# Nonlinear, Microwave and Quantum Photonics with Kerr Microcombs

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Kerr optical frequency combs currently constitute a well-established research area in integrated, nonlinear and quantum photonics. These systems have found a plethora of technological applications, while serving as an excellent platform to investigate fundamental scientific topics such as light-matter interactions, pattern formation in driven-dissipative systems, or entangled twin-photon generation. We here provide a brief overview of the topic, highlight some of the most recent advances, and discuss a few of the main challenges ahead in this field.

Kerr optical frequency combs – of *microcombs* – are sets of equidistant spectral lines that are generated after pumping a high-$Q$ resonator with a continuous-wave laser. These combs have attracted an intense research interest in recent years, as reviewed in refs.[1,2].

The typical platform for microcomb generation is a high-$Q$ resonator, which allows long-lifetime photons to be trapped in its toruslike eigenmodes, and thereby to mutually interact via the nonlinearity of the host medium. The optical cavity is characterized by eigenmodes that are quasi-equidistantly spaced as $\omega_\ell \simeq \ell\Omega_{\rm R}$, where $\Omega_{\rm R}$ is the free spectral range of the resonator, where the integer eigennumber $\ell$ stands for the quantized angular momentum of the intracavity photons ($\simeq \hbar\ell/a$ for a circular resonator of main radius $a$). When a given mode $\ell_0$ is pumped with a laser, it is convenient to consider it as a reference so that the eigenmodes can now be conveniently labeled using the reduced eigennumber $l = \ell - \ell_0$. The purpose of microcomb generation is therefore to pump a unique mode $l = 0$ with a resonant continuous-wave laser, and thereby achieve the efficient excitation the sidemodes $l = \pm 1, \pm 2, \cdots$ via the bulk medium Kerr nonlinearity.

At the experimental level, the first demonstrations involved hyperparametric oscillations in a monolithic whispering-gallery mode resonators, exceited via the degenerate photonic interaction $2\hbar\omega_0 \rightarrow \hbar\omega_l + \omega_{-l}$ where two pump photons of frequency $\omega_0$ are down- and up-converted to $\omega_0 \pm l\Omega_{\rm R}$ via four-wave mixing[3,4]. A major breakthrough occurred in 2007 when it was shown that using a high-$Q$ chipscale microresonator, the four-wave mixing interaction $\hbar\omega_m + \hbar\omega_p \rightarrow \hbar\omega_n + \hbar\omega_l$ could be massively cascaded and yield a large set of equidistantly-space teeth in the spectral domain – a Kerr optical frequency comb[5]. This achievement was timely and appealing because two years prior, the Nobel Prize of Physics had been awarded to John Hall and Theodor Hänsch, "*for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique*". Kerr combs did hold the potential of delivering all the technological promises of optical frequency combs, while providing optimal performance in terms of size, weight, power (SWAP) and scalability through photonic integration.

The main problem after the experimental demonstration of Kerr combs the absence of a modeling framework for the understanding of these combs. While rule-of-thumb guidelines for Kerr comb generation were relatively easy to define, it was however apparent that the theoretical analysis of such a high-dimensional and nonlinear system would not be a trivial endeavor.

Indeed, all the relevant parameters for Kerr comb generation were known and readily measurable. They are namely the laser pump power $P_{\rm L}$ and frequency $\omega_{\rm L}$ generating a input photon flux $\Phi = P_{\rm L}/\hbar\omega_{\rm L}$; the intrinsic and extrinsic cavity loss parameters characterized by their resonance half-linewidths $\kappa_{\rm i}$ and $\kappa_{\rm e}$, respectively; the detuning $\sigma = \omega_{\rm L} - \omega_0$ between the laser and pumped resonance frequency; the second-order (group-velocity) dispersion coefficient $\zeta$; the Kerr coefficient $n_2$ and modal volume $V$ which are conveniently combined into a single nonlinear parameter $g \propto n_2/V$ corresponding to the self-phase modulation frequency shift induced by a single photon. The challenge was then to build a model that would incorporate all these parameters and allow the community to optimize these combs through a deeper understanding of their dynamical properties.

This problem was solved by Chembo and Yu as they introduced a model built *ab initio* from Maxwell's equations[6,7]. They obtained a set coupled mode ordinary differential equations ruling the dynamics of the intracavity fields $a_l(t)$ as:

$$\begin{aligned}\frac{da_l}{dt} &= -\kappa a_l + i\sigma a_l - i\frac{\zeta}{2}l^2 a_l \\ &\quad + ig \sum_{m,n,p} \delta(m-n+p-l)\, a_m a_n^* a_p \\ &\quad + \delta(l)\sqrt{2\eta\kappa\Phi}, \qquad (1)\end{aligned}$$

where $\kappa = \kappa_{\rm i} + \kappa_{\rm e}$ stands for the total losses while the escape ratio $\eta = \kappa_{\rm e}/\kappa$ weights the efficiency of the coupling. The complex-valued slowly-varying amplitudes $a_l(t)$ are here normalized in such a way that $|a_l(t)|^2$ is the instantaneous number of intracavity photons in the mode $l$. The function $\delta(x)$ is the Kronecker delta-function that equals 1 when $x = 0$ and

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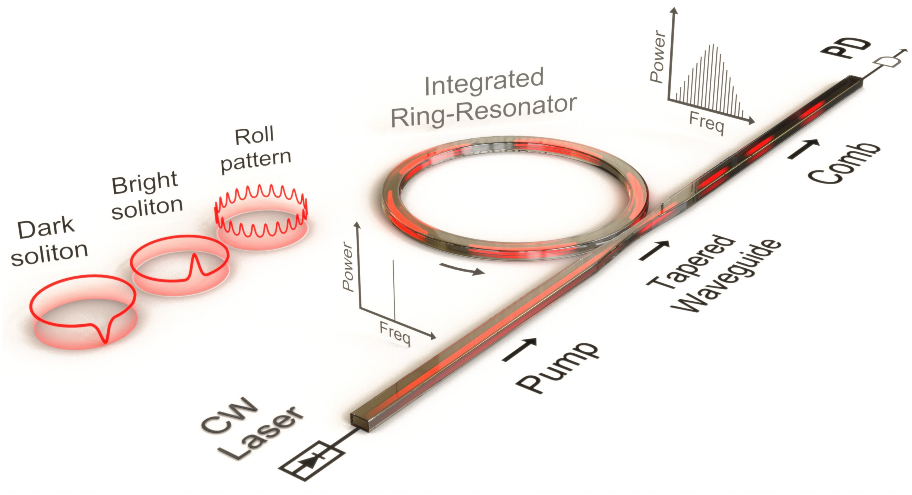

FIG. 1. Schematic representation of microcomb generation. A microresonator is pumped by a resonant single-frequency laser. A spatiotemporal pattern is created inside the cavity via the interplay between laser power, losses, nonlinearity, dispersion, and frequency detuning. This process outputs a train of optical pulses that yield an optical frequency comb in the spectral domain. We have depicted on the left some of the intracavity patterns that can be created, such as dark solitons, bright solitons, or roll patterns.

equals zero otherwise: it indicates that only the mode $l = 0$ is pumped, and imposes the conservation of angular momentum for the interacting photons through $m + p = n + l$. Including higher-order dispersion in this model is achieved with the trivial extension $(\zeta/2)l^2 \to \sum_{k=2}^{+\infty}(\zeta_k/k!)l^k$, which is disregarded in this article for the sake of conciseness.

A few years later, Chembo and Menyuk demonstrated that this set of ordinary ordinary differential equations can be rewritten as a single partial differential equation[8], following

$$\begin{aligned}\frac{\partial A}{\partial t} = & -\kappa A + i\sigma A - i\frac{\zeta}{2}\frac{\partial^2 A}{\partial \theta^2} \\ & + ig|A|^2 A + \sqrt{2\eta\kappa\Phi}\,, \end{aligned} \tag{2}$$

where $A(\theta, t) = \sum_l a_l(t) e^{il\theta}$ is the total intracavity field, and $\theta \in [-\pi, \pi]$ is the azimuthal angle along the closed-path circumference of the resonator. This equation was readily recognized as the Lugiato-Lefever equation (LLE), which is a damped, driven and dissipative version of the nonlinear Schrödinger equation (NLSE). From this breakthrough, it was possible to connect Kerr comb analysis to the vast amount of knowledge that was readily available in the area of of nonlinear fiber optics, where the NLSE had already been extensively studied, and in the area of nonlinear cavity optics, where the LLE had also been investigated in much detail.

The *spectrotemporal* and *spatiotemporal* models represented by Eqs. (1) and (2), respectively, are strictly equivalent: Their complementarity allows one to explore different facets of microcomb science and technology. The spatiotemporal model appears to be the best approach to investigate pattern formation, and identify the type of dynamics that arise depending on the system's parameters, such as soliton or roll patterns. On the other hand, the spectrotemporal model is the best tool when there is a need monitor specific frequency modes, such as for noise analysis. This duality between the spatial and spectral domains – which stems from the fact that $\ell$ and $\theta$ are Fourier-conjugate variables – is graphically emphasized in Fig. 1. These models allow to determine as well dynamics of the microwave signal $M(t) \propto |\sqrt{2\eta\kappa}A - \sqrt{\Phi}|^2$, which is the envelope of the outcoupled spatiotemporal pattern.

As emphasized earlier, Kerr optical frequency combs result from a cascade of four-wave mixing interactions $\hbar\omega_m + \hbar\omega_p \to \hbar\omega_n + \hbar\omega_l$. Consequently, purely quantum phenomena based on the non-classical nature of light can play a significant role in these combs. The semi-classical model of Eq. (1) is an ideal stepstone towards the quantum realm, and most specifically, towards the concept of *quantum microcombs*. Indeed, the spectrotemporal model can be quantized via the process of canonical quantization, which consists in two main steps. The first one is to perfom the transformations $a(t) \to \hat{\mathsf{a}}_l$ and $a^*(t) \to \hat{\mathsf{a}}_l^\dagger$, where the annihilation and creation operators $\hat{\mathsf{a}}_l$ and $\hat{\mathsf{a}}_l^\dagger$ describe the quantum state of each mode $l$, while obeying the usual bosonic commutation rules $[\hat{\mathsf{a}}_l, \hat{\mathsf{a}}_{l'}^\dagger] = \delta_{l,l'}$ and $[\hat{\mathsf{a}}_l, \hat{\mathsf{a}}_{l'}] = [\hat{\mathsf{a}}_l^\dagger, \hat{\mathsf{a}}_{l'}^\dagger] = 0$. The second step is to introduce vacuum fluctuation operators $\hat{\mathsf{V}}_{\mathrm{s},l}(t)$ as Langevin driving terms for every loss mechanism in the microresonator (s = intrinsic or extrinsic) , with $\langle \hat{\mathsf{V}}_{\mathrm{s},l}(t) \rangle = 0$ and $[\hat{\mathsf{V}}_{\mathrm{s},l}(t), \hat{\mathsf{V}}_{\mathrm{s}',l'}^\dagger(t')] = \delta_{\mathrm{s},\mathrm{s}'}\,\delta_{l,l'}\,\delta(t - t')$. The resulting equations ruling the quantum dynamics of the system can therefore be written as[9]

$$\begin{aligned}\frac{d\hat{\mathsf{a}}_l}{dt} = & -\kappa\hat{\mathsf{a}}_l + i\sigma\hat{\mathsf{a}}_l - i\frac{\zeta}{2}l^2\,\hat{\mathsf{a}}_l + \delta(l)\,\sqrt{2\eta\kappa\Phi} \\ & + ig\sum_{m,n,p}\delta(m - n + p - l)\,\hat{\mathsf{a}}_n^\dagger\hat{\mathsf{a}}_m\hat{\mathsf{a}}_p \\ & + \sqrt{2\eta\kappa}\hat{\mathsf{V}}_{\mathrm{e},l} + \sqrt{2(1-\eta)\kappa}\hat{\mathsf{V}}_{\mathrm{i},l}\,. \end{aligned} \tag{3}$$

It is sometimes useful to rewrite this quantum microcomb model under the form of Heisenberg equations

$$\frac{d\hat{\mathsf{a}}_l}{dt} = \frac{1}{i\hbar}[\hat{\mathsf{a}}_l, \hat{\mathsf{H}}] + \sum_{\mathrm{s=e,i}} \left( -\kappa_{\mathrm{s}} \hat{\mathsf{a}}_l + \sqrt{2\kappa_{\mathrm{s}}}\, \hat{\mathsf{V}}_{\mathrm{s},l} \right) \tag{4}$$

where

$$\begin{aligned} \hat{\mathsf{H}} = {} & \hbar \sum_l \left( \sigma - \frac{1}{2}\zeta l^2 \right) \hat{\mathsf{a}}_l^\dagger \hat{\mathsf{a}}_l \\ & + i\hbar\sqrt{2\eta\kappa\Phi} \left( \hat{\mathsf{a}}_0^\dagger - \hat{\mathsf{a}}_0 \right) \\ & - \frac{1}{2}\hbar g \sum_{m,n,p,q} \delta(m-n+p-q)\, \hat{\mathsf{a}}_n^\dagger \hat{\mathsf{a}}_q^\dagger \hat{\mathsf{a}}_m \hat{\mathsf{a}}_p \end{aligned} \tag{5}$$

is the Hamiltonian of the system, with the first term corresponding to the free propagation of the intracavity fields, the second term standing for the laser pump, while the last term describes to the Kerr interaction.

Equations (1), (2) and (3) represent the backbone of microcomb theory, and have been extensively used to achieve an in-depth understanding of microcomb systems.

The successful development of an accurate theoretical framework for microcombs has played a major role that led to outstanding achievements of major technological relevance. Amongst them we can highlight ultra-low phase noise microwave generation[10–15], quantum state generation[16–23], optical telecommunications[24–27], metrology for astronomical measurements[28] spectroscopy[35,36] imaging[34], pptical ranging[29?,30], Precision metrology[31,32], or dual comb technology[33?]. Despite these astounding successes, the field of Kerr comb science and technology is facing several challenges. We would like to highlight an arbitrary selection of three of them below.

The first challenge is related to performance and controllability. Indeed, Kerr combs is today firmly established as a technology that has the potential to revolutionize at least a dozen of distinct technological segments. However, it is well known that for the any industry to switch from one paradigm to the next, consistent demonstration of leapfrog improvement in terms of performance is required (the rule-of-thumb is a factor of at least 10). In most instances, the scientific literature reports results that are definitely encouraging, if nor groundbreaking, but they are not always commanding enough to warrant an irreversible redefinition of existing standards or practices. The vigorous development of Kerr comb research definitely benefits from strong hardware overlap with off-the-shelf components that are readily available from the technologically mature sector of optical fiber telecommunications around 1550 nm. This issue of performance is also connected to the one of controllability, that is, the capability to excite precisely and repeatably optimal types of comb characterized by well-defined spectral properties. While this objective depends on the incremental improvement of our understanding of the experimental constraints surrounding Kerr comb generation, we also anticipate that a research still needs to be undertaken to deepen our understanding of the core Eqs. (1) and (2), as in refs. [XXX]. In search of maximal adequacy between theory and experiments, these models can also be complemented with additional terms accounting for thermal effects and other phenomena such as avoided mode-crossing, parasitic nonlinear interactions, etc. This theoretical effort would allow the community to identify areas of interest in the large parameter space, as well as optimal paths to get there.

The second challenge first one is related to SWAP requirements. Size and weight might seem to be an easy target for microcomb technology, since the resonators have millimeter or micrometer scale. However this viewpoint is misleading because almost all Kerr comb generation platforms are actually table-top experiments. Downsizing these systems (including the laser and eventually a photodetector) to the volume of a typical matchbox is a very difficult endeavor. For example, while crystalline WGM resonators can feature ultra-high $Q$-factors (in excess of a billion at 1550 nm), it is known that achieve a vibration-immune evanescent coupling for the pump laser signal is a major difficulty. However, it has been shown that this problem can be overcome and these resonators can be packaged in a few cube centimeters [Cite Maleki]. On th other hand, chipscale microresonators bring forward other difficulties, related to their lower $Q$-factor, which requires higher pump power and subsequent and thermal management. With respect to this specific issue power, the theoretical model presented in Eq. (1) indicates that Kerr combs can be generated with milliwatt pump power when resonators with billion $Q$-factors are used. However, with very few exceptions (such as ref. Diallo), most experimental work report pump powers typically one or even two orders of magnitude higher, and sometimes require an optical amplifier to boost the pump prior to resonator coupling. This situation is a clear indication that coupling efficiency is one area where the most dramatic improvements are likely to be witnessed in the years to come. A breakthrough along this direction was recently achieved by the research group of Lipson et al., who demonstrated the first ever battery-operated Kerr comb generator.

The third challenge is related to quantum technology. In high-$Q$ microresonators, non-classical states of light can be generated both under and above threshold, corresponding to *spontaneous* and *stimulated* four-wave mixing – this is analogous to the phenomenology of spontaneous and stimulated emission in lasers. In the first case, two photons from the pump are symmetrically up- and down-converted, and these twin-photons (or biphotons) can be frequency-entangled over several tens of cavity modes [? ? ? ? ? ? ?]. The latter case corresponds to two-mode squeezing, where symmetrical side-modes in the frequency domain are excited in a quantum coherent states with quantum correlations below the shot noise level. Eqs. (3) and (4) provide a complete theoretical framework to investigate quantum microcombs and the related technological applications. Indeed, both twin-photon generation and two-mode squeezing have been experimentally evidenced, but as emphasized earlier, progress is still needed to achieve substantially better performances with high SWAP efficiency. From the theoretical viewpoint, a complete framework is still lacking for a self-consistent description of frequency-bin states beyond the few-mode q-dit expansion with quasi-equal probability amplitudes. Along the same line, most quantum descriptions in the Schrödinger picture abide by the Hamiltonian approximation where the quantum system can be described as a pure state, thereby failing to acknowledge intrinsic losses that couple the system to its environment and mandate the use of a density operator formalism. It is noteworthy that microcombs have the potential to be influential in optical quantum computing and quantum communications, owing to the fact that they are based on a room-temperature, chipscale platform with quantum states spanning a high-dimensional Hilbert space.

Microcombs are today firmly established at the center of contemporary nonlinear, microwave and quantum photonics. We can anticipate that as our understanding of these system consolidates with time, research in this field will deliver far-reaching outcomes that will lead microcombs to become ubiquitous in research labs, in various technological settings, and ultimately, in our daily lives.

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