

# A Study of the Transition to Turbulence in a Bed of 67 Spherical Pebbles

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## Abstract

Packed beds are commonly found in many engineering systems and have been widely studied for decades. A relatively new packed bed system is the Pebble Bed Reactor, a type of generation-IV nuclear reactor. Unlike many of the packed beds encountered in chemical and process engineering applications, Pebble Bed Reactors are larger and operate at significantly higher Reynolds numbers. As a result of these differences, there is a very limited amount of information on the detailed flow physics that exist in these complex geometries. This work seeks to contribute to a growing database of flow data for Pebble Bed Reactor systems by performing Direct Numerical Simulations of the flow in an experimental bed of 67 pebbles for a range of conditions. Simulations are performed at a Prandtl number of 0.66 and Reynolds numbers from 300 - 600. These Reynolds numbers are chosen to gain additional knowledge on the spatial development of turbulence in these systems. Analysis of the Turbulent Kinetic Energy, turbulence anisotropy, and Turbulent Heat Flux is performed. Results demonstrate significant development of the TKE across the tested range of Reynolds numbers. Examination of both

the TKE and THF reveal that development first occurs near the center of the bed and propagates radially as the flow moves further into the bed. Notable regions of negative production of turbulent kinetic energy are observed in regions where flow accelerates around pebble contact points. These regions are found to coincide with regions of 1-component turbulence.

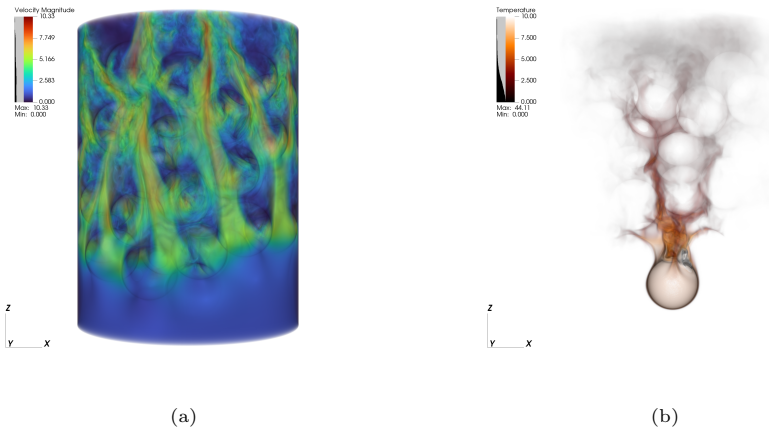
**Keywords:** Packed Beds, Onset of Turbulence, Direct Numerical Simulation, Turbulence Anisotropy

## 1 Introduction

Fluid flow through a packed bed is a critical component of many engineering systems, seen in chemical reactors, water purification, soil systems, and nuclear reactors. Understanding the flow of fluid through a packed bed has been a historically difficult problem due to the geometric complexity in these systems. Many voids with unique shapes and arrangements are present, creating complex flowpaths. Due to the prevalence of packed beds in a variety of engineering applications, however, many researchers aim to gain a better understanding of flow in these systems to improve the safety and efficiency in these porous media.

A large number of works in porous media research have aimed at gaining an understanding of the porosity, pressure drop, and heat transfer in porous media. Early works such as that of Kozeny [1] and Carman [2] studied creeping flow through porous media, as would be found in soils and some chemical reactors. Their contributions led to the derivation of equations to predict the pressure drop through packed beds at these conditions. Ergun later followed up this initial work with a more comprehensive pressure drop equation that accounts for both viscous and inertial losses [3]. Ranz was one of the first to suggest a heat transfer correlation for use in packed beds [4]. Researchers such as Mueller [5] and De Klerk [6] have produced correlations capable of predicting both the average packing fraction and the radial packing fraction profile that exists in packed beds. Many other correlations have improved these early correlations for the pressure drop and heat transfer in packed beds, expanding the range of valid Reynolds numbers, Prandtl numbers, and bed geometries [7–11].

One application of packed beds that has seen growing interest in recent years is the Pebble Bed Reactor (PBR). Designs such as the PBMR-400, Xe-100, KP-FHR, and HTR-PM have been through a wide range of development and deployment. These PBRs use a bed of randomly packed spherical graphite pebbles embedded with nuclear fuel to form the reactor core. There are a number of advantages to using spherical fuel, although they do come with the tradeoff of increased geometric complexity. One important difference between PBRs and the more thoroughly-studied applications of packed beds is that the Reynolds number, defined as  $Re = u_s \rho D_{peb} / \mu$ , in PBRs tends to be much higher, in the range of  $Re = 100's - 10,000's$  compared to  $Re < 100$  for most other applications. This increase in Reynolds number is a result of the necessity to provide a coolant mass flow rate that is high enough to sufficiently cool the reactor considering the substantially higher power density in a nuclear reactor compared to



**Fig. 1** Instantaneous velocity and temperature fields for a bed of 67 pebbles at  $Re = 600$ .

other applications of porous media. As a consequence, the flow is typically highly turbulent, and pressure drag dominates the unrecoverable losses. Furthermore, the pebble diameter to bed diameter ratio is quite high due to the large size of the pebbles encountered in nuclear reactors. This regime, which is quite far from what is typically encountered in chemical and process engineering, has only recently become the subject of detailed fluid dynamics studies.

In fact, obtaining a thorough understanding of the flow physics in a randomly packed pebble bed has historically been difficult due to the random and complex geometry. Many previous studies, both computational and experimental, have focused on analyzing the flow physics in packed beds often with a very small number of spheres or with an orderly arrangement of spheres. Forming a thorough understanding of the flow physics in randomly packed pebble beds is important for the development of lower order models, such as for turbulence modeling with Reynolds-Averaged Navier-Stokes (RANS) models or porous media models used for licensing. Advances in computing power have allowed for Direct Numerical Simulation (DNS) studies in randomly packed pebble beds to be performed. This work will focus on a DNS study of the flow in a packed bed of 67 pebbles, shown in Figure 1, at the onset of turbulence.

Experimental flow studies are difficult due to challenges in making measurements in the complex geometries present in PBRs. Earlier studies often focused on the flow past a single sphere [12]. Recent studies have utilized Particle Image Velocimetry (PIV) to gain a more detailed look at the flow in randomly packed beds [13]. A study by Lee [14] was one of the first to present experimental PIV data for the flow in a randomly packed bed. More recently, work by Nguyen [15, 16] has focused on the use of PIV to analyze the flow in packed beds with modal decomposition. They have also performed an analysis of cross-flow mixing in these geometries.

Computational studies of the flow in packed beds can also prove to be useful due to the ease of extracting data. The work by Shams [17, 18] was one of the first to perform a DNS of the flow in a packed bed. They focused on a small regular arrangement of five

102 pebbles and analyzed the flow and temperature characteristics to provide comparative  
 103 data for turbulence modeling. This work was later expanded on by Fick and Merzari  
 104 [19] who increased the size of the bed and also added a confining wall, as would be found  
 105 in a PBR. They still, however, used a regular arrangement of pebbles. Work by Das  
 106 [20] performed a DNS of a slender bed to examine the pressure drop and heat transfer  
 107 characteristics within the bed. A study by Yildiz [21] analyzed the flow in a bed of  
 108 146 pebbles and compared the flow to a corresponding experiment. A recent work by  
 109 Lanade [22] performed Large Eddy Simulations and RANS simulations for a slender  
 110 experimental bed. Other computational studies by Nguyen and Merzari [23, 24] have  
 111 performed LES simulations of large-scale randomly-packed pebble beds. None of these  
 112 studies have, however, looked in detail at the spatial evolution of turbulence in the bed  
 113 as a function of the Reynolds number. This is an important scientific and engineering  
 114 question, as during transient scenarios in advanced reactors the flow may experience  
 115 a range of conditions including laminarization. As heat transfer during transients is a  
 116 limiting factor in the design and licensing of pebble bed reactors, the effect of Reynolds  
 117 number on the spatial evolution of turbulence must be better understood.

118 Several other studies have been performed to gain insights into the development  
 119 of turbulence in porous media. Early DNS studies by Hill [25, 26] provided insight on  
 120 the development of turbulence in a regular face-centered cubic (FCC) arrangement of  
 121 spheres. These studies focused on the evolution of the flow in the inertial and transi-  
 122 tional regimes. Patil [27] performed an experimental study to analyze the evolution of  
 123 pore flow structures as the Reynolds number was increased. They identified regions of  
 124 tortuous channel flow, recirculating flow, and jet-like flow. More recently, a study by  
 125 He [28] performed a DNS study of an FCC arrangement of pebbles that covered the  
 126 inertial, transitional, and turbulent flow regimes. They performed analysis of the tur-  
 127 bulent kinetic energy budgets, where they found regions of negative production that  
 128 corresponded with large peaks in the pressure diffusion term.

129 This work seeks to add to the understanding of the flow in randomly packed beds  
 130 by performing a DNS for a range of conditions of the flow in a heated slender bed of 67  
 131 pebbles. Detailed comparisons against experimental isothermal data were conducted  
 132 in a previous study [29] and additional validation is conducted in this work. We focus  
 133 the analysis on the spatial evolution of turbulence in the bed as the Reynolds number  
 134 changes. In fact, understanding the factors that influence the flow in packed beds  
 135 can be crucial towards providing insight on how simplified models may be failing to  
 136 reproduce the more complex physics present in these systems. This manuscript details,  
 137 in particular, a DNS study of a bed of 67 randomly-packed pebbles at four Reynolds  
 138 numbers. An overview of these cases is found in Table 1. These Reynolds numbers are  
 139 chosen specifically to gain an understanding of how the budget terms evolve spatially  
 140 as turbulence develops in the flow. Therefore, Reynolds numbers of 300, 400, 500,  
 141 and 600 are chosen to capture the top of the transitional flow regime and the start  
 142 of the turbulent flow regime based on a preliminary scoping study and estimates  
 143 from earlier works [30]. The analysis focuses on the evolution of spatial budgets of  
 144 interest, including the Turbulent Kinetic Energy budgets and the Turbulent Heat Flux  
 145 budgets, as well as the evolution of anisotropy invariants. Analysis is mainly aimed  
 146 at understanding the production and dissipation terms of the budgets. We emphasize



| Case | Number of Pebbles | $D_{bed}/D_{peb}$ | $\epsilon_{avg}$ | Reynolds Number |
|------|-------------------|-------------------|------------------|-----------------|
| 1a   | 67                | 4.4               | 0.48             | 300             |
| 1b   | 67                | 4.4               | 0.48             | 400             |
| 1c   | 67                | 4.4               | 0.48             | 500             |
| 1d   | 67                | 4.4               | 0.48             | 600             |
| 2    | 1                 | 4.4               | —                | 500             |
| 3    | 67                | 4.4               | 0.48             | 500             |
| 4    | 67                | 4.4               | 0.48             | 500             |

**Table 1** Description of all cases used in this work. All cases are simulated with a Prandtl number of 0.66. Case 3 has a different random packing than cases 1 and 4. Case 4 includes an extended pipe at the inlet to allow the flow to develop before entering the bed.

again that this is, to our knowledge, the first study to present a DNS analysis of the transition to turbulence in a randomly-packed bed.

## 2 Methods

### 2.1 Introduction to NekRS

The high-fidelity simulations in this work were performed with NekRS, an open-source GPU-oriented spectral element CFD code developed at Argonne National Lab [31]. NekRS builds off of the well-established open-source code Nek5000 [32], inheriting its existing pre- and post-processing features. It demonstrates excellent scalability [33] and has been demonstrated in performing full-core CFD simulations [34].

The simulations performed in this work use the incompressible, constant-properties Navier-Stokes equations in dimensionless form:

$$\nabla \cdot u^* = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial t^*} + u^* \cdot \nabla u^* = -\nabla p^* + \frac{1}{Re} \nabla^2 u^* \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} + u^* \cdot \nabla T^* = \frac{1}{Pe} \nabla \cdot \nabla T^* + q''' \quad (3)$$

The nondimensionalization of variables follows the following scheme.

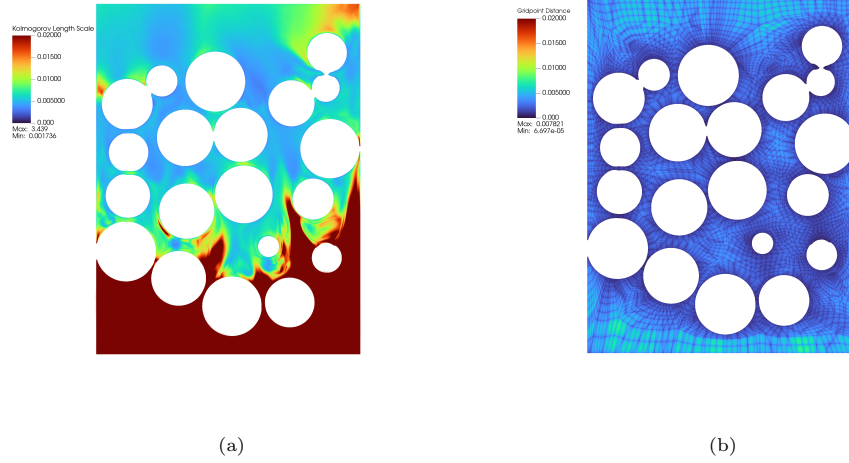
$$x^* = \frac{x}{D_{peb}} \quad (4)$$

$$u^* = \frac{u}{u_{inlet}} \quad (5)$$

$$t^* = \frac{t u_{inlet}}{D_{peb}} \quad (6)$$

$$p^* = \frac{p}{\rho u_{inlet}^2} \quad (7)$$

$$T^* = \frac{T - T_0}{\Delta T_{reference}} \quad (8)$$



**Fig. 2** (a) Calculated Kolmogorov length scale for the  $Re = 600$  case and (b) Calculated distance between mesh gridpoints at polynomial order 9.

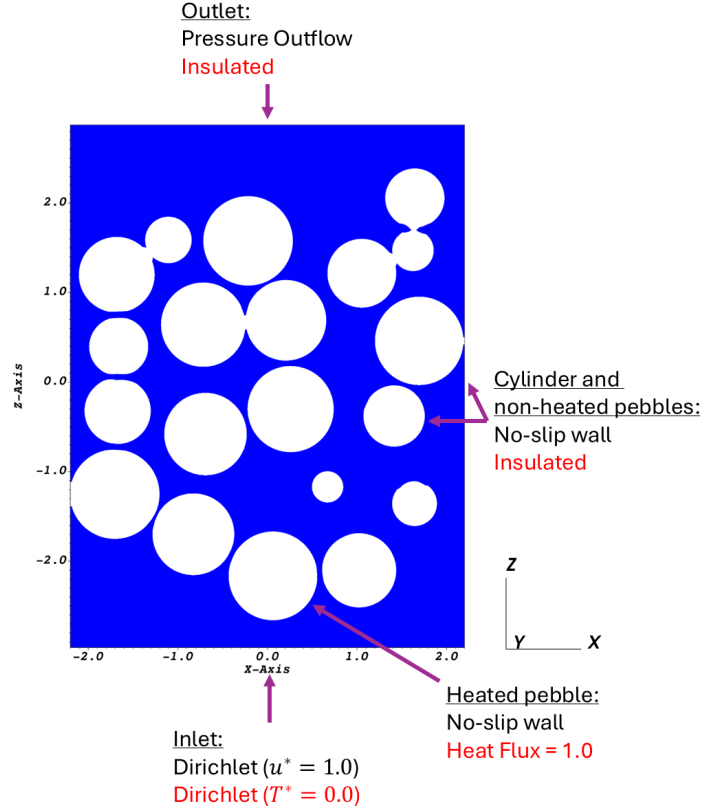
NekRS has been extensively validated for both pebble bed and non-pebble bed applications in a number of previous studies [21, 29, 35, 36]. It is worth noting that the cases in this work are targeting forced convection in PBRs. Therefore, buoyancy effects are not significant and the temperature may be treated as a passive scalar. Future works may examine the TKE and THF for cases in which buoyancy plays an important role, but it is out of the scope of the current work.

## 2.2 Meshing and Case Setup

The pebble centers for this work were retrieved from experimental measurements [37]. In the experiment, acrylic pebbles were packed into an acrylic cylinder with an aspect ratio of 4.4. PIV was then used to measure the pebble centers for a section of 67 pebbles at the top of the bed. The pebble centers were used to generate a mesh for the fluid region using a voronoi cell approach [38]. The mesh was generated sufficiently fine to resolve the Kolmogorov length scales. This comparison is shown in Figure 2 where it is evident that the distance between gridpoints is sufficiently small compared to the Kolmogorov scales. Additionally, the mesh is designed to resolve the boundary layer near the wall with the maximum wall  $y^+$  value calculated to be 0.8342. This mesh resolution is comparable to those that have previously been used for DNS simulation of flow in PBRs [21].

The resulting mesh consisted of 722,043 spectral elements. At the polynomial order of 9 that was used, this results in 722,043,000 total mesh gridpoints. Cases were run on the Summit supercomputer, with each simulation taking roughly 70 hours of simulation time on 270 Nvidia V100 GPU's.

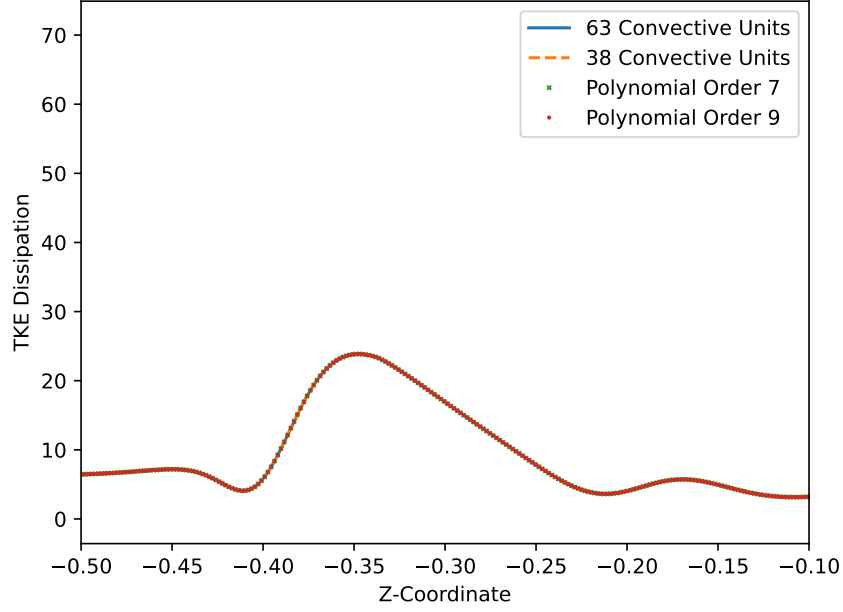
At the inlet of the NekRS cases, a Dirichlet boundary condition was applied for both the velocity and temperature with values of 1.0 and 0.0 respectively. At the outlet, a turbulent outflow boundary condition [39] was applied along with a zero-Neumann



**Fig. 3** Sketch of the computational domain with labeled boundary conditions and coordinate axes for reference.

boundary condition for the temperature. No-slip boundary conditions were applied for the velocity on the pebble surfaces and the reflector wall. A single pebble was heated with a constant heat flux of 1.0. The rest of the pebble surfaces and the reflector wall were given an insulated boundary condition. A domain sketch with labeled boundary conditions and reference coordinates is shown in Figure 3.

The flow was initialized for several flow-through-times before averaging began. The first, second, and third-order statistics were then averaged until time-convergence was achieved. Shown in Figure 4, it is clear that the averaging window is sufficiently long to converge the third-order statistics. Figure 4 also shows the convergence of the mesh, where it is evident that a polynomial order of 9 is sufficient to converge the results. All simulations in this work were therefore run at a polynomial order of 9 with an averaging window of 63 convective units or more.

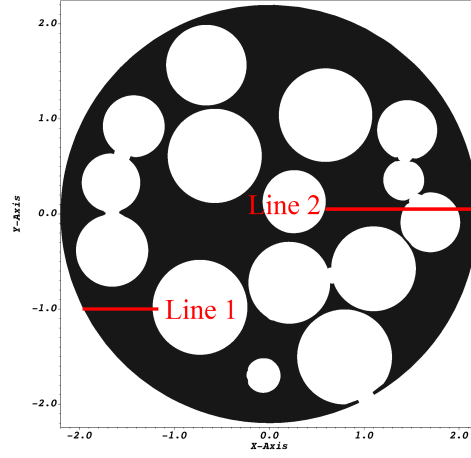


**Fig. 4** Time and mesh convergence for the TKE dissipation in an axial line sample near the center of the domain for case 1d.

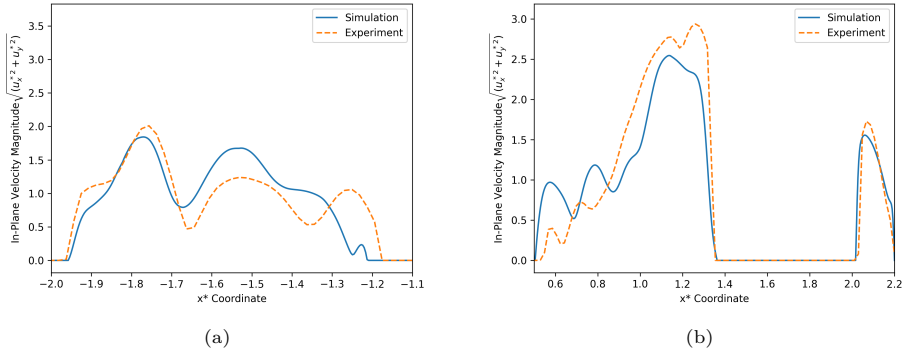
### 3 Validation

It is important to validate the codes and models that are employed when using a numerical tool to generate flow data. The bed of 67 pebbles that is used in this work originated from an experiment at Texas A&M University. In this experiment, the flow velocity and RMS values were measured with PIV in a spanwise sampling plane at  $Re = 600$ . The sampling plane is positioned in the axial center of the bed ( $z^* = 0.0$ ), and measurements occurred over two sampling lines, shown in Figure 5.

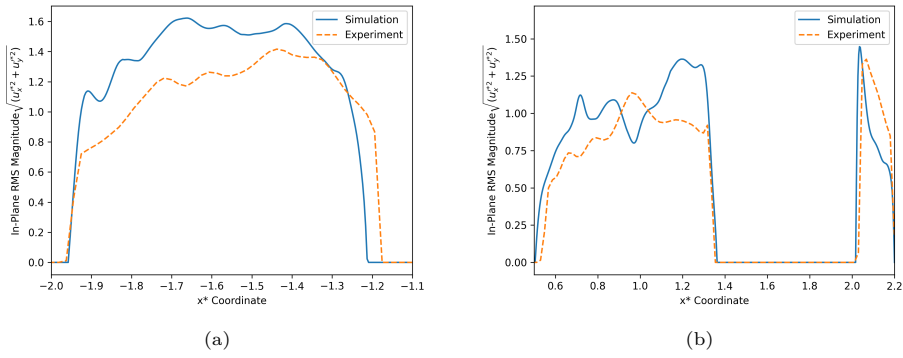
A comparison of the in-plane velocity magnitude between the experimental measurements and the NekRS simulation is shown in Figure 6. Acceptable agreement is achieved between the simulation and experiment. Comparisons of this type are typically difficult due to uncertainties in the measured location of the experimental pebbles, as even small changes can have a large impact on the flow field. A comparison of the in-plane RMS magnitude is also shown in Figure 7. Agreement in the second-order statistics is similar to the first-order statistics. These comparisons are consistent with previous experimental comparisons that have been made for packed beds in [21, 22, 29].



**Fig. 5** Diagram of the PIV sampling plane used in the experiment. The lines for data extraction are shown in red.



**Fig. 6** In-Plane velocity magnitude over two sampling lines for the 67-pebble validation study.



**Fig. 7** In-Plane RMS magnitude over two sampling lines for the 67-pebble validation study.

## 4 Results

This section presents the time-averaged budget results for the TKE and THF. Analysis of the TKE budgets is performed in Section 4.1, with the THF analyzed in Section 4.3

### 4.1 Analysis of the turbulent kinetic energy budget

The turbulent kinetic energy (TKE) is a parameter that describes the mean kinetic energy carried by the eddies in turbulent flow. TKE is typically produced through shear, forcing, or friction. It is then transferred down the energy cascade where it is eventually dissipated by viscous forces in the smallest eddies. The TKE can be determined as half of the trace of the Reynolds stress tensor:

$$k_e = \frac{1}{2} \langle u'_j u'_j \rangle \quad (9)$$

The TKE can be represented as the sum of the individual contributing mechanisms to produce the TKE equation. These terms are known as the budgets of the TKE

$$\begin{aligned} \underbrace{\frac{\partial k_e}{\partial t}}_{\text{TKE Derivative}} + \underbrace{\overline{u_j} \frac{\partial k_e}{\partial x_j}}_{\text{Advection}} = & \underbrace{-\frac{1}{\rho} \frac{\partial \overline{u'_i p'}}{\partial x_j}}_{\text{Pressure Diffusion}} - \underbrace{\frac{1}{2} \frac{\partial \overline{u'_j u'_j u'_i}}{\partial x_i}}_{\text{Turbulent Transport}} + \underbrace{\nu \frac{\partial^2 k_e}{\partial x_j^2}}_{\text{Molecular Transport}} \\ & - \underbrace{\overline{u'_i u'_j} \frac{\partial \overline{u_i}}{\partial x_j}}_{\text{Production}} - \underbrace{\nu \frac{\partial \overline{u'_i}}{\partial x_j} \frac{\partial \overline{u'_i}}{\partial x_j}}_{\text{Dissipation}} \end{aligned} \quad (10)$$

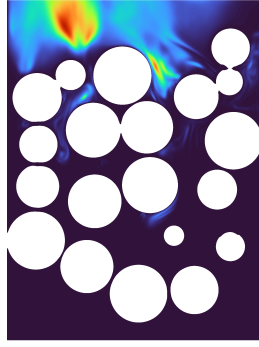
This work will focus on the production and dissipation terms.

The turbulent kinetic energy budgets in a packed bed can provide useful information about the flow physics in various regions of the bed. For this reason, it can be a helpful metric for analyzing differences in the flow at several locations in the bed.

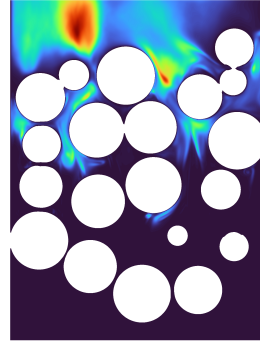
A centerplane slice of the TKE in each of the four cases is presented in Figure 8. Here the development of turbulence with increasing Reynolds number is evident, as the TKE is significantly smaller in Case 1a and increases substantially when moving to Case 1b. Cases 1c and 1d then see further development of the TKE. It would also appear as though it the TKE takes slightly longer to develop in Case 1a, as the values of the TKE remain low until the flow has moved through three layers of pebbles (axial distance of  $3Z^*$  above the bottom pebbles). The TKE appears to develop more substantially lower in the bed for the cases at higher Reynolds numbers, with case 1d developing soon after the flow has moved past the first layer of pebbles.

### 4.2 Discussion on Case Setup

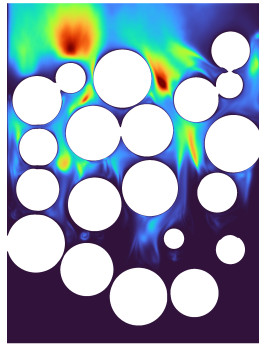
While examining the results of the present study, it is important to consider the effects of the case setup and how it may affect the observed results. For this reason, several other cases were tested to examine the effect on the TKE field. This provides insight on how the turbulent characteristics of these simulations may differ from case to case.



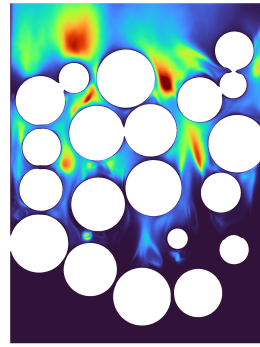
(a)  $Re = 300$  (Case 1a)



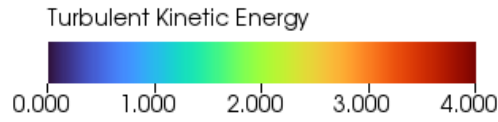
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)



(d)  $Re = 600$  (Case 1d)



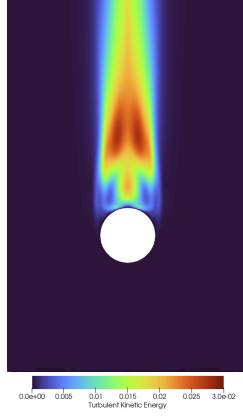
(e)

**Fig. 8** Centerplane slice of the TKE at  $Re = 300$ ,  $400$ ,  $500$ , and  $600$ .

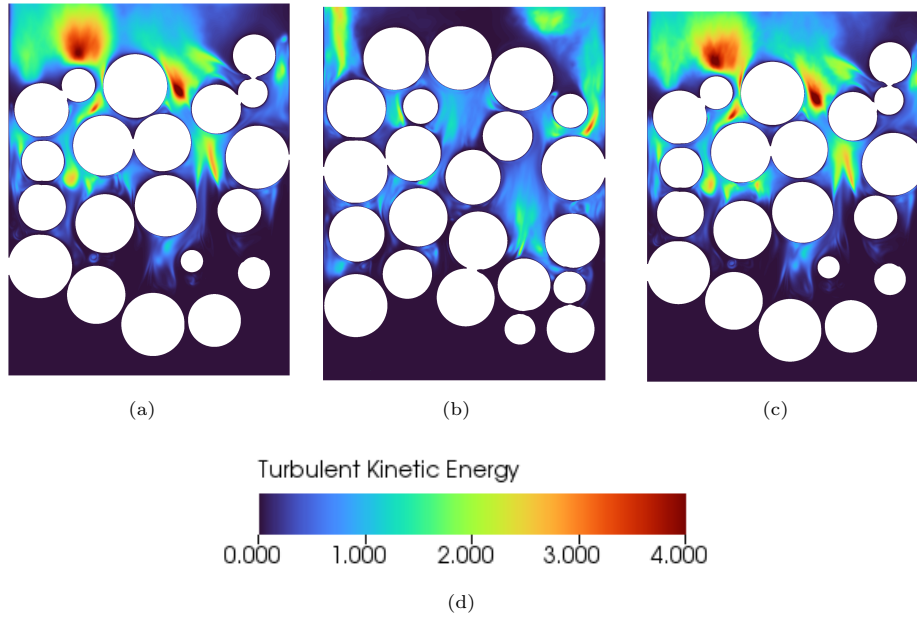
247 Flow around a single sphere at  $Re = 500$ , flow through another bed of 67 pebbles at  
 248  $Re = 500$ , and flow with a turbulent inlet boundary condition at  $Re = 500$  were all  
 249 examined for comparison. The TKE fields for each of these three cases are shown in  
 250 Figures 9 and 10.

251 Beginning with the case of the single sphere at  $Re = 500$ , it is clear that the  
 252 geometry of the randomly packed bed generates significantly more turbulence. There  
 253 is a small amount of TKE in the pebble's wake region, but this is nearly two orders



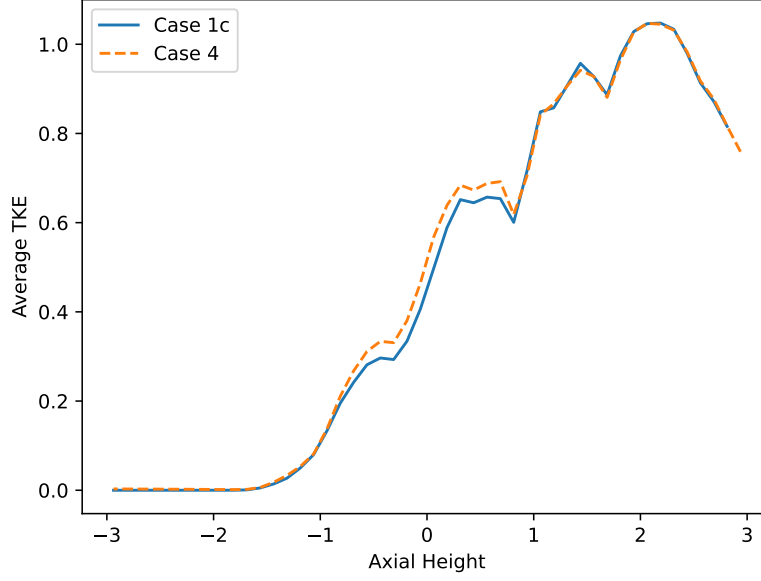


**Fig. 9** TKE field for case 2 of flow around a single sphere at  $Re = 500$ .



**Fig. 10** TKE fields for the cases of (a) flow through Case 1c at  $Re = 500$ , (b) flow through a differently-packed bed of 67 pebbles at  $Re = 500$  (Case 3), and (c) flow through the Case 1 geometry with a turbulent inlet boundary condition at  $Re = 500$  (Case 4).

254 of magnitude lower than the magnitude of the TKE in the comparative case 1c in  
 255 Figure 8. The case of flow through a different packed bed in Figure 10 demonstrates  
 256 that the TKE field is sensitive to the exact geometry of the randomly packed bed.  
 257 The maximum value of the TKE is lower than the values seen in Figure 8. The general  
 258 trends of the TKE, however, align with the observations made in Case 1c, as the peaks  
 259 are once again seen in the wake regions behind pebbles. This implies that although



**Fig. 11** Comparison of axial TKE profiles between Case 1c and Case 4. Case 1c uses a uniform velocity inlet condition, while Case 4 uses a turbulent velocity inlet condition.

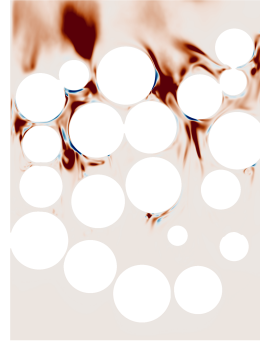
the exact magnitude of the TKE and budget terms may vary based on the random packing, the general trends in the budget terms that are observed in this study will likely be present in other beds of randomly packed spheres. Finally, Figure 10 presents the TKE field when a turbulent inlet condition is used instead of a constant value. This is achieved by extending the inlet to allow turbulence to develop in an open pipe prior to reaching the bed. The TKE field, however, does not appear to be significantly affected by this different inlet condition. This is likely due to the fact that the flow conditions of interest for the transition to turbulence in a packed bed are not at a high enough Reynolds number to produce significant turbulence in the inlet pipe. This comparison is examined more quantitatively in Figure 11 by looking at the average TKE in axial slices in the bed. Here, some differences between the two cases can be observed, particularly toward the start of the bed where the bed with the turbulent inflow condition has slightly higher TKE than case 1c with a uniform inlet velocity. These differences, however, are small and the inlet condition does not appear to affect the TKE further in the bed once the flow is fully developed.

#### 4.2.1 TKE production

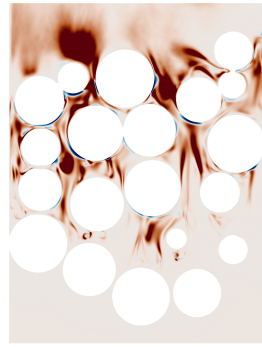
The TKE production in packed beds is particularly interesting, as it has been previously demonstrated to exhibit features not found in canonical cases such as pipe flow or channel flow. The most notable feature of the TKE production is the presence of regions of negative TKE production. These regions have been observed previously in



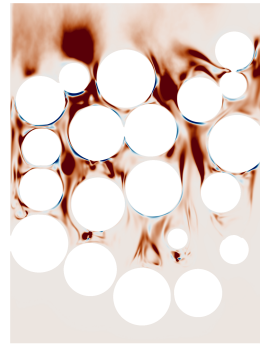
(a)  $Re = 300$  (Case 1a)



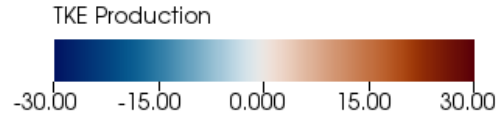
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)



(d)  $Re = 600$  (Case 1d)



(e)

**Fig. 12** Centerplane slice of the TKE production at  $Re = 300, 400, 500$ , and  $600$ .

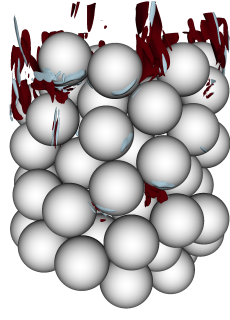
multiple independent studies [21, 22, 29]. Negative TKE production occurs in packed  
beds near regions of particle contact where the flow accelerates around a contact point.  
These can occur near particle-particle and particle-wall contacts. The acceleration of  
the flow suppresses the TKE and temporarily relaminarizes the flow until it decelerates  
into the void region behind the contact point. The work by Yildiz [21] has previously  
linked this negative TKE production to strong anisotropy in these regions as well.

Analysis may first begin by qualitatively examining the TKE production fields, shown in Figure 12. There is clearly a large difference between the production field in Case 1a and 1b, with a much smaller difference as the Reynolds number increases further. For Case 1a, there is a small amount of TKE production in the large void after the first layer of pebbles, but very little production towards the outer regions of the bed. After two or three layers of pebbles, the production field bears resemblance to the production fields in the higher Reynolds number cases. The areas of negative production and the peaks in positive production can be isolated as areas of interest. In many canonical cases, such as channel flow, the production term is positive and acts as a source of TKE. A complex case such as a pebble bed, however, can see this term change sign and become negative, suppressing the TKE rather than strengthening it. In all cases, there are areas of negative production clearly visible on the bottom surfaces of the pebbles where the flow accelerates between pebbles or between pebbles and the cylinder wall. The peaks in positive production, however, tend to occur in large elongated features that are present in the wakes of pebbles where flow decelerates into the void spaces between pebbles.

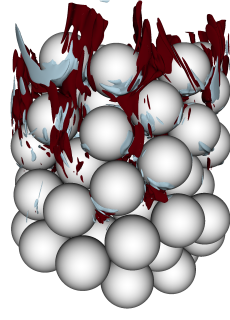
These features can be seen more clearly by examining a 3D visualization of the production in Figure 13. By examining the 3D visualization, these regions of negative production can be seen in greater detail. From this figure, we can see the areas of negative production around the bottom surfaces of the pebbles. For the pebbles near the wall, it can clearly be seen that these negative production regions exist just below where the pebble makes contact with the wall. In these locations, the flow accelerates around the contact point, causing negative TKE production and subsequent laminarization. This is consistent with the findings from [21], [19], and [28]. There also appears to be some large elongated regions of negative TKE production in the large voids that were not previously visible. It is also clear from these visualization that the regions of negative TKE production increase in size as the Reynolds number is increased. The regions of highly-positive TKE production appear just after the flow has passed by a pebble and decelerates into a more open region. This deceleration increases the turbulence in these wake regions and causes large velocity gradients, thus increasing the production of the TKE. These findings support the idea that the flow in packed beds is heavily influenced by inertial effects as the fluid accelerates and decelerates in regions of pebble-wall contact and void space.

#### 4.2.2 TKE dissipation

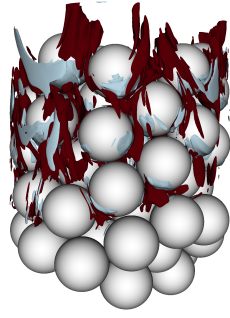
The plots of the dissipation in Figure 14 show similarities to the production plots. The evolution of the TKE dissipation term as the Reynolds number is increased follows the same trends as the TKE and TKE production. It appears as though the dissipation develops slightly slower than the production in the streamwise direction, however. This is particularly evident in the large void region after the first layer of pebbles where elongated peaks appear in the production that do not appear in the dissipation. This can also be seen clearly just after the leftmost pebble in the bottom layer where there is a strong peak of positive production but no strong peak in dissipation. Further examination of the peaks in the dissipation field can help to understand interplay between the various budget terms in several regions of the bed. First, there are clearly



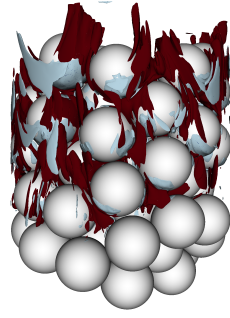
(a)  $Re = 300$  (Case 1a)



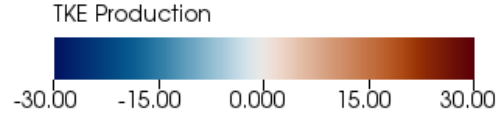
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)



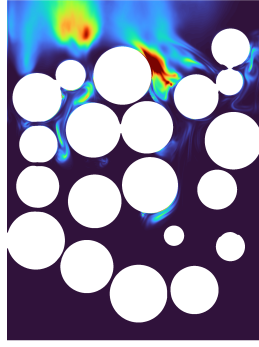
(d)  $Re = 600$  (Case 1d)



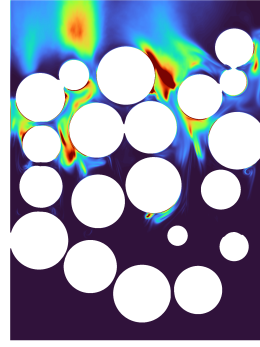
(e)

**Fig. 13** 3D visualization of the TKE production at  $Re = 300, 400, 500$ , and  $600$ .

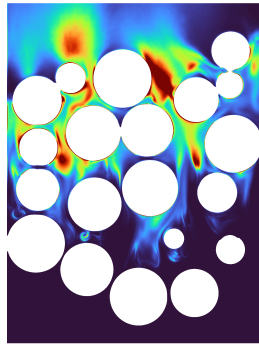
330 peaks in the TKE dissipation close to both the pebble walls and the cylinder wall. This  
 331 is expected, as the dissipation term helps to balance the viscous diffusion term very  
 332 close to the walls. Aside from these regions, the dissipation also peaks in many areas  
 333 where high positive production was observed, namely in the void regions in the wakes  
 334 of the pebbles. This aligns with observations in many other canonical cases, where far  
 335 from the walls the production term is balanced mainly by the dissipation term.



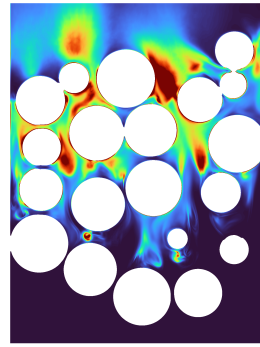
(a)  $Re = 300$  (Case 1a)



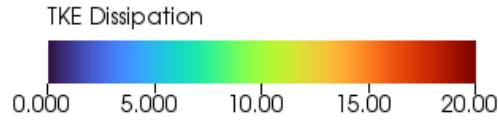
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)



(d)  $Re = 600$  (Case 1d)

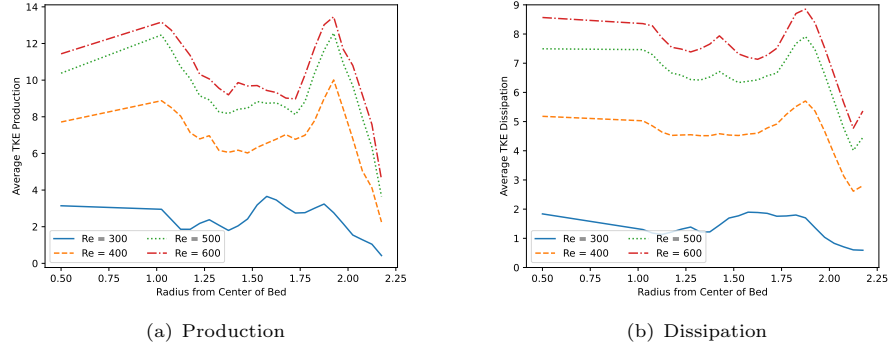


(e)

**Fig. 14** Centerplane slice of the TKE dissipation at  $Re = 300, 400, 500$ , and  $600$ .

### 4.2.3 Porous media radial and axial profiles

The focus of examining detailed aspects of the flow is often for providing improvement to lower fidelity models. In the case of packed beds, porous media models are often used as low fidelity simulation tools. Therefore, this section will explore the budgets of the TKE through a porous media viewpoint. This means that the terms will be spatially averaged in certain regions to provide insight on more general trends of the

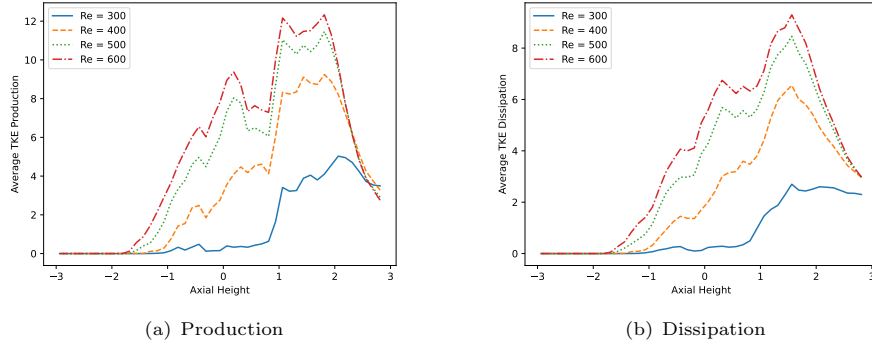


**Fig. 15** Variation of the TKE production and dissipation as a function of the radius from the center of the bed.

budgets that are less sensitive to local changes. The production and dissipation terms will now be averaged in both radial and axial averaging regions. The bed is separated into 24 concentric rings of  $0.05D_{peb}$  width with 1 inner ring of  $1.0D_{peb}$  width for the radial examination, and the bed is separated into 48 cylinders of  $0.125D_{peb}$  height for axial examination.

The radial variation of the TKE production and dissipation is shown in Figure 15. The ring-average values are plotted at the radial midpoint of each ring. It should be noted that as the radius approaches 0, the averaging volume becomes very small, and therefore the specific local effects become more prominent. The innermost ring is therefore chosen to be much larger than other rings to maintain statistical significance. The radial trends in both the production and dissipation appear to be largely the same with a peak closer to the wall around a radius of 1.8. One notable observation, however, is that the near-wall peak in the budget terms appears to shift slightly inward for Case 1a at  $Re = 300$  while the other three cases have this near-wall peak at roughly the same location. This can be explained in by examining the budget terms in Figures 12 and 14. In these figures, it can be seen that for Case 1a, the budget terms first develop in the radial center of the bed and then develop outward radially as the flow continues to move through the bed. This lack of flow development towards the outside of the bed therefore explains the observed shift inwards of the peaks for Case 1a in Figure 15. Also of note is the increase in the dissipation for the outermost ring. This increase in dissipation can be seen visually in Figure 14 with the thin layer of high dissipation along the wall in the top section of the bed. Similarly to how the dissipation peaks near the pebble surfaces, a peak is also present along the cylinder wall.

The axial variation in these terms can provide information on how the flow develops in a packed bed. This axial variation is shown in Figure 16 with ring-average values plotted at the ring's axial midpoint. The trends in the TKE budget terms with the axial height are largely as expected, developing along the streamwise direction, peaking near the top of the bed, and then decreasing in the outlet region. The production in the  $Re = 300$  case appears to develop much more slowly compared to the higher Reynolds number cases. There is very little development until  $Z = 1.0$ , whereas the other cases



**Fig. 16** Variation of the TKE production and dissipation as a function of the axial height in the bed.

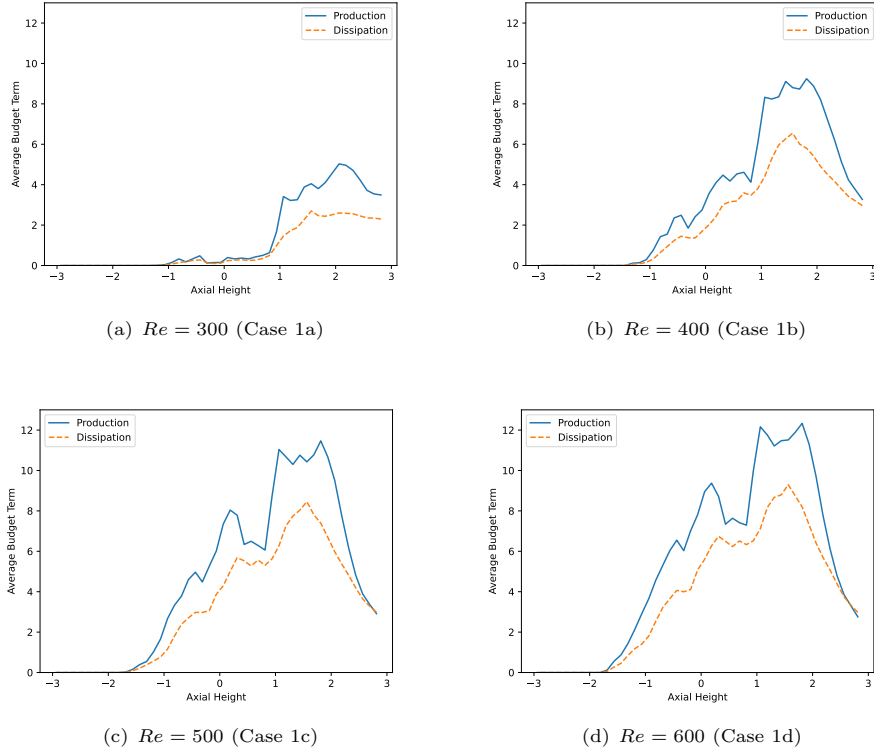
all see significant development near the start of the bed at  $Z = -2.0$ . Nonetheless, all four cases appear to peak at roughly the same axial height.

The axial variation of the production and dissipation are also plotted together for each Reynolds number in Figure 17. Here the lag between the development of the dissipation and the production can be seen as the production grows faster than the dissipation. This was observed visually in Figure 14 as well. The dissipation does still appear to fairly closely follow the production, with the shape of the profile being roughly the same between the two terms.

Previous work by the author has focused on using numerical models to generate an improved porous media pressure drop equation for PBRs [40]. Part of this work involved the generation of modifying term for the form losses that varies with the local porosity. This modification factor was later linked to the amount of local negative TKE production [29]. Understanding the evolution of the negative TKE production with the Reynolds number can therefore be helpful for understanding the point at which the flow is not turbulent enough for this form term to be applied.

As has been done with the other investigations, the bed was separated into rings and the average negative TKE production ( $\langle |P| \rangle$  for  $P < 0$ ) was calculated for each ring. Figure 18 presents the plots of  $1/\langle |P| \rangle$  against the porosity for each Reynolds number. At the lowest Reynolds number in Case 1a, there does not appear to be any clear trend between the negative TKE production and the local porosity. As the Reynolds number increases, this trend appears to develop. Case 1c and 1d exhibit roughly the same trend that has been observed in previous work [29], where there is are increases in the low and high porosity regions with fairly constant negative TKE production in the medium porosity regime. The near-wall point at the highest porosity does not appear to be consistent with the trend, although this can be attributed to the small size of the bed that was studied. Other current work has demonstrated that for a larger bed, the negative TKE production in the near-wall region will follow the increasing trend at high porosity [41].





**Fig. 17** Axial variation of the TKE production and dissipation for Cases 1a-1d.

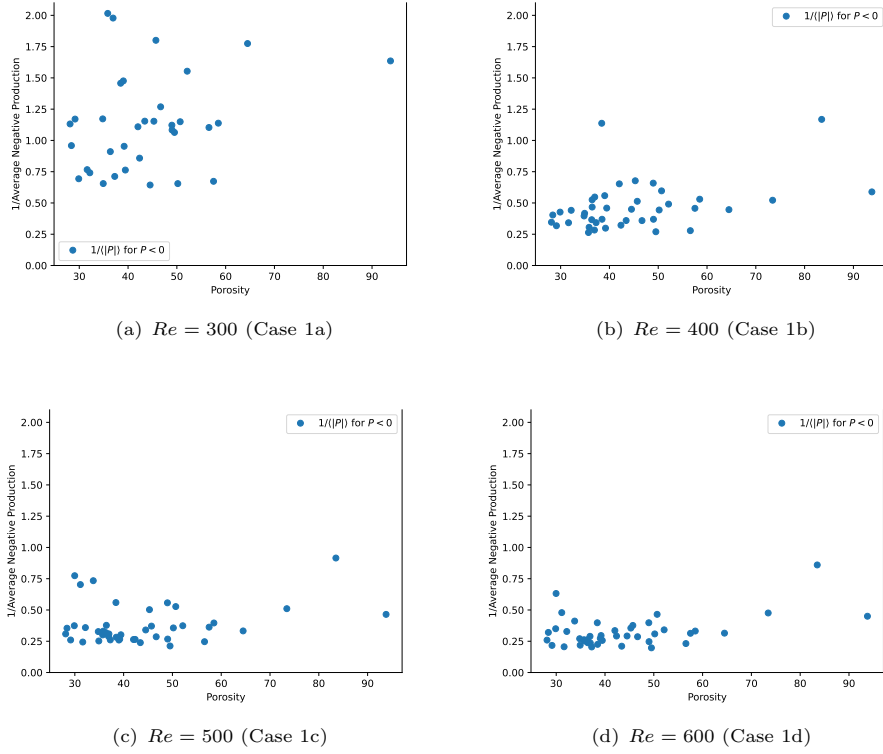
#### 4.2.4 Anisotropy analysis

Examination of the behavior of turbulence in a domain can provide additional insight on the flow physics. One method of examining the turbulence is by performing invariant analysis of the anisotropy tensor. The Reynolds anisotropy tensor is defined as:

$$a_{ij} = \frac{\overline{u'_i u'_j}}{2k_e} - \frac{\delta_{ij}}{3} \quad (11)$$

where  $k_e$  is the turbulent kinetic energy given by Equation 9.

The Lumley turbulence triangle [42] is a common method of mapping and visualizing invariants. Maps such as these provide 2D representations of the invariant properties of the flow. A major shortcoming is that they struggle to portray spatial information. The method utilized in this work was developed by Emomry and Iaccorino [43] to mend this shortcoming of previous methods, where the three states of turbulence can be determined from the eigenvalues of  $a_{ij}$  and mapped to a corner of a barycentric map. This can then provide a color-based representation of the states of turbulence, where one corner of the map is one-component or cigar-shaped turbulence, one corner is two-component or pancake-shaped turbulence, and one corner



**Fig. 18** Average of negative production ( $1/\langle|P|\rangle$  for  $P < 0$ ) versus local ring porosity.

is three-component or sphere-shaped turbulence, also known as isotropic turbulence. All turbulence in the system will therefore be some combination of these three limiting conditions and can be mapped into the barycentric map based on the following weighting:

$$C_{1c} = \lambda_1 - \lambda_2 \quad (12)$$

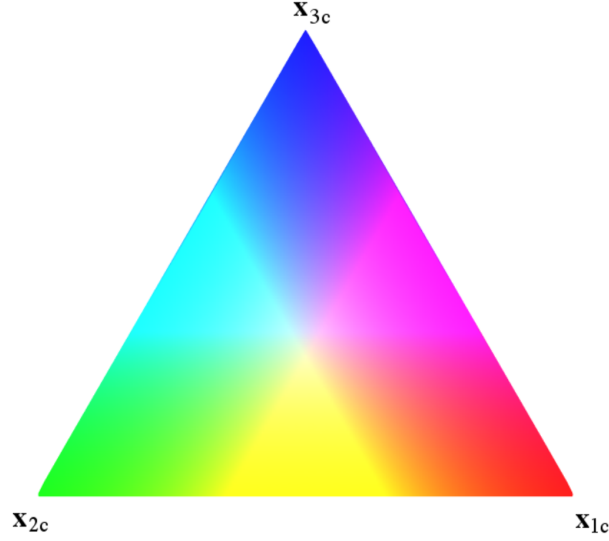
$$C_{2c} = 2(\lambda_2 - \lambda_3) \quad (13)$$

$$C_{3c} = 3\lambda_3 + 1 \quad (14)$$

where  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are the eigenvalues of  $a_{ij}$ . These three eigenvalues sum to zero while the three weights sum to unity. These weights can then be used as the three components of a Red-Green-Blue (RGB) triplet to spatially visualize the anisotropy where red represents one-component turbulence, green represents two-component turbulence, and blue represents three-component turbulence. This map can be further modified to provide slightly different color information using the following modification:

$$C_{ic}^* = (C_{ic} + C_{off})^{C_{exp}} \quad (15)$$

where  $C_{ic}$  are the unmodified weights and  $C_{off}$  and  $C_{exp}$  are the offset and exponent modifiers respectively. This work used  $C_{off} = 0.65$  and  $C_{exp} = 5$  to yield the

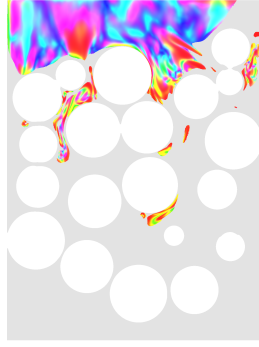


**Fig. 19** Barycentric map used for anisotropy visualization where  $C_{off} = 0.65$  and  $C_{exp} = 5$ .

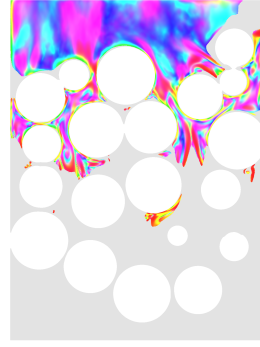
429 barycentric map shown in Figure 19. In this map, one-component, two-component,  
 430 and three-component turbulence still correspond to red, green, and blue respectively,  
 431 but additional colors are added between the corners to provide additional information  
 432 on which combination of turbulence states exist.

433 The anisotropy tensor was calculated for Cases 1a-1d and the invariants were  
 434 visualized using the barycentric map in Figure 19. A downside of the barycentric map  
 435 approach is that since the weights always sum to unity, a color will be assigned even  
 436 to locations where the flow is not turbulent. Since Cases 1a-1d capture the onset  
 437 of turbulence, an additional filter is applied where the colors are only visualized for  
 438 regions where  $k_e > 0.1$ . This therefore removes the misleading visualization of the  
 439 anisotropy in regions where turbulence has not yet developed.

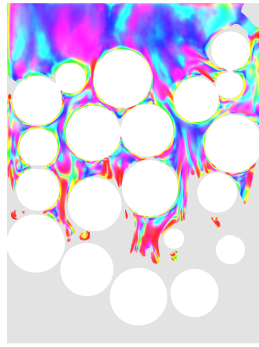
440 The visualizations of the anisotropy componentality are shown in Figure 20. Firstly,  
 441 it can be seen that the turbulence develops mainly as one-component turbulence at  
 442 the entrance of the bed. The filter based on the TKE in these visualizations also  
 443 demonstrate how much sooner the turbulence develops near the exterior of the bed as  
 444 the Reynolds number is increased. Two-component turbulence is mainly found near  
 445 the surfaces of the pebbles. Three-component or isotropic turbulence is then found  
 446 in the void spaces and outlet region in regions that are sufficiently far from either  
 447 the cylinder wall or pebble walls. An interesting observation is that one-component  
 448 turbulence is once again found near pebble contact points. This can be closely linked  
 449 to the previously-observed regions of negative TKE production. As the flow passes  
 450 near pebble contacts, acceleration causes negative TKE production and laminarizes  
 451 the flow. After passing through the contact point, turbulence then redevelops in these  
 452 regions where one-component turbulence is again found. This finding provides further  
 453 evidence on the link between porosity and local flow physics, where the inertial effects



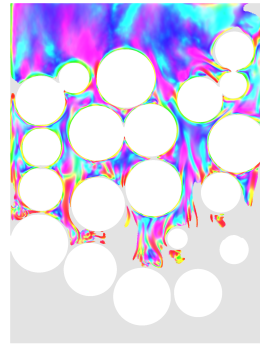
(a)  $Re = 300$  (Case 1a)



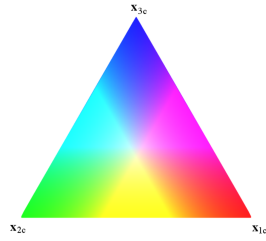
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)



(d)  $Re = 600$  (Case 1d)



(e)

**Fig. 20** Centerplane slice of the visualization of anisotropy componentality in Case 1a-1d. Anisotropy visualization is only shown for areas where  $TKE \geq 0.1$ .

454 in a region are influenced by the number of contact points and void spaces found in  
 455 that region.

### 4.3 Analysis of the turbulent heat flux budget

The turbulent heat flux (THF), similarly to the TKE, is a key parameter for scalar transport in turbulent flow. In addition to molecular diffusion, heat is also transported through the bulk motion of fluid. In the case of turbulent flow, heat transport can also occur due to the interaction of eddies, leading to mixing and enhanced heat transfer. The turbulent heat flux can help quantify the heat transfer throughout the fluid that results from these different mechanisms. The balance of the THF can be represented as the sum of each mechanism, known as the budgets of the THF:

$$\begin{aligned}
 \overline{u_j} \frac{\partial \overline{T' u'_i}}{\partial x_j} = & \underbrace{-\frac{\partial}{\partial x_j} \left( \overline{u'_i u'_j T'} \right)}_{\text{Turbulent Diffusion}} + \underbrace{\frac{\partial}{\partial x_j} \left( \overline{\nu T' \frac{\partial u'_i}{\partial x_j}} + \alpha_d \overline{u'_i \frac{\partial T'}{\partial x_j}} \right)}_{\text{Molecular Diffusion}} - \underbrace{\overline{u'_i u'_j} \frac{\partial \overline{T}}{\partial x_j} - \overline{u'_j T'} \frac{\partial \overline{u'_i}}{\partial x_j}}_{\text{Production}} \\
 - & \underbrace{\frac{\overline{T'} \partial \overline{p'}}{\rho \partial x_i}}_{\text{Temp. Press. Grad. Corr}} - \underbrace{(\alpha_d + \nu) \frac{\partial \overline{T'}}{\partial x_j} \frac{\partial \overline{u'_i}}{\partial x_j}}_{\text{Dissipation}}
 \end{aligned} \tag{16}$$

Analysis of the turbulent heat flux budgets can provide a database of information that may be useful for RANS modeling approached of these systems in the future. This section examines the production and dissipation of the THF in the three component directions.

#### 4.3.1 THF Production

Figures 21-23 show the production of the three components of the THF for increasing Reynolds number. The heated pebble is the bottom-center pebble in this figure. Examination of the THF budgets can be divided into two regions: the region near the heated pebble, and the downstream unheated region.

Generally, the trends in the THF production with the Reynolds number are the same across the X, Y, and Z axes. In all cases, as was seen with the TKE, positive and negative peaks become more pronounced as the Reynolds number is increased. Additionally, these features develop at lower axial heights in the bed at higher Reynolds numbers.

Beginning with the region near the heated pebble, there are strong THF production peaks just behind the heated pebble. This can be attributed to two main factors. Firstly, as was discovered in the examination of the TKE production, the wake region behind pebbles tends to be turbulent, contributing to the production of the THF. Additionally, the temperature gradients near the heated pebble are large, also contributing to the increased THF production.

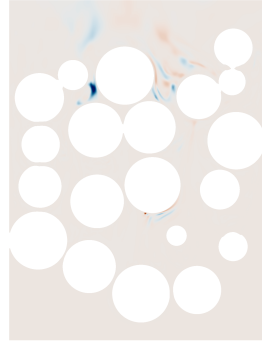
Downstream from the heated pebble, there are still significant elongated peaks in the THF production. Once again, these elongated features align with the observations made in the analysis of the TKE where the same features are present in the wake region behind pebbles. Unlike in the region near the heated pebble, the temperature

488 gradient in these areas is fairly small, indicating that the THF is mainly a result of  
489 the high turbulence in these wake regions. This can also be seen in the voids on the  
490 left and right of the top-center pebble, where strong peaks are once again observed in  
491 the THF production. The peaks in these regions align with the TKE peaks in Figure  
492 8, indicating that the THF production peaks in these voids is likely due to strong  
493 mixing that results from flow deceleration into the voids.

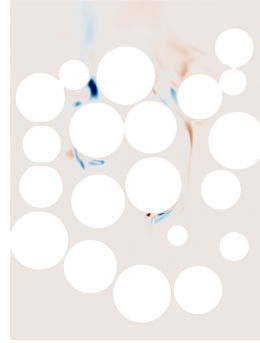
494 One of the primary reasons for only heating a single pebble was to provide some  
495 insight on the evolution of the temperature plume as the Reynolds number is increased.  
496 This can provide insight into the energy dispersion in PBR systems. Figures 21 - 23 can  
497 help provide insight on the evolution of this temperature plume. Beginning with Case  
498 1a at  $Re = 300$ , it can be seen that the THF production has non-trivial values directly  
499 above the heated pebble with very little radial propagation. As the Reynolds number  
500 is increased to 400, there is more radial propagation as more energy is dispersed due to  
501 mixing. The radial propagation is seen further Cases 1c and 1d, where the features of  
502 the THF develop near the outer regions of the bed sooner than in Case 1b. Although  
503 not a focus of improvement for this work, this information can potentially be useful  
504 towards assessment of the effective fluid conductivity closures in porous media models  
505 where the fluid conductivity is increased in an attempt to account for this greater  
506 radial dispersion of energy that arises from flow mixing.

### 507 4.3.2 THF Dissipation

508 The THF dissipation along the three axes is shown in Figures 24-26. In these figures,  
509 there are many takeaways that closely align with those observed in the THF production  
510 plots. Firstly, the strongest peaks in the dissipation occur just downstream from the  
511 heated pebble in the void that experiences large temperature gradients and strong  
512 mixing. There does not appear to be any noteworthy features in the THF dissipation  
513 that have not already been observed in the THF production plots, so these plots may  
514 simply serve as references for future verification and modeling works.



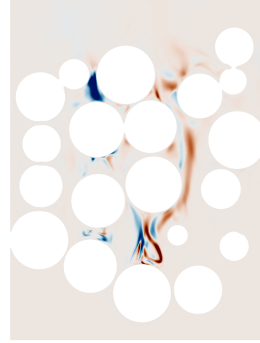
(a)  $Re = 300$  (Case 1a)



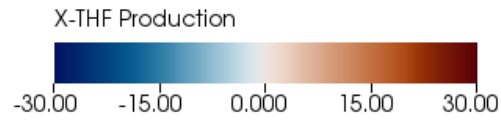
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)

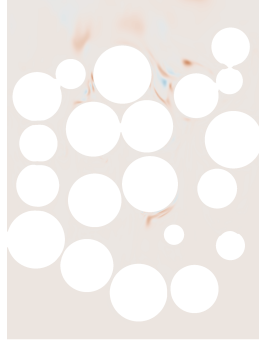


(d)  $Re = 600$  (Case 1d)

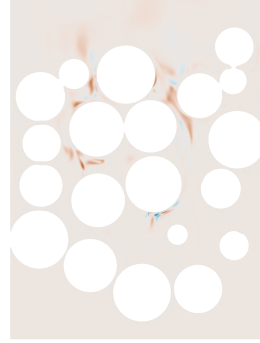


(e)

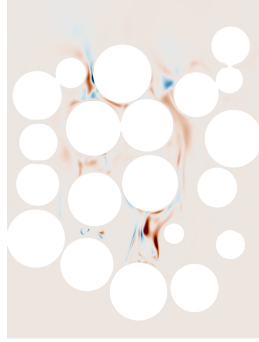
**Fig. 21** Centerplane slice of the streamwise X-THF dissipation at  $Re = 300, 400, 500$ , and  $600$ .



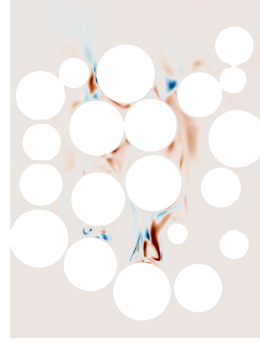
(a)  $Re = 300$  (Case 1a)



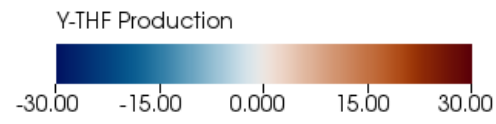
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)



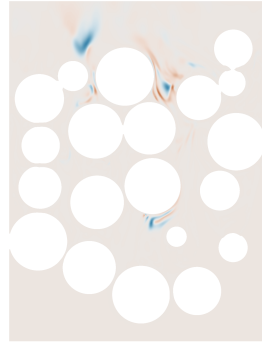
(d)  $Re = 600$  (Case 1d)



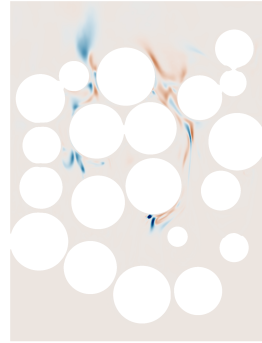
(e)

**Fig. 22** Centerplane slice of the Y-THF dissipation at  $Re = 300, 400, 500$ , and  $600$ .

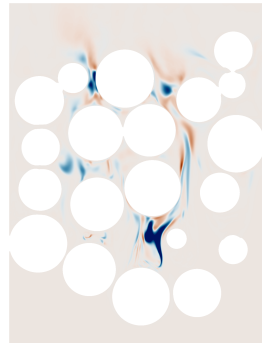




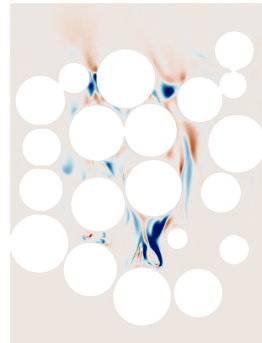
(a)  $Re = 300$  (Case 1a)



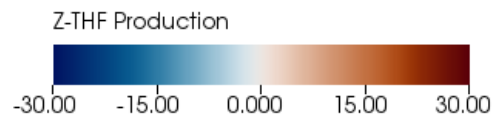
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)

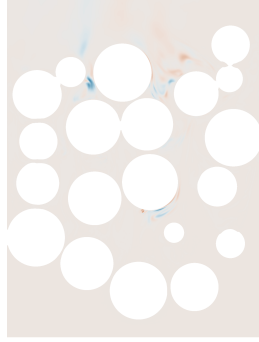


(d)  $Re = 600$  (Case 1d)

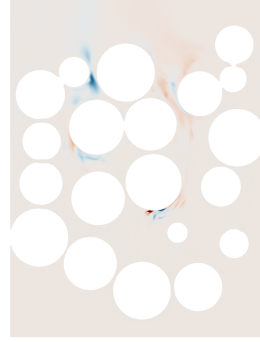


(e)

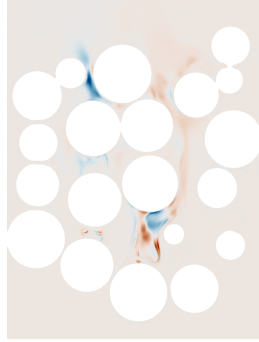
**Fig. 23** Centerplane slice of the Z-THF (streamwise) production at  $Re = 300, 400, 500$ , and  $600$ .



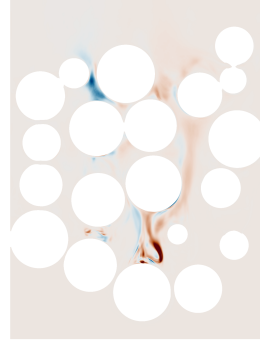
(a)  $Re = 300$  (Case 1a)



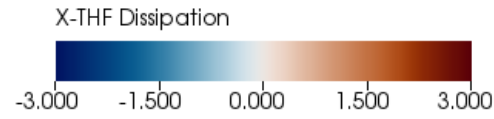
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)

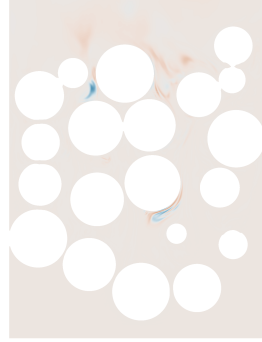


(d)  $Re = 600$  (Case 1d)

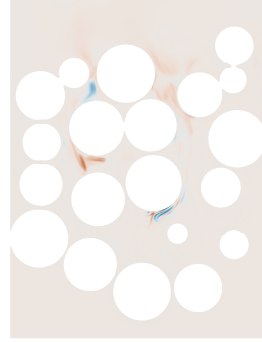


(e)

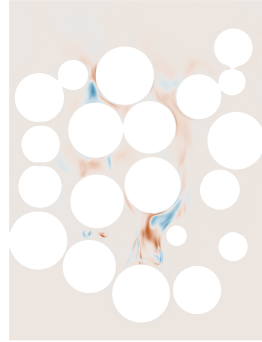
**Fig. 24** Centerplane slice of the X-THF dissipation at  $Re = 300, 400, 500$ , and  $600$ .



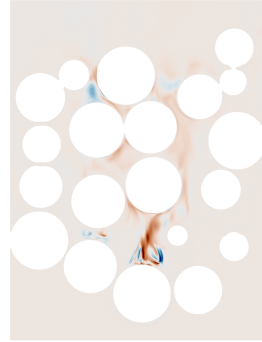
(a)  $Re = 300$  (Case 1a)



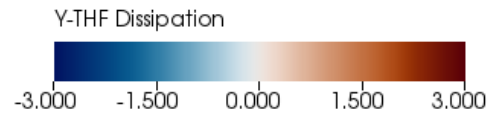
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)

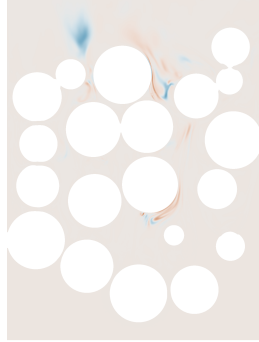


(d)  $Re = 600$  (Case 1d)

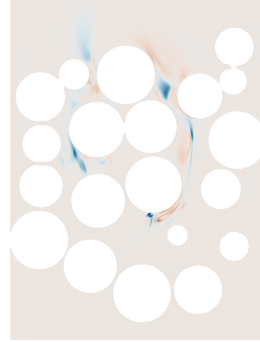


(e)

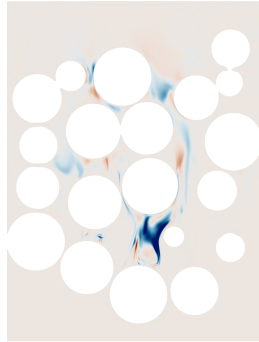
**Fig. 25** Centerplane slice of the Y-THF dissipation at  $Re = 300, 400, 500$ , and  $600$ .



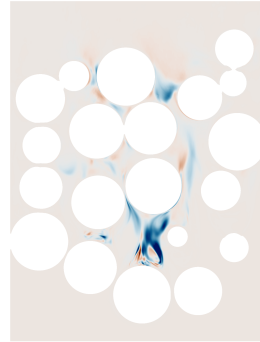
(a)  $Re = 300$  (Case 1a)



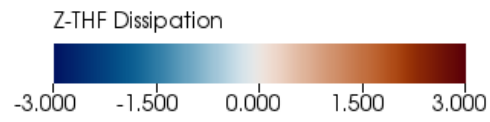
(b)  $Re = 400$  (Case 1b)



(c)  $Re = 500$  (Case 1c)



(d)  $Re = 600$  (Case 1d)



(e)

**Fig. 26** Centerplane slice of the Z- THF (streamwise) dissipation at  $Re = 300, 400, 500,$  and  $600$ .

## 5 Conclusions

This manuscript presented a DNS study of the flow in a small bed of 67 pebbles with an aspect ratio of 4.4. The bed was simulated at Reynolds numbers of 300, 400, 500, and 600, to investigate the transition to turbulence in packed beds. A single heated pebble was included at the bottom center of the bed, and the fluid Prandtl number was 0.66 for all cases. **This study represents one of the first comprehensive DNS studies of transitional effects in a randomly packed pebble bed to our knowledge.**

Analysis of the TKE revealed that the areas of highest TKE occurred in the wake regions behind pebbles. Additionally, the TKE was fairly undeveloped in the  $Re = 300$  case until the flow had passed through 3-4 layers of pebbles. The TKE field appeared to be developed earlier as the Reynolds number was increased. Large differences in the TKE field were observed between the  $Re = 300$  and  $Re = 400$  cases with some significant differences observed between the  $Re = 400$  and  $Re = 500$  cases. The differences between the  $Re = 500$  and  $Re = 600$  cases was fairly minimal, indicating that the flow is approaching the turbulent regime rather than the transitional regime at this point.

Analysis was then performed on the TKE budget, particularly the production and dissipation terms. Observations with both of these terms aligned closely with those made with regards to the TKE, where significant differences were observed between the  $Re = 300$  and 400 cases, with less differences observed between the higher Reynolds number cases.

Analysis of the TKE production revealed a number of interesting findings. Areas of negative TKE production were observed across all cases. These regions of negative TKE production occurred near contact points between two pebbles or between a pebble and the confining wall. In this region, acceleration of the flow serves to inhibit the TKE rather than enhance it, causing negative TKE production and subsequent laminarization. Regions of highly positive TKE were also observed in the wake regions behind pebbles, particularly in the large voids. In these regions, deceleration of the flow in to the open voids causes large gradients and enhances the TKE in these regions. Further analysis of the areas of negative TKE production separated the bed into concentric rings of  $0.05D_{peb}$  width and the average negative production was determined in each ring. This was then compared to the ring's porosity. A trend was observed where the more negative production was found to occur in regions where the porosity was around 0.4. At the high and low porosity rings, there was notably less negative production. This trend was not observed for the  $Re = 300$  case, and the trend was present, although weaker for the  $Re = 400$  case. Analysis of the turbulence anisotropy revealed the presence of highly anisotropic 1-component turbulence near these regions of negative TKE production.

The TKE production and dissipation terms were then examined through a porous media lens, where the bed was separated into several radial and axial regions with the average TKE production and dissipation being calculated for each region. Examination of the radial profile revealed a peak in the production near the wall. This peak was shifted inwards for the  $Re = 300$  case. This is due to the fact that the budget terms in this case develop first in the center of the bed and develop towards the outer region of the bed after the flow has progressed farther in the bed. This radial development

occurs sooner for the higher Reynolds number cases, thus explaining why the peak in the  $Re = 300$  case is closer towards the interior of the bed than the rest of the cases. The axial profiles revealed similar findings, where the production and dissipation in the  $Re = 300$  case took slightly longer in the axial direction to develop.

An investigation into the THF budget was performed with a focus on the production and dissipation, as was done for the TKE. Similarly to the TKE budget, significant differences were observed between the  $Re = 300$  and 400 cases, with fewer differences observed between the higher Reynolds number cases. Given that only a single pebble was heated, analysis of the budgets was divided between the region near the heated pebble, and the region downstream. The THF production and dissipation were large near the heated pebble. This is expected, as the temperature gradient is largest near the heated pebble. This THF production is therefore a product of these large gradients. Away from the heated pebble, elongated features also appeared in the wake regions. Given the low temperature gradients in these locations, these peaks can be mainly attributed to the high mixing in these regions.

Future investigations will incorporate these findings into improved porous media and RANS modeling of pebble beds at conditions relevant to gas cooled nuclear reactors.

## Declarations

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**Ethical Approval** N/A

**Informed Consent** N/A

**Author Contribution** D.R. - Conceptualization, Methodology, Validation, Investigation, Analysis, Visualization, and Writing E.M. - Conceptualization, Writing Review, Supervision, Funding acquisition, Resources T.N. - Data acquisition Y.L. - Methodology, Data acquisition, and Software for the creation of the mesh. P.F. - Supervision Y.H. - Conceptualization, Supervision, and Data acquisition for experimental data

## Nomenclature

|     |                              |
|-----|------------------------------|
| CFD | Computational Fluid Dynamics |
| DNS | Direct Numerical Simulation  |
| FCC | Face-centered cubic          |
| GPU | Graphics Processing Unit     |

|     |                        |                                                  |
|-----|------------------------|--------------------------------------------------|
| 602 | PBR                    | Pebble Bed Reactor                               |
| 603 | PIV                    | Particle Image Velocimetry                       |
| 604 | RANS                   | Reynolds-Averaged Navier-Stokes                  |
| 605 | RMS                    | Root Mean Square                                 |
| 606 | THF                    | Turbulent Heat Flux                              |
| 607 | TKE                    | Turbulent Kinetic Energy                         |
| 608 | $-\ast$                | Nondimensional Value                             |
| 609 | $\alpha_d$             | Thermal diffusivity                              |
| 610 | $\delta_{ij}$          | Kronecker delta                                  |
| 611 | $\Delta T_{reference}$ | Reference temperature difference                 |
| 612 | $\epsilon$             | Porosity                                         |
| 613 | $\mu$                  | Dynamic viscosity                                |
| 614 | $\nu$                  | Kinematic viscosity                              |
| 615 | $\rho$                 | Density                                          |
| 616 | $c_p$                  | Specific heat capacity                           |
| 617 | $D_{bed}$              | Bed diameter                                     |
| 618 | $D_{bed}/D_{peb}$      | Aspect ratio                                     |
| 619 | $D_{peb}$              | Pebble diameter                                  |
| 620 | $k$                    | Thermal conductivity                             |
| 621 | $k_e$                  | Turbulent Kinetic Energy                         |
| 622 | $P$                    | Production of Turbulent Kinetic Energy           |
| 623 | $p$                    | Pressure                                         |
| 624 | $Pe$                   | Peclet number                                    |
| 625 | $Pr$                   | Prandtl number ( $\mu c_p/k$ )                   |
| 626 | $q''$                  | Heat flux                                        |
| 627 | $Re$                   | Reynolds number ( $\frac{\rho u D_{peb}}{\mu}$ ) |
| 628 | $T$                    | Temperature                                      |
| 629 | $t$                    | Time                                             |
| 630 | $T_0$                  | Reference temperature                            |
| 631 | $u$                    | Velocity                                         |
| 632 | $V$                    | Volume                                           |
| 633 | $x$                    | Distance                                         |

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