Lepton flavor violation by three units

Julian Heeck,[1,*] Mikheil Sokhashvili,[1,†] and Anil Thapa[1,2,‡]

[1]*Department of Physics, University of Virginia, Charlottesville, Virginia 22904-4714, USA*
[2]*Physics Department, Colorado State University, Fort Collins, Colorado 80523, USA*

(Received 30 May 2025; accepted 1 September 2025; published 29 September 2025)

The conservation of lepton flavor is a prediction of the Standard Model and is still an excellent approximate symmetry despite our observation of neutrino oscillations. Lepton flavor violation by one or two units has been discussed for decades, with several dedicated experiments exploring the vast model landscape but no discoveries so far. Here, we explore operators and processes that violate at least one lepton flavor by three units and identify testable signatures. In the Standard Model effective field theory, such operators already arise at mass dimension 7 and can be tested through their contributions to Michel parameters in leptonic decays. True neutrinoless charged-lepton flavor violation arises at mass dimension 10 and can realistically only be seen in the tau decay channels $\tau \to eee\bar{\mu}\,\bar{\mu}$ or $\tau \to \mu\mu\mu\bar{e}\,\bar{e}$, for example in Belle II. Testable rates for these tau decays require light new particles and subsequently predict an avalanche of remarkably clean but so-far unconstrained collider signatures.

DOI: 10.1103/2zkx-d3yk

I. INTRODUCTION

The three individual lepton flavors—electron, muon, and tau number—are conserved quantum numbers in the Standard Model (SM) of particle physics that forbid otherwise seemingly allowed decays such as $\mu^- \to e^-\gamma$ [1]. No short-range process violating this symmetry $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$ has ever been observed [2,3], in perfect agreement with the SM, but through *neutrino oscillations*, $\nu_\alpha \to \nu_\beta$, we have learned that these symmetries are at least broken in the neutrino sector. This is a spectacular observation that forces us to extend the SM by new particles to account for nonzero neutrino masses, but by itself does *not* imply testable rates for *charged* lepton flavor violation, since these are typically suppressed by the tiny neutrino masses [4–8]. $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$, or any of its subgroups, then remain excellent *approximate* symmetries in the charged-lepton sector, which could have an entirely different flavor breaking pattern than in the neutrino sector [9,10]. Lepton flavor violation (LFV) by *one* unit, as in $\mu \to e\gamma$, might then be suppressed compared to LFV by *two* units, such as the muonium conversion $\mu^+e^- \to \mu^-e^+$, as recently emphasized more generally in Ref. [11]. Despite the more complicated experimental signatures, it is important to search for LFV by *two* units in order not to miss new physics due to theoretical bias.

Here, we take this argument one step further and investigate whether LFV by *three* units could be detectable. While LFV by one or two units arises already at mass dimension $d = 5$ (together with total lepton number L violation) or $d = 6$ (without L violation) in the Standard Model effective field theory (SMEFT) [12], LFV by *three* units first arises at $d = 7$ and is thus generically more suppressed. Nevertheless, we show below that these operators can lead to potentially observable modifications of muon and tau decays, parametrized by familiar Michel parameters. *Charged* LFV by three units first arises at $d = 10$ and requires UV-complete models with new light particles for testable rates, but could then lead to clean tau decays such as $\tau \to eee\bar{\mu}\,\bar{\mu}$ and $\tau \to \mu\mu\mu\bar{e}\,\bar{e}$ [9], potentially observable at Belle II.

The rest of this article is organized as follows: Sec. II is devoted to $d = 7$ LFV operators, the lowest mass dimension in which LFV by three units occurs; in Sec. III we briefly sketch LFV by three units in $d = 9$. Sec. IV has a long discussion of LFV at $d = 10$, the lowest mass dimension with *charged* LFV by three units, including a detailed discussion of one UV completion. We conclude in Sec. V. Appendixes A, B, and C list UV completions for the $d = 7$, $d = 9$, and $d = 10$ operators, respectively. Appendix D provides a detailed analysis of large electron-positron collider (LEP) limits from $e^+e^- \to \mu^+\mu^-$, relevant for the UV completion in Sec. IV.

*Contact author: heeck@virginia.edu
†Contact author: ms2guc@virginia.edu
‡Contact author: a.thapa@colostate.edu

TABLE I. SM fields and quantum numbers; hypercharge is related to electric charge via $Q = Y + T_3$.

Field	Chirality	Generations	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Q	Left	3	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
u_R	Right	3	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$
d_R	Right	3	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$
L	Left	3	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
l_R	Right	3	$(\mathbf{1}, \mathbf{1}, -1)$
H	Scalar	1	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

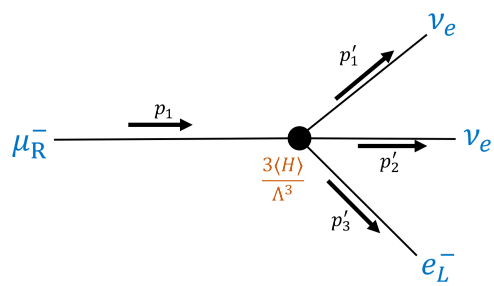

FIG. 1. Muon decay $\mu \to e\nu_e\nu_e$ through the $\Delta L_e = 3 = -3\Delta L_\mu$ LFV operator from Eq. (1).

II. DIMENSION 7

The lowest SMEFT operators violating lepton flavor by three units arise at dimension 7, and are of the simple form

$$\frac{1}{\Lambda^3}\bar{\mu}_R L_e L_e L_e H = \frac{1}{\Lambda^3}(\mu_R^c L_e^i)(L_e^j L_e^k)H^l \epsilon_{ij}\epsilon_{kl}, \qquad (1)$$

or flavor permutations thereof, where the brackets indicate Lorentz singlets and i, j, k, l are $SU(2)_L$ indices. The SM fields and their gauge quantum numbers are collected in Table I. This operator violates $\Delta L_e = 3 = -3\Delta L_\mu$; if only one such operator is added to the SM Lagrangian, the SMEFT scale Λ can be chosen to be real without loss of generality. Tree-level UV completions for this operator are listed in App. A. This operator can be enhanced compared to lower-d LFV using the anomaly-free flavor symmetry $U(1)_{L_e - 4L_\tau + 3L_\mu}$ [9,10], although that symmetry needs to be broken in the neutrino sector to comply with neutrino-oscillation data. Notice that the operator also violates *total* lepton number L by two units, but will *not* lead to any kind of Majorana neutrino masses due to the flavor symmetry. Stated differently, one cannot close loops to turn this operator into a neutrino propagator.

While the operator violates lepton flavor by three units, it does not generate *charged* LFV; after electroweak symmetry breaking, the operator becomes $\langle H\rangle \mu_R^c e_L \nu_e \nu_e/\Lambda^3$ and thus contains two neutrinos. The best experimental signature is likely the induced $\mu \to e\nu_e\nu_e$ decay. Due to the different neutrino flavors, the new operator does not interfere with the SM muon decay contribution and thus *shortens* the muon lifetime. More importantly, the electron distribution will be different from the SM Michel spectrum [13]. In Ref. [14], it has been shown that even LFV and ΔL operators inducing $\mu \to e\nu\nu$ can be mapped onto the usual Michel parameters, but our case is different still since we have two identical neutrinos in the final state. The explicit calculation below shows that our operator can nevertheless be captured in the form of Michel parameters.

The amplitude $\mathcal{M}$ for polarized muon decay $\mu \to e\nu_e\nu_e$ via Eq. (1) (Feynman diagram in Fig. 1) can be written as

$$\mathcal{M} = \frac{3v}{\sqrt{2}\Lambda^3}[\overline{\Sigma_1 v_1} P_R v_1' \overline{\Sigma_3' v_3'} P_R v_2' - \overline{\Sigma_1 v_1} P_R v_2' \overline{\Sigma_3' v_3'} P_R v_1'], \qquad (2)$$

where we defined the Higgs vacuum expectation value $v \simeq 246$ GeV and introduced the spin projection operators Σ_1 and Σ_3' for the muon and the electron, respectively,

$$\Sigma_1 \equiv \frac{1 + \gamma^5 \not{s}_1}{2}, \qquad \Sigma_3' \equiv \frac{1 + \gamma^5 \not{s}_3'}{2}. \qquad (3)$$

Considering neutrinos and electron to be massless, we find

$$\langle |\mathcal{M}|^2 \rangle = \frac{9v^2}{2\Lambda^6}(p_1' \cdot p_2')(p_3' \cdot [p_1 + m_\mu s_1]), \qquad (4)$$

notably independent of the electron polarization, just like in the SM case [15]. This means we have to multiply $\langle |\mathcal{M}|^2 \rangle$ by a factor of two, as we would have gotten the same answer had we not introduced the spin projection of the electron in the first place, but to account for the identical neutrinos, we also divide $\langle |\mathcal{M}|^2 \rangle$ by a factor of two, so nothing changes. The electron four momentum dotted with the muon spin direction vector, $s_1 = (0, \boldsymbol{s})$, is $p_3' \cdot s_1 = P_\mu \cos(\theta) E_3'$, with θ being the angle between electron momentum and muon spin, and muon polarization P_μ. Further defining $x \equiv 2E_3'/m_\mu$, we eventually find the double differential rate for the electron energy and emission direction as

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\,\mathrm{d}\cos\theta} = \frac{9m_\mu^5 v^2}{4096\pi^3\Lambda^6}[1 + P_\mu \cos(\theta)](1 - x)x^2. \qquad (5)$$

For comparison, the SM decay $\mu \to e\nu_\mu\bar{\nu}_e$ takes the form

$$\frac{\mathrm{d}\Gamma_{\mathrm{SM}}}{\mathrm{d}x\,\mathrm{d}\cos\theta} = \frac{G_F^2 m_\mu^5}{192\pi^3}[3 - 2x + P_\mu \cos(\theta)(2x - 1)]x^2. \qquad (6)$$

The final electron-energy distribution is illustrated in Fig. 2; the LFV contribution enhances the number of electrons with energies $E_e \sim m_\mu/3$.

Our additional contribution to $\mu \to e\nu\nu$ can be mapped onto the following Michel parameters, assuming $\Lambda \gg v$:

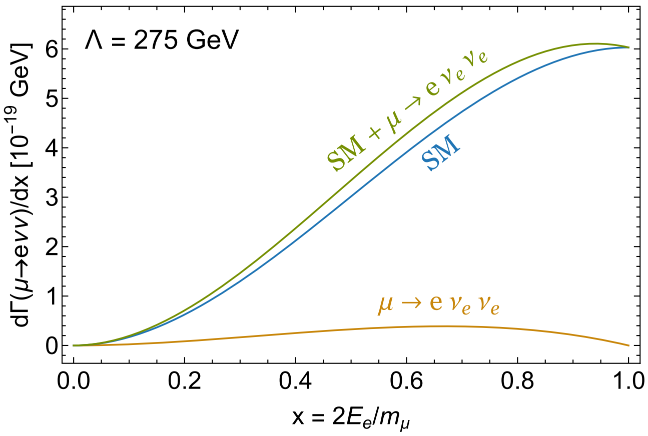

FIG. 2. Electron energy distribution in $\mu \to e\nu\nu$ from both SM and our LFV-by-three-units operator (1) with $\Lambda = 275$ GeV.

$$\rho = \frac{3}{4} - \frac{27}{512\sqrt{2}G_F^3\Lambda^6} + \mathcal{O}\left(\frac{1}{\Lambda^{12}}\right),$$
$$\delta = \frac{3}{4} - \frac{81}{512\sqrt{2}G_F^3\Lambda^6} + \mathcal{O}\left(\frac{1}{\Lambda^{12}}\right),$$
$$\xi = 1 + \frac{9}{64\sqrt{2}G_F^3\Lambda^6} + \mathcal{O}\left(\frac{1}{\Lambda^{12}}\right). \tag{7}$$

Swapping μ_R for τ_R in Eq. (1) similarly gives $\tau \to e\nu_e\nu_e$, and the third possible flavor combination gives $\tau \to \mu\nu_\mu\nu_\mu$, all of which result in the same Michel parameters in the limit of massless final-state leptons.[1]

Experimental limits on the three sets of relevant Michel parameters are listed in Table II; we construct a simple χ^2 function to fit Λ to all three Michel parameters, using $\Delta\chi^2 = 3.84$ to get 95% CL limits via

$$\chi^2(\Lambda) \le \chi^2_{\rm min} + \Delta\chi^2. \tag{8}$$

The actual minimum $\chi^2_{\rm min}$ is achieved for negative Λ^6, so we use the smallest physically allowed value, $\chi^2_{\rm min} = \chi^2(\infty)$, although this gives essentially the same limits, collected in Table III. Despite coming from $d = 7$ operators and relatively unclean signatures, these are relevant SMEFT limits, illustrating that we are already sensitive to LFV by three units. Future precision measurements of muon and tauon decays [16–18] could improve these limits further. The outgoing undetectable neutrinos make it impossible to prove lepton flavor or number violation in these decays, but the correlation of the Michel parameters shown in Eq. (7) would be a strong indication for our operators.

[1]Three other operators arise from flavor exchange *within* each operator, e.g., $\bar{e}_R L_\mu L_\mu L_\mu H$, which now violates $\Delta L_\mu = 3 = -3\Delta L_e$ and gives rise to $\mu^- \to e^-\bar{\nu}_\mu\bar{\nu}_\mu$ with $P_\mu \to -P_\mu$ in Eq. (6).

TABLE II. Experimental Michel-parameter measurements from Refs. [19–22]. Each column lists the measured values of ρ, δ (or $\delta\xi$), and ξ for the respective process indicated in the header.

$\mu \to e\nu\nu$	$\tau \to \mu\nu\nu$	$\tau \to e\nu\nu$
$\rho = 0.74979 \pm 0.00026$	$\rho = 0.763 \pm 0.020$	$\rho = 0.747 \pm 0.010$
$\delta = 0.75047 \pm 0.00034$	$\delta\xi = 0.778 \pm 0.037$	$\delta\xi = 0.734 \pm 0.028$
$\xi P_\mu = 1.0009^{+0.0016}_{-0.0007}$	$\xi = 1.030 \pm 0.059$	$\xi = 0.994 \pm 0.040$

TABLE III. 95% CL limits on the magnitudes of the Wilson coefficients using Eq. (7) and experimental values from [19].

Process	95% CL lower limit on Λ
$\mu \to e\nu\nu$	0.74 TeV
$\tau \to \mu\nu\nu$	0.32 TeV
$\tau \to e\nu\nu$	0.33 TeV

III. DIMENSION 9

The next instances of LFV by three units in the SMEFT arise at $d = 9$. Of the nonderivative operators, one type, $Qu_Ru_RL_eL_eL_e$, violates baryon number [10,23], not under consideration here, while the others again violate $\Delta L = 2$ and still do not induce Majorana neutrino masses on account of the mismatch in flavor violation. The operators involving Higgs fields,

$$\frac{1}{\Lambda^5}\bar{\ell}_R L_e L_e L_e H|H|^2, \qquad \frac{1}{\Lambda^5}\bar{L}_\alpha e_R L_e L_e HHH, \tag{9}$$

can be experimentally tested through Michel parameters, just like the $d = 7$ case above. Purely leptonic operators are of the form $\bar{\ell}_R\bar{\ell}_R LLLL$ and thus involve four charged leptons and two neutrinos. For example, the flavor combination

$$\frac{1}{\Lambda^5}\bar{e}_R\bar{e}_R L_\mu L_\mu L_\mu L_\tau, \tag{10}$$

can be enhanced compared to lower-d LFV using the anomaly-free flavor symmetry $U(1)_{2L_e+3L_\mu-5L_\tau}$ and induces two-neutrino processes such as $\tau \to ee\bar{\mu}\bar{\nu}_\mu\bar{\nu}_\mu$; without taus, the only flavor combination is $\bar{\mu}_R\bar{e}_R L_e L_e L_e L_e$, which induces $\mu \to ee\bar{e}\nu_e\nu_e$. All of these decays mimic SM decays [24] and thus require precision measurements, basically Michel parameter analyses of $\ell \to \ell\ell\ell\nu\nu$. While this can be attempted at Mu3e and Belle II, we do not expect any constraints on Λ that would push it above the electroweak scale, so the SMEFT applicability is questionable.

Finally, the last set of operators involves quarks and comes in the forms $\bar{Q}u_R\bar{\ell}_R LLL$, $\bar{d}_R Q\bar{\ell}_R LLL$, $\bar{d}_R u_R\bar{\ell}_R\bar{\ell}_R LL$, and $\bar{d}_R u_R\bar{L}LLL$, which give rise to processes involving only *one* neutrino, and thus moderately more visible than the other $d = 9$ operators. For example, the flavor combination

$$\frac{1}{\Lambda^5}\bar{d}_R u_R \bar{\mu}_R e_R L_e L_e, \tag{11}$$

yields meson decays such as $\pi^+ \to \mu^- e^+ e^+ \bar{\nu}_e$, constrained to branching ratios below 10^{-6} [19,25]. Kaon or B-meson decays into three charged leptons and one neutrino can also occur, as well as tau decays into two charged leptons, a meson, and a neutrino.

Within the SMEFT, all these operators are too suppressed to generate testable rates, especially considering that the experimental signatures are not as spectacular as other LFV decays, although the meson decays such as $\pi^+ \to \mu^- e^+ e^+ \bar{\nu}_e$ are relatively clean and show flavor violation even without neutrino identification. Still, testable rates require Λ far below the electroweak scale, forcing us to study UV completions of these operators with light neutral particles—potentially light enough to be emitted on-shell in tauon or meson decays—to confidently claim that this is a testable signature. We provide UV completions of the operators (10) and (11) in Appendix B but will refrain from a detailed study of these $d = 9$ operators and instead move on to better-visible $d = 10$ operators.

IV. DIMENSION 10

At mass dimension $d = 10$ in the SMEFT, we finally find operators that violate lepton flavor by three units without also violating baryon or total lepton number. These can give rise to fully visible neutrinoless *charged* LFV. Nonderivative operators all contain one Higgs doublet H and come in three different types, organized by the number of lepton doublets,

$$\bar{L}\,\bar{L}\,\bar{L}\,LLl_R H, \qquad \bar{L}\,\bar{L}\,L\bar{l}_R l_R l_R H, \qquad \bar{L}\bar{l}_R\bar{l}_R l_R l_R l_R H. \tag{12}$$

In terms of their flavor structure, these operators can give LFV by three units in the two varieties

$$\Delta L_\alpha = 3, \qquad \Delta L_\beta = -2, \qquad \Delta L_\gamma = -1, \tag{13}$$

$$\Delta L_\alpha = 3, \qquad \Delta L_\beta = -3, \qquad \Delta L_\gamma = 0, \tag{14}$$

with α, β, γ being three different lepton flavors. Only the first variety can lead to *neutrinoless* lepton decays, namely

$$\tau \to eee\bar{\mu}\,\bar{\mu}, \quad \text{which conserves} \quad U(1)_{L_e+4L_\mu-5L_\tau}, \tag{15}$$

$$\tau \to \mu\mu\mu\bar{e}\,\bar{e}, \quad \text{which conserves} \quad U(1)_{L_\mu+4L_e-5L_\tau}. \tag{16}$$

These two decays are experimentally clean, so Belle II should be able to probe these down to branching ratios of order 10^{-8}, and from the theoretical perspective we can easily make either of these two decays dominant over all other LFV by imposing the appropriate $U(1)$ flavor symmetry; for details see the argument in Refs. [9] or [10]. However, testable rates would require $\Lambda \sim \text{GeV}$ for the SMEFT scale, of questionable validity given that mediator particles with such low masses would have been observed at collider experiments long ago. This makes it necessary to study full UV completions of these LFV operators with light particles to investigate if testable LFV rates are compatible with indirect constraints. We will perform such an analysis below for one particular UV completion to highlight the challenges.

A. UV completion for $\tau \to eee\bar{\mu}\,\bar{\mu}$

We will study the decay channel $\tau \to eee\bar{\mu}\,\bar{\mu}$; switching electron and muon makes little difference. The relevant $d = 10$ operators are listed in Table IV and their UV completions compiled in Appendix C. We will focus on the operator $\bar{L}_\mu \bar{L}_\mu \bar{\tau}_R e_R e_R L_e H$ and its UV completion by the diagram shown in Fig. 3 via two new scalars $S_{1,2}$ and one fermion F; their charges under the SM gauge group and the conserved $U(1)_{L_e+4L_\mu-5L_\tau}$ are given in Table V.

The relevant couplings of the new particles take the form

$$-\mathcal{L} \supset Y_1 \bar{F}_L S_1 e_R + \tilde{Y}_1 \bar{F}_L S_1^\dagger \tau_R + Y_2 \bar{L}_\mu S_2^\dagger e_R + \tilde{Y}_2 \bar{L}_e S_2^\dagger F_R + m_F \bar{F}_L F_R + \mu S_2 H S_1^\dagger + \text{H.c.}, \tag{17}$$

with the imposed $U(1)_{L_e+4L_\mu-5L_\tau}$ forbidding many other terms that would induce LFV by one or two units. μ and m_F are parameters with dimensions of mass, positive without

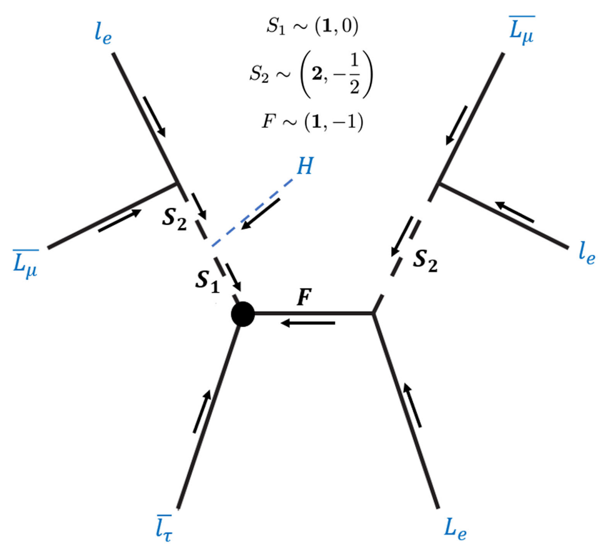

FIG. 3. UV completion of the operator $\bar{L}_\mu \bar{L}_\mu \bar{l}_\tau l_e l_e L_e H$ of interest in Sec. IV A. Arrows indicate the direction of momentum flow; $SU(2)_L \times U(1)_Y$ quantum numbers of the new particles are also displayed.

TABLE IV. Nonderivative $d=10$ operators for $\tau \to eee\bar{\mu}\,\bar{\mu}$.

Generic operator	Operator(s) with flavor indices
$\bar{L}\,\bar{L}\,\bar{L}\,lLLH$	$\bar{L}_\tau\bar{L}_\mu\bar{L}_\mu l_e L_e L_e H$
	$\bar{L}_e\bar{L}_e\bar{L}_e l_\mu L_\mu L_\tau H$
	$\bar{L}_e\bar{L}_e\bar{L}_e l_\tau L_\mu L_\mu H$
$\bar{L}\,\bar{L}\,\bar{l}\,llLH$	$\bar{L}_\tau\bar{L}_\mu\bar{l}_\mu l_e l_e L_e H$
	$\bar{L}_\mu\bar{L}_\mu\bar{l}_\tau l_e l_e L_e H$
	$\bar{L}_e\bar{L}_e\bar{l}_e l_\mu l_\mu L_\tau H$
	$\bar{L}_e\bar{L}_e\bar{l}_e l_\mu l_\tau L_\mu H$
$\bar{L}\,\bar{l}\,\bar{l}\,lllH$	$\bar{L}_e\bar{l}_e\bar{l}_e l_\mu l_\mu l_\tau H$

TABLE V. Charge assignments for our UV completion in Sec. IV A, F and $S_{1,2}$ being new particles.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{L_e+4L_\mu-5L_\tau}$
L_e	**1**	**2**	$-\frac{1}{2}$	1
e_R	**1**	**1**	-1	1
L_μ	**1**	**2**	$-\frac{1}{2}$	4
μ_R	**1**	**1**	-1	4
L_τ	**1**	**2**	$-\frac{1}{2}$	-5
τ_R	**1**	**1**	-1	-5
$F_{L,R}$	**1**	**1**	-1	-2
H	**1**	**2**	$\frac{1}{2}$	0
S_1	**1**	**1**	0	-3
S_2	**1**	**2**	$-\frac{1}{2}$	-3

loss of generality; the Y are dimensionless Yukawa couplings. Together, these couplings give rise to the desired $\tau \to eee\bar{\mu}\,\bar{\mu}$ via Fig. 3, but of course more than that, as the new particles induce various non-LFV processes. F and S_2 in particular carry SM gauge charges and should thus be heavier than $m_Z/2$ to avoid egregious contributions to Z decays. This is true even for the neutral component of S_2 because it remains a complex scalar carrying $U(1)_{L_e+4L_\mu-5L_\tau}$ charge, i.e., is not split into two real scalars as in other two-Higgs-doublet models [26]. S_1 on the other hand is an SM *singlet* and could be much lighter, which is the key to enhance $\tau \to eee\bar{\mu}\,\bar{\mu}$. In fact, the biggest rate enhancement is achieved when S_1 is lighter than the tauon, splitting the five-body decay into the four-body decay $\tau \to ee\bar{\mu}S_1$ followed by near-instantaneous $S_1 \to e\bar{\mu}$.

To calculate the relevant rates, we note that $\mu S_2 H S_1^\dagger$ mixes S_1 and S_2^0 after electroweak symmetry breaking, with mixing angle

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{2}\mu v}{m_{S_2^0}^2 - m_{S_1}^2}\right) \simeq \frac{\mu v}{\sqrt{2}m_{S_2^0}^2}, \tag{18}$$

working in the small-angle limit for simplicity. The neutral mass eigenstates $s_{1,2}$,

$$s_1 \equiv \cos\theta S_1 + \sin\theta S_2^0, \tag{19}$$

$$s_2 \equiv -\sin\theta S_1 + \cos\theta S_2^0, \tag{20}$$

are the relevant scalars for our tau decay, while the charged component of S_2 is assumed to be heavy enough to be ignored for the time being. F and s_2 are heavy enough to be integrated out, yielding the four-body tau decay rate

$$\Gamma(\tau \to ee\bar{\mu}s_1) = \frac{2|\tilde{Y}_2\tilde{Y}_1 Y_2|^2}{m_{s_2}^4 m_F^2}\frac{m_\tau^7}{2949120\pi^5} f\left(\frac{m_{s_1}^2}{m_\tau^2}\right) \tag{21}$$

with function

$$f(x) = 1 + \frac{155}{4}x + 20x^2 - 55x^3 - 5x^4 + \frac{1}{4}x^5 + 15x(1+4x+2x^2)\log x. \tag{22}$$

The scalar s_1 subsequently decays with rate

$$\Gamma(s_1 \to e\bar{\mu}) \simeq \frac{m_{s_1}|Y_2|^2\theta^2}{16\pi} \simeq \frac{\mu^2|Y_2|^2 v^2 m_{s_1}}{32\pi m_{s_2}^4}. \tag{23}$$

s_1 carries a typical momentum of $m_\tau/4$ in the tau rest frame, which gives a boosted decay length of

$$L_{\text{avg}}(s_1) = \frac{m_\tau/4}{m_{s_1}\Gamma(s_1 \to e\bar{\mu})}. \tag{24}$$

In most experiments, the tau rest frame is not identical to the lab frame, but we neglect this distinction here for simplicity. The goal is to maximize $\Gamma(\tau \to ee\bar{\mu}s_1)$ while keeping $L_{\text{avg}}(s_1)$ short enough so that s_1 decays *inside* the detector. For this, we need to study limits on the relevant masses and couplings. Since the tau decay rate of Eq. (21) scales with $m_{s_2}^{-4}$, it is desirable to make the neutral S_2 component as light as possible.

B. Limits

$s_2 \sim S_2^0$ has the Y_2 coupling to electrons and muons, which in particular induces an $\bar{e}e\bar{\mu}\mu$ four-lepton operator that conserves lepton flavor and number, but is still subject to experimental constraints:

(i) It contributes to the anomalous magnetic moments of muon and electron, $\Delta a_{\mu,e}$, which in the limit $m_\ell \ll m_{S_2^0}$ read as [27]

$$\Delta a_\ell = \frac{2m_\ell^2|Y_2|^2}{96\pi^2 m_{S_2^0}^2}, \tag{25}$$

the factor of 2 arising from the degenerate real scalar and pseudoscalar inside the complex S_2^0. Comparing this with the experimental values $\Delta a_e = (-87 \pm 36) \times 10^{-14}$ [28–30] and $\Delta a_\mu = (2.49 \pm 0.48) \times 10^{-9}$ [31,32] gives the limits

$$|Y_2| < 4.1\left(\frac{m_{S_2^0}}{100\ \mathrm{GeV}}\right), \qquad \text{from } \Delta a_e, \tag{26}$$

$$|Y_2| < 1.45\left(\frac{m_{S_2^0}}{100\ \mathrm{GeV}}\right), \qquad \text{from } \Delta a_\mu. \tag{27}$$

Here, we took the 5σ CL range of the measured values to derive these limits, indicating that the maximum allowed values for a given mediator mass beyond which new physics contributions would exceed the experimentally measured values. Given the current mismatch between experimental measurement [31,32] and SM prediction [33,34] for Δa_μ, the limit should be taken with a grain of salt. In our case, Δa_ℓ gives weaker limits than LEP anyway, so the details are less important.

(ii) Integrating out S_2^0 gives the four-fermion operator

$$\mathcal{L}_{\rm eff} = -\frac{|Y_2|^2}{2m_{S_2^0}^2}(\bar{e}_R\gamma^\alpha e_R)(\bar{\mu}_L\gamma_\alpha\mu_L) \tag{28}$$

after performing a Fierz transformation. Contact interactions such as this one modify $e^+e^- \to \mu^+\mu^-$ and have been constrained by LEP [35]. Specifically, the above interaction corresponds to the $\Lambda^-_{LR,\mu\mu}$ parameter [36], which is the least constrained of all $ee\mu\mu$ contact interactions by far, with Ref. [35] quoting 2.2 TeV. In Appendix D, we explain in detail that this is due to an approximate degeneracy in the SM $+\,\Lambda^-_{LR,\mu\mu}$ cross section in the narrow energy region used in the LEP fit, and that the inclusion of low-$\sqrt{s}$ data from TRISTAN [37] lifts the degeneracy and enforces the much stronger limit $\Lambda^-_{LR,\mu\mu} > 8$ TeV, or

$$m_{S_2^0}/|Y_2| > 1.6\ \mathrm{TeV}. \tag{29}$$

In any case, this limit is only valid for S_2 masses far above LEP energies, not exactly the region of interest for us. Instead, we calculate the full cross section $e^+e^- \to \mu^+\mu^-$ and fit it to the same cross section and forward-backward asymmetry data that underlies the contact-interaction limits to find the exclusion curve in Fig. 4, which reduces to the $\Lambda^-_{LR,\mu\mu}$ limit for large masses but flattens out to $Y_2 \simeq 0.1$ for light S_2^0. Together with the Z-decay limit, $m_{S_2^0} > m_Z/2$, this is stronger than the magnetic-moment constraints and even future MUonE [38] sensitivity to deviations in $\mu e \to \mu e$ scattering from such scalars [39].

(iii) $f\bar{f} \to Z^* \to (S_2^0 \to \mu^+e^-)(\bar{S}_2^0 \to \mu^-e^+)$ could provide excellent bounds for $m_{S_2^0} < 100$ GeV (LEP-II) or $m_{S_2^0} < \mathcal{O}(\mathrm{TeV})$ (LHC), although we are not aware of such searches.[2] Given the clean signature and large cross section for this process, we will conservatively assume that this would have been observed; evading such limits is possible for $m_{S_2^0} > m_h + m_{S_1}$, since then the decay $S_2^0 \to hS_1$ becomes kinematically allowed and can dominate over $S_2^0 \to \mu^+e^-$ if $\mu \gg \sqrt{2}m_{S_2^0}Y_2$ (or $\theta \gg Y_2 v/m_{S_2^0}$), which is doable even in the small-θ regime. This will severely weaken LEP and LHC constraints on S_2^0, likely opening up parameter space around $Y_2 \sim 0.1$, $m_{S_2^0} \gtrsim m_h$. Again, dedicated searches for this kind of neutral scalar could drastically improve limits, so we encourage our experimental colleagues to look for this clean signature. Notice that this mass hierarchy also eliminates $h \to s_1 s_2$.

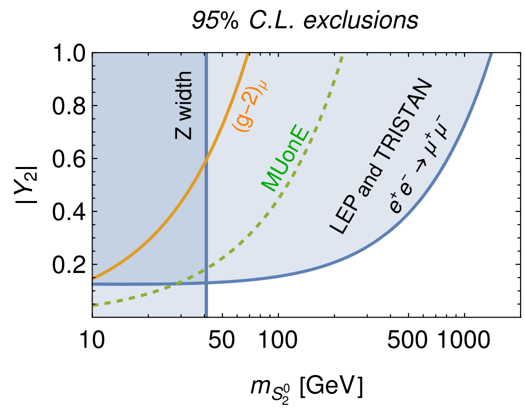

FIG. 4. Limits on the complex neutral scalar S_2^0 with μ_L–e_R coupling Y_2, see text for details.

The charged component of S_2 decays almost exclusively via $S_2^- \to e^-\bar{\nu}_\mu$ and does not couple to quarks, unlike the typical charged scalars under scrutiny at LHC [42]. It does mimic a left-handed selectron $\tilde{e}_L$ in the massless-neutralino limit in existing searches for supersymmetric particles [43,44]. The ATLAS analysis of the LHC Run-2 data [43] provides a lower bound of $m_{S_2^+} > 550$ GeV at 95% CL. This does not directly impact the tau decay of interest to us, since the diagram shown in Fig. 3 does not involve S_2^-, but a large mass splitting within the S_2 doublet will contribute to the oblique parameters, notably T [45,46]. This effect arises at loop level and potentially interferes with other new physics in the electroweak sector, so we will not take this limit into account here.

The light complex singlet $s_1 \sim S_1$ inherits a coupling to the Z boson from the mixing with S_2^0, which makes it available in Z decays,

$$\mathrm{BR}(Z \to s_1\bar{s}_1) \simeq \frac{3\theta^4}{86}. \tag{30}$$

[2] Aside from different lepton flavors, similar signatures have been proposed in Refs. [40,41].

The total Z width agrees very well with the SM prediction [19], which can be translated into a 2σ upper bound of 2×10^{-3} on any non-SM Z branching ratio, enforcing $|\theta| < 0.5$, easily satisfied in the small-mixing-angle regime. A dedicated search for $Z\to s_1\bar{s}_1\to\mu^+\mu^-e^+e^-$, typically with displaced s_1 vertex, could vastly improve this limit.

Lastly, the new charged fermion F was implicitly taken to be heavier than S_2^0 above, because that hierarchy maximizes the tau decay of Eq. (21). Notice that F does not mix with the SM leptons on account of the $U(1)_{L_e+4L_\mu-5L_\tau}$ symmetry. It can decay into eS_1 through the Y_1 coupling, which does not feed into $\tau\to ee\bar{\mu}s_1$, or into $\tau\bar{S}_1$ via $\tilde{Y}_1$, or into L_eS_2 via $\tilde{Y}_2$. The decays into S_1 will lead to displaced-vertex signatures, the decay $F^-\to e^-(S_2^0\to\mu^+e^-)$ will be prompt. We are not aware of searches for such final states, although they should be fantastically clean. Since S_1 will decay displaced, it might appear as missing energy, rendering the signature similar to right-handed stau searches, $pp\to\tilde{\tau}_R\tilde{\tau}_R\to\tau\tau+\not{E}$. Recent ATLAS limits exclude stau masses below 350 GeV, assuming 100% decays into $\tau+\not{E}$ [47]. The limits on our fermion will likely be stronger, since the Drell-Yan cross section into fermions is roughly a factor 4 larger than into scalars, albeit with a more complicated relationship near threshold. Once again we encourage dedicated searches for this clean setup, especially $pp\to F^-F^+$ followed by the decay chain $F^-\to e^-(S_2^0\to\mu^+e^-)$. For now, we assume that order-one couplings of F with $m_F\sim 350$ GeV are currently allowed.

C. Discussion

We will take the following benchmark point, which satisfies current collider constraints:

$$m_{s_2}=140\ \text{GeV},\qquad m_F=350\ \text{GeV},\qquad \theta=0.2,$$
$$Y_2=0.1,\qquad \tilde{Y}_1=\tilde{Y}_2=3,\tag{31}$$

with m_{s_1} varying from $m_\mu+m_e$ (to allow for $s_1\to e\bar{\mu}$) to $m_\tau-m_\mu-2m_e$ (to enable $\tau\to ee\bar{\mu}s_1$). The resulting branching ratio for the LFV-by-three-units tauon decay is shown in Fig. 5. For very light s_1, near the threshold for it to decay into μe, the tau branching ratio can be of order 10^{-9}, potentially in reach of Belle II. Dedicated collider searches for the new particles could probe the same region of parameter space. Since s_1 is very light here, its decay length (in the tau rest frame) is fairly long, see Fig. 6, although still below a nanometer, rendering the tau decay $\tau\to eee\bar{\mu}\,\bar{\mu}$ effectively prompt.

Let this suffice as an example for the challenges involved in generating a large $\tau\to eee\bar{\mu}\,\bar{\mu}$ rate; other operators and UV completions might fare better or worse, our intent is not to promote one particular model but rather to emphasize that LFV-by-three-units signatures are not entirely hopeless but can be probed at colliders and τ factories.

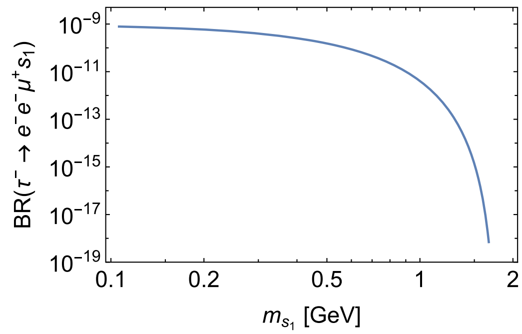

FIG. 5. Branching ratio $\mathrm{BR}(\tau\to ee\bar{\mu}s_1)$ as function of m_{s_1} for the benchmark values from Eq. (31). Together with the near-prompt decay $s_1\to e\bar{\mu}$ (see Fig. 6), this gives the LFV-by-three-units decay $\tau\to eee\bar{\mu}\,\bar{\mu}$.

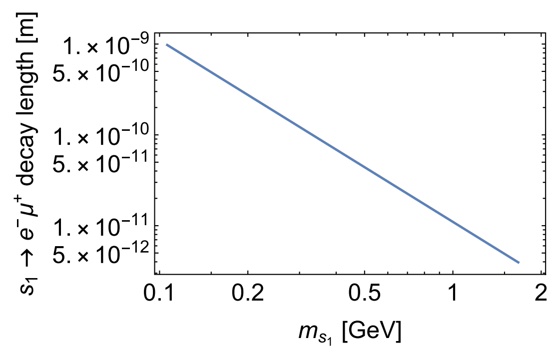

FIG. 6. Estimated decay length of s_1 following the tau decay $\tau\to ee\bar{\mu}s_1$ as a function of m_{s_1}, using the benchmark values from Eq. (31).

V. CONCLUSIONS

Searches for lepton flavor violation are as old as the discovery of the muon and have steadily improved ever since, but we have yet to discover signs of it. This is all the more surprising since neutrino oscillations have proven beyond doubt that lepton flavor is violated in nature. It appears as though lepton flavor is at least an excellent *approximate* symmetry, neutrino masses aside. This makes possible different flavor breaking patterns in the neutrino and charged lepton sector, which could give rise to rather unusual signatures, for example flavor violation by *three* units. In this article, we have shown that it is indeed theoretically possible to heavily suppress flavor violation by one or two units, making three units the dominant channel. In the SMEFT, such operators arise first at dimension 7, but come together with total lepton number violation, forcing neutrinos into any and all such processes. Still, $\ell_\alpha\to\ell_\beta\nu_\beta\nu_\beta$ provide testable signals of such operators, conveniently parametrized through Michel parameters. *Charged* lepton flavor violation by three units first arises at mass dimension 10, with golden modes $\tau\to eee\bar{\mu}\,\bar{\mu}$ or $\tau\to\mu\mu\mu\bar{e}\,\bar{e}$, but with absurdly suppressed rates within the SMEFT framework. We provide tree-level UV completions for all operators of interest in this study, and investigate one example in which a new particle is lighter than the tau, enabling the parametrically faster decay $\tau \to ee\bar{\mu}(s \to e\bar{\mu})$. In this model, tau decay rates in reach of Belle II seem possible, but the same model can also be investigated at the LHC in a variety of clean signatures ranging from $pp \to Z^* \to S_2^0 \bar{S}_2^0 \to \mu^+ e^- \mu^- e^+$ to $pp \to \gamma^*/Z^* \to F^- F^+ \to e^- S_2^0 e^+ S_2^0 \to e^- \mu^+ e^- e^+ \mu^- e^-$. We encourage our experimental colleagues to have a look at these novel signatures, which should be useful beyond our model example and might reveal particles overlooked so far.

We have made excessive use of Renato Fonseca's *Mathematica* packages Sym2Int [48] and GroupMath [49].

ACKNOWLEDGMENTS

J. H. thanks Martin Gruenewald and Dimitri Bourilkov for their patient correspondence regarding Ref. [35]. This work was supported by the U.S. Department of Energy under Grant No. DE-SC0007974.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: UV COMPLETIONS FOR $d = 7$ OPERATORS

Here, we list all tree-level UV completions for the $d = 7$ operators $\bar{l}LLLH$ of interest in Sec. II. Notice that we list *all* UV completions, even though not all of them give the LFV-by-three-units decays we are interested in, because they can come with unwelcome antisymmetric couplings: the scalar $S \sim (\mathbf{1}, \mathbf{1}, 1)$ couples to $\bar{L}^c L$, which is *antisymmetric* in flavor space, meaning it does not contain the desired $L_e L_e$ coupling for Eq. (1). For this, we need to use the triplet scalar $S \sim (\mathbf{1}, \mathbf{3}, 1)$, which has a *symmetric* coupling to $\bar{L}^c L$.

This appendix consists of two sections, each starting with a Feynman diagram of a given tree-level topology. Diagrams are followed by several tables containing information about gauge numbers of mediating particles. Each row contains a set of gauge quantum numbers in $SU(3)_C \times SU(2)_L \times U(1)_Y$ space corresponding to each mediating particle. Table captions display which layout of external particles allows for the rows of mediators listed in the respective tables.

1. First topology (Fig. 7)

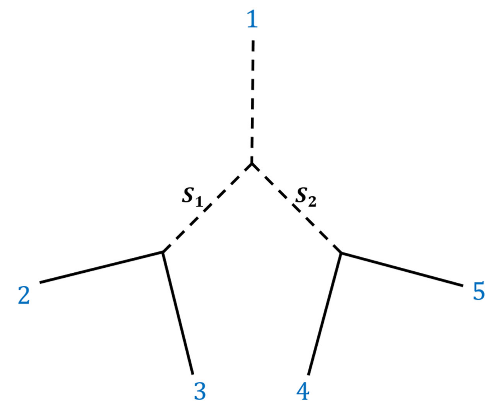

FIG. 7. Feynman diagram for Table VI.

TABLE VI. $1 \to H$, $2 \to \bar{l}$, $3 \to L$, $4 \to L$, $5 \to L$.

S_1	S_2
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

2. Second topology (Fig. 8)

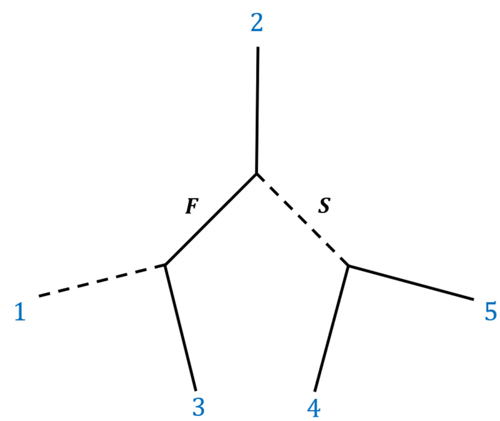

FIG. 8. Feynman diagram for Tables VII–IX.

TABLE VII. $1 \to H$, $2 \to \bar{l}$, $3 \to L$, $4 \to L$, $5 \to L$.

F	S
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE VIII. $1 \to H$, $2 \to L$, $3 \to \bar{l}$, $4 \to L$, $5 \to L$.

F	S
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE IX. $1 \to H$, $2 \to L$, $3 \to L$, $4 \to \bar{l}$, $5 \to L$.

F	S
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

APPENDIX B: UV COMPLETIONS FOR $d = 9$ OPERATORS

Here, we list tree-level UV completions for the $d = 9$ operators of interest in Sec. III. Notation and organization follow Appendix A.

1. First topology (Fig. 9)

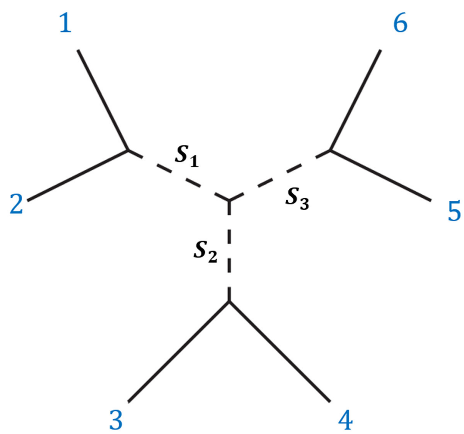

FIG. 9. Feynman diagram for the given topology. The corresponding tables are Tables X–XIII.

a. $\bar{l}\bar{l}LLLL$

TABLE X. $1 \to \bar{l}$, $2 \to \bar{l}$, $3 \to L$, $4 \to L$, $5 \to L$, $6 \to L$.

S_1	S_2	S_3
$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, -1)$
$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, -1)$

TABLE XI. $1 \to \bar{l}$, $2 \to L$, $3 \to \bar{l}$, $4 \to L$, $5 \to L$, $6 \to L$.

S_1	S_2	S_3
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$

b. $\bar{d}\bar{l}ulLL$

TABLE XII. $1 \to l$, $2 \to u$, $3 \to \bar{l}$, $4 \to \bar{d}$, $5 \to L$, $6 \to L$.

S_1	S_2	S_3
$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{4}{3})$	$(\mathbf{1}, \mathbf{1}, -1)$

TABLE XIII. $1 \to l$, $2 \to u$, $3 \to \bar{l}$, $4 \to L$, $5 \to \bar{d}$, $6 \to L$.

S_1	S_2	S_3
$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\bar{\mathbf{3}}, \mathbf{2}, -\frac{1}{6})$

2. Second topology (Fig. 10)

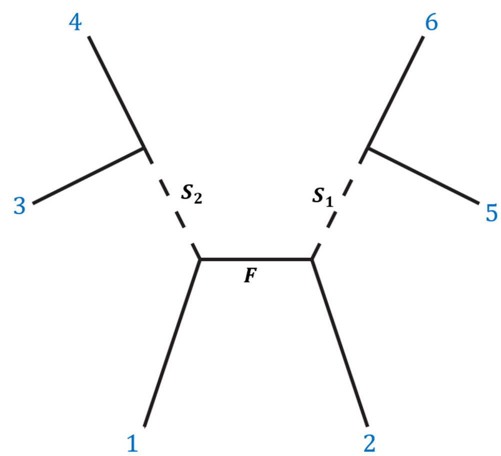

FIG. 10. Feynman diagram for the given topology. The corresponding tables are Tables XIV–XXIX.

a. $\bar{l}\bar{l}LLLL$

TABLE XIV. $1 \to \bar{l}$, $2 \to \bar{l}$, $3 \to L$, $4 \to L$, $5 \to L$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE XV. $1 \to \bar{l}$, $2 \to L$, $3 \to \bar{l}$, $4 \to L$, $5 \to L$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE XVI. $1 \to \bar{l}$, $2 \to L$, $3 \to L$, $4 \to L$, $5 \to \bar{l}$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE XVII. $1 \to L$, $2 \to L$, $3 \to \bar{l}$, $4 \to \bar{l}$, $5 \to L$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$
$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$

TABLE XVIII. $1 \to L$, $2 \to L$, $3 \to \bar{l}$, $4 \to L$, $5 \to \bar{l}$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

b. $\bar{d}\,\bar{l}\,ulLL$

TABLE XIX. $1 \to l$, $2 \to u$, $3 \to \bar{l}$, $4 \to \bar{d}$, $5 \to L$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	$(\mathbf{3}, \mathbf{1}, -\frac{4}{3})$

TABLE XX. $1 \to l$, $2 \to u$, $3 \to \bar{l}$, $4 \to L$, $5 \to \bar{d}$, $6 \to L$.

S_1	F	S_2
$(\bar{\mathbf{3}}, \mathbf{2}, -\frac{1}{6})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE XXI. $1 \to l$, $2 \to u$, $3 \to \bar{d}$, $4 \to L$, $5 \to \bar{l}$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{3}, \mathbf{2}, \frac{7}{6})$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$

TABLE XXII. $1 \to l$, $2 \to u$, $3 \to L$, $4 \to L$, $5 \to \bar{l}$, $6 \to \bar{d}$.

S_1	F	S_2
$(\bar{\mathbf{3}}, \mathbf{1}, \frac{4}{3})$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE XXIII. $1 \to \bar{l}$, $2 \to \bar{d}$, $3 \to l$, $4 \to u$, $5 \to L$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{1}, -1)$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$

TABLE XXIV. $1 \to \bar{l}$, $2 \to L$, $3 \to l$, $4 \to u$, $5 \to \bar{d}$, $6 \to L$.

S_1	F	S_2
$(\bar{\mathbf{3}}, \mathbf{2}, -\frac{1}{6})$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$

TABLE XXV. $1 \to \bar{d}$, $2 \to \bar{l}$, $3 \to l$, $4 \to u$, $5 \to L$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$

TABLE XXVI. $1 \to \bar{d}$, $2 \to L$, $3 \to l$, $4 \to u$, $5 \to \bar{l}$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$

TABLE XXVII. $1 \to L$, $2 \to \bar{l}$, $3 \to l$, $4 \to u$, $5 \to \bar{d}$, $6 \to L$.

S_1	F	S_2
$(\bar{\mathbf{3}}, \mathbf{2}, -\frac{1}{6})$	$(\bar{\mathbf{3}}, \mathbf{2}, \frac{5}{6})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$

TABLE XXVIII. $1 \to L$, $2 \to \bar{d}$, $3 \to l$, $4 \to u$, $5 \to \bar{l}$, $6 \to L$.

S_1	F	S_2
$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\bar{\mathbf{3}}, \mathbf{2}, \frac{5}{6})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$

TABLE XXIX. $1 \to L$, $2 \to L$, $3 \to l$, $4 \to u$, $5 \to \bar{l}$, $6 \to \bar{d}$.

S_1	F	S_2
$(\bar{\mathbf{3}}, \mathbf{1}, \frac{4}{3})$	$(\bar{\mathbf{3}}, \mathbf{2}, \frac{5}{6})$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$

APPENDIX C: UV COMPLETIONS FOR $d = 10$ OPERATORS

Here, we list tree-level UV completions for the $d = 10$ operators of interest in Sec. IV. Notation and organization follow Appendix A.

1. First topology (Fig. 11)

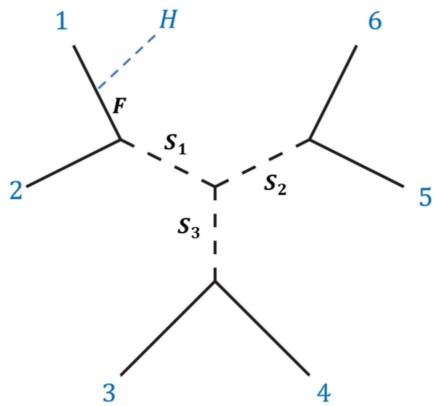

FIG. 11. Feynman diagram for the cases where H is attached to an external leg. The corresponding tables are Tables XXX–XLV.

a. $\bar{L}\bar{L}\bar{L}lLLH$

TABLE XXX. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	S_3	F
$(\mathbf{1}, \mathbf{4}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE XXXI. $1 \to \bar{L}$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	S_3	F
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE XXXII. $1 \to L$, $2 \to L$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	S_3	F
$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$

TABLE XXXIII. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	S_3	F
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

b. $\bar{L}\bar{L}\bar{l}llLH$

TABLE XXXIV. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	S_3	F
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE XXXV. $1 \to \bar{L}$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	S_3	F
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE XXXVI. $1 \to \bar{l}$, $2 \to L$, $3 \to l$, $4 \to l$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	S_3	F
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$

TABLE XXXVII. $1 \to \bar{l}, 2 \to L, 3 \to l, 4 \to \bar{L}, 5 \to l, 6 \to \bar{L}$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$

TABLE XXXVIII. $1 \to l, 2 \to \bar{L}, 3 \to l, 4 \to \bar{L}, 5 \to \bar{l}, 6 \to L$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE XXXIX. $1 \to l, 2 \to l, 3 \to \bar{L}, 4 \to \bar{L}, 5 \to \bar{l}, 6 \to L$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE XL. $1 \to L, 2 \to \bar{l}, 3 \to l, 4 \to l, 5 \to \bar{L}, 6 \to \bar{L}$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{3},0)$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},0)$

TABLE XLI. $1 \to L, 2 \to \bar{l}, 3 \to l, 4 \to \bar{L}, 5 \to l, 6 \to \bar{L}$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$

c. $\bar{L}\bar{l}\bar{l}lllH$

TABLE XLII. $1 \to \bar{L}, 2 \to l, 3 \to l, 4 \to l, 5 \to \bar{l}, 6 \to \bar{l}$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},1)$

TABLE XLIII. $1 \to \bar{l}, 2 \to \bar{l}, 3 \to l, 4 \to l, 5 \to l, 6 \to \bar{L}$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{2},\frac{5}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$

TABLE XLIV. $1 \to l, 2 \to \bar{L}, 3 \to l, 4 \to l, 5 \to \bar{l}, 6 \to \bar{l}$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE XLV. $1 \to l, 2 \to l, 3 \to l, 4 \to \bar{L}, 5 \to \bar{l}, 6 \to \bar{l}$.

S_1	S_2	S_3	F
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

2. Second topology (Fig. 12)

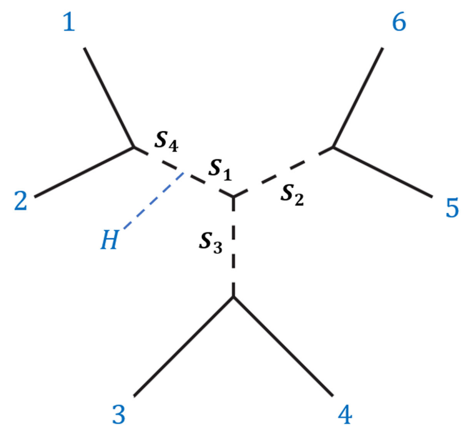

FIG. 12. Feynman diagram for the cases where H is attached to an internal mediator. The corresponding tables are Tables XLVI–LVI.

a. $\bar{L}\bar{L}\bar{L}lLLH$

TABLE XLVI. $1 \to \bar{L}, 2 \to \bar{L}, 3 \to l, 4 \to \bar{L}, 5 \to L, 6 \to L$.

S_1	S_2	S_3	S_4
$(\mathbf{1},\mathbf{4},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE XLVII. $1 \to l, 2 \to \bar{L}, 3 \to \bar{L}, 4 \to \bar{L}, 5 \to L, 6 \to L$.

S_1	S_2	S_3	S_4
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE XLVIII. $1 \to L$, $2 \to L$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$

b. $\bar{L}\bar{L}\bar{l}llLH$

TABLE XLIX. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE L. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LI. $1 \to \bar{l}$, $2 \to L$, $3 \to l$, $4 \to l$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

TABLE LII. $1 \to \bar{l}$, $2 \to L$, $3 \to l$, $4 \to \bar{L}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

TABLE LIII. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$

c. $\bar{L}\bar{l}\bar{l}llH$

TABLE LIV. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LV. $1 \to \bar{l}$, $2 \to \bar{l}$, $3 \to l$, $4 \to l$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{2}, \frac{5}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, 2)$

TABLE LVI. $1 \to l$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	S_3	S_4
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$

3. Third topology (Fig. 13)

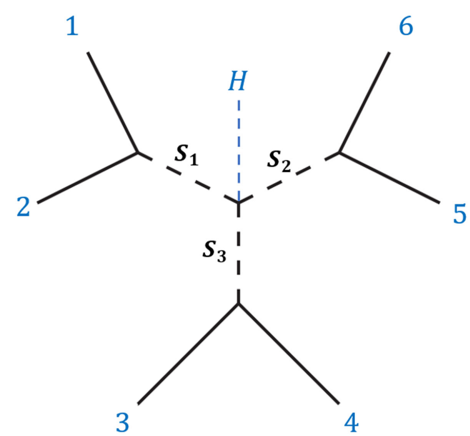

FIG. 13. Feynman diagram for the cases where H is attached to a central vertex of mediators. The corresponding tables are Tables LVII–LX.

a. $\bar{L}\bar{L}\bar{L}lLLH$

TABLE LVII. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	S_3
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{1}, 1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 1)$

b. $\bar{L}\bar{L}\bar{l}llLH$

TABLE LVIII. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	S_3
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE LIX. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	S_3
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

c. $\bar{L}\bar{l}\bar{l}lllH$

TABLE LX. $1 \to l$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	S_3
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

4. Fourth topology (Fig. 14)

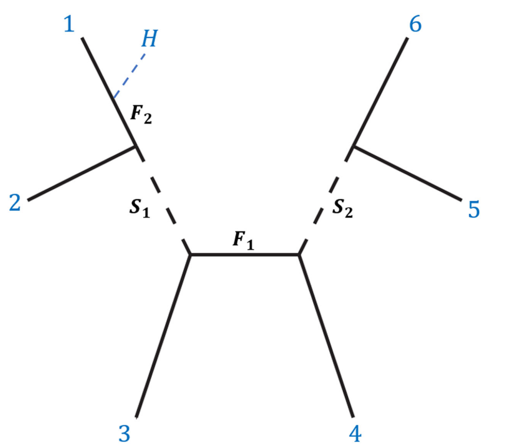

FIG. 14. Feynman diagram for the cases where H is attached to an external leg. The corresponding tables are Tables LXI–CII.

a. $\bar{L}\bar{L}\bar{L}lLLH$

TABLE LXI. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to l$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{4},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},-2)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},-2)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},-2)$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},1)$

TABLE LXII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to L$, $4 \to L$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{4},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},1)$

TABLE LXIII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{4},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{4},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE LXIV. $1 \to \bar{L}$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{4},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE LXV. $1 \to \bar{L}$, $2 \to l$, $3 \to L$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{4},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE LXVI. $1 \to L$, $2 \to L$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{4},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},0)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},0)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},0)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{1},0)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},0)$

TABLE LXVII. $1 \to L$, $2 \to L$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{1}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 0)$

TABLE LXVIII. $1 \to L$, $2 \to L$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{4}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$

TABLE LXIX. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LXX. $1 \to l$, $2 \to \bar{L}$, $3 \to L$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{4}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

b. $\bar{L}\bar{L}\bar{l}llLH$

TABLE LXXI. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to \bar{l}$, $4 \to L$, $5 \to l$, $6 \to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{5}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{5}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE LXXII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE LXXIII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to L$, $4 \to \bar{l}$, $5 \to l$, $6 \to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE LXXIV. $1 \to \bar{L}$, $2 \to l$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE LXXV. $1 \to \bar{L}$, $2 \to l$, $3 \to \bar{l}$, $4 \to L$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE LXXVI. $1 \to \bar{L}$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE LXXVII. $1 \to \bar{L}$, $2 \to l$, $3 \to L$, $4 \to \bar{l}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE LXXVIII. $1 \to \bar{l}$, $2 \to L$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to l$, $6 \to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$

TABLE LXXIX. $1 \to \bar{l}$, $2 \to L$, $3 \to \bar{L}$, $4 \to l$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$

TABLE LXXX. $1 \to \bar{l}$, $2 \to L$, $3 \to l$, $4 \to \bar{L}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$

TABLE LXXXI. $1 \to \bar{l}$, $2 \to L$, $3 \to l$, $4 \to l$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$

TABLE LXXXII. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LXXXIII. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{l}$, $4 \to L$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LXXXIV. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LXXXV. $1 \to l$, $2 \to \bar{L}$, $3 \to L$, $4 \to \bar{l}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LXXXVI. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LXXXVII. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LXXXVIII. $1 \to l$, $2 \to l$, $3 \to L$, $4 \to \bar{l}$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 2)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE LXXXIX. $1 \to L$, $2 \to \bar{l}$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to l$, $6 \to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$

TABLE XC. $1 \to L$, $2 \to \bar{l}$, $3 \to \bar{L}$, $4 \to l$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$

TABLE XCI. $1 \to L$, $2 \to \bar{l}$, $3 \to l$, $4 \to \bar{L}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 0)$

TABLE XCII. $1 \to L$, $2 \to \bar{l}$, $3 \to l$, $4 \to l$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{3}, 0)$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{1}, 0)$

c. $\bar{L}\bar{l}\bar{l}lllH$

TABLE XCIII. $1\to\bar{L}$, $2\to l$, $3\to\bar{l}$, $4\to\bar{l}$, $5\to l$, $6\to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},1)$

TABLE XCIV. $1\to\bar{L}$, $2\to l$, $3\to l$, $4\to l$, $5\to\bar{l}$, $6\to\bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},1)$

TABLE XCV. $1\to\bar{l}$, $2\to\bar{l}$, $3\to\bar{L}$, $4\to l$, $5\to l$, $6\to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},\frac{5}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},-3)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$

TABLE XCVI. $1\to\bar{l}$, $2\to\bar{l}$, $3\to l$, $4\to\bar{L}$, $5\to l$, $6\to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},\frac{5}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$

TABLE XCVII. $1\to\bar{l}$, $2\to\bar{l}$, $3\to l$, $4\to l$, $5\to l$, $6\to\bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},\frac{5}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$

TABLE XCVIII. $1\to l$, $2\to\bar{L}$, $3\to\bar{l}$, $4\to\bar{l}$, $5\to l$, $6\to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE XCIX. $1\to l$, $2\to\bar{L}$, $3\to l$, $4\to l$, $5\to\bar{l}$, $6\to\bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE C. $1\to l$, $2\to l$, $3\to\bar{L}$, $4\to l$, $5\to\bar{l}$, $6\to\bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CI. $1\to l$, $2\to l$, $3\to\bar{l}$, $4\to\bar{l}$, $5\to l$, $6\to\bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CII. $1\to l$, $2\to l$, $3\to l$, $4\to\bar{L}$, $5\to\bar{l}$, $6\to\bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{2},\frac{5}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

5. Fifth topology (Fig. 15)

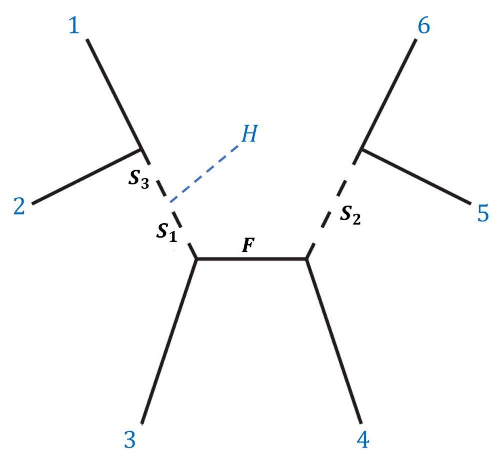

FIG. 15. Feynman diagram for the cases where H is attached to an internal scalar. The corresponding tables are Tables CIII–CXXXII.

a. $\bar{L}\bar{L}\bar{L}lLLH$

TABLE CIII. $1\to\bar{L}$, $2\to\bar{L}$, $3\to\bar{L}$, $4\to l$, $5\to L$, $6\to L$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{4},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},-2)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},-2)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},-2)$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},1)$

TABLE CIV. $1\to\bar{L}$, $2\to\bar{L}$, $3\to L$, $4\to L$, $5\to l$, $6\to\bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{4},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},1)$

TABLE CV. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{4},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{4},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CVI. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{4},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CVII. $1 \to l$, $2 \to \bar{L}$, $3 \to L$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{4},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CVIII. $1 \to L$, $2 \to L$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{4},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{1},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},-1)$

TABLE CIX. $1 \to L$, $2 \to L$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{4},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{1},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},-1)$

TABLE CX. $1 \to L$, $2 \to L$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{4},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{4},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-1)$

b. $\bar{L}\bar{L}\bar{l}llLH$

TABLE CXI. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to \bar{l}$, $4 \to L$, $5 \to l$, $6 \to l$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{5}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{5}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CXII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CXIII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to L$, $4 \to \bar{l}$, $5 \to l$, $6 \to l$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},1)$

TABLE CXIV. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXV. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{l}$, $4 \to L$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXVI. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXVII. $1 \to l$, $2 \to \bar{L}$, $3 \to L$, $4 \to \bar{l}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXVIII. $1 \to \bar{l}$, $2 \to L$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to l$, $6 \to l$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CXIX. $1 \to \bar{l}$, $2 \to L$, $3 \to \bar{L}$, $4 \to l$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CXX. $1 \to \bar{l}$, $2 \to L$, $3 \to l$, $4 \to \bar{L}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CXXI. $1 \to \bar{l}$, $2 \to L$, $3 \to l$, $4 \to l$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CXXII. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{1},-2)$
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},-2)$

TABLE CXXIII. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-2)$
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-2)$

TABLE CXXIV. $1 \to l$, $2 \to l$, $3 \to L$, $4 \to \bar{l}$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},2)$	$(\mathbf{1},\mathbf{1},-2)$
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},-2)$

c. $\bar{L}\bar{l}\bar{l}lllH$

TABLE CXXV. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{l}$, $4 \to \bar{l}$, $5 \to l$, $6 \to l$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXXVI. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXXVII. $1 \to \bar{l}$, $2 \to \bar{l}$, $3 \to \bar{L}$, $4 \to l$, $5 \to l$, $6 \to l$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},\frac{5}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},-3)$	$(\mathbf{1},\mathbf{1},2)$

TABLE CXXVIII. $1 \to \bar{l}$, $2 \to \bar{l}$, $3 \to l$, $4 \to \bar{L}$, $5 \to l$, $6 \to l$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},\frac{5}{2})$	$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},2)$

TABLE CXXIX. $1 \to \bar{l}$, $2 \to \bar{l}$, $3 \to l$, $4 \to l$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},\frac{5}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},2)$

TABLE CXXX. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},-2)$

TABLE CXXXI. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to \bar{l}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F	S_3
$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-2)$

TABLE CXXXII. $1 \to l$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F	S_3
$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{2}, \frac{5}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$

6. Sixth topology (Fig. 16)

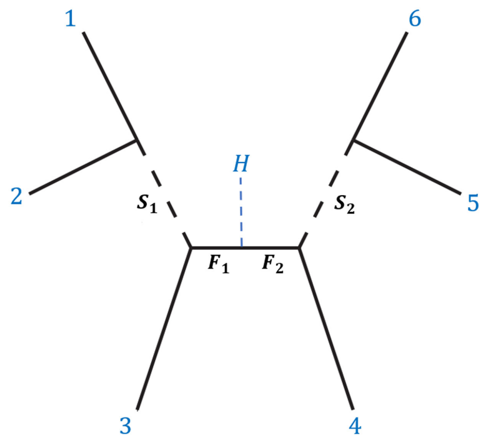

FIG. 16. Feynman diagram for the cases where H is attached to a fermion mediator. The corresponding tables are Tables CXXXIII–CLVI.

a. $\bar{L}\bar{L}\bar{L}lLLH$

TABLE CXXXIII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to l$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{4}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, -2)$
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, -2)$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, -2)$
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{2}, -\frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, -2)$

TABLE CXXXIV. $1 \to \bar{L}$, $2 \to l$, $3 \to L$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{4}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

TABLE CXXXV. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE CXXXVI. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE CXXXVII. $1 \to l$, $2 \to \bar{L}$, $3 \to L$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{4}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

TABLE CXXXVIII. $1 \to \bar{L}$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{4}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

b. $\bar{L}\bar{L}\bar{l}llLH$

TABLE CXXXIX. $1 \to l$, $2 \to l$, $3 \to L$, $4 \to \bar{l}$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{5}{2})$	$(\mathbf{1}, \mathbf{3}, 2)$
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{5}{2})$	$(\mathbf{1}, \mathbf{1}, 2)$

TABLE CXL. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXLI. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CXLII. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXLIII. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{l}$, $4 \to L$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$

TABLE CXLIV. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CXLV. $1 \to l$, $2 \to \bar{L}$, $3 \to L$, $4 \to \bar{l}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CXLVI. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CXLVII. $1 \to \bar{L}$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to L$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CXLVIII. $1 \to \bar{L}$, $2 \to l$, $3 \to \bar{L}$, $4 \to l$, $5 \to L$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CXLIX. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to L$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CL. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{3},1)$
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$

c. $\bar{L}\bar{l}\bar{l}lllH$

TABLE CLI. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to \bar{l}$, $5 \to \bar{L}$, $6 \to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CLII. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CLIII. $1 \to l$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{1},3)$	$(\mathbf{1},\mathbf{2},\frac{5}{2})$

TABLE CLIV. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CLV. $1 \to \bar{L}$, $2 \to l$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\mathbf{2},\frac{3}{2})$	$(\mathbf{1},\mathbf{1},1)$

TABLE CLVI. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to \bar{l}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{1},-2)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

7. Seventh topology (Fig. 17)

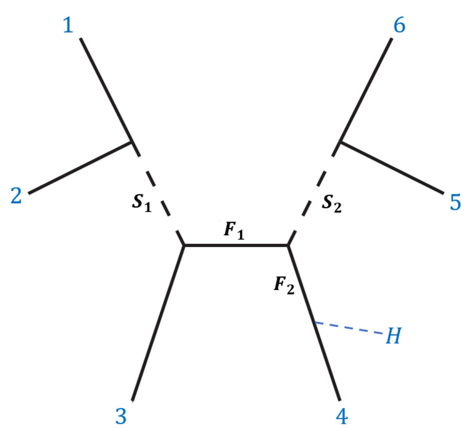

FIG. 17. Feynman diagram for the cases where H is attached to an external leg that connects to a fermion mediator. The corresponding tables are Tables CLVII–CLXXX.

a. $\bar{L}\bar{L}\bar{L}lLLH$

TABLE CLVII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to l$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{4},-\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-2)$
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-2)$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{3},-2)$
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-2)$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	$(\mathbf{1},\mathbf{1},-2)$

TABLE CLVIII. $1 \to \bar{L}$, $2 \to l$, $3 \to L$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{4},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CLIX. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{4},-\frac{1}{2})$
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CLX. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{4},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

TABLE CLXI. $1 \to l$, $2 \to \bar{L}$, $3 \to L$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{4},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$

TABLE CLXII. $1 \to \bar{L}$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{4},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	$(\mathbf{1},\mathbf{1},-1)$	$(\mathbf{1},\mathbf{1},0)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

b. $\bar{L}\bar{L}\bar{l}llLH$

TABLE CLXIII. $1 \to l$, $2 \to l$, $3 \to L$, $4 \to \bar{l}$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{5}{2})$	$(\mathbf{1}, \mathbf{3}, 2)$
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{5}{2})$	$(\mathbf{1}, \mathbf{1}, 2)$

TABLE CLXIV. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE CLXV. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to L$, $5 \to \bar{L}$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

TABLE CLXVI. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE CLXVII. $1 \to l$, $2 \to \bar{L}$, $3 \to \bar{l}$, $4 \to L$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, -1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, -1)$

TABLE CLXVIII. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE CLXIX. $1 \to l$, $2 \to \bar{L}$, $3 \to L$, $4 \to \bar{l}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

TABLE CLXX. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to L$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE CLXXI. $1 \to \bar{L}$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to L$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE CLXXII. $1 \to \bar{L}$, $2 \to l$, $3 \to \bar{L}$, $4 \to l$, $5 \to L$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE CLXXIII. $1 \to \bar{L}$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to L$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{3}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{3}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

TABLE CLXXIV. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to L$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{3}, 1)$
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

c. $\bar{L}\bar{l}\bar{l}lllH$

TABLE CLXXV. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to \bar{l}$, $5 \to \bar{L}$, $6 \to l$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

TABLE CLXXVI. $1 \to l$, $2 \to \bar{L}$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE CLXXVII. $1 \to l$, $2 \to l$, $3 \to l$, $4 \to \bar{L}$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{1}, 3)$	$(\mathbf{1}, \mathbf{2}, \frac{5}{2})$

TABLE CLXXVIII. $1 \to l$, $2 \to l$, $3 \to \bar{L}$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE CLXXIX. $1 \to \bar{L}$, $2 \to l$, $3 \to l$, $4 \to l$, $5 \to \bar{l}$, $6 \to \bar{l}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 2)$	$(\mathbf{1}, \mathbf{2}, \frac{3}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$

TABLE CLXXX. $1 \to l$, $2 \to l$, $3 \to \bar{l}$, $4 \to \bar{l}$, $5 \to l$, $6 \to \bar{L}$.

S_1	S_2	F_1	F_2
$(\mathbf{1}, \mathbf{1}, -2)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$(\mathbf{1}, \mathbf{1}, 1)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

APPENDIX D: LIMITS FROM LEP AND TRISTAN

The neutral scalars in some of our UV completions couple to leptons and thus modify leptonic scattering data. For example, the scalar from Sec. IV A will modify $e^+e^- \to \mu^+\mu^-$ scattering, experimentally studied at LEP. If the scalar mass exceeds the LEP center-of-mass energy of around 200 GeV, the contribution can be parametrized through a four-lepton contact interaction, exactly the scenario investigated and constrained in LEP's legacy article, Ref. [35], which contains combined measurements from the four LEP experiments ALEPH, DELPHI, L3, and OPAL, using center-of-mass data away from the Z pole, between $\sqrt{s} = 130$ GeV and 207 GeV, totaling 3 fb^{-1} in luminosity.

In particular, LEP [35] provides limits on four-fermion contact interactions of the form [36]

$$\mathcal{L}_{\mathrm{eff}} = \frac{g^2}{(1+\delta_{ef})\Lambda_\pm^2} \sum_{i,j=L,R} \eta_{ij} \bar{e}_i \gamma_\mu e_i \bar{f}_j \gamma^\mu f_j \tag{D1}$$

that continue to be relevant. Here, $g^2/(4\pi)$ is set to 1 for simplicity, leaving the scale of the contact interaction, $\Lambda_\pm$ as the only parameter. Different chiralities are encoded through η, which takes on values ± 1 or 0, a negative sign typically indicating destructive interference with the SM contribution in the process $e^+e^- \to f^+f^-$, using the mostly-minus space-time metric. The particular benchmark models shown in Table CLXXXI are constrained by LEP in

TABLE CLXXXI. η_{ij} choices for different contact interaction models.

Model	η_{LL}	η_{RR}	η_{LR}	η_{RL}
$\mathrm{LL}^\pm$	± 1	0	0	0
$\mathrm{RR}^\pm$	0	± 1	0	0
$\mathrm{VV}^\pm$	± 1	± 1	± 1	± 1
$\mathrm{AA}^\pm$	± 1	± 1	∓ 1	∓ 1
$\mathrm{LR}^\pm$	0	0	± 1	0
$\mathrm{RL}^\pm$	0	0	0	± 1
$\mathrm{V0}^\pm$	± 1	± 1	0	0
$\mathrm{A0}^\pm$	0	0	± 1	± 1
$\mathrm{A1}^\pm$	± 1	∓ 1	0	0

Ref. [35] for $f = e, \mu, \tau, d, u$, and combinations. We will only be interested in the muon and tau final states here.

The differential cross sections for $e^+e^- \to \mu^+\mu^-$ and $\tau^+\tau^-$ through electroweak interactions and the contact interactions of Eq. (D1) take the form [50]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{\pi\alpha^2}{2s}\left\{(|\mathcal{A}_{LR}^{ef}(s)|^2 + |\mathcal{A}_{RL}^{ef}(s)|^2)\frac{t^2}{s^2} + (|\mathcal{A}_{LL}^{ef}(s)|^2 + |\mathcal{A}_{RR}^{ef}(s)|^2)\frac{u^2}{s^2}\right\}, \tag{D2}$$

where $s = 4E_{\mathrm{beam}}^2$ is the center-of-mass energy, $t = -s(1-\cos\theta)/2$, and $s + t + u = 0$ since we treat all fermions as massless. The individual dimensionless helicity amplitudes are [50]

$$\mathcal{A}_{ij}^{ef}(s) = Q_e Q_f + c_i^e c_j^f \chi(s) + \eta_{ij}\frac{s}{\alpha\Lambda^2}, \tag{D3}$$

where $\chi(s) = s/(s - M_Z^2 + \mathrm{i}M_Z\Gamma_Z)$ encodes the Z-boson propagators, Q_f the charge of fermion f (e.g., $Q_e = -1$), and $c_j^f = (T_j^f - s_W^2 Q_f)/(s_W c_W)$ the Z-boson coupling to fermion f (e.g., $c_L^e = (-\frac{1}{2} + s_W^2)/(s_W c_W)$ and $c_R^e = (0 + s_W^2)/(s_W c_W)$). Radiative corrections are neglected for simplicity.

The SM limit, which describes the LEP data well [35], arises as $\Lambda \to \infty$, so SM-compatible data generically leads to a lower bound on Λ, in the ballpark of TeV in the LEP case. Interestingly, for some models of Table CLXXXI, the destructive interference case allows for a qualitatively different SM limit with *finite* Λ. Take the LR^- case in $e^+e^- \to \mu^+\mu^-$: only $\mathcal{A}_{LR}^{e\mu}(s) = 1 + c_L^e c_R^\mu \chi(s) - s/(\alpha\Lambda^2)$ deviates from the SM, but by solving

$$1 + c_L^e c_R^\mu \chi(s) - \frac{s}{\alpha\Lambda^2} = -(1 + c_L^e c_R^\mu \chi(s)) \tag{D4}$$

for Λ we obtain *exactly* the same differential cross section as in the SM, as we effectively only flip the sign of one amplitude term. At a fixed center-of-mass energy s, this scenario is indistinguishable from the SM and opens up an allowed region where Λ is not bounded from below but rather by how close it has to be to the value from Eq. (D4). In the above example, this value is $\Lambda = 2.2$ TeV for $\sqrt{s} = 207$ GeV. This region of parameter space can only be excluded by combining data at different energies. For example, at $\sqrt{s} = 183$ GeV, the solution for Eq. (D4) is at $\Lambda = 2.0$ TeV, so it is impossible to satisfy the cancelation over the entire LEP range. However, it just so happens that the data can still be *approximately* satisfied in this cancelation regime, leaving a viable region near this Λ even after combining cross-section data from several s. This is exactly what happened in the LEP analysis of Ref. [35], explaining the surprisingly low quoted bound of $\Lambda_{\mu\mu}^{LR,-} = 2.2$ TeV. After clarifying how this number should be interpreted—not as a lower limit—we will show that this region of parameter space can be excluded entirely by combining LEP data with measurements from lower-s experiments.

Reference [35] provides total cross sections σ and forward–backward asymmetries A_{FB} for $e^+e^- \to \mu^+\mu^-$ and $\tau^+\tau^-$ for many center-of-mass bins. To fit the theoretical cross section from Eq. (D2) to data, the parameter Λ is replaced by $\epsilon = \Lambda^{-2}$ and a $\chi^2(\epsilon)$ function is constructed. For positive η, i.e., in the constructive interference case, ϵ has to be positive, while in the negative η case, ϵ is can be considered to be negative. The 95% CL limits on $\Lambda_\pm$ are then derived through a probability distribution $\propto \exp[-\frac{1}{2}\chi^2(\epsilon)]$ [52] by solving

$$\frac{\int_0^{\pm\Lambda_\pm^{-2}} d\epsilon \, \exp\left[-\frac{1}{2}\chi^2(\epsilon)\right]}{\int_0^{\pm\infty} d\epsilon \, \exp\left[-\frac{1}{2}\chi^2(\epsilon)\right]} = 0.95. \tag{D5}$$

Despite the crudeness of the used cross section (neglecting radiative corrections) and likelihood function, this method manages to reproduce the Λ limits from Ref. [35], collected in Table CLXXXII, to within 10%, thus clearly capturing the overall picture.

In most cases, $\chi^2(\epsilon)$ has two minima, one around $\epsilon = 0$, and one for negative ϵ that corresponds to the sign-flipped amplitude scenario outlined above. The global minimum is always the one near zero, and in most cases the local minimum at negative ϵ has a far larger χ^2 due to the combination of different $\sqrt{s}$ data and can thus be ignored. However, in the LR case for muons, the two minima only differ by $\Delta\chi^2 \simeq 5.2$, not enough to exclude the sign-flip case outright, as illustrated in Fig. 18. Due to the near degeneracy, the above procedure for obtaining 95% CL limits described above yields $\Lambda_{\mu\mu}^{LR,-} \simeq 2.2$ TeV. While this matches the number from Ref. [35] (reproduced in Table CLXXXII), it illustrates that the *interpretation* of this number is slightly different from the other cases, seeing as the Λ range from 2.5 TeV to 8 TeV has an extremely low probability (or large χ^2), not captured by the simple lower limit. Instead, we could treat the two minima separately and state that there are two disconnected allowed regions, roughly

$$2.1 \text{ TeV} \lesssim \Lambda_{\mu\mu}^{LR,-} \lesssim 2.2 \text{ TeV}, \qquad 9 \text{ TeV} \lesssim \Lambda_{\mu\mu}^{LR,-}, \tag{D6}$$

with the one near 2 TeV being disfavored at the 97.8% CL due to the larger χ^2.

The sign-flip scenario survives in the LR$^-$ (= RL$^-$) case despite fitting to data from $\sqrt{s} = 130$ GeV to 207 GeV, which could have broken this degeneracy. Indeed, earlier LEP combinations [53] do not list 2.2 TeV in the muon channel, but rather 7.9 TeV, although the tau limit is still at 2.2 TeV. The energy range covered in LEP is thus seemingly too narrow to exclude the LR$^-$ sign-flip scenario with certainty. Luckily, adding data from lower-$\sqrt{s}$ experiments such as TRISTAN [37] ($\sqrt{s} = 57.8$ GeV) can lift the

TABLE CLXXXII. The 95% confidence limits on the scale, $\Lambda^\pm$, for constructive (+) and destructive interference (−) with the SM, for the contact interaction models discussed in the text. Results are given for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$, reproduced from Ref. [35]. In bold brackets, we give the lower limit we obtain by combining LEP and TRISTAN [37,51] data, as explained in the text.

Model	$\Lambda_{\mu\mu}^-$	$\Lambda_{\mu\mu}^+$	$\Lambda_{\tau\tau}^-$	$\Lambda_{\tau\tau}^+$
LL	9.8	12.2	9.1	9.1
RR	9.3	11.6	8.7	8.7
VV	16.3	18.9	13.8	15.8
AA	13.4	16.7	14.1	11.4
LR	2.2 (**8**)	9.1	2.2 (**5**)	7.7
RL	2.2 (**8**)	9.1	2.2 (**5**)	7.7
V0	13.5	16.9	12.6	12.5
A0	12.1	12.6	8.9	12.1
A1	4.5	5.8	3.9	4.7

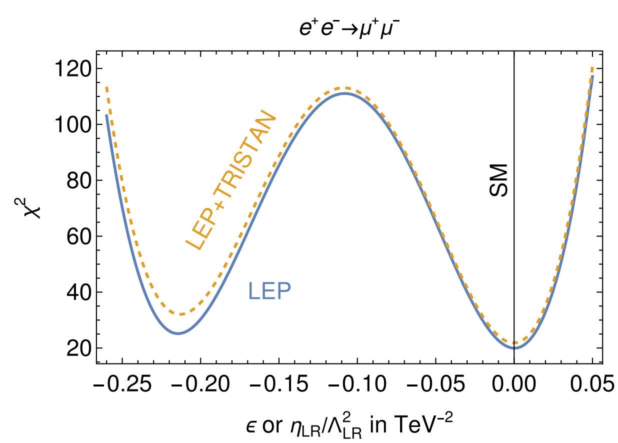

FIG. 18. $\chi^2(\epsilon)$ for $\Lambda_{\mu\mu}^{LR}$. The blue curve is for LEP data [35] only, the dashed orange one includes low-$\sqrt{s}$ data from TRISTAN [37], which lifts the flipped-amplitude local minimum compared to the SM one at $\epsilon = 0$.

second minimum sufficiently ($\Delta\chi^2 \simeq 10$, see Fig. 18) to confidently exclude the flipped-sign case and only leaves the lower bound $9\ \text{TeV} \lesssim \Lambda_{\mu\mu}^{LR,-}$. Since the Λ limits obtained via the above simplified method are systematically larger than the LEP limits, we propose a lower limit of 8 TeV on $\Lambda_{\mu\mu}^{LR,-}$, in line with the previous LEP combination [53]. The limits on other $\mu\mu$ contact interactions are not affected by adding the TRISTAN results.

The tauon case is similar, here we can reproduce all LEP limits to better than 10% except $\Lambda_{\tau\tau}^{LR,+}$, which comes out 20% larger than LEPs. In any case, the LR^- case once again features two minima, this time even more degenerate, $\Delta\chi^2 \simeq 3.9$. Adding TRISTAN data [51] again lifts the second minimum sufficiently to exclude the sign-flip case and provides a conservative lower limit on $\Lambda_{\tau\tau}^{LR,-}$ of 5 TeV.

By combining LEP and TRISTAN data, all amplitude sign-flip scenarios can be excluded, with the exception of A1^-. Here, the sign-flip value for Λ is around 6 TeV, *above* the quoted limits. The data is not able to resolve the two different minima, resulting in one broad minimum that gives rise to weak limits, as can be seen in Table CLXXXII. Improving this bound requires experiments with larger s.

Having rederived the four-lepton contact-interaction limits from LEP, we can go beyond it and fit our neutral-scalar scenario to the data. We calculate the cross section and forward–backward asymmetry for $e^+e^- \to \mu^+\mu^-$ in our model at tree level and fit to the same data as above, obtaining a χ^2 function as a function of the Yukawa coupling and scalar mass. After including TRISTAN data in the LEP set, this gives an upper limit on the Yukawa for a given mass, shown in Fig. 4. For large masses, this matches the contact-interaction limit from $\Lambda_{\mu\mu}^{LR,-}$, slightly different since our limits are now derived for two parameters rather than one.

---

[1] B. Pontecorvo, Neutrino experiments and the problem of conservation of leptonic charge, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967).

[2] L. Calibbi and G. Signorelli, Charged lepton flavour violation: An experimental and theoretical introduction, Riv. Nuovo Cimento **41**, 71 (2018).

[3] S. Davidson, B. Echenard, R. H. Bernstein, J. Heeck, and D. G. Hitlin, Charged lepton flavor violation, arXiv:2209.00142.

[4] S. T. Petcov, The processes $\mu \to e + \gamma$, $\mu \to e + \bar{e}$, $\nu' \to \nu + \gamma$ in the Weinberg-Salam model with neutrino mixing, Sov. J. Nucl. Phys. **25**, 340 (1977); **25**, 698(E) (1977); Yad. Fiz. **25**, 1336(E) (1977).

[5] S. M. Bilenky, S. T. Petcov, and B. Pontecorvo, Lepton mixing, $\mu \to e + \gamma$ decay and neutrino oscillations, Phys. Lett. B **67**, 309 (1977).

[6] W. J. Marciano and A. I. Sanda, Exotic decays of the muon and heavy leptons in gauge theories, Phys. Lett. **67B**, 303 (1977).

[7] B. W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, Muon and electron number nonconservation in a $V - A$ gauge model, Phys. Rev. Lett. **38**, 937 (1977); **38**, 1230(E) (1977).

[8] B. W. Lee and R. E. Shrock, Natural suppression of symmetry violation in gauge theories: Muon- and electron-lepton-number nonconservation, Phys. Rev. D **16**, 1444 (1977).

[9] J. Heeck, Interpretation of lepton flavor violation, Phys. Rev. D **95**, 015022 (2017).

[10] T. Hambye and J. Heeck, Proton decay into charged leptons, Phys. Rev. Lett. **120**, 171801 (2018).

[11] J. Heeck and M. Sokhashvili, Lepton flavor violation by two units, Phys. Lett. B **852**, 138621 (2024).

[12] G. Isidori, F. Wilsch, and D. Wyler, The standard model effective field theory at work, Rev. Mod. Phys. **96**, 015006 (2024).

[13] L. Michel, Interaction between four half spin particles and the decay of the μ meson, Proc. Phys. Soc. London Sect. A **63**, 514 (1950).

[14] P. Langacker and D. London, Analysis of muon decay with lepton number nonconserving interactions, Phys. Rev. D **39**, 266 (1989).

[15] A. Rubbia, *Phenomenology of Particle Physics* (Cambridge University Press, Cambridge, England, 2022), p. 6.

[16] M. Aiba *et al.*, Science case for the new high-intensity muon beams HIMB at PSI, arXiv:2111.05788.

[17] N. Shimizu (Belle Collaboration), Measurement of tau Michel parameters, Proc. Sci. FPCP2017 (**2017**) 027.

[18] D. A. Bodrov, Measurement of the Michel parameter ξ' in tau-lepton decays at the super charm-tau factory, Phys. At. Nucl. **84**, 212 (2021).

[19] S. Navas *et al.* (Particle Data Group Collaboration), Review of particle physics, Phys. Rev. D **110**, 030001 (2024).

[20] J. P. Alexander *et al.* (CLEO Collaboration), Determination of the Michel parameters and the tau-neutrino helicity in tau decay, Phys. Rev. D **56**, 5320 (1997).

[21] A. Heister *et al.* (ALEPH Collaboration), Measurement of the Michel parameters and the nu/tau helicity in tau lepton decays, Eur. Phys. J. C **22**, 217 (2001).

[22] R. Bayes *et al.* (TWIST Collaboration), Experimental constraints on left-right symmetric models from muon decay, Phys. Rev. Lett. **106**, 041804 (2011).

[23] R. M. Fonseca, M. Hirsch, and R. Srivastava, $\Delta L = 3$ processes: Proton decay and the LHC, Phys. Rev. D **97**, 075026 (2018).

[24] A. Flores-Tlalpa, G. López Castro, and P. Roig, Five-body leptonic decays of muon and tau leptons, J. High Energy Phys. 04 (2016) 185.

[25] V. A. Baranov, I. V. Kisel, S. M. Korenchenko, N. A. Kuchinsky, D. A. Mzhaviya, and Z. B. Tsamalaidze, On the decay $\pi^+ \to \mu^- + 2e^+ + \nu$, Sov. J. Nucl. Phys. **54**, 790 (1991).

[26] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, Phys. Rep. **516**, 1 (2012).
[27] L. Lavoura, General formulae for f(1)− > f(2) gamma, Eur. Phys. J. C **29**, 191 (2003).
[28] D. Hanneke, S. Fogwell, and G. Gabrielse, New measurement of the electron magnetic moment and the fine structure constant, Phys. Rev. Lett. **100**, 120801 (2008).
[29] T. Aoyama, T. Kinoshita, and M. Nio, Revised and improved value of the QED tenth-order electron anomalous magnetic moment, Phys. Rev. D **97**, 036001 (2018).
[30] R. H. Parker, C. Yu, W. Zhong, B. Estey, and H. Müller, Measurement of the fine-structure constant as a test of the standard model, Science **360**, 191 (2018).
[31] D. P. Aguillard *et al.* (Muon g-2 Collaboration), Measurement of the positive muon anomalous magnetic moment to 0.20 ppm, Phys. Rev. Lett. **131**, 161802 (2023).
[32] D. P. Aguillard *et al.* (Muon g-2 Collaboration), Detailed report on the measurement of the positive muon anomalous magnetic moment to 0.20 ppm, Phys. Rev. D **110**, 032009 (2024).
[33] T. Aoyama *et al.*, The anomalous magnetic moment of the muon in the standard model, Phys. Rep. **887**, 1 (2020).
[34] G. Colangelo *et al.*, Prospects for precise predictions of a_μ in the standard model, arXiv:2203.15810.
[35] S. Schael *et al.* (ALEPH, DELPHI, L3, OPAL, LEP Electroweak Collaborations), Electroweak measurements in electron-positron collisions at W-boson-pair energies at LEP, Phys. Rep. **532**, 119 (2013).
[36] E. Eichten, K. D. Lane, and M. E. Peskin, New tests for quark and lepton substructure, Phys. Rev. Lett. **50**, 811 (1983).
[37] M. Miura *et al.* (VENUS Collaboration), Precise measurement of the $e^+e^- \to \mu^+\mu^-$ reaction at $\sqrt{s} = 57.77$ GeV, Phys. Rev. D **57**, 5345 (1998).
[38] G. Abbiendi *et al.* (MUonE Collaboration), Measuring the leading hadronic contribution to the muon g-2 via μe scattering, Eur. Phys. J. C **77**, 139 (2017).
[39] P. S. B. Dev, W. Rodejohann, X.-J. Xu, and Y. Zhang, MUonE sensitivity to new physics explanations of the muon anomalous magnetic moment, J. High Energy Phys. 05 (2020) 053.
[40] S. Iguro, Y. Omura, and M. Takeuchi, Testing the 2HDM explanation of the muon g-2 anomaly at the LHC, J. High Energy Phys. 11 (2019) 130.
[41] Y. Afik, P. S. B. Dev, A. Soni, and F. Xu, Probing the muon (g-2) anomaly at the LHC in final states with two muons and two taus, Phys. Lett. B **843**, 138032 (2023).
[42] G. Aad *et al.* (ATLAS Collaboration), ATLAS searches for additional scalars and exotic Higgs boson decays with the LHC Run 2 dataset, Phys. Rep. **1116**, 184 (2025).
[43] G. Aad *et al.* (ATLAS Collaboration), Search for electroweak production of charginos and sleptons decaying into final states with two leptons and missing transverse momentum in $\sqrt{s} = 13$ TeV pp collisions using the ATLAS detector, Eur. Phys. J. C **80**, 123 (2020).
[44] A. M. Sirunyan *et al.* (CMS Collaboration), Search for supersymmetry in final states with two oppositely charged same-flavor leptons and missing transverse momentum in proton-proton collisions at $\sqrt{s} = 13$ TeV, J. High Energy Phys. 04 (2021) 123.
[45] M. E. Peskin and T. Takeuchi, A new constraint on a strongly interacting Higgs sector, Phys. Rev. Lett. **65**, 964 (1990).
[46] M. E. Peskin and T. Takeuchi, Estimation of oblique electroweak corrections, Phys. Rev. D **46**, 381 (1992).
[47] G. Aad *et al.* (ATLAS Collaboration), Search for electroweak production of supersymmetric particles in final states with two τ-leptons in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS detector, J. High Energy Phys. 05 (2024) 150.
[48] R. M. Fonseca, The Sym2Int program: Going from symmetries to interactions, J. Phys. Conf. Ser. **873**, 012045 (2017).
[49] R. M. Fonseca, GroupMath: A *Mathematica* package for group theory calculations, Comput. Phys. Commun. **267**, 108085 (2021).
[50] H. Kroha, Compositeness limits from e^+e^- annihilation revisited, Phys. Rev. D **46**, 58 (1992).
[51] C. Velissaris *et al.* (AMY Collaboration), Measurements of cross-section and charge asymmetry for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ at $\sqrt{s} = 57.8$ GeV, Phys. Lett. B **331**, 227 (1994).
[52] J. Abdallah *et al.* (DELPHI Collaboration), Measurement and interpretation of fermion-pair production at LEP energies above the Z resonance, Eur. Phys. J. C **45**, 589 (2006).
[53] M. Gruenewald *et al.* (LEP, ALEPH, DELPHI, L3, OPAL, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavor Group Collaborations), A combination of preliminary electroweak measurements and constraints on the Standard Model, arXiv:hep-ex/0312023.