagator accurate to the two loop approximation. The cancellation which occur to make this possible are complex and their presence is a good check on the consistency of our approximation technique.

We shall generate all approximations to the Greens function self consistently through the one field function

$$\phi_\alpha(x;J) \equiv \frac{<0\sigma_1|\phi_\alpha(x)|0\sigma_2>}{<0\sigma_1|0\sigma_2>} \quad . \tag{2.4}$$

For example, the two field function can be found by functional differentiation either of $\phi_\alpha(x;J)$ or an approximation to it since

$$G_{\alpha\beta}(x,y;J) = \frac{\delta}{\delta J_\beta(y)} \phi_\alpha(x;J) = i \frac{\langle 0\sigma_1 | (\phi_\alpha(x)\phi_\beta(y))_+ | 0\sigma_2 \rangle}{\langle 0\sigma_1 | 0\sigma_2 \rangle} \tag{2.5}$$

$$- i\phi_\alpha(x;J)\, \phi_\beta(y;J)$$

we can similarly obtain all other Greens functions. From the field equation (2.2) $\phi_\alpha(x;J)$ is found to satisfy the exact equation

$$(-\partial^2 - m_o^2)\, \phi_\alpha(x;J) + g_o\left(\frac{\delta}{i\delta J_\beta(x)}\right)^2 \phi_\alpha(x;J)$$

$$+ 2g_o\, \phi_\beta(x;J)\left(\frac{\delta}{i\delta J_\beta(x)}\, \phi_\alpha(x;J)\right)$$

$$+ g_o\, \phi_\alpha(x;J)\, \frac{\delta}{i\delta J_\beta(x)}\, \phi_\beta(x;J) + g_o\, \phi_\beta^{\,2}(x;J)\, \phi_\alpha(x;J) + J_\alpha(x) = 0 \tag{2.6}$$

and the n-point Green's function satisfies the equation obtained by functional differentiating Eq. (2.6) n-1 times.

We shall now proceed to develop an approximation scheme for $\phi_\alpha(x;J)$ in the form $\phi_\alpha(x;J) = \sum_{n=0}^{\infty} \phi^{(n)}_\alpha(x;J)$ such that the nth order approximation consists of all diagrams with n closed loops. This scheme develops very easily from (2.6). If the natural first approximation of neglecting functional derivatives is made so as to obtain the equation

$$(-\partial^2 - m_o^2)\, \phi^{(o)}_\alpha(x;J) + g_o[\phi_\beta(x;J)]^2\, \phi^{(o)}_\alpha(x;J) + J_\alpha(x) = 0 \quad , \tag{2.7}$$

it is easily found by iterative solution that (2.7) is the tree approximation to equation (2.6). Now we may generalize this prescription to generate the quantity $\phi^{(n)}_\alpha(x;J)$. In equation (2.6) everywhere that $\phi_\alpha(x;J)$ appears replace it by the quantity $\sum_{m=0}^{n} \phi^{(m)}_\alpha(x;J)$. Keep terms of the form

$$\phi_\rho^{(a_1)}(x;J)\cdots\phi_\sigma^{(a_p)}(x;J)\, \frac{\delta^m}{\delta J_\alpha(x;J)\cdots\delta J_\beta(x;J)}\, \phi_\gamma^{(\ell)}(x;J) \text{ if } a_1 + \cdots + a_p + m + \ell = n.$$

Using the equation obtained in this manner for $\phi_\alpha^{\,n}\ (x;J)$ we find that $\phi_\alpha^{\,n}(x;J)$ satisfies the equation

$$F^{(n)}_{(1)}(\phi^{(n)}_\alpha(x;J)) \equiv [(-\partial^2 - m_o^{\,2} + g_o(\phi^{(o)}_\lambda)^2)\,\delta_{\beta\alpha} + 2g_o\,\phi^{(o)}_\beta\,\phi^{(o)}_\alpha]\phi^{(n)}_\beta$$

$$+ g_o \sum_{m+p+\lambda=n} (\phi^m_\beta\,\phi^p_\beta)\,\phi^\lambda_\alpha + g_o\left(\frac{\delta}{i\delta J_\beta}\right)^2 \phi^{(n-2)}_\alpha \tag{2.8}$$

$$+ 2g_o \sum_{m+p=n} \phi^{(m)}_\beta \frac{\delta}{i\delta J_\beta}\phi^{(p-1)}_\alpha + g_o \sum_{m+\lambda=n} \phi^{(\lambda)}_\alpha \frac{\delta}{i\delta J_\beta}\phi^{(m-1)}_\beta$$

$$+ \delta_{no} J_\alpha = 0$$

From (2.8) $\phi^{(n)}_\beta$ may be determined from $\phi^{(m)}_\beta$ with $m<n$. Now it is easily established by induction that $\phi^{(n)}_\beta(x;J)$ is the n loop part of $\phi(x;J) = \sum_{n=0}^{\infty} \phi^{(n)}_\beta(x;J)$. To do this we observe that $\frac{\delta^m}{\delta J_\alpha(x)\ldots\delta J_\lambda(x)}\ \phi^{(o)}_\beta(x;J)$ is an m loop quantity. Induction and Equation (2.8) shows that $\phi^{(n)}_\beta(x;J)$ is an n loop quantity, and that in general $\frac{\delta^m}{\delta J_\alpha(x)\ldots\delta J_\lambda(x)}\ \phi^n_{\ \beta}(x;J)$ is an $m+n$ loop quantity. All the results in this paper will be extracted in one way or other from Equation (2.8). It is, for example, not difficult to derive from (2.8) the general expression for the propagator function

$$G_{\alpha\beta}(x,y;J) = \sum_{m=0}^{\infty} G^{(m)}_{\alpha\beta}(x,y;J) = \sum_{m=0}^{\infty} \frac{\delta}{\delta J_\beta(y)}\ \phi^{(m)}_\alpha(x;J) \tag{2.9}$$

but we shall be content here with the order by order forms as needed.

We shall summarize our approximation scheme in the following. We have a set of equations

$$F_{(1)}^{(n)}(\Phi_{\alpha_1}^{(n)}(x_1;J)) = 0 \tag{2.10}$$

The n-loop, ℓ-point Green's function

$$G^{(n)}_{\alpha_1\cdots\alpha_\ell}(x_1\ldots x_\ell;J) \equiv \frac{\delta^{\ell-1}}{\delta J_{\alpha_2}(x_2)\ldots\delta J_{\alpha_\ell}(x_\ell)}\,\Phi_{\alpha_1}^{(n)}(x_1;J)$$

satisfies

$$F_{(\ell)}^{(n)}(G^{(n)}_{\alpha_1\cdots\alpha_\ell}(x_1\ldots x_\ell;J))$$

$$\equiv \frac{\delta^{\ell-1}}{\delta J_{\alpha_2}(x_2)\ldots\delta J_{\alpha_\ell}(x_\ell)}\,F_{(1)}^{(n)}(\Phi_{\alpha_1}^{(n)}(x_1;J)) = 0 \tag{2.11}$$

From Eq. (2.11) $G^{(n)}_{\alpha_1\cdots\alpha_\ell}(x_1\ldots x_\ell;J)$ is given in turn by terms of the form

$$\prod_i G^{(i)}_{\alpha_1\cdots\alpha_m}(x_1\ldots x_m;J) \qquad \text{with } \Sigma i = n$$

It is easy to see that by solving all equations $F_{(m)}^{(i)} = 0$, $i+m \leq n+\ell$ and $i \leq n$. We will find $G_{\alpha_1\cdots\alpha_\ell}$ uniquely and systematically to n loops.

III. Zero Order Green's Functions

In this section we will demonstrate our technique on the no loop Greens functions. Differentiation of Equation (2.7) or (2.8) yield

$$\{[-\partial_x^2 - m_o^2 + g_o\,\phi^{(o)2}(x;J)]\,\delta_{\alpha\lambda} + 2g_o\,\phi^{(o)}_{\alpha}(x;J)\,\phi^{(o)}_{\lambda}(x;J)\}\,G^{(o)}_{\lambda\beta}(x,y;J)$$
$$= -\,\delta_{\alpha\beta}\,\delta^4(x-y) \tag{3.1}$$

As can be seen from Equation (2.8) in order to obtain the two loop propagator we will need the quantities $\frac{\delta G^{(0)}}{\delta J}$ and $\frac{\delta^2}{\delta J \delta J} G^{(0)}$. These are readily obtained from (3.1), but first we will analyze the consequences of (2.7) and (3.1) when the source is co-ordinate independent.

In the case that $J_\alpha(x) = C_\alpha$ where C_α does not depend on space time, $\phi_\alpha(x;J)$ must also be coordinate independent and we write

$$\phi^{(0)}_\alpha(x;C) \equiv \eta_\alpha \tag{3.2}$$

Equation (2.7) becomes

$$[- m_o{}^2 + g_o\, \eta^2]\, \eta_\alpha = - C_\alpha \tag{3.2a}$$

which if $\eta_\alpha \neq 0$ becomes

$$- m_o{}^2 + g_o \eta^2 = - \frac{C_{\alpha_1}}{\eta_{\alpha_1}} \equiv - \gamma \tag{3.3}$$

In the special case that there is no intrinsic symmetry breaking at all but $\eta_{\alpha_1} \neq 0$ for some α_1 the above equation becomes the Goldstone consistency condition

$$m_o{}^2 = g_o\, \eta^2 . \tag{3.4}$$

Functional differentiation of Equation (2.7) results in the lowest approximation propagator satisfying the equation

$$\begin{aligned}[][(- \partial_x^2 - m_o{}^2 + g_o(\phi^{(0)}(x;J))^2\, \delta_{\alpha\lambda} \\ + 2g_o\, \phi^{(0)}_\alpha(x;J)\, \phi^{(0)}_\lambda(x;J)]\, G^{(0)}_{\lambda\beta}(x,y;J) = - \delta_{\alpha\beta}\delta(x-y)\end{aligned} \tag{3.5}$$

In the constant external field limit this equation may be directly inverted to yield

$$G^{(o)}_{\lambda\beta}(p;C) = \frac{1}{-p^2+\gamma}\left[\delta_{\lambda\beta} - \frac{\eta_\lambda\eta_\beta}{\eta^2}\right]$$

$$+ \frac{1}{-p^2+\gamma-2g_o\eta^2}\,\frac{\eta_\lambda\eta_\beta}{\eta^2} \equiv \pi_p\left[\delta_{\lambda\beta} - \frac{\eta_\lambda\eta_\beta}{\eta^2}\right] \tag{3.6}$$

$$+ \sigma_p\,\frac{\eta_\lambda\eta_\beta}{\eta^2} \equiv G^{(o)}_{\pi\pi}(p;C) + G^{(o)}_{\sigma\sigma}(p;C)$$

This Green's function is pictorially represented in co-ordinate space by Figure 1. The notation in the final identity is chosen so as to correspond with the physical identification of the particle poles when the field has four components. Note that the coefficient of π_p is orthogonal to η_λ. Thus if we choose η_λ to lie on a coordinate axis that axis determines the component of the field whose vacuum expectation is non-vanishing and if we call this component η_σ and all the other components η_π, we may set $\eta_\pi = 0$ without loss of generality. For calculational purposes it will be useful to introduce the vertex function

$$\frac{\delta}{\delta J_\gamma(z)}\, G_{\alpha\beta}(x,y;J) = \Gamma_{\alpha\beta\gamma}(x,y,z;J) \tag{3.7}$$

It is easily found in the constant field limit that

$$\Gamma^{(o)}_{\alpha\beta\gamma}(x,y,z;C) = -2g_o \int \frac{d^4k_1}{i(2\pi)^4}\,\frac{d^4k_2}{i(2\pi)^4}\, e^{-ik_1(x-y)}\, e^{-ik_2(x-z)}$$

$$\Big[\sigma_{1+2}\,\pi_1\,\pi_2\,\eta_\alpha\Big(\delta_{\beta\gamma} - \frac{\eta_\beta\eta_\gamma}{\eta^2}\Big) + \pi_{1+2}\,\sigma_1\,\pi_2\,\eta_\beta\Big(\delta_{\alpha\gamma} - \frac{\eta_\alpha\eta_\gamma}{\eta^2}\Big) \tag{3.8}$$

$$+ \pi_{1+2}\,\pi_1\,\sigma_2\,\eta_\gamma\Big(\delta_{\alpha\beta} - \frac{\eta_\alpha\eta_\beta}{\eta^2}\Big) + 3\sigma_{1+2}\,\sigma_1\,\sigma_2\,\frac{\eta_\alpha\eta_\beta\eta_\gamma}{\eta^2}\Big]$$

where we have further simplified the notation so that for example $\pi_1 \equiv \pi_{k_1}$ etc. This Greens function is graphically represented by Figure 2.

In addition we shall need the expression for

$$\Lambda^{(0)}_{\alpha\beta\gamma\lambda}(x,y,z,w;C) \equiv \left[\frac{\delta^2}{\delta J_\lambda(w)\ \delta J_\gamma(z)}\ G^{(0)}{}_{\alpha\beta}(x,y;J)\right]_{J=C}$$

which is essentially the two body scattering amplitude in the tree approximation. This complicated quantity has the relatively simple graphical representation given in Figure (3) while the messy analytic expression is given in Appendix A.

IV. One Loop Greens Functions

The one loop Greens functions have been worked out elsewhere[2] but for completeness and as an illustration of our methods we will derive the results from Equation (2.8). We find that $\phi^{(1)}_\alpha(x;J)$ satisfies the equation

$$\begin{aligned} &\{(-\partial^2 - m_o^2 + g_o(\phi^{(0)}(x;J))^2\}\ \delta_{\beta\alpha} + 2g_o\ \phi^{(0)}_\beta(x;J)\ \phi^{(0)}_\alpha(x;J)\}\ \phi^{(1)}_\beta(x;J) \\ &+ 2g_o\ \phi^{(0)}_\beta \frac{\delta}{i\delta J_\beta(x)}\ \phi^{(0)}_\alpha(x;J) + g_o\phi^{(0)}_\alpha(x;J)\ \frac{\delta}{i\delta J_\beta(x)}\ \phi^{(0)}{}_\beta(x;J) = 0 \end{aligned} \tag{4.1}$$

Letting the source become the constant C_α it is clear from (4.1) that since $\phi^{(1)}$ is expressed in terms of $\phi^{(0)}$ that $\phi^{(1)}_\alpha \propto \eta_\alpha$ or

$$\phi^{(1)}_\alpha(x;C) \equiv C^{(1)}\eta_\alpha \tag{4.2}$$

By the same argument and the exploitation of Equation (2.8) it is in fact clear that in general we may write

$$\phi_\alpha(x;C) = (1 + C^{(1)} + C^{(2)} + \cdots C^{(i)} + \cdots)\eta_\alpha\quad . \tag{4.3}$$

Returning to Equation (4.2) we see that $C^{(1)}$ may be determined very easily from Equation (4.1) by setting $\phi^{(1)}$ equal to a constant in the equation and using (2.9) and (3.6). We thus find that

$$C^{(1)}\eta_\alpha = \frac{g_o\,\eta_\alpha}{+\,m_o^2 - 3g_o\eta^2}\,(3B_\sigma^{(1)} + (N-1)\,B_\pi^{(1)}) \tag{4.4}$$

Here we have introduced the integrated notation for the propagators

$$B_\sigma^{(1)} = \int \frac{d^4k}{i(2\pi)^4}\,\sigma_k \qquad \text{and} \tag{4.5}$$

$$B_\pi^{(1)} = \int \frac{d^4k}{i(2\pi)^4}\,\pi_k \tag{4.6}$$

Of course, N is the number of components of the field ϕ_α. This equation may be represented graphically in Figure 4.

From variational differentiation of Equation (4.1) we find that $G^{(1)}_{\alpha\gamma}(x,y;J)$ satisfies the equation

$$\begin{aligned}
&(-\partial_x^2 - m_o^2 + g_o\phi^{(0)2}(x;J))G^{(1)}_{\alpha\gamma}(x,y;J) + 2g_o\phi^{(0)\alpha}(x;J)\phi^{(0)}_\beta(x;J)G^{(1)}_{\beta\gamma}(x,y;J)\\
&+ 2g_o\,G^{(0)}_{\beta\gamma}(x,y;J)\,\frac{G^{(0)}_{\alpha\beta}(x,x;J)}{i} + g_oG^{(0)}_{\alpha\gamma}(x,y;J)\,\frac{G^{(0)}_{\beta\beta}(x,x;J)}{i}\\
&+ 2g_o\phi^{(0)}_\beta(x;J)\left(\frac{\delta}{\delta J_\gamma(y)}\,\frac{G^{(0)}_{\alpha\beta}(x,x;J)}{i}\right) + g_o\phi^{(0)}_\alpha(x;J)\left(\frac{\delta}{\delta J_\gamma(y)}\,\frac{G^{(0)}_{\beta\beta}(x,x;J)}{i}\right)\\
&+ 2g_o\,G^{(0)}_{\beta\gamma}(x,y;J)\phi^{(0)}_\alpha(x;J)\phi^{(1)}_\beta(x;J) + 2g_o\phi^{(0)}_\beta(x;J)\,G^{(0)}_{\beta\gamma}(x,y;J)\phi^{(1)}_\alpha(x;J)\\
&+ 2g_o\phi^{(0)}_\beta(x;J)\phi^{(1)}_\beta(x;J)\,G^{(0)}_{\alpha\gamma}(x,y;J) = 0
\end{aligned} \tag{4.7}$$

We are concerned with solving this equation in the constant external field limit. In that case we may insert Equation (3.8) into (4.7) and through simple algebra (using (3.3) and (3.6)) find

$$\begin{aligned}
G^{(1)}_{\alpha\beta}(k^2, C) &= g_o\,\pi_k^2\,Z_\pi^{(1)}(k)\,[\delta_{\alpha\beta} - \frac{\eta_\alpha\eta_\beta}{\eta^2}]\\
&\quad + g_o\sigma_k^2\,Z_\sigma^{(1)}(k)\frac{\eta_\alpha\eta_\beta}{\eta^2}
\end{aligned} \tag{4.8}$$

where

$$Z_\pi^{(1)}(k) = (N+1)\, B_\pi^{(1)} + B_\sigma^{(1)} + 2C^{(1)}\eta^2$$
$$+ 4(g_o\eta^2) \int \frac{d^4k_1}{i(2\pi)^4}\, \pi_1\, \sigma_{1-k} \tag{4.9}$$

and

$$Z_\sigma^{(1)}(k) = (N-1)\, B_\pi^{(1)} + 3B_\sigma^{(1)} + 6C^{(1)}\eta^2$$
$$+2(g_o\eta^2)\int \frac{d^4k_1}{i(2\pi)^4}\,[(N-1)\,\pi_1\,\pi_{1-k} + 9\,\sigma_1\,\sigma_{1-k}] \tag{4.10}$$

To this order, of course, the complete unrenormalized propagator is

$$G = (G^{(0)}(k^2, C) + G^{(1)}(k^2, C)) \tag{4.11}$$

The subscripts π and σ on the Z's indicate the particle associated with this part of the propagator. Thus, it is convenient to write

$$G_{\alpha\beta}(k^2;C) = G_\pi(k^2;C)\,[\delta_{\alpha\beta} - \frac{\eta_\alpha\eta_\beta}{\eta^2}]$$
$$+ G_\sigma(k^2;C)\,\frac{\eta_\alpha\eta_\beta}{\eta^2} \quad , \tag{4.12}$$

Accurate to the one loop approximation it follows that

$$G_\pi^{-1} = -p^2 + m_\pi^2 - g_o z_\pi^{(1)}(p^2) \;, \quad m_\pi^2 = \gamma$$

and

$$G_\sigma^{-1} = -p^2 + m_\sigma^2 - g_o z_\sigma^{(1)}(p^2) \;, \quad m_\sigma^2 = \gamma - 2g_0\eta^2$$

If we make the definitions[2]

$$B_{xy}(p^2) = i\int \frac{d^4k}{(2\pi)^4}\;[\frac{1}{-(p-k)^2 + \mu_x^2}][\frac{1}{-k^2 + \mu_y^2}]$$

and $$\overline{B}_{xy}(p^2) = B_{xy}(p^2) - B_{xy}(0)$$

the above propagators can be rewritten in the form (for N = 4)

$$G_\pi^{-1}(p^2) = -p^2 + m_\pi^2\{1 - \frac{3g_o}{m_\sigma^2}[B_\pi^{(1)} + B_\sigma^{(1)}]\} + 4g_o^2\eta^2\,\overline{B}_{\sigma\pi}(p^2) \qquad (4.13)$$

and

$$G_\sigma^{-1}(p^2) = -p^2 + m_\sigma^2 - \frac{3g_o}{m_\sigma^2}[m_\sigma^2 + 6g_o\eta^2][B_\pi^{(1)} + B_\sigma^{(1)}] \qquad (4.14)$$

$$+ 6g_o^2\eta^2 B_{\pi\pi}(0) + 18g_o^2\eta^2 B_{\sigma\sigma}(0)$$

$$+ 6g_o^2\eta^2[\overline{B}_{\pi\pi}(p^2) + 3\overline{B}_{\sigma\sigma}(p^2)]$$

It is straightforward to verify using (3.3) that when C = 0 but $\eta_\alpha \neq 0$ then $G_\pi^{-1}(p^2 = 0) = 0$. This is the Goldstone theorem. From the condition that $G_\pi^{-1}(M_\pi^2) = 0$ and $G_\sigma^{-1}(M_\sigma^2 - i\Gamma_\sigma) = 0$ where Γ_σ is the σ width it follows that

$$M_\pi^2 = m_\pi^2\{1 - \frac{3g_o}{m_\sigma^2}[B_\pi^{(1)} + B_\sigma^{(1)}]\} + 4g_o^2\eta^2\,\overline{B}_{\sigma\pi}(M_\pi^2) \qquad (4.15)$$

and

$$M_\sigma^2 = m_\sigma^2 - \frac{3g_o}{m_\sigma^2}[m_\sigma^2 + 6g_o\eta^2][B_\pi^{(1)} + B_\sigma^{(1)}]$$

$$+ \mathrm{Re}\,[6g_o^2\eta^2 B_{\pi\pi}(0) + 18g_o^2\eta^2 B_{\sigma\sigma}(0)]$$

$$+ 6g_o^2\eta^2[\overline{B}_{\pi\pi}(M_\sigma^2) + 3\overline{B}_{\sigma\sigma}\, M_\sigma^2)]$$

These above forms allow us to make contact with B. W. Lee's expression. For example, using (4.15) the pion propagator may be rewritten as

$$G_\pi^{-1}(p^2) = -p^2 + M_\pi^2 + 4g_o^2\eta^2[B_{\sigma\pi}(p^2) - B_{\sigma\pi}(M_\pi^2)]$$

The wave function renormalization calculated from this is the residue of the pole at $p^2 = M_\pi^{\ 2}$ and hence is

$$Z_\pi^{\ -1} = - \left. \frac{\partial G_\pi^{\ -1}(p^2)}{\partial p^2} \right|_{p^2 = M_\pi^{\ 2}} = 1 - 4g_o^{\ 2}\eta^2 \left. \frac{\partial B_{\sigma\pi}(p^2)}{\partial p^2} \right|_{p^2 = M_\pi^{\ 2}} \tag{4.17}$$

Since when multiplying a one loop quantity $g_o^{\ 2}\eta^2 = g^2F^2$ in Lee notation, the equivalence of the renormalized functions is established. The procedure with $G_\sigma^{\ -1}$ is essentially identical. It is also of some interest to see how the consistency condition (3.2a) looks when rewritten in terms of $F \equiv [1 + C^{(1)}]\eta$. With some simple algebra it follows that

$$F\mu_o^{\ 2} = C + gz_1(F^2)F - 3igZ_1F \left[\int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2 - \mu_\sigma^{\ 2}} + \frac{1}{k^2 - \mu_\pi^{\ 2}}\right]\right] \tag{4.18}$$

This is not quite B. W. Lee's form. However, in our formulation which is not normal ordered the first order mass correction in the chiral symmetric limit does not vanish but is given as

$$\mu^2 = \mu_o^{\ 2} - 3g_o[B_\pi^{(1)} + B_\sigma^{(1)}]_{\mu_\pi^{\ 2} = \mu_\sigma^{\ 2} = \mu^2}.$$

so the above equation may be written as

$$F\mu^2 = C + gz_1(F^2)F - 3igz_1F \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2 - \mu_\sigma^{\ 2}} - \frac{1}{k^2 - \mu^2} + \frac{1}{k^2 - \mu_\pi^{\ 2}} - \frac{1}{k^2 - \mu^2}\right]$$

which corresponds to B. W. Lee's result.

V. The Propagators in the Two Loop Approximation

We have now established the mechanism to calculate the propagators of the σ model in the two loop approximation. From Equation (2.8) it is easily found that $\phi_\alpha^{(2)}(x;J)$ satisfies

$$
\begin{aligned}
&[-\partial_x^2 - m_o^2 + g_o\phi^{(0)2}(x;J)]\phi_\alpha^{(2)}(x;J) + 2g_o\phi_\beta^{(0)}(x;J)\phi_\beta^{(2)}(x;J)\phi_\alpha^{(0)}(x;J) \\
&- 2g_o i\,\phi^{(1)}_\beta(x;J)\;G^{(0)}_{\alpha\beta}(x,x;J) - g_o i\,\phi^{(1)}_\alpha(x;J)G^{(0)}_{\beta\beta}(x,x;J) \\
&- 2g_o i\,\phi^{(0)}_\beta(x;J)\;G^{(1)}_{\alpha\beta}(x,x;J) \qquad (5.1) \\
&- g_o i\,\phi^{(0)}_\alpha(x;J)\;G^{(1)}_{\beta\beta}(x,x;J) - g_o\frac{\delta}{\delta J_\beta(x)}G^{(0)}_{\alpha\beta}(x,x;J) \\
&+ g_o\phi^{(1)2}(x;J)\phi^{(0)}_\alpha(x;J) + 2g_o\,\phi^{(0)}_\beta(x;J)\;\phi^{(1)}_\beta(x;J)\;\phi^{(1)}_\alpha(x;J) = 0
\end{aligned}
$$

writing $\phi^{(2)}_\alpha = C^{(2)}\eta_\alpha$ and using the results for the lower orders it follows that

$$
\begin{aligned}
C^{(2)} = \frac{1}{m_o^2 - 3g_o\eta^2}\;&[3g_o\eta^2\,C^{(1)2} + C^{(1)}g_o\,[(N-1)\,B_\pi^{(1)} + 3B_\sigma^{(1)}] \\
&+ g_o^2\,[(N-1)B_\pi^{(2)} + 3B_\sigma^{(2)}] \qquad (5.2) \\
&+ 2g_o^2\int\frac{d4k_1}{i(2\pi)4}\frac{d4k_2}{i(2\pi)4}[3\sigma_1\sigma_2\sigma_{1+2} + (N-1)\pi_1\pi_2\pi_{1+2}]
\end{aligned}
$$

where the new notations

$$B_\sigma^{(2)} = \int\frac{d4k}{i(2\pi)4}\,\sigma_k^2\,Z_\sigma^{(1)}(k) \qquad (5.3)$$

and

$$B_\pi^{(2)} = \int\frac{d4k}{i(2\pi)4}\,\pi_k^2\,Z_\pi^{(1)}(k) \qquad (5.4)$$

have been introduced. This result is illustrated in Figure 8.

Using Equation (5.1) we now also find that $G^{(2)}_{\alpha\gamma}$ satisfies the equation:

$$\begin{aligned}
&(-\partial_x^2 - m_o^2 + g_o\,\phi^{(0)2}(x;J)\; G^{(2)}_{\alpha\gamma}(x,y;J) + 2g_o\phi^{(0)}_{\beta}(x;J)\phi^{(0)}_{\alpha}(x;J)G^{(2)}_{\beta\gamma}(x,y;J) \\
&+ 2g_o G^{(1)}_{\beta\gamma}(x,y;J)\,\frac{G^{(0)}_{\alpha\beta}(x,x;J)}{i} + 2g_o\phi^{(1)}_{\beta}(x;J)\,(\frac{\delta}{\delta J_\gamma(y)}\,\frac{G^{(0)}_{\alpha\beta}(x,x;J)}{i}) \\
&+ g_o\, G^{(1)}_{\alpha\gamma}(x,y;J)\,\frac{G^{(0)}_{\beta\beta}(x,x;J)}{i} + g_o\phi^{(1)}_{\alpha}(x;J)\,(\frac{\delta}{\delta J_\gamma(y)}\,\frac{G^{(0)}_{\beta\beta}(x,x;J)}{i} \\
&+ 2g_o\, G^{(0)}_{\beta\gamma}(x,y;J)\,\frac{G^{(1)}_{\alpha\beta}(x,x;J)}{i} + 2g_o\,\phi^{(0)}_{\beta}(x;J)\,(\frac{\delta}{\delta J_\gamma(y)}\,\frac{G^{(1)}_{\beta\alpha}(x,x;J)}{i}) \\
&+ g_o\, G^{(0)}_{\alpha\gamma}(x,y;J)\,\frac{G^{(1)}_{\beta\beta}(x,x;J)}{i} + g_o\,\phi^{(0)}_{\alpha}(x;J)\,(\frac{\delta}{\delta J_\gamma(y)}\,\frac{G^{(1)}_{\beta\beta}(x,x;J)}{i}) \\
&+ g_o\,\phi^{(1)2}(x;J)\; G^{(0)}_{\alpha\gamma}(x,y;J) + 2g_o\,\phi^{(1)}_{\beta}(x;J)\;\phi^{(0)}_{\alpha}(x;J)\; G^{(1)}_{\beta\gamma}(x,y;J) \\
&+ 2g_o\,\phi^{(1)}_{\beta}(x;J)\;\phi^{(1)}_{\alpha}(x;J)\; G^{(0)}_{\beta\gamma}(x,y;J) + 2g_o\,\phi^{(0)}_{\beta}(x;J)G^{(1)}_{\beta\gamma}(x,y;J)\phi^{(1)}_{\alpha}(x;J) \\
&+ 2g_o\,\phi^{(0)}_{\beta}(x;J)\;\phi^{(1)}_{\beta}(x;J)G^{(1)}_{\alpha\gamma}(x,y;J) + 2g_o\,\phi^{(0)}_{\beta}(x;J)\phi^{(2)}_{\alpha}(x;J)G^{(0)}_{\beta\gamma}(x,y;J) \\
&+ 2g_o G^{(0)}_{\beta\gamma}(x,y;J)\;\phi^{(2)}_{\beta}(x;J)\;\phi^{(0)}_{\alpha}(x;J) + 2g_o G^{(0)}_{\alpha\gamma}(x,y;J)\phi^{(2)}_{\beta}(x;J)\phi^{(0)}_{\beta}(x;J) \\
&- g_o(\frac{\delta}{\delta J_\gamma(y)}\;\frac{\delta}{\delta J_\beta(x)}\; G^{(0)}_{\alpha\beta}(x,x;J)) = 0
\end{aligned} \tag{5.5}$$

All the quantities needed to solve this equation have been previously evaluated except

$$\frac{\delta}{\delta J_\gamma(z)}\; G^{(1)}_{\alpha\beta}(x,y;J) \equiv \Gamma^{(1)}_{\alpha\beta\gamma}(x,y,z;J) \quad . \tag{5.6}$$

By differentiating Equation (4.7) and then taking the constant source limit this becomes

$$\Gamma^{(1)}_{\alpha\ \beta\gamma}(x,y,z;C) = g_o \int \frac{d^4k_3}{(2\pi)^4} \frac{d^4k_1}{(2\pi)^4}\ e^{-ik_3(x-z)}\ e^{-ik_1(x-y)}$$
$$\times [X_1 n^\gamma(\delta_{\alpha\beta} - \frac{n_\alpha n_\beta}{n^2}) + X_2 n^\alpha(\delta_{\beta\gamma} - \frac{n_\beta n_\gamma}{n^2})$$
$$+ X_3\ n^\beta(\delta_{\alpha\gamma} - \frac{n_\alpha n_\gamma}{n^2}) + X_4\ \frac{n^\alpha n^\beta n^\gamma}{n^2}] \qquad (5.7)$$

where

$$X_3(1,3) = \pi_3\ \sigma_1\ \pi_{3+1}\ \{2C^{(1)} + 2\ g_o\ \pi_3\ Z_\pi^{(1)}(3) + 2g_o\ \sigma_1\ Z_\sigma^{(1)}(1)$$
$$+ 2g_o\ \pi_{3+1}\ Z_\pi^{(1)}(3+1)$$
$$+ \frac{2g_o}{i} \int \frac{d^4k_2}{(2\pi)^4} [3\sigma_2\ \sigma_{1+2} + (N+1)\ \pi_2\ \pi_{1+2} + 2\pi_2\ \sigma_{3+2}$$
$$+ 2\sigma_2\ \pi_{3+1+2}$$
$$+ (g_o n^2)(12\sigma_2\ \pi_{3+2}\ \sigma_{2-1} + 4\pi_2\ \sigma_{3+2}\ \pi_{2-1})]\}$$

and

$$X_4(1,3) = \sigma_3\ \sigma_1\ \sigma_{3+1}\ \{6C^{(1)} + 6g_o\ \sigma_3\ Z_\sigma^{(1)}(3) + 6g_o\ \sigma_1\ Z_\sigma^{(1)}(1) + 6g_o\sigma_{3+1}Z_\sigma^{(1)}(3+1)$$
$$+ 2g_o \int \frac{d^4k_2}{i(2\pi)^4} [9\sigma_2\ \sigma_{3+2} + (N-1)\ \pi_2\ \pi_{3+2} + 9\sigma_2\ \sigma_{1+2}$$
$$+ (N-1)\ \pi_2\ \pi_{1+2} + 9\sigma_2\ \sigma_{3+1+2} + (N-1)\ \pi_2\ \pi_{3+1+2}$$
$$+ 4(g_o\ n^2)(27\sigma_2\ \sigma_{3+2}\ \sigma_{2-1} + (N-1)\ \pi_2\ \pi_{3+2}\ \pi_{2-1})]\}$$

The diagrams for this function are displayed in Figure 7.

All the necessary calculation has now been done in order to find $G^{(2)}_{\alpha\beta}(k^2;C)$ which we will represent in the form

$$G^{(2)}_{\alpha\beta}(k^2;C) = g_o^2 [\pi_k^2 Z_\pi^2(k)(\delta_{\alpha\beta} - \frac{n_\alpha n_\beta}{n^2}) + \sigma_k^2 Z_\sigma^{(2)}(k) \frac{n_\alpha n_\beta}{n^2}] \qquad (5.8)$$

straightforward but tedious calculation yields the following expressions for $Z_\pi^2(k)$ and $Z_\sigma^{(2)}(k)$:

$$\begin{aligned} Z_\pi^{(2)}(k) = {} & \frac{C^{(1)^2} n^2}{g_o} \\ & + \frac{2C^{(2)} n^2}{g_o} \\ & + \pi_o Z_\pi^{(1)}(k)^2 \\ & + (N+1) B_\pi^{(2)} + B_\sigma^{(2)} \\ & + 8C^{(1)} n^2 \int \frac{d^4k_1}{i(2\pi)^4} \pi_1\sigma_{1-k} \\ & + 8(g_o n^2) \int \frac{d^4k_1}{i(2\pi)^4} \frac{d^4k_2}{i(2\pi)^4} [(N+1)\pi_1\pi_2 \sigma_{1+k} \pi_{1+k+2} \\ & + 3 \pi_1\sigma_2\sigma_{1+k} \sigma_{1+k+2} \\ & + 2\sigma_1\sigma_2 \pi_{1+k} \pi_{1+k+2} \\ & + \sigma_1 \pi_{1+k} \sigma_2 \pi_{k+2} \\ & + 2 \int \frac{d^4k_1}{i(2\pi)^4} \frac{d^4k_2}{i(2\pi)^4} [(N+1) \pi_1\pi_2 \pi_{1+k+2} \\ & + \pi_1\sigma_2 \sigma_{1+k+2}] \end{aligned} \qquad (5.9)$$

$$+ 4(g_o n^2) \int \frac{d^4k_1}{i(2\pi)^4} \; \sigma_1 \pi_{k+1} \, [\sigma_1 Z_\sigma^{(1)}(1) + \pi_{k+1} Z_\pi^{(1)}(k+1)]$$

$$+ (2g_o n^2)^2 \int \frac{d^4k_1}{i(2\pi)^4} \frac{d^4k_2}{i(2\pi)^4} \; 4\sigma_1 \pi_{k+1} (3 \sigma_2 \pi_{k+2} \sigma_{2-1}$$

$$+ \pi_2 \sigma_{k+2} \pi_{2-1})$$

$$Z_\sigma^{(2)}(k) = 6 \frac{C^{(2)} n^2}{g_o} \tag{5.10}$$

$$+ 3 \frac{C^{(1)^2} n^2}{g_o}$$

$$+ \sigma_o Z_\sigma^{(1)}(k)^2$$

$$+ (N-1) B_\pi^{(2)} + 3 \beta_\sigma^{(2)}$$

$$+ 4C^{(1)} n^2 \int \frac{d^4k_1}{i(2\pi)^4} [9\sigma_1\sigma_{1-k} + (N-1) \pi_1\pi_{1-k}]$$

$$+ 4g_o n^2 \int \frac{d^4k_1}{i(2\pi)^4} [9\sigma_1\sigma_{k+1} \sigma_1 Z_\sigma^{(1)}(1) + (N-1) \pi_1\pi_{k+1} Z_\pi^{(1)}(1)\pi_1]$$

$$+ \int \frac{d^4k_1}{i(2\pi)^4} \frac{d^4k_2}{i(2\pi)^4} \{ 6 \sigma_1\sigma_2\sigma_{k+1+2}$$

$$+ 2(N-1) \sigma_1 \pi_2 \pi_{k+1+2}$$

$$+ (g_o n^2)[54 \sigma_1\sigma_{k+1} \sigma_2 \sigma_{+2}$$

$$+ 12(N-1) \sigma_1\sigma_{k+1} \pi_2\pi_{k+2}$$

$$+ 2(N^2-1) \pi_1\pi_{k+1} \pi_2 \pi_{k+2}$$

$$+ 216 \sigma_1\sigma_{k+1} \sigma_2\sigma_{1+2}$$

$$+ 24(N-1)\ \sigma_1\sigma_{k+1}\ \pi_2\pi_{1+2}$$

$$+ 16(N-1)\ \pi_1\pi_{k+1}\ \pi_2\sigma_{k+1+2}\ ]$$

$$+ 8(g_o\eta^2)^2[81\sigma_1\sigma_{1+k}\sigma_2\sigma_{2+k}\ \sigma_{2-1}$$

$$+ 6(N-1)\ \sigma_1\sigma_{k+1}\ \pi_2\pi_{k+2}\ \pi_{2-1}$$

$$+ (N-1)\pi_1\ \pi_{k+1}\ \pi_2\ \pi_{k+2}\ \sigma_{2-1}\ ]\}$$

The graphical representations of these functions are given in Figure 9 and 10.

Accurate to the two loop approximation we may write the unrenormalized propagator function as

$$G_{\lambda\beta}(p^2;C) = [\pi_p^{-1} - g_o\ Z_\pi^{(1)}(p) - g_o^2[Z_\pi^{(2)}(p) - \pi_p(Z_\pi^{(1)}(p))^2]]^{-1}[\delta_{\lambda\beta} - \frac{\eta_\lambda\eta_\beta}{\eta^2}]$$

$$+ [\sigma_p^{-1} - g_o Z_\sigma^{(1)}(p) - g_o^2[Z_\sigma^{(2)}(p) - \sigma_p(Z_\sigma^{(1)}(p)^2]]^{-1}\frac{\eta_\lambda\eta_\beta}{\eta^2} \qquad (5.11)$$

$$\equiv G_\pi^{-1}[\delta_{\lambda\beta} - \frac{\eta_\lambda\eta_\beta}{\eta^2}] + G_\sigma^{-1}\ \frac{\eta_\lambda\eta_\beta}{p^2}$$

Tedious but straightforward calculation establishes that

$$G_{\lambda\beta}(p^2;0)\ \Big|_{p\to 0} \propto \frac{1}{p^2}[\delta_{\lambda\beta} - \frac{\eta_\lambda\eta_\beta}{\eta^2}] \ .$$

Consequently, the Goldstone theorem is valid in this order. This serves as a delicate test of the consistency of our technique. $G_{\lambda\beta}(p;C)$ may be expressed in terms of renormalized quantities in the usual way. For example, from (5.11) it follows that

$$M_\pi^2 = \mu_\pi^2 - g_o Z_\pi^{(1)}(M_\pi) - g_o^2[Z_\pi^{(2)}(M_\pi) - \pi_{M_\pi}(Z_\pi^{(1)}(M_\pi))^2]$$

and

$$Z_\pi^{-1} = -\left[\frac{d}{dp^2} G_\pi^{-1}\right]_{p^2=M_\pi^2} = 1 + g_o \left[\frac{d}{dp^2} Z_\pi^{(1)}(p^2)\right]_{p^2=M_\pi^2}$$

$$+ g_o^2 \frac{d}{dp^2} \left[Z_\pi^2(p) - \bar{\pi}_p (Z_\pi^{(1)}(p))^2\right]_{p^2=M_\pi^2} .$$

Similar expressions can be derived for the σ propagator taking into account that this particle is unstable. The explicit form of these quantities is not very informative except that it can be established that the divergent parts of M_π^2 and M_σ^2 and Z_π^{-1} and Z_σ^{-1} are identical so that the renormalized Lagrangian may be written so as to maintain chiral symmetry except for the linear breaking term if the coupling constant renormalizations are identical for all interactions. The calculations of the two loop approximation vertices and scattering amplitudes, the coupling constant renormalizations, and the numerical values of the σ width are in progress and will be presented in another paper.

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APPENDIX

$$A^{(0)}_{\alpha\beta\gamma\delta}(x,y,z,w;C)$$

$$= 4g_0^2\int \frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}\, e^{-ik_2(x-y)}\, e^{-ik_3(x-z)}\, e^{-ik_4(x-w)}$$

$$\times \Big\{\pi_{2+3+4}\,\pi_4\,\sigma_{2+3}\,\pi_3\pi_2\,\eta^2(\delta_{\alpha\delta}-\frac{\eta_\alpha\eta_\delta}{\eta^2})(\delta_{\gamma\beta}-\frac{\eta_\gamma\eta_\beta}{\eta^2})$$

$$+\pi_{2+3+4}\,\sigma_4\,\pi_{2+3}\sigma_3\pi_2\,\eta_\gamma\eta_\delta\,(\delta_{\alpha\beta}-\frac{\eta_\alpha\eta_\beta}{\eta^2})$$

$$+\pi_{2+3+4}\,\sigma_4\pi_{2+3}\pi_3\sigma_2\,\eta_\beta\eta_\delta\,(\delta_{\alpha\gamma}-\frac{\eta_\alpha\eta_\gamma}{\eta^2})$$

$$+\sigma_{2+3+4}\,\pi_4\pi_{2+3}\sigma_3\pi_2\,\eta_\alpha\eta_\gamma\,(\delta_{\delta\beta}-\frac{\eta_\delta\eta_\beta}{\eta^2})$$

$$+\sigma_{2+3+4}\pi_4\pi_{2+3}\pi_3\sigma_2\,\eta_\alpha\eta_\beta\,(\delta_{\gamma\delta}-\frac{\eta_\gamma\eta_\delta}{\eta^2})$$

$$+3\sigma_{2+3+4}\sigma_4\sigma_{2+3}\pi_3\pi_2\,\eta_\alpha\eta_\delta\,(\delta_{\gamma\beta}-\frac{\eta_\gamma\eta_\beta}{\eta^2})$$

$$+3\pi_{2+3+4}\pi_4\sigma_{2+3}\sigma_3\sigma_2\,\eta_\beta\eta_\gamma\,(\delta_{\alpha\delta}-\frac{\eta_\alpha\eta_\delta}{\eta^2})$$

$$+9\sigma_{2+3+4}\sigma_4\sigma_{2+3}\sigma_3\sigma_2\,\frac{\eta_\alpha\eta_\beta\eta_\gamma\eta_\delta}{\eta^2}\Big\}$$

$+$ two cyclic part

$$+ 2g_0 \int \frac{d^4k_2}{(2\pi)^4} \frac{d^4k_3}{(2\pi)^4} \frac{d^4k_4}{(2\pi)^4} e^{-ik_2(x-y)} e^{-ik_3(x-z)} e^{-ik_4(x-w)}$$

$$\times \Big\{ \pi_{2+3+4}\, \pi_4 \pi_3 \pi_2 \, (\delta_{\alpha\beta} - \frac{\eta_\alpha \eta_\beta}{\eta^2})(\delta_{\gamma\delta} - \frac{\eta_\gamma\eta_\delta}{\eta^2})$$

$$+ \pi_{2+3+4}\, \pi_4 \pi_3 \pi_2 \, (\delta_{\alpha\gamma} - \frac{\eta_\alpha \eta_\gamma}{\eta^2})(\delta_{\beta\delta} - \frac{\eta_\beta\eta_\delta}{\eta^2})$$

$$+ \pi_{2+3+4}\, \pi_4 \pi_3 \pi_2 \, (\delta_{\alpha\delta} - \frac{\eta_\alpha \eta_\delta}{\eta^2})(\delta_{\beta\gamma} - \frac{\eta_\beta\eta_\gamma}{\eta^2})$$

$$+ \pi_{2+3+4}\, \sigma_4 \sigma_3 \pi_2 \, \frac{\eta_\gamma \eta_\delta}{\eta^2} (\delta_{\alpha\beta} - \frac{\eta_\alpha\eta_\beta}{\eta^2})$$

$$+ \pi_{2+3+4}\, \sigma_4 \pi_3 \sigma_2 \, \frac{\eta_\beta \eta_\delta}{\eta^2} (\delta_{\alpha\gamma} - \frac{\eta_\alpha\eta_\gamma}{\eta^2})$$

$$+ \sigma_{2+3+4}\, \pi_4 \pi_3 \sigma_2 \, \frac{\eta_\alpha \eta_\beta}{\eta^2} (\delta_{\gamma\delta} - \frac{\eta_\gamma\eta_\delta}{\eta^2})$$

$$+ \sigma_{2+3+4}\, \pi_4 \sigma_3 \pi_2 \, \frac{\eta_\alpha \eta_\gamma}{\eta^2} (\delta_{\beta\delta} - \frac{\eta_\beta\eta_\delta}{\eta^2})$$

$$+ \pi_{2+3+4}\, \pi_4 \sigma_3 \sigma_2 \, \frac{\eta_\beta \eta_\gamma}{\eta^2} (\delta_{\alpha\delta} - \frac{\eta_\alpha\eta_\delta}{\eta^2})$$

$$+ \sigma_{2+3+4}\, \sigma_4 \pi_3 \pi_2 \, \frac{\eta_\alpha \eta_\delta}{\eta^2} (\delta_{\beta\gamma} - \frac{\eta_\beta\eta_\gamma}{\eta^2})$$

$$+ 3\sigma_{2+3+4}\, \sigma_4 \sigma_3 \sigma_2 \, \frac{\eta_\alpha \eta_\beta \eta_\gamma \eta_\delta}{(\eta^2)^2} \Big\}$$

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FIGURE CAPTIONS

Fig. 1 Zero Order One and Two Point Function

Fig. 2 Zero Order Three Point Function

Fig. 3 Zero Order Four Point Function

Fig. 4 First Order One Point Function

Fig. 5 First Order σ propagator

Fig. 6 First Order π propagator

Fig. 7 First Order Three Point Function

Fig. 8 Second Order One Point Function

Fig. 9 Second Order π propagator

Fig. 10 Second Order σ propagator

FIG. 1

FIG. 2

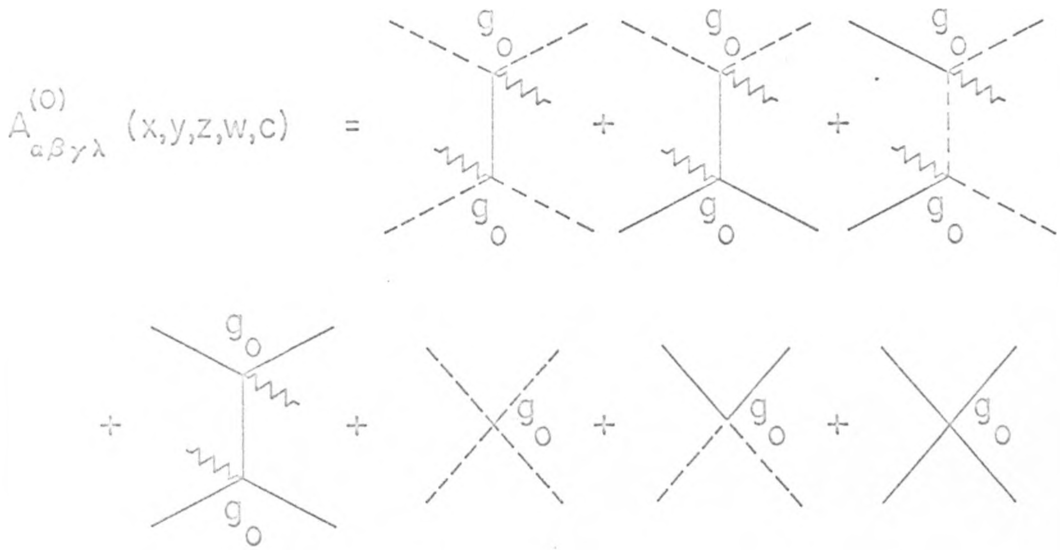

FIG. 3

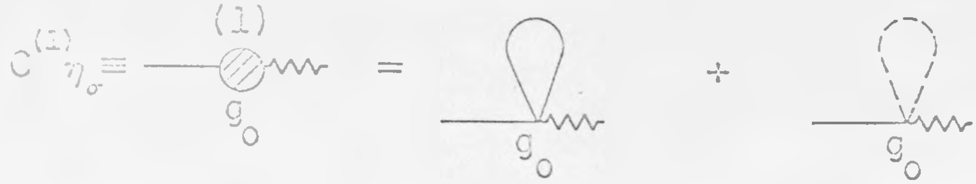

FIG. 4

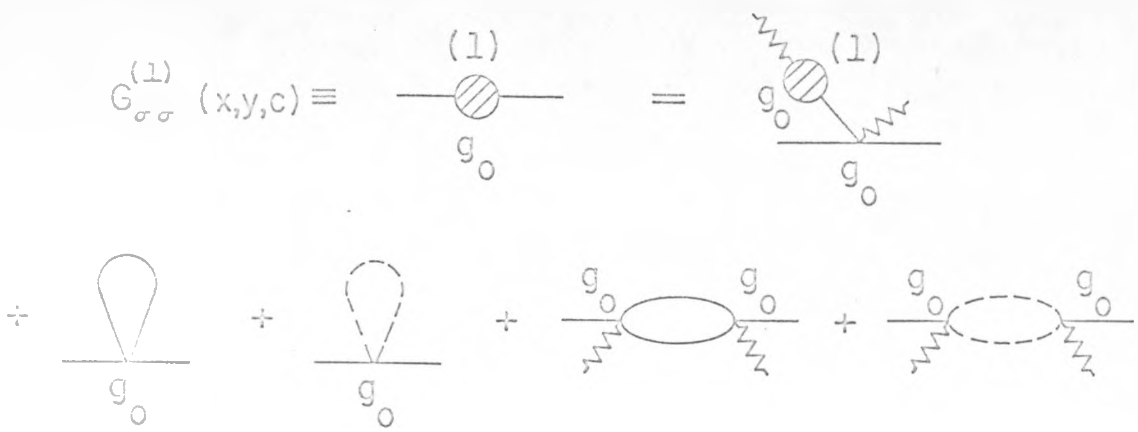

FIG. 5

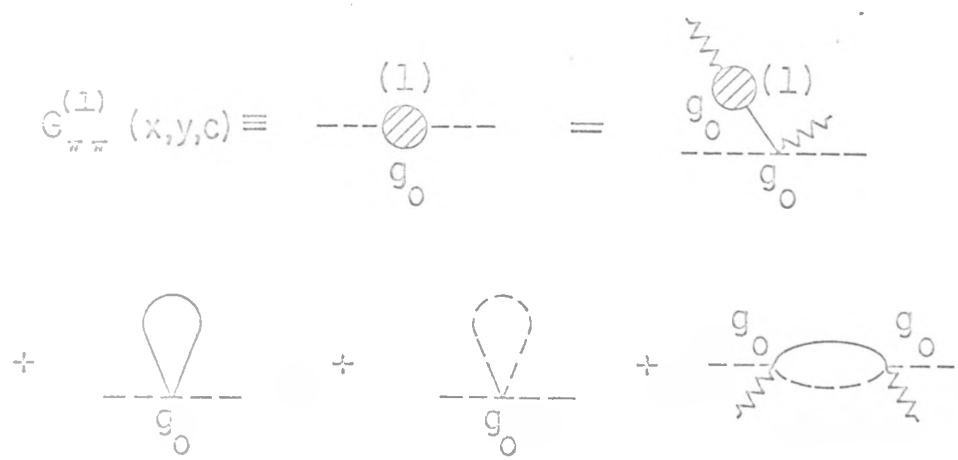

FIG. 6

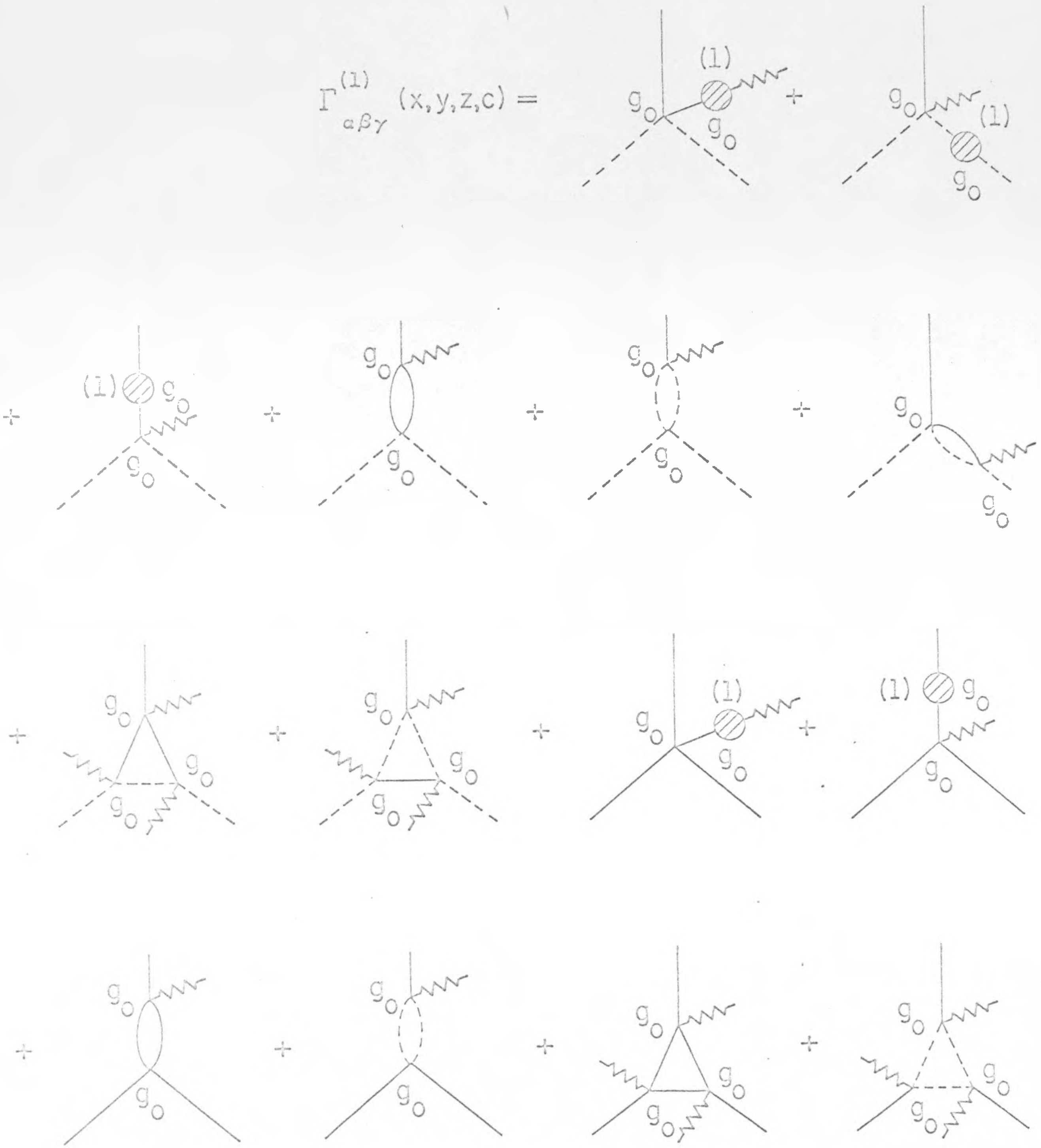

FIG.7

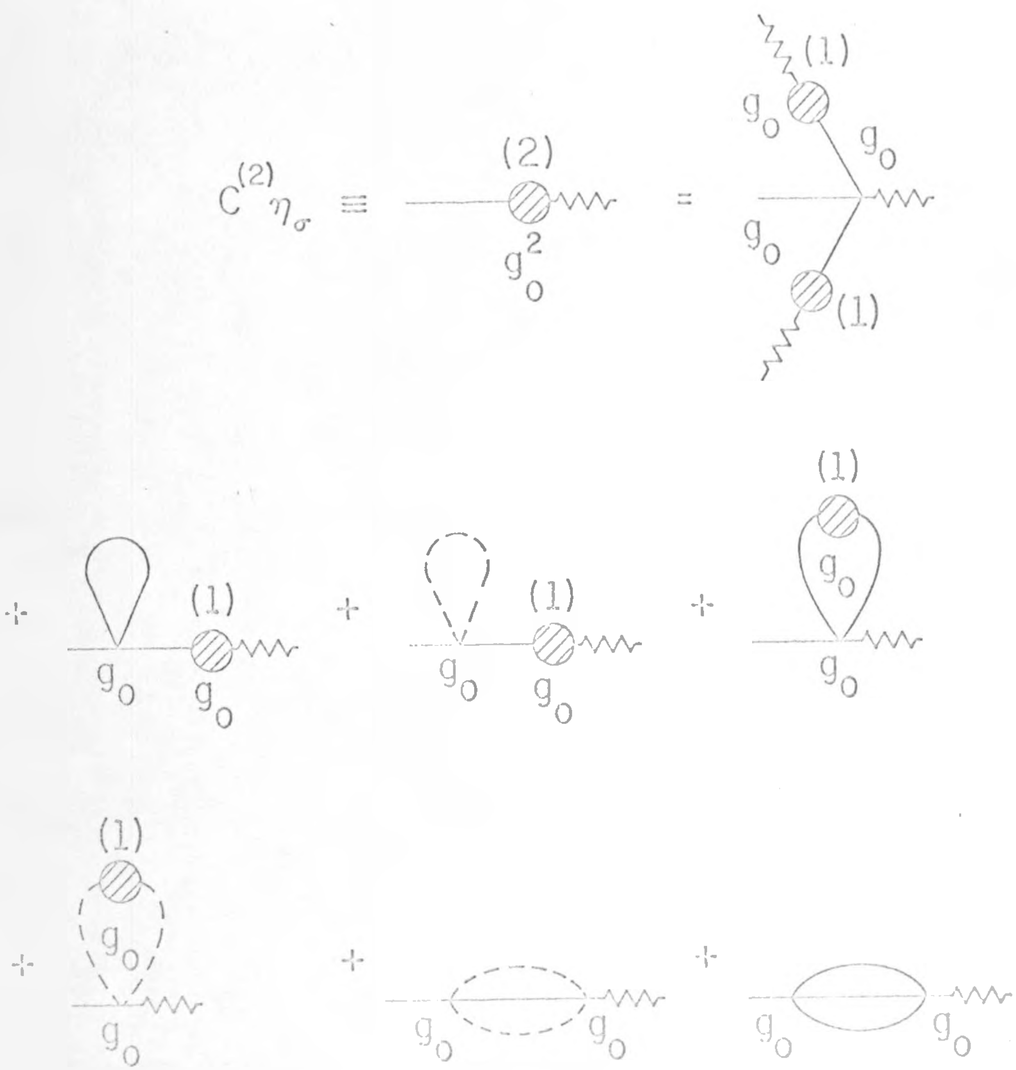

FIG. 8

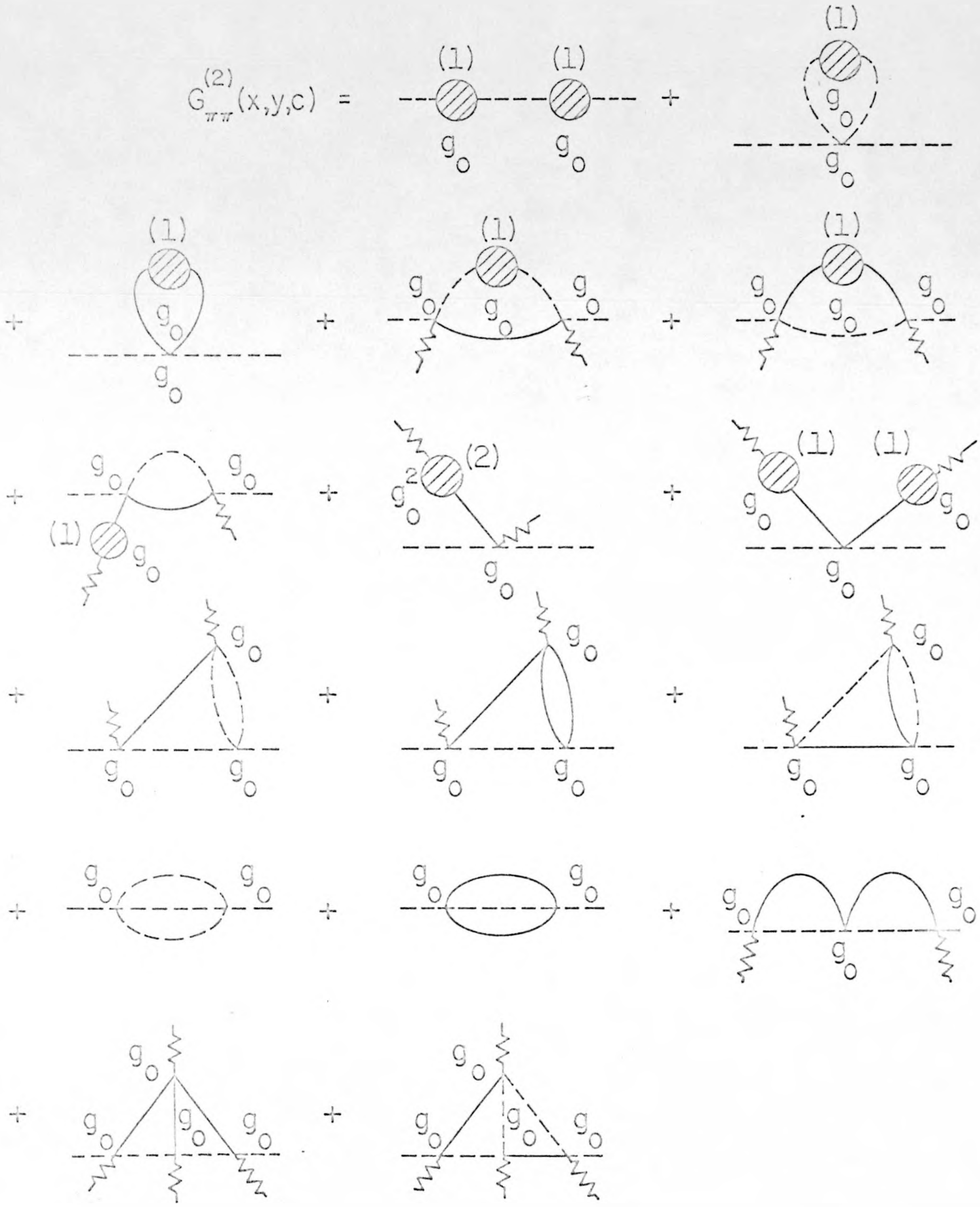

FIG. 9

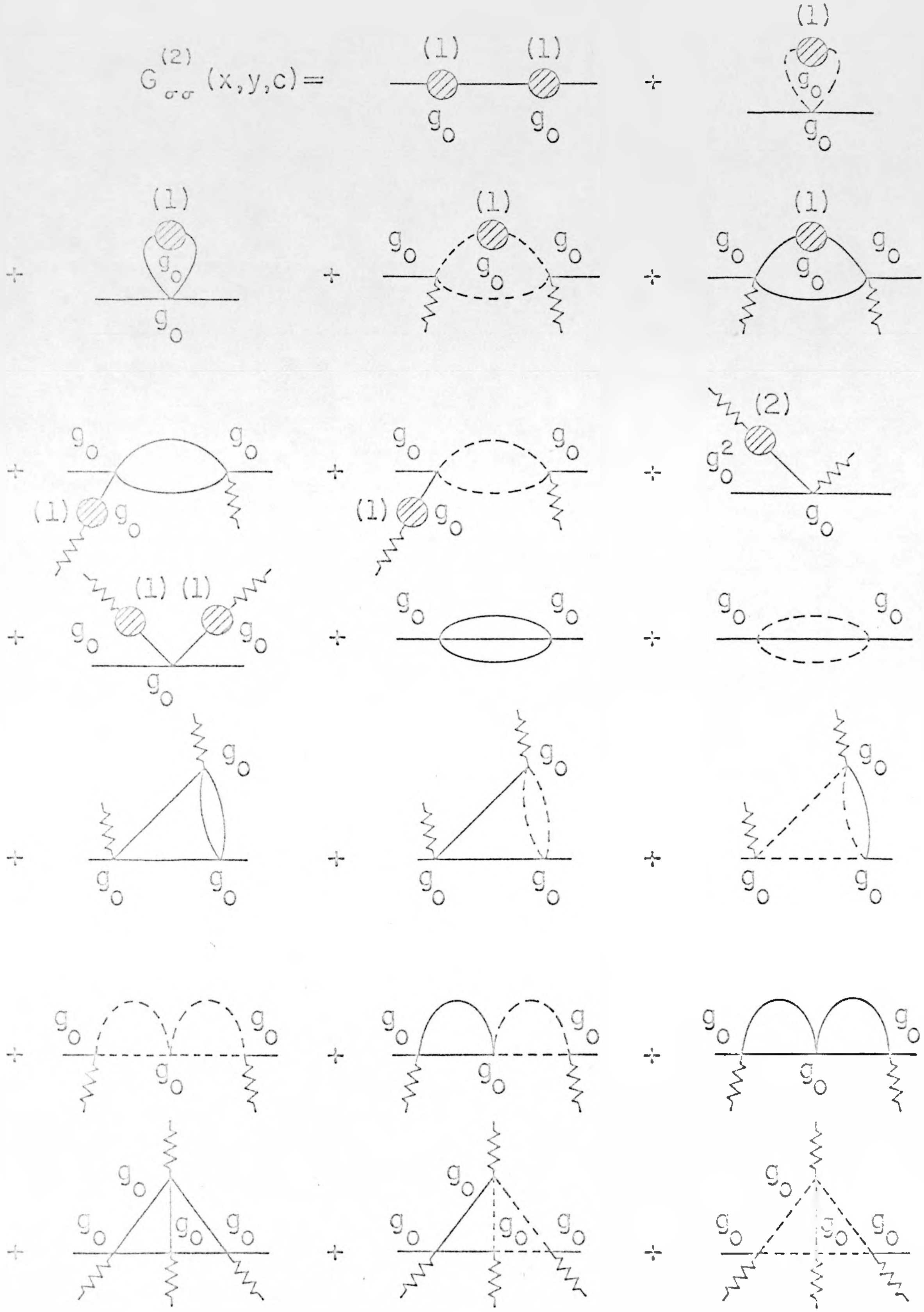

FIG. 10