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Imprints of a supercooled phase transition in the gravitational wave spectrum from a cosmic string network

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Abstract: A network of cosmic strings (CS), if present, would continue emitting gravitational waves (GW) as it evolves throughout the history of the Universe. This results in a characteristic broad spectrum making it a perfect source to infer the expansion history. In particular, a short inflationary period caused by a supercooled phase transition would cause a drop in the spectrum at frequencies corresponding to that event. However, the impact on the spectrum is similar to the ones caused by an early matter-dominated era or from particle production, making it difficult to disentangle these different physical origins. We point out that, in the case of a short inflationary period, the GW spectrum receives an additional contribution from the phase transition itself. This leads to a characteristic imprint of a peak on top of a wide plateau both visible at future GW observatories.

Keywords: Cosmology of Theories BSM, Early Universe Particle Physics, Phase Transitions in the Early Universe

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1 Introduction

Both the Standard Model of Particle Physics (SM) and the Standard ΛCDM Cosmological Model describe with remarkable accuracy a vast amount of observational data on the fundamental interactions of elementary particles [1] and on the dynamics and composition of the Universe [2]. Nevertheless, given the large number of unexplained free parameters, this framework is better viewed as an effective description, one that is incomplete at the fundamental level. For instance, explaining the matter-antimatter asymmetry or the nature of the dark matter requires extending the SM, possibly with new particles featuring additional interactions. These additional degrees of freedom might induce deviations from the standard early universe evolution [3]. Well known examples are an early matter dominated period before the universe became radiation dominated, or a kination era when the background equation-of-state is more stiff than that of radiation [4, 5].

In this respect, a completely new avenue towards the observation of our universe was recently opened with the first detection of GWs by the LIGO/Virgo collaboration [6–10]. This experimental breakthrough, in turn, has helped to accelerate the development of many planned future GW observatories with vastly improved sensitivities [11–21]. Furthermore, while all confirmed detections to date are associated with transient astrophysical sources, hope rises to probe the stochastic backgrounds of gravitational waves (SGWB) that may be generated by early Universe processes [22]. Importantly, the observation of a SGWB originating in the early Universe would enable us to explore the history of the universe before the epoch of Big-Bang Nucleosynthesis (BBN) and, therefore, to study new physics at high energies beyond the reach of particle physics laboratory-based experiments [22]. Nevertheless, we should keep in mind that the populations of compact objects originating the currently observed events will also produce their own GW foregrounds making the detection of any primordial component more challenging [23].

Another result that has energised the theoretical community is the recent observation by pulsar-timing arrays (PTAs) [24–27] of a common spectrum process which could be the first indicator of the upcoming detection of a stochastic GW background. This background could potentially be of primordial origin [28–38] with the foremost candidate being a network of cosmic strings that is the focus of the present work [39, 40].

The latest PTA data that has been recently released shows strong evidence that the signal is caused by GWs [41–47]. Nevertheless, standard cosmic strings provide a poor fit to the observed frequency dependence of the signal, which is better described by e.g. models with smaller string intercommutation probability [48–50]. Fortunately, the interplay between supercooling and string network dynamics that we will discuss in this paper carries over directly to these generalized scenarios [48].

The fact that the SGWB produced by cosmic strings in standard cosmology is scale-invariant [51–69] makes it an excellent tool to probe the expansion of the early Universe [70–79], since deviations from the standard evolution would leave characteristic imprints on the otherwise largely featureless SGWB.

An important example is provided by very strong phase transitions leading to a period of supercooling, as expected in particle physics models involving confinement [80–93] or quasi conformal models [90, 94–106]. Supercooling causes a significant modification of the expansion rate as the field undergoing the transition is trapped in the false vacuum state whose energy dominates the expansion, thus leading to a new short period of thermal inflation. If a network of cosmic strings were present during the phase transition, this period of accelerated expansion would cause a dip in the strings' SGWB spectrum. Such a feature, however, is similar to the changes that would be caused by an epoch of early matter domination [71, 73, 75], which makes it difficult to disentangle these different physical origins. On the other hand, it should be emphasized that supercooled phase transitions are themselves very strong sources of GWs. In this paper, we propose a smoking gun signature of a period of supercooling utilising the SGWB from local cosmic strings. Its uniqueness comes from our ability to observe both the GW background produced by the phase transition at the end of the supercooling period as well as the effect that it would have on the spectrum generated by the network of cosmic strings.

The paper is organized as follows: in section 2 we define the basic connection between the phase transition (PT) parameters and the length of the thermal inflation period. In section 3 we outline the computation of the GW spectrum produced by cosmic strings and the impact that a period of supercooling will have on it. Section 4 is devoted to the signal produced by the PT at the end of the supercooling period. Finally, in section 5 we discuss the results and we conclude in section 6.

2 Supercooled PT and thermal inflation

Our first goal is to establish the connection between the parameters describing a first order PT and the length of the inflationary epoch it causes. The first key parameter of the transition is its strength which in the case of strong transitions can be defined as the ratio of vacuum energy density to the radiation background [107]

$$\alpha = \rho_V/\rho_{R*}, \tag{2.1}$$

where the subscript $*$ indicates the time of the transition. We will be mostly interested in cases where the vacuum energy dominates the expansion for a significant amount of time. The length of the inflationary period is measured by the usual number of efolds,

$$N_e = \log\left(a_*/a_V\right), \tag{2.2}$$

where the subscript V refers to the moment when the vacuum energy begins to dominate the expansion. Using the fact that $\rho_R \propto a^{-4}$ while ρ_V remains constant we can easily relate eq. (2.1) and eq. (2.2):

$$\alpha = \mathrm{e}^{4N_e}, \tag{2.3}$$

which provides the link between the strength of the transition and the number of e-folds of inflation it causes.

The second key parameter describing the transition is the temperature reached as the universe is reheated $T_{\rm reh}$. We will assume instantaneous reheating for simplicity, meaning the temperature can be related to the vacuum energy density in the usual way $T_{\rm reh} \propto \rho_V^{1/4}$. However, in practice, we will keep $T_{\rm reh}$ as a free parameter and, given that it is the same as the temperature when the period of supercooling began, it can also be understood as the moment when the evolution first became non-standard. On the other hand, the strength α of the preceding transition (see eq. (2.3)) dictates the duration of that non-standard period.

Finally, one last important parameter characterizing the transition is its time scale β, which is usually introduced through an approximation of the vacuum decay rate

$$\Gamma \propto \mathrm{e}^{\beta(t-t_*)}. \tag{2.4}$$

The value of β is usually computed by differentiating the decay rate calculated in the particular model under consideration. More generally, in classically scale invariant models featuring such strong transitions [100, 108] this parameter roughly asymptotes to $\beta/H \approx 10$ as the strength of the transition grows [109]. We take this as our benchmark value.

3 Effects of supercooling on the gravitational wave emission from a cosmic string network

Scenarios of physics beyond the standard model commonly predict the existence of additional global or local symmetries. The spontaneous breaking of such symmetries in the early universe might lead to the formation of a network of cosmic strings via the Kibble mechanism [110–113].

Numerical simulations have established that, as long as the scale factor of the universe grows as a power law, a network of cosmic strings reaches a scaling regime [114–116], characterized by a constant mean velocity of long strings $\bar{v}$ and a characteristic correlation length L that remains constant with respect to the Hubble horizon d_H. The correlation length is defined by [113]:

$$\rho_\infty \equiv \frac{\mu}{L^2}\,, \tag{3.1}$$

where μ is the string tension, and ρ_∞ is the energy density of long strings in the network, which form a Brownian random walk on large scales [117]. As the universe expands, the correlation length stretches with the scale factor, which tends to increase the energy density of long strings compared to the radiation background. But this increase is compensated by energy losses associated with the production of small loops that form when long strings cross each other, and the network evolves towards the scaling regime [53]. Once formed, the loops decouple from the network and start to decay. For strings generated from the breaking of a local symmetry, which we focus on, the decay proceeds primarily through the emission of gravitational waves. When the expansion of the universe changes, e.g. from a radiation dominated to a matter dominated epoch, the network enters a transient regime until it settles into the new scaling regime.

The evolution of the long-string network borne out by simulations is well approximated by the velocity-dependent one-scale (VOS) model [118–122]. In this model, the parameters L and $\bar{v}$ that characterize the string network evolve according to [119, 120]:

$$\frac{\mathrm{d}L}{\mathrm{d}t} = (1+\bar{v}^2)\,HL + \frac{\tilde{c}\bar{v}}{2} \tag{3.2}$$

$$\frac{\mathrm{d}\bar{v}}{\mathrm{d}t} = (1-\bar{v}^2)\left[\frac{k(\bar{v})}{L} - 2H\,\bar{v}\right], \tag{3.3}$$

where the momentum parameter

$$k(\bar{v}) = \frac{2\sqrt{2}}{\pi}(1-\bar{v}^2)(1+2\sqrt{2}\bar{v}^3)\left(\frac{1-8\bar{v}^6}{1+8\bar{v}^6}\right), \tag{3.4}$$

measures the deviation from straight strings with $k(\bar{v}) = 1$, and $\tilde{c} \simeq 0.23$ describes closed loop formation [120]. It is well known that scale invariant solutions of the form $L = \xi t$ (i.e. $L \propto 1/H$) and constant $\bar{v}$ exist for the system (3.3) when the scale factor is a power law.

On the other hand, during the epoch of supercooling, the strings are exponentially stretched and the network is diluted due to the accelerated expansion of the universe [73]. The network departs from the scaling regime and the production of loops is suppressed. We follow ref. [78], and we use a simplified picture of supercooling and secondary reheating together with the VOS model to estimate the dilution of cosmic strings and their subsequent evolution.

More specifically, we assume at early times radiation domination $\rho \propto a^{-4}$ up until the Hubble parameter reaches $H_V = \Delta V/3M_{\rm Pl}^2$. The energy density of the universe is then dominated by vacuum energy while the scale factor grows by a fixed number of efolds N_e (see eq. (2.2)) until the phase transition is completed and standard radiation domination resumes. The energy scale during this period is fixed by our choice of the reheating temperature $T_{\rm reh} = (30/\pi^2 g_* \Delta V)^{1/4}$, assuming for simplicity that the reheating is instantaneous. In this inflating background, we track the evolution of the network using

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the VOS equations (3.2)–(3.4) beginning with the standard scaling initial conditions in the early radiation dominated period before supercooling sets in. During the supercooling period, the network is diluted, with the correlation length growing beyond the Hubble scale $HL \gg 1$ and the velocity dropping $\bar{v} \ll 1$. Once radiation domination resumes, the correlation length follows $L \propto a$ as the network energy regrows until it eventually reaches again the scaling solution. In contrast to the case of strings diluted by inflation, here we expect that the string network will always grow back to scaling relatively quickly and the low frequency part of the spectrum will remain unchanged [78].

The network of cosmic strings acts as a long-lasting source of gravitational waves from the time of their production until today. The emission is dominated by the closed string loops that are formed when long strings in the network intersect [53]. Gravitational waves emitted by oscillating loops at different epochs generate a SGWB over a large frequency range, and events such as phase transitions leave imprints on the resulting GW spectrum. This makes the SGWB an invaluable tool to gain insight into modifications of the expansion of the universe induced by high-energy physics effects [70, 71].

To determine the SGWB from the cumulative emission of closed loops we note that recent simulations of Nambu-Goto string networks observe that long strings can be separated into a population of large and non-relativistic loops, with initial size $l_i = \alpha_L L(t_i)$ and $\alpha_L \approx 0.37$ such that in radiation domination we have $l_i \approx \alpha t_i$ with $\alpha \approx 0.1$, as well as a collection of smaller and highly relativistic loops. The energy in the smaller loops is mostly in the form of kinetic energy that redshifts away. Hence, large loops are the main source of the SGWB. Roughly a fraction $\mathcal{F}_\alpha \sim 0.1$ of the total energy is transferred into loops [123–127], and we include an additional reduction factor $f_r = \sqrt{2}$ that accounts for the fraction of energy lost into peculiar velocities of loops [72]. After their formation at time t_i, a loop will oscillate and gradually shorten due to energy losses to GWs:

$$l(t) = \alpha_L L(t_i) - \Gamma G\mu(t - t_i), \tag{3.5}$$

where $\Gamma \approx 50$ is the total rate of GW emission [125, 128]. To compute the spectrum we will need to sum the emission from all normal modes of a closed loop. We will start from the fundamental one whose total emission can be expressed as [72, 121]:

$$\Omega^{(1)}_{\rm GW}(f) = \frac{16\pi}{3f}\frac{\mathcal{F}_\alpha}{f_r}\frac{G\mu^2}{H_0^2}\frac{\Gamma}{\alpha_L\zeta(q)}\int_{t_F}^{t_0} d\tilde{t}\,\frac{1}{\alpha_L\dot{L}(t_i)+\Gamma G\mu}\frac{\tilde{c}v(t_i)}{L(t_i)^4}\left[\frac{a(\tilde{t})}{a(t_0)}\right]^5\left[\frac{a(t_i)}{a(\tilde{t})}\right]^3\Theta(t_i - t_F) \tag{3.6}$$

where we integrate over the emission time $\tilde{t}$, and the time of formation of contributing loops t_i is found using eq. (3.5) taking into account that loops with size $l(\tilde{t}, f) = \frac{2}{f}\frac{a(\tilde{t})}{a(t_0)}$ emit at frequency f. We also added $\zeta(q) = \sum_k q^{-q}$ which assuming emission by cusps gives $\zeta(4/3) \approx 3.6$ and ensures the power in all modes sums to Γ. Finally, t_F is the initial time at which the network first reached scaling after its formation.

Assuming the emission is dominated by cusps, the contribution from higher modes can be expressed through that the first one:

$$\Omega^{(k)}_{\rm GW}(f) = k^{-4/3}\,\Omega^{(1)}_{\rm GW}(f/k)\,. \tag{3.7}$$

The total abundance is then a sum of the emission from all modes, which we approximate as:

$$\Omega_{GW}(f) \simeq \sum_{k=1}^{N} \Omega_{GW}^{(k)} + \int_{N+1}^{\infty} dk\, \Omega_{GW}^{(k)}\,, \tag{3.8}$$

where above $N = 10^3$ we approximate the sum with an integral which leads to accurate predictions with no significant impact on the computation time [78].

4 Gravitational Waves from Phase Transitions

There are several sources of GWs associated with a phase transition that have been discussed in the literature [91, 129]. Firstly the bubble wall collisions [130–133] featured in extremely supercooled scenarios [107, 108, 132–134]. Secondly, the motion of the fluid shells propelled by the growing bubbles [135–139]. Finally a possible contribution from turbulence produced as the plasma flow becomes non-linear [140–143].

Since we focus exclusively on strongly supercooled phase transitions, we can safely assume that the wall velocity is close to the speed of light, as this would be true even for much weaker transitions [144–148]. Assuming that the strength of the transition is large $\alpha \gg 1$, we do not need to diligently compute the energy budget [100, 108, 149] in order to check what fraction of energy will be deposited into each of the sources. This is because in this regime the fluid shells propelled by the bubbles behave as relativistic shocks [150]. These are very peaked and continue propagating at relativistic velocities. Their energy also dissipates at the same rate as that of the scalar field gradients after the collision and, as a result, the GW spectrum they produce is identical to the one produced by bubble wall collisions [151]. The spectrum takes the form [133, 151]:

$$\Omega_{\rm GW}^{\rm PT}(f)h^2 = 1.67\times10^{-5}\left(\frac{100}{g_*(T_{\rm reh})}\right)^{\frac{1}{3}}\left(\frac{\beta}{H}\right)^{-2}\left(\frac{\alpha}{1+\alpha}\right)^{2}\frac{A\,(a+b)^c}{\left[b\left(\frac{f}{f_p}\right)^{-\frac{a}{c}}+a\left(\frac{f}{f_p}\right)^{\frac{b}{c}}\right]^c}\,, \tag{4.1}$$

where, $a = b = 2.4$, $c = 4.0$ and $A = 5.13\times10^{-2}$. The peak frequency is given by

$$f_{\rm p} = 1.65\times10^{-5}{\rm Hz}\left(\frac{T_{\rm reh}}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\frac{0.71}{2\pi}\left(\frac{\beta}{H}\right)\,, \tag{4.2}$$

and g_* is the number of relativistic degrees of freedom that we approximate using the results of [152].

The signal-to-noise ratio we refer to in the results that follow is computed in the standard way

$$\mathrm{SNR} \equiv \left[\tau\int_{f_{\rm min}}^{f_{\rm max}}\left(\frac{\Omega_{\rm sw}(f)}{\Omega_{\rm noise}(f)}\right)^2 {\rm d}f\right]^{1/2}\,, \tag{4.3}$$

where we take the noise curve for a given experiment and assume the duration of each mission to be $\tau = 4$ years.

5 Resulting GW spectra

Finally, we can put together the GW spectra computed for cosmic strings in section 3 and from the phase transition ending the supercooling inflationary period discussed in section 4. We contrast these with the predicted sensitivities of LISA [153, 154], SKA [13], AION-1km [17], AEDGE [18], AEDGE+ [21], and ET [11, 12] experiments, as well as the currently running LIGO/Virgo/KAGRA [155–158].

Figure 1 shows the combined spectra. In all the panels we set the tension of the strings to $G\mu = 10^{-10}$ inspired by the NANOGrav data [39] and the time scale of the phase transition is $\beta/H = 10$ as a benchmark for supercooled scenarios [109]. The solid black line is the cosmic string spectrum with no period of supercooling, while the increasingly red lines with longer dashing show the combined spectra with a period of supercooling from $N_e = 0.5$ to $N_e = 12$ efolds, which covers what is achievable in typical models [83, 102]. The left panel shows the results for reheating temperature of $T_{\rm reh} = 10\,\text{GeV}$ while the right one $T_{\rm reh} = 10^3\,\text{GeV}$. The temperature controls the frequency of the features in the spectrum as both the PT signal [91, 129] and the frequency at which the cosmic string plateau is modified [70, 71] grow linearly with temperature. The modified string spectra at high frequency follow a $f^{-\frac{1}{3}}$ power-law. The peak above the string spectrum comes from the phase transition ending the supercooling. As expected, for all cases with non-negligible supercooling the peak height reaches a nearly constant value as in these cases the GW source at the PT is using already the entirety of the energy budget and eq. (4.1) reaches its large α limit.

The effect of supercooling on the string network is much more complex. This is because the inflationary period not only redshifts the previously generated spectrum according to the VOS description, but it also dilutes the network of strings. Depending on how much the network is diluted, it can take a significant amount of time for the density to grow back to the scaling solution [78]. It is only after that time that loops will be produced at the same rate as before the inflationary period and their decay into GWs, which is the main source for the spectrum, will also go back to the scaling result. Due to this fact, we see that longer periods of inflation shift to lower frequencies the return of the spectrum to its standard cosmology form (black solid lines) while the reheating temperature remains fixed. In fact since the return to scaling defines when the string spectrum goes back to its standard form there is a degeneracy between reheating and number of efolds. If we increase the number of efolds of thermal inflation increasing the dilution but also increase the reheating temperature to give strings more time to grow back into scaling at the same time their spectrum will remain unchanged. In order to quantify this dependence we recall that the scaling of string energy when chopping becomes unimportant during inflation $\rho_\infty \propto a^{-2}$ while after inflation $\rho_\infty/\rho_R \propto a^2$. This means the number of efolds spent by the network in inflation is the same as the number of efolds needed after it for the strings to grow back. Using this fact and $a \propto T^{-1}$ we see that $N_e \propto \log T^2_{\rm reh}$ and changing the number of efolds according to this formula as we vary the reheating temperature we would always find the same GW spectrum. Luckily this degeneracy in our scenario is broken by the phase transition spectra which are very insensitive to the exact length of the inflationary period while their frequency shifts with reheating temperature.

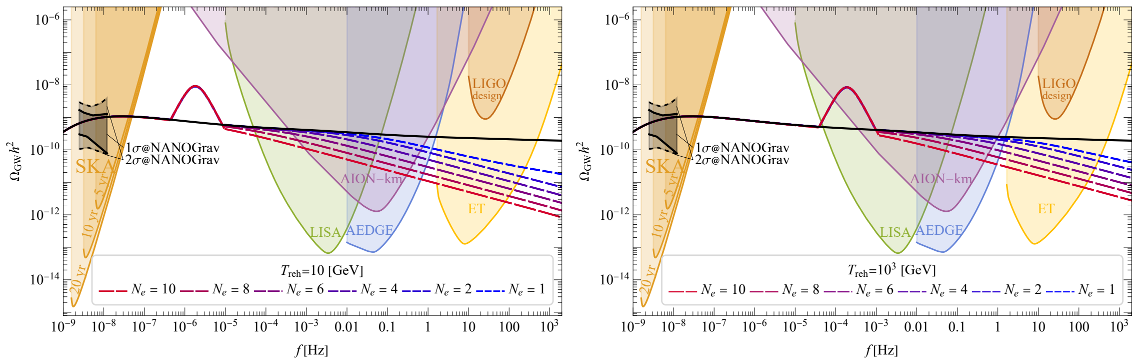

Figure 1. GW spectra from a supercooled phase transition and cosmic strings. The thick black line shows the cosmic string signal with no supercooled phase transition, while the coloured dashed lines show both sources for the number of efolds of supercooling indicated. The bump above the black line is the contribution from the transition itself, while the falling lines at higher frequencies illustrate how the string spectrum is impacted by the short inflation period associated with supercooling. For the strings, we used $G\mu = 10^{-10}$ while for the PT $\beta/H = 10$ in this example.

Figure 2 shows parts of the parameter space in which our signals will be visible in upcoming experiments with $SNR > 10$ (see eq. (4.3)). Here we fix $\beta/H = 10$ and vary the reheating temperature $T_{\rm reh}$ as well as the tension of the string network $G\mu$. The grey bands show the values giving a one and two sigma fit to the NANOGRav data [39]. The two panels show the dependence of the results on the amount of supercooling; we fix $N_e = 10$ in the upper panel and $N_e = 1$ in the lower one. The solid and dashed contours indicate where a given experiment would observe the PT spectrum and the modified cosmic string spectrum respectively. By modified spectrum here we mean one that can be distinguished from the result obtained assuming standard cosmological expansion. In practice, when computing the integral in (4.3) we include only the parts of the spectrum where the amplitude deviates from the standard one (black lines in figure 1) by at least 10% as an example of a conservative threshold.

Regions, where both solid and dashed contours overlap in figure 2, indicate where we would see a smoking gun signal of supercooling by observing both the PT spectrum and the modification of the cosmic string spectrum. The prospects grow with the amount of supercooling, since the increasingly diluted string network retains the modifications at lower frequencies that are radiated later as the network slowly comes back to scaling after dilution.

6 Conclusion

Gravitational waves provide a pathway to test new high energy physics phenomena that took place in the early pre-BBN stages of the evolution of the universe. We considered a particularly well-motivated non-standard cosmological era known as supercooling or secondary inflation, and we studied its effects on the SGWB generated by a network of cosmic strings. Cosmic strings have a reasonably well understood GW emission spectrum. In

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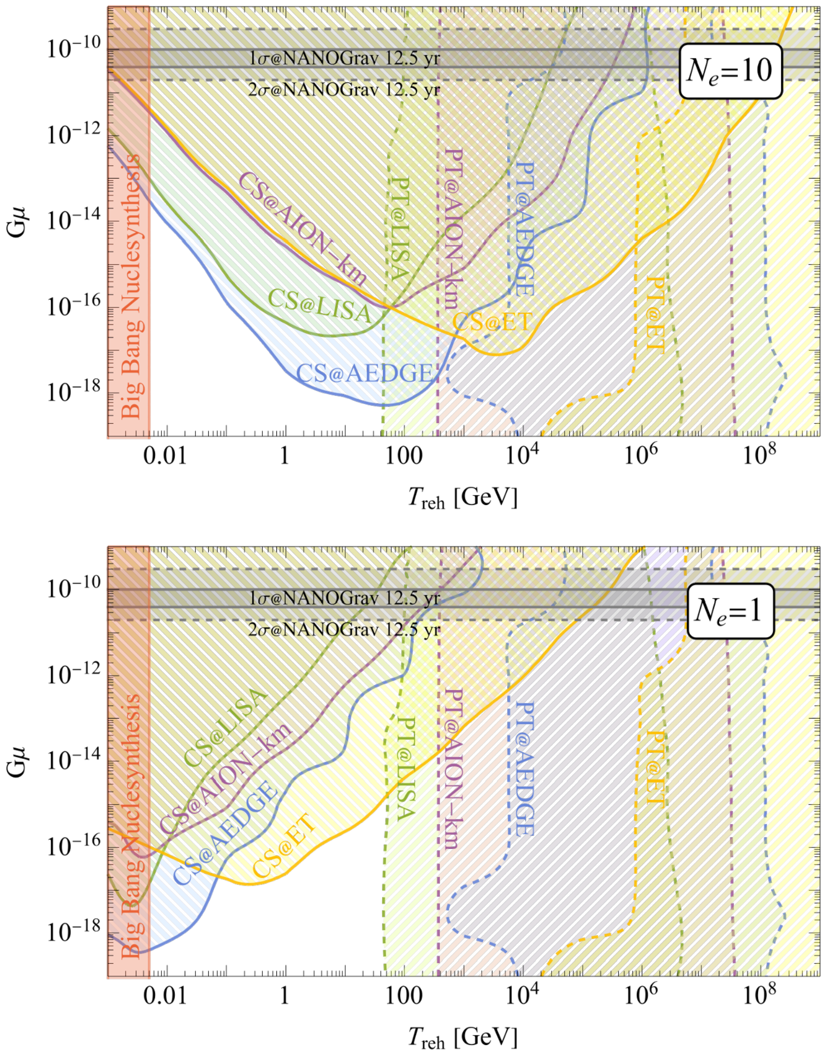

Figure 2. Part of the parameter space where our modified signals will be observed with $SNR > 10$. We separate the calculation into the modified cosmic string tail (solid contours) and the phase transition contributions (dashed lines). We take into account only modified parts of the spectrum where the amplitude is at least 10% away from the standard string spectrum with a given $G\mu$. For this example, we used $\beta/H = 10$ for the PT, while the length of supercooling is taken to be $N_e = 10$ (top panel) and $N_e = 10$ (bottom panel).

particular, the emission during the standard radiation-dominated era results in a GW spectrum with a long flat plateau at high frequency. We can use this featureless spectrum to propagate through it various assumed cosmological histories and thus probe them with observational data.

In this paper, we have worked out in detail the structure imprinted on this SGWB by an epoch of supercooling. We find that such an early period of thermal inflation leaves a characteristic descending cosmic string plateau plus the peak produced at the end of supercooling. This can be used to distinguish thermal inflation from any other mechanism that can suppress the SGWB from strings at high frequencies, such as an early matter domination period (see figure 1). The reason is that supercooling will forcibly end in a first order phase transition, whereas the other contributions might not.

For typical phase transition parameters with $\beta/H = 10$ involving large supercooling, we found that detection prospects depend crucially on the length of the supercooling period if we aim to find spectra from both sources. For a very short thermal inflation period with $N_e = 1$ the combined data from future experiments will probe the features of the spectrum with string tension above $G\mu = 10^{-15}$ and only if the reheating temperature is around $T_{reh} = 100\,\mathrm{GeV}$. For larger string tensions close to the ones fitting the NANOGrav signal the detection range extends to $T_{reh} \in [10^2, 10^5]\,\mathrm{GeV}$. For a long period of supercooling $N_e = 10$ the detection prospects reach down to $G\mu = 10^{-18}$ and extend to $T_{reh} \in [10^2, 10^8]\,\mathrm{GeV}$ for strong string spectra close to PTA limits (see figure 2). Crucially, these include both the emission from the strings and phase transition itself which can be used as a smoking gun to distinguish an epoch of supercooling from other modifications of the early universe evolution such as an era of early matter domination.

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