

## Explaining Snowball-in-Hell Phenomena in Heavy-Ion Collisions Using a Novel Thermodynamic Variable

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A loosely bound hadronic molecule produced by a relativistic heavy-ion collision has been described as a “snowball in hell” since it emerges from a hadron resonance gas whose temperature is orders of magnitude larger than the binding energy of the molecule. This remarkable phenomenon can be explained in terms of a novel thermodynamic variable called the “contact” that is conjugate to the binding momentum of the molecule. The production rate of the molecule can be expressed in terms of the contact density at the kinetic freeze-out of the hadron resonance gas. It approaches a nonzero limit as the binding energy goes to 0.

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**Introduction**—A molecule in a medium whose temperature is much larger than the molecule’s binding energy is expected to disassociate almost immediately due to scattering with other particles. However, loosely bound hadronic molecules *have* been observed in heavy-ion collisions and they seem to emerge from a hadron resonance gas whose temperature is orders of magnitude larger than the binding energy. This problem has been referred to as the “snowball in hell” puzzle [1]. The phrase comes from the English idiom “snowball’s chance in hell,” which refers to an event that is extremely unlikely.

According to the standard model of relativistic heavy-ion collisions [2,3], a sufficiently central collision produces a region of *quark-gluon plasma* that expands and cools and then transitions to a *hadron resonance gas*. The hadron resonance gas expands and cools until *kinetic freeze-out*, when it has become so dilute that momentum distributions no longer change. After kinetic freeze-out, the hadrons free stream to the detectors. One might expect a hadronic molecule whose binding energy is much smaller than the energy scales at kinetic freeze-out to have a snowball’s chance in hell of surviving in the expanding hadron resonance gas.

Several types of loosely bound hadronic molecules have been observed in heavy-ion collisions. The STAR

collaboration has observed the antideuteron [4] and the hypertriton [5] in Au-Au collisions at the Relativistic Heavy Ion Collider. The ALICE collaboration has observed the antideuteron [6] and the hypertriton [7] in Pb-Pb collisions at the Large Hadron Collider (LHC). The CMS collaboration has reported evidence for the charm-meson molecule  $\chi_{c1}(3872)$  in Pb-Pb collisions at the LHC [8].

Previous theoretical efforts to understand the production of loosely bound nuclei in heavy-ion collisions have been reviewed in Refs. [9,10]. A nonexhaustive list of recent efforts can be found in Refs. [11–20]. In simple *thermal models*, hadrons are produced in thermal equilibrium at the transition from the quark-gluon plasma to the hadron resonance gas [9]. Thermal models have been remarkably successful in describing the rapidity distributions of loosely bound nuclei as well as ordinary hadrons [21], but they cannot be easily applied to transverse momentum distributions. In simple *coalescence models*, the momentum distribution of a molecule is proportional to the product of the momentum distributions of its constituents [22]. The transverse momentum distributions of a molecule can be described by a sufficiently complicated coalescence model that depends on the detailed properties of the molecule, e.g., Refs. [23,24]. Other models rely on dynamically generating the loosely bound states, such as the *minimal spanning tree* [15] or *stochastic reactions* [20].

The *contact* is a thermodynamic variable relevant to systems whose constituents can form loosely bound molecules. It was first introduced in cold-atom physics less than 20 years ago. Here, we point out that the production rate of a loosely bound hadronic molecule in a heavy-ion collision is determined by the *contact density* at kinetic freeze-out. We show how this observation can be exploited to predict the multiplicity of the molecule.

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*Contact*—The *contact* was introduced by Shina Tan in 2005 in the context of the *strongly interacting Fermi gas* [25,26], which consists of fermions with two spin states that interact only through an S-wave scattering length. Tan derived a number of universal relations involving the contact that apply to any state of the system: few body or many body, homogeneous or trapped, ground state or thermal, equilibrium or time dependent. The contact plays a central role in many experimental probes of ultracold atoms [27]. The first experimental verifications of Tan’s universal relations using ultracold atoms were carried out in 2010 [28].

The contact can be defined for any system that includes particles with an S-wave scattering length  $a$  that is large compared to the range of their interactions. The few-body physics of these particles has universal aspects that are completely determined by  $a$  [29]. We consider a system that includes two types of particles labeled by  $\sigma = 1, 2$  with masses  $m_1$  and  $m_2$ , reduced mass  $m_{12}$ , and large scattering length  $a$ . If  $a > 0$ , the two particles form a loosely bound molecule  $X$  with binding energy  $|E_X| = 1/(2m_{12}a^2)$ . The molecule has a universal wave function  $\exp(-r/a)/r$  for  $r$  much larger than the range. Its constituents therefore have a large mean separation  $r_X = a/2$ .

In a many-body system, one might expect momentum distributions to fall off exponentially at large momentum  $q$ , like the Fermi-Dirac or Bose-Einstein distributions. Tan’s *large-momentum relation* states that the momentum distributions  $f_\sigma(\mathbf{q})$  of the particles with a large scattering length have power-law tails [25]:

$$f_\sigma(\mathbf{q}) \longrightarrow C/q^4, \quad \sigma = 1, 2. \quad (1)$$

The coefficient  $C$ , which is the same for both particles, is the *contact*. We have normalized  $f_\sigma(\mathbf{q})$  so the total number of particles of type  $\sigma$  is  $\int (d^3q/(2\pi)^3)f_\sigma(\mathbf{q})$ . Note that the contact  $C$ , which has the dimensions of a momentum, can take into account short-distance aspects of the interactions between the particles.

Tan’s *adiabatic relation* expresses the contact in terms of a derivative of the total energy  $E$  of the system with respect to the binding momentum  $\gamma = 1/a$  at fixed entropy [26]:

$$C = -8\pi m_{12} \left( \frac{\partial E}{\partial \gamma} \right)_S. \quad (2)$$

Thus  $-(C/8\pi m_{12})d\gamma$  is the work done on the system by a small change in the scattering length. The adiabatic relation implies that the contact is, up to a normalization factor, the extensive thermodynamic variable conjugate to  $\gamma$ . In ultracold atoms,  $\gamma$  can be controlled experimentally by tuning the magnetic field to near a Feshbach resonance [30]. If we insert the energy  $E_X = -\gamma^2/2m_{12}$  for a single molecule into Eq. (2), we find that the contact for a single molecule  $X$  is  $C_X = 8\pi\gamma$ . The contact density for a locally homogeneous

system can be expressed as a derivative of the pressure at fixed temperature:

$$C = 8\pi m_{12} \left( \frac{\partial P}{\partial \gamma} \right)_T. \quad (3)$$

In the dilute limit, the contact density is the sum of the contact for each particle and bound cluster weighted by their number densities. Since the only bound cluster in the strongly interacting Fermi gas is the loosely bound molecule, the contact density reduces to a single term given by the product of  $C_X$  and the molecule number density  $\mathbf{n}_X$ :

$$C = 8\pi\gamma\mathbf{n}_X. \quad (4)$$

This relation holds for any sufficiently dilute system including particles with a small binding momentum  $\gamma$ .

*Virial expansion*—If a system is sufficiently dilute, its thermodynamic variables can be calculated using the *virial expansion*. We consider a homogeneous system in which the two particles with large scattering length  $a$  are in thermal equilibrium at temperature  $T = 1/\beta$  and in chemical equilibrium with number densities  $\mathbf{n}_\sigma$  determined by chemical potentials  $\mu_\sigma$ . If the number densities  $\mathbf{n}_\sigma$  are sufficiently low, the thermodynamic variables have virial expansions in powers of the fugacities  $z_\sigma = \exp(\beta\mu_\sigma) = \mathbf{n}_\sigma(2\pi/m_\sigma T)^{3/2}$ .

The leading term in the virial expansion for the pressure from the interactions between the particles of types 1 and 2 has the form

$$P_{12} = 2T(m_{12}T/\pi)^{3/2}b_{12}z_1z_2. \quad (5)$$

The interaction virial coefficient  $b_{12}$  is a function of  $\gamma/\sqrt{m_{12}T}$  only. It can be deduced from the virial coefficient for identical particles with 2-body phase shifts calculated by Beth and Uhlenbeck in 1937 [31]:

$$b_{12} = \sqrt{2}e^{\beta\gamma^2/2m_{12}}\theta(\gamma) - \frac{\sqrt{2}}{\pi} \int_0^\infty dp \frac{\gamma}{\gamma^2 + p^2} e^{-\beta p^2/2m_{12}}. \quad (6)$$

The contact density can be obtained by differentiating the pressure as in Eq. (3) and then canceling the  $\delta(\gamma)$  term by a subtraction term obtained by differentiating the integral. The leading term in the virial expansion is [32]

$$C = \frac{16}{\pi} (m_{12}T)^2 z_1 z_2 F(\gamma/\sqrt{2m_{12}T}), \quad (7)$$

where the dimensionless function  $F(w)$  is

$$F(w) = \frac{2w}{\sqrt{\pi}} \left( \pi e^{w^2} \theta(w) + \int_0^\infty dx \frac{x^2}{1+x^2} e^{-x^2 w^2} \right). \quad (8)$$

It has an expansion in powers of  $w$ :  $F(w) = 1 + \sqrt{\pi}w + \dots$  if  $w > 0$ .

*Expanding hadron resonance gas*—To derive a relation between the molecule number density and the contact density, we use a toy model for the hadronic system produced by the heavy-ion collision. At a proper time  $\tau$  after the collision, our toy model is a locally homogeneous system at a temperature  $T(\tau)$  and a volume  $V(\tau)$  [33]. At the transition from quark-gluon plasma to the hadron resonance gas, the system is in thermal and chemical equilibrium at a temperature  $T_{\text{ch}} \approx 156$  MeV [34]. After the transition, the system can be described by a decreasing temperature  $T(\tau)$ , an increasing volume  $V(\tau)$ , and a chemical potential  $\mu_h(\tau)$  for each hadron  $h$ . Each of the pions  $\pi^+$ ,  $\pi^0$ , and  $\pi^-$  has decreasing number density  $\mathbf{n}_\pi(\tau)$ . The kinetic-freeze-out temperature  $T_{\text{kf}}$  depends on the center-of-mass energy of the colliding ions and on the centrality of the collision. After kinetic freeze-out, the short-lived resonances decay and resonances are no longer created by collisions. The hadron resonance gas near and after kinetic freeze-out can alternatively be described by a hadron gas consisting only of stable or nearly stable hadrons. The hadron gas can be obtained from the hadron resonance gas by integrating out the short-lived hadron resonances in favor of their decay products.

At the proper time  $\tau_{\text{kf}}$  of kinetic freeze-out, the temperature of the hadron gas is  $T_{\text{kf}}$  and the total pion number density is  $3\mathbf{n}_{\pi\text{kf}}$ . After kinetic freeze-out, the volume  $V(\tau)$  continues to increase. The shapes of the momentum distributions of the hadrons remain the same as at kinetic freeze-out, where they are determined by  $T_{\text{kf}}$  and the hadron chemical potentials  $\mu_{h\text{kf}}$ . The number density  $\mathbf{n}_h(\tau)$  of hadron  $h$  decreases in proportion to  $1/V(\tau)$ . The ratio  $\mathbf{n}_h(\tau)/\mathbf{n}_\pi(\tau)$  of the number densities of the hadron and a pion therefore remains fixed and must be equal to the ratio of the multiplicities  $dN/dy$  observed at the detector.

*Evolution of the contact density*—Ordinary hadrons have strong nuclear interactions whose range is comparable to or shorter than the inverse pion mass  $1/m_\pi = 1.41$  fm. They are therefore essentially noninteracting after kinetic freeze-out. The constituents of a loosely bound hadronic molecule  $X$  are exceptions. After kinetic freeze-out, they decouple from the pions and other hadrons but they continue to interact with each other through their small binding momentum  $\gamma_X$ . They can be described by an effective field theory near the unitary renormalization-group fixed point defined by  $\gamma_X = 0$ . This RG fixed point is a nonrelativistic conformal field theory [35]. The contact density  $\mathcal{C}_X(\tau)$  associated with  $X$  is the expectation value of an operator with scaling dimension 4 [36], so it decreases as  $V(\tau)^{-4/3}$  or equivalently  $\mathbf{n}_\pi(\tau)^{4/3}$ . This anomalous scaling behavior with exponent  $4/3$  continues until a time  $\tau_X$  when there is a crossover to the conventional scaling behavior with exponent 1:

$$\mathcal{C}_X(\tau) = \mathcal{C}_X(\tau_{\text{kf}})[\mathbf{n}_\pi(\tau)/\mathbf{n}_\pi(\tau_{\text{kf}})]^{4/3}, \quad \tau_{\text{kf}} < \tau \lesssim \tau_X. \quad (9)$$

In the dilute limit, the contact density is given by Eq. (4):  $\mathcal{C}_X(\tau) = 8\pi\gamma_X\mathbf{n}_X(\tau)$ , where  $\mathbf{n}_X(\tau)$  is the molecule number density. The conventional scaling behavior required by the dilute limit is proportional to  $1/V(\tau)$  or equivalently  $\mathbf{n}_\pi(\tau)$ :

$$\mathcal{C}_X(\tau) = \mathcal{C}_X(\tau_X)[\mathbf{n}_\pi(\tau)/\mathbf{n}_\pi(\tau_X)], \quad \tau \gtrsim \tau_X. \quad (10)$$

Equation (10) is the familiar statement that as the volume  $V(\tau)$  of a system of noninteracting particles increases, their number densities scale as  $1/V(\tau)$ . Equation (9) is the analogous statement for interacting particles in a quantum field theory near a nontrivial renormalization-group fixed point. Densities scale in proportion to  $V(\tau)$  raised to the appropriate anomalous dimension. After inserting Eq. (10) for  $\mathcal{C}_X(\tau)$  and then using Eq. (9) for  $\mathcal{C}_X(\tau_X)$ , the molecule number density for  $\tau > \tau_X$  reduces to

$$\mathbf{n}_X(\tau) = \frac{1}{8\pi\gamma_X}\mathcal{C}_X(\tau_{\text{kf}})\left(\frac{\mathbf{n}_\pi(\tau_X)}{\mathbf{n}_\pi(\tau_{\text{kf}})}\right)^{1/3}\frac{\mathbf{n}_\pi(\tau)}{\mathbf{n}_\pi(\tau_{\text{kf}})}. \quad (11)$$

As an estimate of the crossover time  $\tau_X$  in the hadron resonance gas, we take the time when the mean distance  $r_\pi(\tau)$  to the nearest pion exceeds the mean separation  $r_X = 1/(2\gamma_X)$  of the constituents of the molecule by a numerical factor:  $r_\pi(\tau_X) = [\Gamma(\frac{4}{3})(4\pi)^{-1/3}/\kappa]r_X$ . We expect the coefficient of  $r_X$  to be roughly 1, but we will treat  $\kappa$  as a phenomenological parameter. The mean pion distance in a homogeneous system with uniform pion number density  $\mathbf{n}_\pi(\tau)$  is  $r_\pi(\tau) = \Gamma(\frac{4}{3})[4\pi\mathbf{n}_\pi(\tau)]^{-1/3}$  [37]. The pion number density at  $\tau_X$  then reduces to  $\mathbf{n}_\pi(\tau_X) = (2\kappa\gamma_X)^3$ . This simple expression explains why we chose the complicated expression for  $r_\pi(\tau_X)$  above. Given that  $\mathbf{n}_\pi(\tau)$  after kinetic freeze-out scales roughly as  $1/\tau^3$ , the crossover time  $\tau_X$  scales as  $1/\gamma_X$ . Since the ratio of the number densities of  $X$  and  $\pi$  for  $\tau > \tau_X$  is equal to the ratio of their multiplicities, the multiplicity of the molecule is

$$dN_X/dy = \frac{\kappa}{4\pi}\left(\mathcal{C}_{X\text{kf}}/\mathbf{n}_{\pi\text{kf}}^{4/3}\right)dN_\pi/dy, \quad (12)$$

where  $\mathcal{C}_{X\text{kf}}$  and  $\mathbf{n}_{\pi\text{kf}}$  are the contact density and pion number density at kinetic freeze-out. This expression for the multiplicity of the molecule is the primary result of our Letter. It does not depend on the toy model used in its derivation. Note that  $dN_X/dy$  depends on  $\gamma_X$  only through the contact density at kinetic freeze-out. Since  $\mathcal{C}_{X\text{kf}}$  has a nonzero limit as  $\gamma_X \rightarrow 0$ , the multiplicity in Eq. (12) is nonzero in that limit. This disagrees with the intuition that the production rate of a loosely bound molecule should go to 0 as its binding energy goes to zero.

To illustrate the application of Eq. (12) for the molecule multiplicity, we approximate the contact density  $\mathcal{C}_{X\text{kf}}$  at kinetic freeze-out by the leading term in the virial expansion for its constituents in Eq. (7) evaluated at  $T = T_{\text{kf}}$ . The number densities  $\mathbf{n}_1$  and  $\mathbf{n}_2$  of the constituents are those in

a hadron gas in which short-lived resonances have been integrated out. The multiplicity of a molecule  $X$  with binding momentum  $\gamma_X$  is then

$$dN_X/dy = 32\pi\kappa f_X F(\gamma_X/\sqrt{2m_{12}T_{\text{kf}}}) \times \left(\frac{m_{12}\mathbf{n}_{\pi\text{kf}}^{4/3}}{m_X^3 T_{\text{kf}}^2}\right)^{1/2} \frac{dN_1/dy dN_2/dy}{dN_\pi/dy}. \quad (13)$$

Since  $m_h \gg T_{\text{kf}}$ , we have used the Boltzmann approximation to express the fugacity for a constituent hadron  $h$  as  $z_h = \mathbf{n}_h(\tau_{\text{kf}})(2\pi/m_h T_{\text{kf}})^{3/2}/(2s_h + 1)$ , where  $s_h$  is the hadron spin. The factor  $f_X$  in Eq. (13) is the fraction of the scattering channels for the two constituents that have the small binding momentum  $\gamma_X$ . We have replaced the ratios  $\mathbf{n}_h(\tau)/\mathbf{n}_\pi(\tau)$  by the corresponding ratios of multiplicities. We have also replaced  $m_1 + m_2$  by the mass  $m_X$  of the molecule.

*Estimation of  $\kappa$* —The deuteron ( $d$ ) is a proton-neutron ( $pn$ ) bound state with spin 1, isospin 0, and a relatively small binding energy 2.225 MeV. The fraction of  $np$  scattering channels that are resonant is  $f_d = 3/8$ . The ALICE collaboration has measured the production of the deuteron (and antideuteron) in Pb-Pb collisions at the center-of-mass energy per nucleon  $\sqrt{s_{NN}} = 2.76$  TeV [38]. The mean deuteron multiplicity  $dN_d/dy$  in the 0–10% bin of the centrality of the collision is  $(9.8 \pm 1.6) \times 10^{-2}$ . We will use this result to obtain an order-of-magnitude estimate of the parameter  $\kappa$  in Eq. (12).

The mean proton multiplicity  $dN_p/dy$  (which is equal to that of the neutron by isospin symmetry) and the mean pion ( $\pi^+$  or  $\pi^-$ ) multiplicity  $dN_\pi/dy$  were measured in the 0%–5% and 5%–10% centrality bins in Ref. [39]. The temperature  $T_{\text{kf}}$  at kinetic freeze-out can be estimated through blast-wave fits to the transverse momentum distributions of  $\pi$ ,  $K$ , and  $p$ . For Pb-Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV, a fit that also allows for a pion chemical potential  $\mu_{\pi\text{kf}}$  has given  $T_{\text{kf}} = 78.3 \pm 1.6$  MeV and  $\mu_{\pi\text{kf}} \approx 90$  MeV [40]. Upon inserting these results into Eq. (13) and solving for  $\kappa$ , we obtain  $\kappa_d = 0.18 \pm 0.04$ , where the errors from multiplicities have been combined in quadrature.

It is useful to have quantitative estimates of the relevant length scales. The mean pion distance at kinetic freeze-out with  $T_{\text{kf}} = 78.3$  MeV and  $\mu_{\pi\text{kf}} = 90$  MeV is  $r_{\pi\text{kf}} = 1.61$  fm. This is a little smaller than the mean separation of the constituents of the deuteron inferred from its binding energy:  $r_d = 2.16$  fm. Our estimate for  $\kappa_d$  implies that the mean pion distance at the deuteron crossover time is  $r_\pi(\tau_d) \approx 2.1r_d$ .

*Multiplicities*—Equation (12) implies that the ratio of the multiplicities of two loosely bound molecules is just the ratio of the corresponding contact densities at kinetic freeze-out. If the contact density is approximated by the leading term in the virial expansion in Eq. (7), the ratio for a

loosely bound molecule  $X$  and the deuteron is

$$\frac{dN_X/dy}{dN_d/dy} = \frac{f_X}{f_d} \left(\frac{m_d^3 m_{12}}{m_X^3 m_{pn}}\right)^{1/2} \times \frac{F(\gamma_X/\sqrt{2m_{12}T_{\text{kf}}})}{F(\gamma_d/\sqrt{2m_{pn}T_{\text{kf}}})} \frac{(dN_1/dy)(dN_2/dy)}{(dN_p/dy)^2}. \quad (14)$$

The contact density at kinetic freeze-out enters only through the ratio of the functions  $F(w)$ . The function in the numerator approaches 1 as  $\gamma_X \rightarrow 0$ .

The hypertriton ( $^3_\Lambda\text{H}$ ) is a  $pn\Lambda$  bound state with spin  $\frac{1}{2}$  that is essentially a molecule composed of a deuteron and the strange baryon  $\Lambda$ . The fraction of  $d\Lambda$  scattering channels that are resonant is  $f_{^3_\Lambda\text{H}} = 1/3$ . The  $\Lambda$  separation energy has been measured in emulsion experiments [41] and in heavy-ion collisions [7,42]. The average of the existing measurements of the  $\Lambda$  separation energy is  $148 \pm 40$  keV [43]. The ALICE collaboration has measured the production rate of the hypertriton (and anti-hypertriton) in Pb-Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV [44]. The mean multiplicity in the 0–10% centrality bin multiplied by the branching fraction of the hypertriton into  $^3\text{He}\pi^-$  is  $(3.67 \pm 0.74) \times 10^{-5}$ , where the errors have been added in quadrature. The mean hypertriton multiplicity in that bin can be predicted by inserting the multiplicities for  $p$ ,  $d$ , and  $\Lambda$  from Refs. [39], [38], and [45] into Eq. (14) along with  $T_{\text{kf}} = 78.3$  MeV. In our approximation for the contact density, we ignore the fact that the deuteron number density at kinetic freeze-out may not be well-defined. It may be possible instead to express the contact density associated with the hypertriton in terms of the contact density associated with the deuteron, which is well-defined at kinetic freeze-out. Our prediction for the hypertriton multiplicity is  $(10.4 \pm 3.9) \times 10^{-5}$ , where the errors from multiplicities have been combined in quadrature. The prediction is insensitive to the  $\Lambda$  separation energy. Our prediction multiplied by the 25% branching fraction into  $^3\text{He}\pi^-$  [46] is consistent with the ALICE result to within the errors.

The  $\chi_{c1}(3872)$  is a loosely bound charm-meson molecule discovered in 2003 [47]. The difference between its mass and the threshold for the charm-meson pair  $D^{*0}\bar{D}^0$  is  $-50 \pm 93$  keV [48]. Its quantum numbers  $J^{PC} = 1^{++}$  [49] imply that its constituents are the linear combination  $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$ . The fraction of  $D^{*0}\bar{D}^0$  and  $D^0\bar{D}^{*0}$  scattering channels that are resonant is  $f_X = 1/2$ . The CMS collaboration has presented evidence for the production of  $\chi_{c1}(3872)$  in Pb-Pb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV [8]. In order to use Eq. (14) to predict the multiplicity of  $\chi_{c1}(3872)$ , we need the multiplicities of its charm-meson constituents, which have not been measured. They have however been predicted for Pb-Pb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV using the statistical hadronization model with charm quarks

(SHMc) [50]. The mean  $D^{*0}$  multiplicity should by isospin symmetry be equal to that for  $D^{*+}$ :  $2.4 \pm 0.4$  in the 0-10% centrality bin. The mean multiplicity for  $D^0$  before  $D^*$  decays can be inferred by isospin symmetry from the SHMc predictions for  $D^+$  and  $D^{*+}$ :  $1.9 \pm 0.5$  in the 0-10% bin. The mean proton multiplicity in the 0-10% bin is given in Ref. [51]. The mean deuteron multiplicity in the 0-10% bin can be obtained from Fig. 4 of Ref. [52]:  $dN_d/dy = (11.9 \pm 0.4) \times 10^{-2}$ . Inserting these results into Eq. (14) along with  $T_{\text{kf}} = 78.3$  MeV, our prediction for the mean multiplicity of  $\chi_{c1}(3872)$  in the 0–10% bin is  $(23.4 \pm 7.8) \times 10^{-5}$ , where the errors from multiplicities have been combined in quadrature. The predicted multiplicity is insensitive to the  $\chi_{c1}(3872)$  binding energy.

**Conclusions**—We have shown that the production rate of a loosely bound hadronic molecule in relativistic heavy-ion collisions can be determined from the contact density of the resulting hadron gas at kinetic freeze-out using Eq. (12). We illustrated the application of that equation by approximating the contact density by the leading term in the virial expansion for the constituents. The resulting expression for the ratio of the multiplicities of a loosely bound hadronic molecule and the deuteron is given in Eq. (14). We then used this equation to calculate the multiplicities of the hypertriton and the  $\chi_{c1}(3872)$ . The predicted hypertriton multiplicity is consistent with the measured value to within the errors.

The contact density can be calculated using any model for the hadron gas in which the loosely bound molecule is generated dynamically by the large scattering length of its constituents. An important step would be to improve on our simple approximation using the virial expansion to obtain a more accurate description of the contact density in the hadron gas. Our approximation for the contact density does not take into account 3-body effects. The most important 3-body effects involve pions, since they are the most abundant constituents of the hadron gas. Our approximation for the contact density could be improved by using an effective field theory near the unitary fixed point that includes pions as well as the constituents of the molecule [53,54]. The contact density can be calculated as an expansion in the fugacities of the heavy constituents and in the coupling constant for their interactions with pions.

We have related the abundance of a loosely bound 2-body hadronic molecule in heavy-ion collisions to the contact density at kinetic freeze-out. In addition to the loosely bound 2-body molecule, there may be loosely bound 3-body molecules, in which case the system may have universal properties determined by the 3-body contact [55]. It would be interesting to see if these universal properties can be applied to the abundance of loosely bound 3-body hadronic molecules, such as  $\text{H}^3$  and  $\text{He}^3$  nuclei, in heavy-ion collisions.

Given our approximation for the contact density, we calculated the multiplicities of loosely bound molecules.

Our methods can be extended to calculate the transverse momentum distribution. The blast-wave fits of hadron momentum distributions determine not only  $T_{\text{kf}}$  but also the phase-space distribution of the hadronic fluid. The transverse momentum distribution of a molecule can be predicted by assuming it has a Maxwell-Boltzmann distribution in the co-moving frame of the expanding fluid at the crossover time.

Our methods can be applied to other loosely bound hadronic molecules, such as  $T_{cc}^+(3875)$ , a charm-meson molecule with constituents  $D^{*+}D^0$  and binding energy  $273 \pm 62$  keV discovered in 2021 [56]. They can also be applied to the production of loosely bound molecules in other fields of physics. Cold-atom physics provides systems that are theoretically pristine. The scattering length can be controlled experimentally and used to make the binding energy of a molecule arbitrarily small. Systems in which an atomic gas escapes from a trapping potential can be engineered with exquisite experimental control. They should allow quantitative studies of the role played by the contact in the production of snowballs from hell.

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