RESEARCH ARTICLE

Enhanced material identification via momentum-integrated muon scattering tomography [version 1; peer review: 2 approved with reservations]

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Abstract

Background

Cosmic ray muons, originating from interactions in the upper atmosphere, possess high energy and unique penetrative capabilities suitable for non-traditional radiographic inspection. This study explores their application in various fields such as nuclear fuel cask monitoring, nuclear reactor imaging, and archaeology, leveraging the principle of multiple Coulomb scattering for imaging dense materials. While muon scattering tomography has shown promise, accurately measuring muon momentum remains challenging.

Methods

This research introduces the Momentum Integrated Point-of-Closest Approach (mPoCA) algorithm, integrating muon momentum data into the traditional Point-of-Closest Approach (PoCA) framework. Utilizing the Cherenkov muon spectrometer, renowned for precise muon momentum estimation, the mPoCA algorithm offers a novel imaging approach.

Results

Simulations conducted with GEANT4 evaluate the mPoCA algorithm's performance against the standard PoCA method, demonstrating superior image resolution and enhanced material identification capabilities, particularly in distinguishing materials like uranium and lead.

Open Peer Review

Approval Status ? ?

	1	2
version 1 29 Apr 2024	? view	? view

1. **Ryuichi Nishiyama**, University of Tokyo, Tokyo, Japan

2. **László Balázs**, HUN-REN Wigner Research Centre for Physics High Energy Experimental Particle and Heavy Ion Physics, Budapest, Hungary

Any reports and responses or comments on the article can be found at the end of the article.

Conclusions

These findings underscore the potential of the mPoCA algorithm for advancing muon scattering tomography applications.

Keywords
Cosmic ray muons, muon tomography, image reconstruction, nuclear material accountancy, nuclear nonproliferation

This article is included in the Research from Advances in Nuclear Nonproliferation Technology and Policy Conferences (ANTPC) collection.

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Author roles: Bae J: Methodology, Software, Visualization, Writing – Original Draft Preparation; **Montgomery R**: Project Administration, Supervision, Writing – Review & Editing; **Chatzidakis S**: Formal Analysis, Project Administration, Supervision, Validation, Writing – Review & Editing

Competing interests: No competing interests were disclosed.

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Introduction

Cosmic ray muons, produced in the upper atmosphere, cascade towards Earth with high energy, offering unique capabilities for non-traditional radiographic inspection. Their penetrative power is especially beneficial for imaging and monitoring dense materials, making them ideal for various applications,[1] including nuclear fuel cask monitoring, nuclear reactor imaging,[2–7] geological surveys,[8–10] homeland security,[11,12] and archaeology.[13,14] The principle of multiple Coulomb scattering, which approximates muon deflection angles through a Gaussian distribution, plays a crucial role in such imaging techniques. This distribution's key characteristics hinge on the material's density and the muon's momentum. Incorporating muon momentum into muon scattering tomography has shown to significantly improve its efficacy.[15–17] Nonetheless, accurately measuring muon momentum in practical settings remains a challenge, often requiring the use of extensive magnetic or time-of-flight spectrometers.

This research presents the Momentum Integrated Point-of-Closest Approach (mPoCA) algorithm, a significant advancement in muon scattering tomography that incorporates muon momentum data into the traditional Point-of-Closest Approach (PoCA) framework. Utilizing the innovative Cherenkov muon spectrometer,[18–21] which is celebrated for its precise muon momentum estimation with a resolution of ± 0.5 GeV/c across a broad momentum spectrum (0.1 to 10.0 GeV/c) and achieving around 90% accuracy, the mPoCA algorithm offers a novel approach to imaging. Through a series of simulations conducted with GEANT4,[22] a particle transport code, this study evaluates the mPoCA algorithm against the standard PoCA algorithm in scenarios involving both a realistic cosmic muon spectrum and mono-energetic 3 GeV muons. The findings highlight the mPoCA algorithm's superior image resolution and improved material identification capabilities, showcasing its potential to distinguish between materials like uranium and lead, which pose challenges for the conventional PoCA method.

Imaging algorithm for muon scattering tomography

In real-world scenarios, predicting muon scattering angle and path within a medium is challenging because of its inherently random nature of the process. To address this practical obstacle, a PoCA algorithm was developed.[23] However, utilizing the PoCA algorithm comes with two key limitations. Firstly, it provides only a single scattering point, and secondly, it assumes straight incoming and outgoing muon trajectories, regardless of the actual muon path.

To detect cosmic ray muons, two sets of muon trackers are typically deployed above and below the target objects. This setup enables the measurement of both incoming and outgoing muon trajectories, essential for implementing the PoCA algorithm. The measured muon positions in terms of Cartesian coordinates at four muon detectors are:

$$p_i = (x_i, y_i, z_i) \quad i = 1, 2, 3, \text{and } 4. \tag{1}$$

The unit vectors of incoming and outgoing muon path, $\boldsymbol{u_1}$ and $\boldsymbol{u_2}$, are given by:

$$\boldsymbol{u_1} = \frac{p_2 - p_1}{\|p_2, p_1\|} \quad \text{and} \quad \boldsymbol{u_2} = \frac{p_4 - p_3}{\|p_4, p_3\|}. \tag{2}$$

Both unit vectors are extrapolated and sets of extrapolated line segments, L_1 and L_2:

$$L_1 = \boldsymbol{k_1} \cdot \boldsymbol{u_1} \quad \text{and} \quad L_2 = \boldsymbol{k_2} \cdot \boldsymbol{u_2}, \tag{3}$$

$$\boldsymbol{k_1} = \left[k_{1,x}, k_{1,y}, k_{1,z}\right]^T; \quad \boldsymbol{k_2} = \left[k_{2,x}, k_{2,y}, k_{2,z}\right]^T, \tag{4}$$

The determination of the closest distance between two straight lines involves identifying the minimum distance between two points, l_1 and l_2, on lines, L_1 and L_2:

$$D = \min \|l_{1,i}, l_{2,i}\| \quad i = 1, \ldots, N, \tag{5}$$

where D is the distance and N is the number of points on L_1 and L_2. A PoCA point is defined as the closest point from L_1 and L_2, which is a midpoint of l_1 and l_2:

$$P(x, y, z) = \frac{1}{2}[l_{1,m}(x, y, z) + l_{2,n}(x, y, z)], \tag{6}$$

where $l_{1,m}$ and $l_{2,n}$ are the points where $\|l_{1,m}, l_{2,n}\|$ equals the line distance, D. To estimate muon scattering angle from the trajectories of incoming and outgoing muons, the directions of travel along the x-z and y-z planes for both incoming and outgoing muon trajectories are crucial components of this estimation.

$$\theta^{\text{in}}_{\text{xz-plane}} = \text{atan}\left(\frac{|p_{2,z} - p_{1,z}|}{|p_{2,x} - p_{1,x}|}\right), \tag{7}$$

$$\theta^{\text{out}}_{\text{xz-plane}} = \text{atan}\left(\frac{|p_{4,z} - p_{3,z}|}{|p_{4,x} - p_{3,x}|}\right), \tag{8}$$

$$\theta^{\text{in}}_{\text{yz-plane}} = \text{atan}\left(\frac{|p_{2,z} - p_{1,z}|}{|p_{2,y} - p_{1,y}|}\right), \tag{9}$$

$$\theta^{\text{out}}_{\text{yz-plane}} = \text{atan}\left(\frac{|p_{4,z} - p_{3,z}|}{|p_{4,y} - p_{3,y}|}\right). \tag{10}$$

The spatial muon scattering angle, θ, can be estimated using computed scattering angles seen in Eqs. (11)–(13):

$$\theta \approx \sqrt{\Delta\theta^2_{xz} + \Delta\theta^2_{yz}}, \tag{11}$$

$$\Delta\theta_{xz} = |\theta^{\text{in}}_{\text{xz-plane}} - \theta^{\text{out}}_{\text{xz-plane}}|, \tag{12}$$

$$\Delta\theta_{yz} = |\theta^{\text{in}}_{\text{yz-plane}} - \theta^{\text{out}}_{\text{yz-plane}}|. \tag{13}$$

The computed spatial muon scattering angles are associated with a designated PoCA voxel, and these values are utilized for imaging purposes. In instances where two or more spatial muon scattering angles are registered within a PoCA voxel, the average scattering angle is employed:

$$\theta = \frac{1}{N_{Voxel}} \sum \theta_i, \quad i = 1, 2, \ldots, N_{Voxel}, \tag{14}$$

where N_{Voxel} is the number of recorded scattering angles in a PoCA voxel.

Results

Momentum integrated PoCA algorithm

To incorporate muon momentum information, p_μ, into a PoCA algorithm, momentum is further associated with a PoCA point and scattering angle. The complete mPoCA vector encompasses five values: the three Cartesian components of a PoCA point (P_x, P_y, and P_z), the reconstructed spatial muon scattering angle, and the estimated muon momentum:

$$\boldsymbol{X_i} = \left[P_x, P_y, P_z, \theta, p_\mu\right]_i \quad i = 1, \ldots, N_\mu, \tag{15}$$

where N_μ is the total number of muons. Estimated muon momenta are classified into a finite number of momentum groups, N_p. To facilitate the reconstruction of an image of the target object, the volume of interest is discretized into $N_X \times N_Y \times N_Z$ voxels, with the designated PoCA point represented as $V_{l,m,n}$. Consequently, the muon events, $\boldsymbol{X_i}$, are decomposed with respect to momentum and PoCA voxel:

$$\boldsymbol{X^T} = \left[\vec{X}^T_1, \vec{X}^T_2, \ldots, \vec{X}^T_{N_p-1}, \vec{X}^T_{N_p}\right], \tag{16}$$

$$\vec{X}_j = \left[\begin{pmatrix} V_{l,m,n}(1) \\ \theta_1 \end{pmatrix} \begin{pmatrix} V_{l,m,n}(2) \\ \theta_2 \end{pmatrix} \cdots \begin{pmatrix} V_{l,m,n}(L) \\ \theta_L \end{pmatrix}\right]_j, \tag{17}$$

where L represents the length of X_j. Since the expected muon scattering angle distribution is contingent on muon momentum, each transpose of $\boldsymbol{X_j}$ (denoted as $\boldsymbol{X_j^T}$) will contain a unique scattering angle distribution. Following the decomposition process of $\boldsymbol{X_i}$, data for all muon events will be recorded, comprising (1) a designated voxel as PoCA, (2) scattering angle, and (3) momentum group information:

$$\boldsymbol{Y_{i,j}} = [V_{l,m,n}, \theta_i, \pi(j)], \tag{18}$$

where θ_i represents the i-th muon event in the j-th muon momentum group, denoted as $\pi(j)$. For image reconstruction, materials can be categorized based on their atomic number (Z), as the radiation length (X_0) is dependent on

Z. All materials are classified into M_N material classes using M_N-1 threshold scattering angles, denoted as θ_{th}. These threshold scattering angles are estimated by determining the intersection of two expected scattering angle distributions of anticipated material classes:

$$f(\boldsymbol{\theta_{th}};k,\pi(j)) = f(\boldsymbol{\theta_{th}};k+1,\pi(j)), \quad k = 1,\ldots,M_N - 1 \tag{19}$$

where $f(\theta_{th}; k, \pi(j))$ represents the anticipated muon scattering angle distribution for the material class k when the muon momentum group is $\pi(j)$. By comparing the measured muon scattering angle to the threshold angles, the most likely material class can be determined. This process is then repeated for all momentum groups. Consequently, each muon event will be associated with a PoCA voxel and an expected material class. Thus, the muon event vector can be redefined to replace the angle with the material class. The imaging process using the mPoCA algorithm in muon scattering tomography is completed by assigning a distinct color for each material class within all PoCA voxels.

GEANT4 modeling and simulation

A computational simulation model was designed using GEANT4 to assess the advantages of the mPoCA algorithm in material identification applications. In these simulations, four cylindrical materials with dimensions (both radius and height are 5 cm) made of (1) aluminum, (2) steel, (3) lead, and (4) uranium, were evenly arranged within the air. Two muon detectors measuring 50 cm × 50 cm were positioned above and below the samples. A class of materials that could be identified using an absolute colormap. For image reconstruction, MATLAB (algorithm development) and ParaView (image rendering) were employed.[24,25] Our study explored four distinct scenarios to assess muon tomography's effectiveness in material identification application. (1) PoCA algorithm with 3 GeV mono-energetic muons

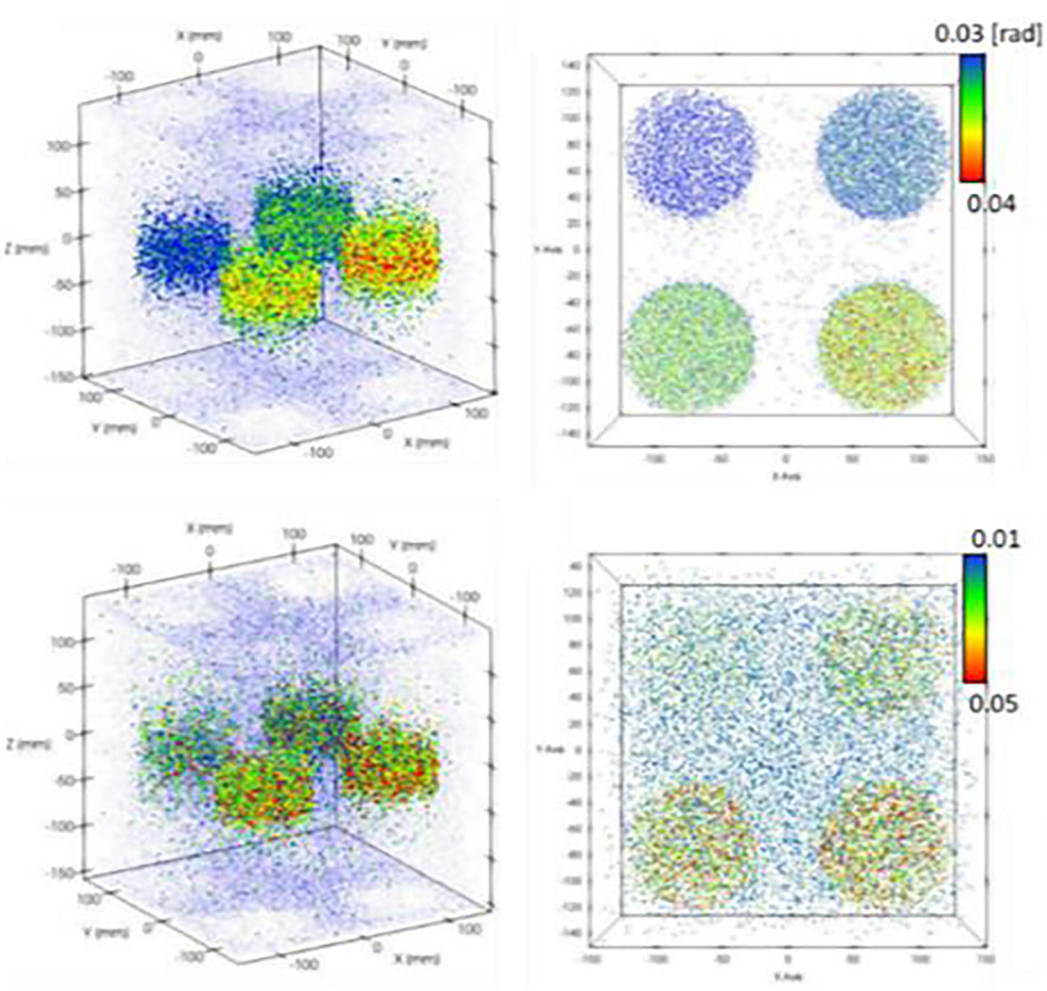

Figure 1. 3D and cross-sectional visualization of four cylindrical materials: aluminum, steel, lead, and uranium. PoCa algorithm was used with 10^5 mono-energy muons (top) and cosmic muons (bottom).

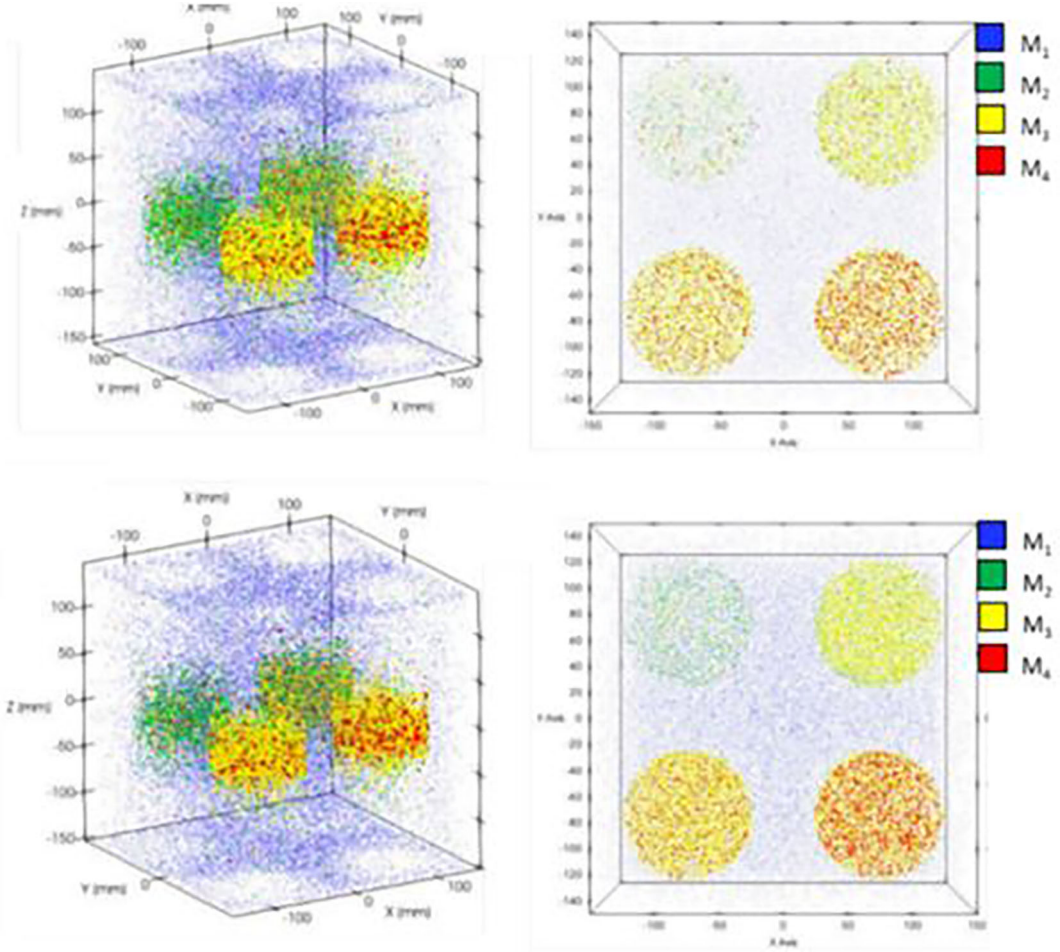

Figure 2. 3D and cross-sectional visualization of four cylindrical materials: aluminum, steel, lead, and uranium. mPoCA algorithm was used with $5x10^4$ (top) and 10^5 (bottom) cosmic ray muons.

($N_\mu = 10^5$), (2) PoCA algorithm with a cosmic muon spectrum ($N_\mu = 10^5$), (3) mPoCA algorithm with a cosmic muon spectrum ($N_\mu = 5\times10^4$), (4) mPoCA algorithm with a cosmic muon spectrum ($N_\mu = 10^5$). Our findings highlighted a key difference between the original PoCA and mPoCA algorithms. The PoCA algorithm employs a continuous color map to differentiate scattering angles, while mPoCA algorithm assigns specific colors to each material class, facilitating clearer material identification. This distinction is visually represented in Figures 1 and 2. We observed that tungsten and steel could not be distinguished using the PoCA algorithm with cosmic muon spectrum. However, in the fourth scenario using mPoCA, we successfully classified materials into three categories—heavy, light solids, and fluids—using the colors red, yellow, and blue, respectively. Further, our simulation results with the mPoCA algorithm introduced a color-coded material classification: blue for gases or liquids (atomic number $Z < 11$), green for light structural materials ($Z \approx 20$), yellow for heavy structural or shielding materials ($Z \approx 74$), and red for potential special nuclear materials ($Z < 90$). This system, especially when combined with muon momentum information, markedly improved our ability to differentiate between materials, including those with similar densities like lead and uranium, which are challenging to distinguish without this data. Notably, using mono-energetic muons narrowed the cattering angle distribution, significantly enhancing material classification accuracy. The experiment with a reduced muon count demonstrated that the mPoCA algorithm could significantly improve image resolution, allowing for the straightforward identification of all materials tested.

Conclusion

This research introduces an improved method in muon scattering tomography through the development of a momentum integrated PoCA algorithm. This innovative approach enhances the original PoCA by incorporating muon momentum information, obtained via a multi-layer pressurized gas Cherenkov muon spectrometer which enables the classification of the cosmic ray muon spectrum, ranging from 0.1 to over 5.0 GeV/c, into six distinct groups with a precision of

$\pm$ 0.5 GeV/c. In this work, sample materials were classified based on their atomic number into four categories: gases or liquids ($Z < 11$), light structural materials ($Z \approx 20$), heavy structural or shielding materials ($Z \approx 74$), and potential special nuclear materials ($Z > 90$). This classification was visually represented through the assignment of distinct colors to each material. To evaluate the effectiveness of the mPoCA algorithm, we conducted comprehensive GEANT4 simulations, reconstructing images of various materials—aluminum, steel, lead, and uranium—under four specific scenarios: using 10^5 mono-energetic 3 GeV muons, 10^5 muons within a cosmic muon spectrum via PoCA, 5×10^4 muons, and 10^5 muons within a cosmic muon spectrum via mPoCA. The mPoCA algorithm significantly enhanced imaging resolution and material classification capability. Unlike the original PoCA algorithm, which struggled to differentiate between lead and uranium due to their similar scattering ranges (0.01–0.05 rad), mPoCA's refined analysis could clearly identify all tested materials, including air, aluminum, steel, lead, and uranium. Our findings demonstrate the substantial potential of the mPoCA algorithm to advance the field of muon scattering tomography. By leveraging a high-resolution, fieldable muon spectrometer, the mPoCA algorithm opens new avenues for accurately identifying and classifying materials hidden within structures, a capability of paramount importance for nuclear accountancy and security measures.

Data and software code availability

The datasets generated and/or analyzed in the current study are available from Zenodo (DOI: 10.5281/zenodo.10975757) and the data analysis codes can be found in the same directory (file name: MMST_Dataset_J.Bae.xlsx) and an open-source programming language, *GNU Octave*, can be used to run the provided MATLAB data analysis codes with any necessary modifications.

Source code available from: https://github.com/Junghyun-Bae-Purdue/bae43

Archived source code at time of publication: https://doi.org/10.5281/zenodo.10975757.[26]

Acknowledgments

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Open Peer Review

Current Peer Review Status: ? ?

Version 1

Reviewer Report 07 June 2024

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László Balázs
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Enhanced material identification via momentum-integrated muon scattering tomography
Review
Tomography based on muon scattering, supplemented by muon momentum determination, offers the possibility of an approximate determination of the composition of the materials under investigation. The authors who have been involved in the development of this method have published several articles on the principle and the implementation possibilities of the method. This article is a short summary of simulation studies on the selectivity of the method.
A few comments:
p3
It would be useful to mark the vectors with uniformly bold characters (e.g. measuring points p etc.)
The normalizing factor for unit vectors should be the norm of the difference between the two vectors (eq 2)
What are the **k** vectors here? It would be more appropriate to specify the line segments with two vectors and an affine parameter. (eq. 3) (instead of dot product ?)
*l*1 and *l*2 are also vectors and should be denoted accordingly.
p4
It would be useful to define the PoCA voxel in one sentence.
Eq 15. The momentum is marked in the same character (p) as the measurement points.
p5
It would be useful to specify and illustrate the distribution functions in Equation 15 for two of the materials under study. This is a key element for the separability of materials.
Although the effect under study is visible, the quality of the figures is poor and captions are missing.
p6
It would be useful to indicate which muon spectrum was used in the simulation of cosmic muons.
It would also be useful to briefly discuss the likelihood of errors in material classification.
Summary

The article can be considered as a brief introduction to the research topic indicated. I suggest improvements and correction considering the above suggestions.
I also note, the more detailed article with similar figures has been published by the authors. (Bae J, et al., 2024 [Ref 1])

References

1. Bae J, Montgomery R, Chatzidakis S: Nuclear material accountancy using momentum-informed muon scattering tomography. *Annals of Nuclear Energy*. 2024; **197**. Publisher Full Text

Is the work clearly and accurately presented and does it cite the current literature?
Partly

Is the study design appropriate and does the work have academic merit?
Partly

Are sufficient details of methods and analysis provided to allow replication by others?
Partly

If applicable, is the statistical analysis and its interpretation appropriate?
Partly

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: I am a geophysicist working in muon tomography and related data processing

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

Reviewer Report 08 May 2024

https://doi.org/10.21956/nuclscitechnolopenres.18821.r27304

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Ryuichi Nishiyama
University of Tokyo, Tokyo, Japan

The submitted manuscript "**Enhanced material identification via momentum-integrated muon scattering tomography**" by Bae et al. proposed a new analysis method incorporating muon momentum for muon scattering radiography and demonstrated the idea by applying it to Geant4 simulated data. I recommend approval status after the authors correct with respect to the following minor issues.

INTRODUCTION
1st paragraph:
"This distribution's key characteristics hinge on the material's density and the muon's momentum." This sentence is not clear to me. Please rewrite it.

Equation (4):
Please define k_1 and k_2 so that readers can imagine how the unit vectors are extrapolated.

RESULTS
Momentum integrated PoCA algorithm:
Equation (15) is very simple and easy to follow. However, equations (16) and (17) are confusing for me. The confusion arises because X_i in eq. (15) is defined for each muon whereas X_j in eqs. (16) and (17) is defined for each momentum group. I wish that the author presents a more explicit and comprehensive formulation on this issue. The author states "L represents the length of X_j". However, this should be rephrased as "L represents the number of muon trajectories registered as j-th momentum group". In addition, please explain Y_{i,j} in a more explicit manner, as it is unclear how to construct Y_{i,j} from X_j in the current manuscript. Equation (19) is also unclear. It is unclear what is f, probability density function of scattering angle? I wish that the author shows an example of actual threshold values estimated for the mPoCA demonstration in the next subsection.

GEANT4 modeling and simulation:
It seems the simulations (3) and (4) are no different, because their difference is just x2 of muon statistics. Please explain the difference if there is. This simulation section needs more description on the considered instrument and its error (position and momentum resolution).
Typos:
"red for potential special nuclear materials (Z < 90)" --> "(Z >90)"
"narrowed the cattering angle" --> "scattering angle"

Is the work clearly and accurately presented and does it cite the current literature?
Partly

Is the study design appropriate and does the work have academic merit?
Partly

Are sufficient details of methods and analysis provided to allow replication by others?
Partly

If applicable, is the statistical analysis and its interpretation appropriate?
Partly

Are all the source data underlying the results available to ensure full reproducibility?

Partly

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Geophysics including muography and other methods

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.