Comparison of second-order spectral line widths formulae

C. A. Iglesias, T. A. Gomez

November 14, 2023

High Energy Density Physics

Comparison of second-order spectral line widths formulae

CARLOS A. IGLESIAS
Lawrence Livermore National Laboratories
P.O. Box 808, Livermore, CA 94550, USA

THOMAS A. GOMEZ
Department of Astrophysical and Planetary Science
University of Colorado
2000 Colorado Avenue, Boulder, CO 80305 USA

Abstract

Frequently used second-order spectral line width formulae from the projector operator and kinetic theory methods have not been formally compared previously. It is shown that a systematic second-order expansion of the projection operator expression including initial correlations agrees with the second-order kinetic theory result. The agreement assumes a common approximation in the projector operator method that introduces a screened radiator-perturber interaction to account for neglected electron-electron correlations. On the other hand, it is shown that the usual width expression from the projection operator approach neglecting initial correlations differs from kinetic theory. The differences, however, are third order in the radiator-perturber interaction. These comparisons suggest using the more compact width expression from kinetic theory, which systematically accounts for screening of the radiator-perturber interaction and initial correlations, as the starting point for second-order calculations.

E-mail: iglesias1@llnl.gov (C. A. Iglesias)

1. Introduction

Spectral lines provide a valuable plasma diagnostic tool [Griem (1974)] and various methods have been developed to compute line shapes [Gigosos (2014); Gomez et al. (2022)]. The theories often express the line width and shift as a mean field term plus a dynamic contribution describing momentum transfer between the radiator internal states and the perturbing plasma. The mean field leads to level shifts and is all order in the radiator-perturber interaction. The dynamic part contributes shifts and widths and has second order as the lowest non-vanishing term.

Using only the lowest order terms is expected to be valid near line center where the radiator-perturber interaction is weak. Nevertheless, due to their practicability second-order theories are extensively used [e.g., Calisti et al. (1990); Mancini et al. (1991)]. In view of the uncertainty, second-order theories were tested using an all-order scattering theory approach that reported good agreement for charged radiators but had concerns for neutrals [Gomez et al. (2021)]. The problem with neutral radiators is that the strong collision treatment in the Born approximation to scattering does not conserve unitarity. Thus, the validity of second-order theories can be extended with appropriate strong collision cutoffs [Iglesias (2016)]. Incidentally, second-order theories also provide physical insight to line broadening processes.

This work revisits frequently used second-order expressions for the line width from the projection operator [Smith & Hooper (1967)] and kinetic theory formalisms [Hussey et al. (1975)]. Although these methods were developed about half a century ago, no detailed comparisons exist in the literature. The present analysis introduces several approximations common in spectral line broadening by plasmas. Specifically, the system has a single radiator (emitting or absorbing atom). The plasma is factored into independent subsystems: an electron gas in the external potential of the radiator net electric charge and quasi-static ions interacting via an electron-screened potential [Iglesias & Dufty (1983)]. The kinetic theory relies on the equilibrium BBGKY hierarchy; thus, the system is in thermal equilibrium at a local temperature. The radiator center of mass motion and collisional broadening are assumed independent. Finally, exchange between the perturber and radiator bound electrons is neglected.

The dipole radiation line shape function at photon energy $\hbar\omega$ for the assumed system can be written in the form (suppressing the ion contribution to simplify notation) [Smith & Hooper (1967); Hussey et al. (1975)]

$$I(\omega) = -\pi^{-1}\,\mathrm{Re}\int_{o}^{\infty} dt e^{i\omega t} Tr_{a,e}\left\{\rho\vec{d}\cdot\vec{d}(t)\right\}$$
$$= -\pi^{-1}\,\mathrm{Im}\, Tr_{a}\left\{\vec{d}\cdot\frac{1}{\omega - L(a) - M(a,\omega)} f(a)\vec{d}\right\} \tag{1.1}$$

where Tr_a and Tr_e respectively denote a trace over the radiator internal states and the electron gas degrees of freedom, $\vec{d}$ is the radiator dipole operator, and

$$L(a) = \hbar^{-1}\left[H(a),\cdots\right] \tag{1.2}$$

with $H(a)$ the Hamiltonian for the isolated radiator internal states and the square brackets a commutator. The reduced distribution function for the radiator internal states is defined by

$$f(a) = Tr_{e}\rho \tag{1.3}$$

with ρ the density matrix for the radiator-electron gas system. The operator $M(a,\omega)$ represents the perturbing electron gas effects on the radiator. It contains a mean field term producing level shifts, which are similar in the projection operator and kinetic theory approaches. The present emphasis, however, is on widths given by the imaginary part of $M(a,\omega)$.

The kinetic theory approach is considered in Section 2 and the projection operator method in Section 3. Comparisons are provided in Section 4 with conclusions in the last Section 5. In the past, widths formulae have often been expressed with additional approximations (e.g., no-lower-state broadening) significantly simplifying the formulae. Since an additional goal is to present the complete second-order line widths formulas from these two formalisms, numerous details are included in appendices.

2. Kinetic theory

The second-order dynamic contributions from kinetic theory are [Hussey et al. (1975)],

$$M^{KT}(a,\omega) = n_e Tr_1\left\{L_s(a1;\Delta\omega)\frac{1}{\omega - L(a) - L(1) + i\varepsilon} f_o(a) f_o(1) L_s(a1) f_o^{-1}(a)\right\} \tag{2.1}$$

where n_e is the free electron number density, the trace is over a single perturber degrees of freedom, a and 1 denote radiator and perturbing electron variables, respectively, and ε is a positive infinitesimal. The Liouville operator for the perturbing electron is

$$L(1) = \hbar^{-1}\left[H(1),\cdots\right] \tag{2.2}$$

with $H(1)$ the Hamiltonian for the perturbing electron, which includes the kinetic energy and the interaction with the net radiator charge. A success of the kinetic theory was a systematic derivation of statically and dynamcally screened radiator-electron interactions due to electron-electron correlations; respectively, $L_s(a1)$ and $L_s(a1;\Delta\omega)$. It has been noted that the full dynamic screening has a small effect on the line shape [Hussey et al. (1977); Boercker, Lee & Rogers (1983)]. Thus, taking the vanishing detuning frequency limit

$$\begin{aligned} L_s(a1;\Delta\omega \to 0) &= L_s(a1) \\ &= \hbar^{-1}\left[V_s(a1),\cdots\right] \end{aligned} \tag{2.3}$$

with $V_s(a1)$ a statically screened perturber interaction with the radiator internal states. The reduced distribution function for the perturbing electron accounts for the net radiator charge but is zeroth order in $V_s(a1)$. The matrix elements between continuum eigenstates are

$$\begin{aligned} \langle \vec{k}' | f_o(1) | \vec{k} \rangle &= \lambda_T^3 \langle \vec{k}' | e^{-H(1)/T_e} | \vec{k} \rangle \\ &= \lambda_T^3 e^{-\hbar^2 k^2/2m_e T_e} \delta(\vec{k}' - \vec{k}) \end{aligned} \tag{2.4}$$

where the thermal de Broglie wavelength

$$\lambda_T = \sqrt{2\pi\hbar^2/m_e T_e} \tag{2.5}$$

with m_e and T_e the electron mass and temperature in energy units [O'Brien & Hooper (1974)]. The matrix elements for the zeroth-order radiator reduced distribution function are

$$\begin{aligned} \langle \mu' | f_o(a) | \mu \rangle &= f_o(\mu)\delta_{\mu'\mu} \\ &\propto e^{-E_\mu/T_e}\delta_{\mu'\mu} \end{aligned} \tag{2.6}$$

where the proportionality constant will not be required and E_μ is the energy of the isolated radiator eigenstate $|\mu\rangle$.

2.1 Matrix elements

The Liouville operators in Eq. (2.1) are conveniently treated through double atom [Baranger (1958)] or tetradic notation [Smith & Hooper (1967)],

$$\langle \mu | M(a,\omega) y(a) | \nu \rangle = \sum_{\mu'\nu'} M_{\mu\nu,\mu'\nu'}(\omega) y_{\mu'\nu'} \tag{2.1.1}$$

with $y(a)$ an operator in the Hilbert space of the radiator internal states. After lengthy but straightforward algebra, the matrix elements of the width and shift operator are

$$M^{KT}_{\mu\nu,\mu'\nu'}(\omega)=\frac{n_e}{\hbar}\int d\vec{k}_1\int d\vec{k}_2\left\{\sum_\tau\left[\delta_{\nu'\nu}f_o(k_2)\frac{\langle\mu\vec{k}_1|V_s|\tau\vec{k}_2\rangle\langle\tau\vec{k}_2|V_s|\mu'\vec{k}_1\rangle}{\hbar\Delta\omega_{\tau\nu}+E_{k_1}-E_{k_2}+i\varepsilon}\frac{f_o(\tau)}{f_o(\mu')}\right.\right.$$
$$\left.+\delta_{\mu'\mu}f_o(k_1)\frac{\langle\nu\vec{k}_1|V_s|\tau\vec{k}_2\rangle\langle\tau\vec{k}_2|V_s|\nu'\vec{k}_1\rangle}{\hbar\Delta\omega_{\mu\tau}+E_{k_2}-E_{k_1}+i\varepsilon}\right] \tag{2.1.2}$$
$$\left.-\langle\mu\vec{k}_1|V_s|\mu'\vec{k}_2\rangle\langle\nu'\vec{k}_2|V_s|\nu\vec{k}_1\rangle\left[\frac{f_o(k_1)f_o(\mu)f_o^{-1}(\mu')}{\hbar\Delta\omega_{\mu\nu'}+E_{k_2}-E_{k_1}+i\varepsilon}+\frac{f_o(k_2)}{\hbar\Delta\omega_{\mu'\nu}+E_{k_1}-E_{k_2}+i\varepsilon}\right]\right\}$$

where Greek letters represent isolated radiator internal eigenstates, k's perturber continuum eigenstates with energy $E_k=\hbar^2k^2/2m_e$, and the detuning frequency is given by

$$\hbar\Delta\omega_{\mu\nu}=\hbar\omega-E_\mu+E_\nu \tag{2.1.3}$$

The results in Eq. (2.1.2) assumed the identity operator

$$\int d\vec{k}_2|\vec{k}_2\rangle\langle\vec{k}_2|=1 \tag{2.1.4}$$

which neglects bound states for charged radiators; this effectively becomes a minimum impact parameter cutoff common to second-order theories [O'Brien & Hooper (1974)].

2.2 Line width

The dynamic shift and width contributions are given respectively by the real and imaginary parts of $M(a,\omega)$. Using the identities

$$\frac{1}{x\pm i0^+}=\frac{\text{p.v.}}{x}\mp i\pi\delta(x) \tag{2.2.1}$$

with p.v. the Cauchy principal value, yields for the width $\Gamma=\operatorname{Im}M$. Applying the results in Appendices A and B yields

$$\Gamma^{KT}_{\mu\nu,\mu'\nu'}(\omega)=\frac{n_e\lambda_T^3}{\hbar}\left(\frac{e^2}{a_o}\right)\sum_\lambda\frac{\pi}{(2\lambda+1)}\left\{\sum_{\gamma_\tau J_\tau}\left[\delta_{\nu'\nu}\delta_{J_\mu J_{\mu'}}\delta_{M_\mu M_{\mu'}}\frac{(-1)^{J_\mu-J_\tau}f_o(\tau)}{(2J_\mu+1)f_o(\mu')}\gamma^{(\lambda)}_{\mu\tau,\tau\mu'}(-\Delta\omega_{\tau\nu})\right.\right.$$
$$\left.+\delta_{\mu'\mu}\delta_{J_\nu J_{\nu'}}\delta_{M_\nu M_{\nu'}}\frac{(-1)^{J_\nu-J_\tau}}{2J_\nu+1}\gamma^{(\lambda)}_{\nu\tau,\tau\nu'}(-\Delta\omega_{\mu\tau})\right] \tag{2.2.2}$$
$$\left.-\left[\frac{f_o(\mu)}{f_o(\mu')}\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}(-\Delta\omega_{\mu\nu'})+\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}(-\Delta\omega_{\mu'\nu})\right]\Theta^{(\lambda)}_{\mu\mu',\nu'\nu}\right\}$$

with e the elemental electric charge, Bohr radius a_o, $\Theta^{(\lambda)}_{\mu\mu',\nu'\nu}$ a sum over products of 3-j symbols defined in Eq. (A.2.6) that requires $M_\mu - M_{\mu'} = M_\nu - M_{\nu'}$, and thermal average in Appendix B

$$\begin{aligned}\gamma^{(\lambda)}_{\alpha\beta,\alpha'\beta'}(\omega>0)&=\int_o^\infty\frac{dk}{k_f}e^{-\hbar^2k^2/2m_eT_e}D^{(\lambda)}_{\alpha\beta,\alpha'\beta'}(k,k_f)\\ \gamma^{(\lambda)}_{\alpha\beta,\alpha'\beta'}(\omega<0)&=e^{-\hbar|\omega|/T_e}\gamma^{(\lambda)}_{\alpha\beta,\alpha'\beta'}(|\omega|)\end{aligned} \tag{2.2.3}$$

with $k_f=\sqrt{k^2+2m_e\omega/\hbar}$. The λ sum is due to the partial wave expansion of the radiator-perturber interaction and $D^{(\lambda)}_{\alpha\beta,\alpha'\beta'}(k_1,k_2)$ in Eq. (A.2.7) is a product of continuum electron radial matrix elements of the λ term summed over the perturbing electron orbital angular momenta. The width expression in Eq. (2.2.2) reproduces previous results [Junkel et at. (2000)] except they only considered the first term and omitted the atomic distributions.

3. Projection operator

An additional approximation often introduced when applying the projector operator method is an ideal Boltzmann electron gas interacting with the radiator through a screened potential to account for the neglected electron-electron correlations. With this approximation, the projection operator second-order dynamic contributions are derived in Appendix C and give

$$\begin{aligned}M^{PO}(a,\omega)&=n_eTr_1\left\{L_s(a1)\frac{1}{\omega-L(a)-L(1)+i\varepsilon}L_s(a1)f_o(1)\right\}\\ &+n_eTr_1\left\{L_s(a1)\frac{1}{\omega-L(a)-L(1)+i\varepsilon}f_1(a1)f_o^{-1}(a)\Delta\omega\right\}\end{aligned} \tag{3.1}$$

The first term is the usual second-order expression [Smith & Hooper (1967)]. The second term is a consequence of retaining initial correlations and involves the first order correction, $f_1(a1)$, to the radiator-electron reduced distribution function. In principle there are additional terms but because of the neglected electron-electron correlations all terms of $O(n_e^2)$ cancel. The corrections in Eq. (3.1) agree with previous results [Gomez et al. (2018); Comment].

3.1 Matrix elements

To help with the comparisons below, the matrix elements between isolated radiator eigenstates are split into two parts corresponding to the two terms in Eq. (3.1),

$$M^{PO}(a,\omega) = M^{PO_1}(a,\omega) + M^{PO_2}(a,\omega) \tag{3.1.1}$$

Explicitly,

$$\begin{aligned} M^{PO_1}_{\mu\nu,\mu'\nu'}(\omega) &= \frac{n_e}{\hbar}\int d\vec{k}_1 \int d\vec{k}_2 \\ &\times \Bigg\{ f_o(k_1) \sum_\tau \left[\delta_{\nu'\nu} \frac{\langle \mu\vec{k}_1 | V_s | \tau\vec{k}_2 \rangle \langle \tau\vec{k}_2 | V_s | \mu'\vec{k}_1 \rangle}{\hbar\Delta\omega_{\tau\nu} + E_{k_1} - E_{k_2} + i\varepsilon} + \delta_{\mu'\mu} \frac{\langle \nu\vec{k}_1 | V_s | \tau\vec{k}_2 \rangle \langle \tau\vec{k}_2 | V_s | \nu'\vec{k}_1 \rangle}{\hbar\Delta\omega_{\mu\tau} + E_{k_2} - E_{k_1} + i\varepsilon} \right] \\ &- \langle \mu\vec{k}_1 | V_s | \mu'\vec{k}_2 \rangle \langle \nu'\vec{k}_2 | V_s | \nu\vec{k}_1 \rangle \left[\frac{f_o(k_2)}{\hbar\Delta\omega_{\mu\nu'} + E_{k_2} - E_{k_1} + i\varepsilon} + \frac{f_o(k_2)}{\hbar\Delta\omega_{\mu'\nu} + E_{k_1} - E_{k_2} + i\varepsilon} \right] \Bigg\} \end{aligned} \tag{3.1.2}$$

These four terms are the familiar second-order results neglecting initial correlations [Smith & Hooper (1967)]. The second term has

$$\begin{aligned} M^{PO_2}_{\mu\nu,\mu'\nu'}(\omega) &= \frac{n_e}{\hbar}\int d\vec{k}_1 \int d\vec{k}_2 \\ &\times \Bigg\{ \delta_{\nu'\nu} f_o(k_2) \sum_\tau \left[\frac{\langle \mu\vec{k}_1 | V_s | \tau\vec{k}_2 \rangle \langle \tau\vec{k}_2 | V_s | \mu'\vec{k}_1 \rangle}{\hbar\Delta\omega_{\tau\nu} + E_{k_1} - E_{k_2} + i\varepsilon} \frac{f_o(\tau)}{f_o(\mu')} \left(\frac{1 - e^{\left(E_\tau - E_{\mu'} + E_{k_2} - E_{k_1}\right)/T_e}}{E_\tau - E_{\mu'} + E_{k_2} - E_{k_1}} \right) \right] \\ &- f_o(k_1) \frac{\langle \mu\vec{k}_1 | V_s | \mu'\vec{k}_2 \rangle \langle \nu'\vec{k}_2 | V_s | \nu\vec{k}_1 \rangle}{\hbar\Delta\omega_{\mu\nu'} + E_{k_2} - E_{k_1} + i\varepsilon} \frac{f_o(\mu)}{f_o(\mu')} \left(\frac{1 - e^{\left(E_\mu - E_{\mu'} + E_{k_1} - E_{k_2}\right)/T_e}}{E_\mu - E_{\mu'} + E_{k_1} - E_{k_2}} \right) \Bigg\} \hbar\Delta\omega_{\mu'\nu'} \end{aligned} \tag{3.1.3}$$

with initial correlation corrections and matrix elements of $f_1(a1)$ in Appendix A. Although more explicit here, this is the same results derived in Ref. [17].

3.2 Line width

Using the results in Appendices A and B yields the width from the projection operator method. The widths corresponding to the two terms in Eq. (3.1.1) are

$$\Gamma^{PO_1}_{\mu\nu,\mu'\nu'}(\omega)=\frac{\pi n_e\lambda_T^3}{\hbar}\left(\frac{e^2}{a_o}\right)\sum_\lambda\frac{1}{2\lambda+1}\left\{\sum_{\gamma_\tau J_\tau}\left[\delta_{\nu'\nu}\delta_{J_\mu J_{\mu'}}\delta_{M_\mu M_{\mu'}}\frac{(-1)^{J_\mu-J_\tau}}{(2J_\mu+1)}\gamma^{(\lambda)}_{\mu\tau,\tau\mu'}(\Delta\omega_{\tau\nu})\right.\right.$$
$$\left.\left.+\delta_{\mu'\mu}\delta_{J_\nu J_{\nu'}}\delta_{M_\nu M_{\nu'}}\frac{(-1)^{J_\nu-J_\tau}}{2J_\nu+1}\gamma^{(\lambda)}_{\nu\tau,\tau\nu'}(-\Delta\omega_{\mu\tau})\right]-\left[\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}(\Delta\omega_{\mu\nu'})+\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}(-\Delta\omega_{\mu'\nu})\right]\Theta^{(\lambda)}_{\mu\mu',\nu'\nu}\right\} \tag{3.2.1}$$

the usual width expression neglecting initial correlations and

$$\Gamma^{PO_2}_{\mu\nu,\mu'\nu'}(\omega)=\frac{\pi n_e\lambda_T^3}{\hbar}\left(\frac{e^2}{a_o}\right)\left(1-e^{\hbar\Delta\omega_{\mu'\nu}/T_e}\right)\sum_\lambda\frac{1}{2\lambda+1}$$
$$\times\left\{\frac{\delta_{\nu'\nu}\delta_{J_\mu J_{\mu'}}\delta_{M_\mu M_{\mu'}}}{(2J_\mu+1)}\sum_{\gamma_\tau J_\tau}(-1)^{J_\mu-J_\tau}\frac{f_o(\tau)}{f_o^{-1}(\mu')}\gamma^{(\lambda)}_{\mu\tau,\tau\mu'}(-\Delta\omega_{\tau\nu})\right. \tag{3.2.2}$$
$$\left.-\frac{f_o(\mu)}{f_o^{-1}(\mu')}\Theta^{(\lambda)}_{\mu\mu',\nu'\nu}\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}(-\Delta\omega_{\mu\nu'})\right\}$$

containing the initial correlation corrections that modify the first and third term in Eq. (3.2.1).

4. Comparison

This Section first contacts past work by comparing widths from the kinetic theory and the projection operator without initial correlations. This is followed by comparisons of the kinetic theory and the complete second-order projection operator widths.

4.1 Γ^{KT} and Γ^{PO_1}

The second-order dynamic contributions to the line width in Eqs. (2.2.2) and the no initial correlations part in (3.2.1) are similar but not identical. Firstly, the kinetic theory width contains ratios of the form

$$f_o(\alpha)f_o^{-1}(\beta)=e^{-(E_\alpha-E_\beta)/T_e} \tag{4.1.1}$$

Secondly, there is an opposite asymmetry with respect to the detuning frequency. Both differences occur in the first and third terms of those expressions. That is,

$$
\begin{aligned}
\left[\Gamma^{PO_1}_{\mu\nu,\mu'\nu'}(\omega)-\Gamma^{KT}_{\mu\nu,\mu'\nu'}(\omega)\right]_{1^{st}\,term} &\propto \delta_{\nu'\nu}\left[\gamma^{(\lambda)}_{\mu\tau,\tau\mu'}\left(\Delta\omega_{\tau\nu'}\right)-f_o(\tau)f_o^{-1}(\mu')\gamma^{(\lambda)}_{\mu\tau,\tau\mu'}\left(-\Delta\omega_{\tau\nu'}\right)\right]\\
&=\delta_{\nu'\nu}\left[1-e^{\left(E_{\mu'}-E_\tau-\hbar\Delta\omega_{\tau\nu'}\right)/T_e}\right]\gamma^{(\lambda)}_{\mu\tau,\tau\mu'}\left(\Delta\omega_{\tau\nu'}\right)\\
&\propto O\left(\hbar\Delta\omega_{\mu'\nu'}V_s^2(a1)\right)
\end{aligned}
\tag{4.1.2}
$$

which used the detail balance property in Eq. (B.6). Similarly for the third term,

$$
\begin{aligned}
\left[\Gamma^{PO_1}_{\mu\nu,\mu'\nu'}(\omega)-\Gamma^{KT}_{\mu\nu,\mu'\nu'}(\omega)\right]_{3^{rd}\,term} &\propto \gamma^{(\lambda)}_{\mu\mu',\nu'\nu}\left(\Delta\omega_{\mu\nu'}\right)-f_o(\mu)f_o^{-1}(\mu')\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}\left(-\Delta\omega_{\mu\nu'}\right)\\
&=\left[1-e^{\left(E_{\mu'}-E_\mu-\hbar\Delta\omega_{\mu\nu'}\right)/T_e}\right]\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}\left(\Delta\omega_{\mu\nu'}\right)\\
&\propto O\left(\hbar\Delta\omega_{\mu'\nu'}V_s^2(a1)\right)
\end{aligned}
\tag{4.1.3}
$$

These differences are at least third order in the radiator-perturber interaction (γ's are second-order and the detuning $\Delta\omega$ is also a measure of the interaction).

4.2 Γ^{KT} and Γ^{PO}

Collecting results from Eqs. (3.2.1) and (3.2.2) together with Eq. (2.2.2) write

$$
\begin{aligned}
&\Gamma^{PO}_{\mu\nu,\mu'\nu'}(\omega)-\Gamma^{KT}_{\mu\nu,\mu'\nu'}(\omega)=\frac{\pi n_e\lambda_T^3}{\hbar}\left(\frac{e^2}{a_o}\right)\sum_\lambda\frac{1}{(2\lambda+1)}\\
&\times\left\{\delta_{\nu'\nu}\delta_{J_\mu J_{\mu'}}\delta_{M_\mu M_{\mu'}}\sum_{\gamma_\tau J_\tau}\frac{(-1)^{J_\mu-J_\tau}}{2J_\mu+1}\left[\gamma^{(\lambda)}_{\mu\tau,\tau\mu'}\left(\Delta\omega_{\tau\nu}\right)-e^{\hbar\Delta\omega_{\mu'\nu'}/T_e}\frac{f_o(\tau)}{f_o(\mu')}\gamma^{(\lambda)}_{\mu\tau,\tau\mu'}\left(-\Delta\omega_{\tau\nu}\right)\right]\right.\\
&\left.-\left[\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}\left(\Delta\omega_{\mu\nu'}\right)-e^{\hbar\Delta\omega_{\mu'\nu'}/T_e}\frac{f_o(\mu)}{f_o(\mu')}\gamma^{(\lambda)}_{\mu\mu',\nu'\nu}\left(-\Delta\omega_{\mu\nu'}\right)\right]\Theta^{(\lambda)}_{\mu\mu',\nu'\nu}\right\}=0
\end{aligned}
\tag{4.2.1}
$$

and the differnce vanishes after applying the detailed balance property in Eq. (B.6). Thus, both approaches formaly lead to the same line widths.

5. Conclusion

The spectral line width expansions to second order in the radiator-electron interaction from the projection operator and kinetic theory formalisms were revisited. It was shown that a systematic treatment of the projection operator expression *including* initial correlations agrees with kinetic theory. On the other hand, the commonly used second-order projection operator width neglecting initial correlations (factorization of density matrix) leads to differences with the kinetic theory result. The differences, however, are third order in the radiator-electron interaction and therefore can be deemed negligible where the second-order theory is valid.

It is emphasized that the agreement assumed the *ad hoc* introduction of a screened radiator-electron interaction in the projection operator method. This approximation is extraneous in the kinetic theory approach since the electron-electron correlations systematically shield the interactions. In fact, the kinetic theory justifies to a large degree the use of ideal quasi-particles with screened radiator-electron interactions [Hussey et a. (1975)]. Furthermore, the kinetic theory second-order width formula accounts for initial correlations. With these advantages, it is sensible to use the more compact formula from kinetic theory as a starting point for second-order width calculations.

Appendix A. ***Radiator-electron interaction***

The perturber interaction with the radiator internal states can be written in the form

$$V_s(a1) = e^2 \sum_{\lambda=0} \sum_{q=-\lambda}^{\lambda} \frac{4\pi}{(2\lambda+1)} Y^*_{\lambda q}(\hat{r}_1) \sum_b V^{(\lambda)}(r_b, r_1) Y_{\lambda q}(\hat{r}_b) \tag{A.1}$$

where $Y_{\ell m}$ is a spherical harmonic, the sum b is over radiator bound electrons, and e is the elemental electric charge. For example, a screened Coulomb interaction has coefficients

$$V^{(\lambda)}(r_b, r_1) = \frac{(2\lambda+1)}{\sqrt{r_b r_1}} I_{\lambda+\frac{1}{2}}(\kappa r_<) K_{\lambda+\frac{1}{2}}(\kappa r_>) - \delta_{\lambda 0} \frac{e^{-\kappa r_1}}{r_1} \tag{A.2}$$

where I and K are modified Bessel functions and κ is an inverse screening length [Abramowitz & Stegun (1972)]. Here, $r_<$ and $r_>$ are the smaller and larger of $|\vec{r}_1|$ and $|\vec{r}_b|$ where $\vec{r}_1$ and $\vec{r}_b$ are the positions of the perturber and bound radiator electron b, respectively, $\hat{r}$ is a unit vector in the direction of vector $\vec{r}$, and the origin of the coordinate system is at the radiator nucleus. The oft made dipole approximation is briefly discussed at the end of this appendix.

A.1 Perturber wavefunctions

The spinless, non-relativistic, one-electron picture is assumed for the continuum electrons moving in a central potential $\chi e U(r)$ with χe the net radiator elecric charge. The wavefunction for the perturbing electron is expresses as a sum over partial waves [Johnston (1967)]

$$\langle \vec{r} | \vec{k} \rangle = (kr)^{-1} \sum_{\ell=0} \sum_{m=-\ell}^{\ell} i^{\ell} e^{i\sigma_\ell(k)} w_{k\ell}(r) Y^*_{\ell m}(\hat{r}) Y_{\ell m}(\hat{k}) \tag{A.1.1}$$

with $\sigma_\ell(k)$ the elastic scattering phase shift and the reduced radial wavefunction satisfies the Schrodinger equation

$$\left\{ \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - 2\chi \frac{m_e e^2}{\hbar^2} U(r) + k^2 \right\} w_{k\ell}(r) = 0 \tag{A.1.2}$$

with boundary conditions are

$$\begin{aligned} & w_{k\ell}(r \to 0) \propto r^{\ell+1} \\ & w_{k\ell}(r \to \infty) = \sqrt{\frac{2}{\pi}} \sin\left(kr - \frac{\ell\pi}{2} + \sigma_\ell \right) \end{aligned} \tag{A.1.3}$$

with k-scale normalization.

$$\int_{o}^{\infty} dx\, w_{k'\ell}(x)\, w_{k\ell}(x) = \delta(k'-k) \tag{A.1.4}$$

A.2 Matrix elements

Substituting the expansion in Eq. (A.1) and performing the angular integrations get for the matrix elements

$$\begin{aligned}\left\langle \alpha\vec{k}_1 \left| V_s(a1) \right| \beta\vec{k}_2 \right\rangle &\rightarrow \frac{e^2}{k_1 k_2} \sum_{\lambda q} \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} i^{\ell_2-\ell_1} e^{i\left(\sigma_{\ell_2}-\sigma_{\ell_1}\right)} Y^*_{\ell_1 m_1}(\hat{k}_1)\, Y_{\ell_2 m_2}(\hat{k}_2) \\ &\times (-1)^{J_\alpha - M_\alpha + q - m_1} \begin{pmatrix} \ell_1 & \lambda & \ell_2 \\ -m_1 & -q & m_2 \end{pmatrix} \begin{pmatrix} J_\alpha & \lambda & J_\beta \\ -M_\alpha & q & M_\beta \end{pmatrix} T^{(\lambda)}_{\alpha\beta}(k_1\ell_1, k_2\ell_2)\end{aligned} \tag{A.2.1}$$

Here, k's represent electron continuum eigenstates and Greek letters radiator internal eigenstates $\left|\alpha\right\rangle = \left|\zeta_\alpha J_\alpha M_\alpha\right\rangle$ with J and M the total angular momentum and magnetic quantum numbers, and ζ denotes all remaining quantum numbers to identify the state. To simplify notation, sums over orbital angular momentum start at zero with obvious restrictions for the associated magnetic numbers. The result in Eq. (A.2.1) introduced

$$T^{(\lambda)}_{\alpha\beta}(k_1\ell_1, k_2\ell_2) = \sqrt{(2\ell_1+1)(2\ell_2+1)} \begin{pmatrix} \ell_1 & \lambda & \ell_2 \\ 0 & 0 & 0 \end{pmatrix} \int_{o}^{\infty} dr\, w_{k_1\ell_1}(r)\, w_{k_2\ell_2}(r)\, A^{(\lambda)}_{\alpha\beta}(r) \tag{A.2.2}$$

with reduced matrix elements of the Wigner-Eckart theorem

$$A^{(\lambda)}_{\alpha\beta}(r_1) = \left\langle \zeta_\alpha J_\alpha \left\| \sum_b V^{(\lambda)}(r_b, r_1)\, C^{(\lambda)}(\hat{x}_b) \right\| \zeta_\beta J_\beta \right\rangle \tag{A.2.3}$$

where $C^{(\lambda)}(\hat{x})$ is the renormalized spherical harmonic [Cowan (1981)].

Products of matrix elements integrated over the continuum electron solid angles simplify to

$$\begin{aligned}\int d\Omega_{\hat{k}_1} \int d\Omega_{\hat{k}_2} \left\langle \alpha\vec{k}_1 \left| V_s \right| \beta\vec{k}_2 \right\rangle \left\langle \nu\vec{k}_2 \left| V_s \right| \mu\vec{k}_1 \right\rangle &= \frac{e^4}{k_1^2 k_2^2} \sum_\lambda (2\lambda+1)^{-1} \\ &\times \sum_{\ell_1\ell_2} T^{(\lambda)}_{\alpha\beta}(k_1\ell_1, k_2\ell_2)\, T^{(\lambda)}_{\nu\mu}(k_1\ell_1, k_2\ell_2)\, \Theta^{(\lambda)}_{\alpha\beta,\nu\mu}\end{aligned} \tag{A.2.5}$$

with

$$\Theta^{(\lambda)}_{\mu\mu',\nu'\nu} = (-1)^{J_\mu - J_{\nu'}} \sum_q \begin{pmatrix} J_\mu & \lambda & J_{\mu'} \\ -M_\mu & q & M_{\mu'} \end{pmatrix} \begin{pmatrix} J_{\nu'} & \lambda & J_\nu \\ -M_{\nu'} & -q & M_\nu \end{pmatrix} \tag{A.2.6}$$

that vanishes unless $M_\mu - M_{\mu'} = M_\nu - M_{\nu'}$. These results used the orthogonality of the spherical harmonics and properties of 3-j symbols.

For the sum over intermediate radiator states again apply properties of 3-j symbols; that is, set $\beta = \nu = \tau$ in Eq. (A.2.5) and sum over M_τ. Collecting results and defining

$$D^{(\lambda)}_{\alpha\beta,\alpha'\beta'}(k_1,k_2) = \sum_{\ell_1\ell_2} T^{(\lambda)}_{\alpha\beta}(k_1\ell_1,k_2\ell_2)\,T^{(\lambda)}_{\alpha'\beta'}(k_1\ell_1,k_2\ell_2) \tag{A.2.7}$$

write

$$\begin{aligned}&\sum_{M_\tau}\int d\Omega_{\hat{k}_1}\int d\Omega_{\hat{k}_2}\,\langle \mu,k_1\ell_1 m_1|V_s|\tau,k_2\ell_2 m_2\rangle\langle \tau,k_2\ell_2 m_2|V_s|\mu',k_1\ell_1 m_1\rangle \\ &\quad = \delta_{J_\mu J_{\mu'}}\delta_{M_\mu M_{\mu'}}\frac{(-1)^{J_\mu - J_\tau}}{2J_\mu+1}\left(\frac{e^4}{k_1^2k_2^2}\right)\sum_\lambda (2\lambda+1)^{-1} D^{(\lambda)}_{\mu\tau,\tau\mu'}(k_1,k_2)\end{aligned} \tag{A.2.8}$$

This result reproduces Eq. (15) of Woltz & Hooper (1984) for hydrogenic radiators and Coulomb interaction except for pre-factors associated with the definition of the electron radial wavefunctions (for $U(r)=1/r$, $w_{k\ell}(r)\to\sqrt{2/\pi}F_\ell(kr)$ with F the Coulomb wavefunction [Abramowitz & Stegun (1972)]) and overall normalization of the electron distribution (the factor $n_e\lambda_T^3$). Similarly, write

$$\int d\Omega_{\hat{k}_1}\int d\Omega_{\hat{k}_2}\,\langle \mu\vec{k}_1\ell_1 m_1|V_s|\mu'\vec{k}_2\rangle\langle \nu'\vec{k}_2|V_s|\nu\vec{k}_1\rangle = \left(\frac{e^4}{k_1^2k_2^2}\right)\sum_\lambda \frac{\Theta^{(\lambda)}_{\mu\mu',\nu'\nu}}{(2\lambda+1)} D^{(\lambda)}_{\mu\mu',\nu'\nu}(k_1,k_2) \tag{A.2.9}$$

for the interference terms.

A.3 Density matrix correction

The subsections above deal with the usual terms found in the width and shift operator. The expression in Eq. (3.1) has a correction to the density matrix [Boercker & Iglesias (1984)],

$$f_1(a1) = -f_o(a)f_o(1)\int_o^{\hbar/T_e} d\tau\, e^{[H(a)+H(1)]\tau/\hbar}V_s(a1)\,e^{-[H(a)+H(1)]\tau/\hbar} \tag{A.3.1}$$

with matrix elements

$$\langle \alpha\vec{k}_1|f_1(a1)|\beta\vec{k}_2\rangle = \hbar f_o(\alpha)f_o(k_1)\left[\frac{1-e^{(\hbar\omega_\alpha-\hbar\omega_\beta+E_{k_1}-E_{k_2})/T_e}}{\hbar\omega_\alpha-\hbar\omega_\beta+E_{k_1}-E_{k_2}}\right]\langle \alpha\vec{k}_1|V_s(a1)|\beta\vec{k}_2\rangle \tag{A.3.2}$$

These terms can then make use of the interaction matrix elements results above.

A.4 Dipole approximation

The dipole approximation restricts Eq. (A.1) to $\lambda = 1$ and $|\vec{x}_1| > |\vec{x}_b|$ for all *b*. The screened Coulomb interaction also requires $\kappa|\vec{x}_b| \ll 1$ [Iglesias (2021)]. Then the screened interaction simplifies to

$$\begin{aligned} V_s(a1) &\to \frac{4\pi e^2}{3}\sum_{q=-1}^{1} Y_{1q}^*(\hat{r}_1)\sum_b V^{(1)}(r_b,r_1)Y_{1q}(\hat{r}_b) \\ &= \vec{d}\cdot\vec{r}_1\left(e\frac{\partial}{\partial r_1}\frac{e^{-\kappa r_1}}{r_1}\right) = -\vec{d}\cdot\vec{E}(r_1) \end{aligned} \tag{A.4.1}$$

with $\vec{E}(r_1)$ the electric field at the radiator nucleus due to an electron at $\vec{r}_1$. Accordingly, for a screened Coulomb interaction

$$\begin{aligned} T_{\alpha\beta}^{(\lambda)}(k_1\ell_1,k_2\ell_2) &\propto \langle\alpha\|\vec{d}\|\beta\rangle\delta_{\ell_2,\ell_1\pm1}(-1)^{\ell_1+\ell_>}\sqrt{\ell_>}\left\{\int_o^\infty dr\, w_{k_1\ell_1}(r)\left(\frac{\partial e^{-\kappa r}}{\partial r}\right)w_{k_2\ell_2}(r)\right\} \\ &= \langle\alpha\|\vec{d}\|\beta\rangle\delta_{\ell_2,\ell_1\pm1}(-1)^{\ell_1+\ell_>}\sqrt{\ell_>}\,T_{\ell_1\ell_2}(k_1,k_2) \end{aligned} \tag{A.4.2}$$

defining $T_{\ell_1\ell_2}(k_1,k_2)$ and $\ell_> = Max[\ell_1,\ell_2]$. This leads to

$$D_{\alpha\beta,\alpha'\beta'}^{(\lambda)}(k_1,k_2) \propto g_{ff}(k_1,k_2) = \frac{2\sqrt{3}}{\pi k_1 k_2}\sum_{\ell=0}^{\infty}\left[(\ell+1)T_{\ell,\ell+1}^2(k_1,k_2) + \ell T_{\ell,\ell-1}^2(k_1,k_2)\right] \tag{A.4.3}$$

the free-free Gaunt factor in the acceleration form [Johnston (1967)]. After some algebra together with the change of variables $E = \hbar^2 k_1^2/2m_e$, get

$$\begin{aligned} \gamma(\omega>0) &\propto \int_o^\infty \frac{dk_1}{T_e}k_1 e^{-\hbar^2k_1^2/2m_eT_e}\int_o^\infty dk_2 k_2 g_{ff}(k_1,k_2)\delta\left(\hbar\omega + \frac{\hbar^2k_1^2}{2m_e} - \frac{\hbar^2k_2^2}{2m_e}\right) \\ &= \int_o^\infty \frac{dE}{T_e}e^{-E/T_e}g_{ff}\left(k_1,\sqrt{k_1+\frac{2m_e\omega}{\hbar}}\right) = \left\langle g_{ff}^{abs}(|\omega|)\right\rangle \end{aligned} \tag{A.4.4}$$

the thermally averaged inverse bremsstrahlung Gaunt factor for absorption of a photon with energy $\hbar\omega$ [Johnston (1967)]. The steps above outline an alternative derivation for the connection between inverse bremsstrahlung and line widths [O'Brien & Hooper (1974)].

Appendix B. ***Thermal average***

Noting that $D(k_2,k_1)=D(k_1,k_2)$ (suppressing superscripts and subscripts), there are two dimensionless integrals describing the thermal averages over perturber degrees of freedom in the second-order widths from the two formalisms in Sections 2 and 3. These are given by

$$\gamma(\omega)=e^2 a_o\int_o^\infty dk_1 e^{-\hbar^2k_1^2/2m_eT_e}\int_o^\infty dk_2 D(k_1,k_2)\delta\left(\hbar\omega+\frac{\hbar^2k_1^2}{2m_e}-\frac{\hbar^2k_2^2}{2m_e}\right) \tag{B.1}$$

$$\xi(\omega)=e^2 a_o\int_o^\infty dk_1 e^{-\hbar^2k_1^2/2m_eT_e}\int_o^\infty dk_2 D(k_1,k_2)\delta\left(\hbar\omega-\frac{\hbar^2k_1^2}{2m_e}+\frac{\hbar^2k_2^2}{2m_e}\right) \tag{B.2}$$

with Bohr radius $a_o=\hbar^2/m_e e^2$ and $D(k_1,k_2)$ in atomic units. It follows that

$$\xi(\omega)=\gamma(-\omega) \tag{B.3}$$

It is convenient to treat the cases $\omega>0$ and $\omega<0$ separately.

For $\omega>0$, use the δ-function in Eq. (B.1) to first perform the integral over k_2 to get

$$\begin{aligned}\gamma(\omega>0)&=e^2 a_o\int_o^\infty dk_1 e^{-\hbar^2k_1^2/2m_eT_e}\int_o^\infty dk_2 D(k_1,k_2)\delta\left(\hbar|\omega|+\frac{\hbar^2k_1^2}{2m_e}-\frac{\hbar^2k_2^2}{2m_e}\right)\\ &=\int_o^\infty dk_1 e^{-\hbar^2k_1^2/2m_eT_e}\int_o^\infty \frac{dk_2}{k_2}D(k_1,k_2)\left[\delta\left(k_2-k_1-\frac{2m_e|\omega|}{\hbar}\right)+\delta\left(k_2+k_1+\frac{2m_e|\omega|}{\hbar}\right)\right]\\ &=\int_o^\infty dk\, e^{-\hbar^2k^2/2m_eT_e}\frac{D^{(\lambda)}\left(k,\sqrt{k^2+\frac{2m_e|\omega|}{\hbar}}\right)}{\sqrt{k^2+\frac{2m_e|\omega|}{\hbar}}}\end{aligned} \tag{B.4}$$

For $\omega<0$, use the δ-function in Eq. (B.1) to first perform the integral over k_1 to get

$$\begin{aligned}\gamma(\omega<0)&=e^2 a_o\int_o^\infty dk_1 e^{-\hbar^2k_1^2/2m_eT_e}\int_o^\infty dk_2 D(k_1,k_2)\delta\left(\frac{\hbar^2k_1^2}{2m_e}-\frac{\hbar^2k_2^2}{2m_e}-\hbar|\omega|\right)\\ &=e^{-\hbar|\omega|/T_e}\gamma(|\omega|)\end{aligned} \tag{B.5}$$

Collecting results write

$$\gamma(-\omega)=e^{-\hbar\omega/T_e}\gamma(\omega) \tag{B.6}$$

satisfying detailed balance.

Appendix C. ***Projector operator method width and shift operator***

The second-order dynamic contributions to the line width and shift operator from the projection operator method typically neglect initial correlations [Smith & Hooper (1967); O'Brien & Hooper (1974)]. The second order results can be obtained following Dufty (1969) without neglecting initial correlations. To proceed, let

$$Tr_a\vec{d}\cdot Tr_e\left(\frac{1}{\Delta\omega-L_e-L_I}\right)\rho\vec{d}=Tr_a\vec{d}\cdot\left(\frac{1}{\Delta\omega-M\left(a,\omega\right)}\right)\rho_a\vec{d} \tag{C.1}$$

with $\Delta\omega=\omega-L\left(a\right)$, $\rho_a=Tr_e\rho$ with ρ the density matrix for the radiator-electron gas system,

$$\begin{aligned}L_e&=\sum_{j=1}^{N_e}L\left(j\right)=\hbar^{-1}\sum_{j=1}^{N_e}\left[H\left(j\right),\cdots\right]\\ L_I&=\sum_{j=1}^{N_e}L_s\left(aj\right)=\hbar^{-1}\sum_{j=1}^{N_e}\left[V_s\left(aj\right),\cdots\right]\end{aligned} \tag{C.2}$$

Adding an expansion parameter, Eq. (C.1) is rewrite as

$$Tr_e\left(\frac{1}{\Delta\omega-L_e-\lambda L_I}\right)\rho\rho_a^{-1}=\frac{1}{\Delta\omega-M\left(a,\omega\right)} \tag{C.3}$$

Expanding both sides of Eq. (C.2) and equating like terms in λ yields $M\left(a,\omega\right)$.

C.1 Second-order expansion

The expansions for the expressions in the LHS of Eq. (C.3) in the parameter λ are given by

$$\begin{aligned}&\left(\Delta\omega-L_e-\lambda L_I\right)^{-1}=R_o\left[1+\lambda L_IR_o+\lambda^2L_IR_oL_IR_o+O\left(\lambda^3\right)\right]\\ &\quad\rho=\rho_a^{(0)}\rho_e^{(0)}+\lambda\rho_{ae}^{(1)}+\lambda^2\rho_{ae}^{(2)}+O\left(\lambda^3\right)\\ &\quad\rho_a^{-1}=\frac{1}{\rho_a^{(0)}}\left[1-\lambda Tr_e\,\rho_{ae}^{(1)}/\rho_a^{(0)}+\lambda^2\left[\left(Tr_e\,\rho_{ae}^{(1)}/\rho_a^{(0)}\right)^2-Tr_e\,\rho_{ae}^{(2)}/\rho_a^{(0)}\right]+O\left(\lambda^3\right)\right]\end{aligned} \tag{C.1.1}$$

with $R_o=\left(\Delta\omega-L_e\right)^{-1}$. Substituting these expansions yields

$$\begin{aligned}Tr_e\left(\frac{1}{\Delta\omega-L_e-\lambda L_I}\right)\rho\rho_a^{-1}&=R_a+R_a\left[\lambda Tr_e\left\{L_I\rho_e^{(0)}\right\}+\lambda^2Tr_e\left\{L_IR_oL_I\rho_e^{(0)}\right\}\right]R_a\\ &+\lambda^2R_a\left[Tr_e\left\{L_IR_o\,\rho_{ae}^{(1)}/\rho_a^{(0)}\right\}-Tr_e\left\{L_I\rho_e^{(0)}\right\}R_aTr_e\left\{\rho_{ae}^{(1)}/\rho_a^{(0)}\right\}\right]+O\left(\lambda^3\right)\end{aligned} \tag{C.1.2}$$

The results used

$$\begin{aligned} &Tr_e R_o g(a,e) = R_a Tr_e g(a,e) \\ &R_o \rho_e^{(0)} y(a) = \rho_e^{(0)} R_a y(a) \end{aligned} \tag{C.1.3}$$

where $L_e \rho_e^{(0)} = 0$ because of the assumed thermal equilibrium, $g(a,e)$ depends on both radiator internal states and electron gas variables, $y(a)$ only depends on radiator internal states, and $R_a = \Delta\omega^{-1}$. The last two second-order terms in Eq. (C.1.2) come from the initial correlations.

For the RHS of Eq. (C.3) assume

$$M = M^{(0)} + \lambda M^{(1)} + \lambda^2 M^{(2)} + O(\lambda^3) \tag{C.1.4}$$

and take advantage of the operator identity

$$\frac{\partial A^{-1}}{\partial \lambda} = -A^{-1} \frac{\partial A}{\partial \lambda} A^{-1} \tag{C.1.5}$$

to form the Taylor series

$$(\Delta\omega - M)^{-1} \approx \left(\Delta\omega - M^{(0)}\right)^{-1} \left\{1 + \left[\lambda M^{(1)} + \lambda^2 \left(M^{(2)} + M^{(1)} R_a M^{(1)}\right)\right]\left(\Delta\omega - M^{(0)}\right)^{-1}\right\} \tag{C.1.6}$$

Equating like powers of λ in Eqs. (C.1.2) and (C.1.6) yields

$$\begin{aligned} M^{(0)} &= 0 \\ M^{(1)} &= Tr_e\left\{L_I \rho_e^{(0)}\right\} \\ M^{(2)} &= Tr_e\left\{L_I R_o L_I \rho_e^{(0)}\right\} - Tr_e\left\{L_I \rho_e^{(0)}\right\} R_a Tr_e\left\{L_I \rho_e^{(0)}\right\} \\ &\quad + Tr_e\left\{L_I R_o \, \rho_{ae}^{(1)} / \rho_a^{(0)}\right\}\Delta\omega - Tr_e\left\{L_I \rho_e^{(0)}\right\} R_a Tr_e\left\{\rho_{ae}^{(1)} / \rho_a^{(0)}\right\}\Delta\omega \end{aligned} \tag{C.1.7}$$

The zeroth and first order terms reproduce previous results [Smith & Hooper (1967)]. The second-order term has initial correlation corrections that agree with Gomez et al. (2021).

C.2 Reduced distributions

The reduced distribution functions for the radiator and plasma quasi-particles are given by

$$\begin{aligned} n_e^s f_o(1\cdots s) &= N_e(N_e - 1)\cdots(N_e - s) Tr_{s+1\cdots N_e} \rho_e^{(0)} \\ &= n_e^s \Pi_{j=1}^s f_o(j) \\ n_e^s f_1(a1\cdots s) &= N_e(N_e - 1)\cdots(N_e - s) Tr_{s+1\cdots N_e} \rho_{ae}^{(1)} \\ &= n_e^s \sum_{i=1}^{s} f_1(ai) \Pi_{\substack{j=1 \\ j\neq i}}^s f_o(j) \end{aligned} \tag{C.2.1}$$

with N_e the number of free electrons and n_e the free electron number density. Substituting into the first order term yields

$$M^{(1)}(a,\omega) = n_e Tr_1 \left\{ L_s(a1) f_o(1) \right\} \tag{C.2.2}$$

the usual first order mean field term, except here the interaction is screened.

The four second-order terms in Eq. (C.1.7) are considered individually. The first gives

$$\begin{aligned} Tr_e\left\{ L_I R_o L_I \rho_e^{(0)} \right\} &= \sum_{ij} Tr_e \left\{ L_s(ai) \frac{1}{\Delta\omega - \sum_k L(k)} L_s(aj) \rho_e^{(0)} \right\} \\ &= N_e Tr_1 \left\{ L_s(a1) \frac{1}{\Delta\omega - L(1)} L_s(a1) Tr_{2\cdots N_e} \rho_e^{(0)} \right\} \\ &\quad + N_e(N_e - 1) Tr_1 \left\{ L_s(a1) Tr_{2\cdots N_e} \left\{ \frac{1}{\Delta\omega - L(2)} L_s(a2) \rho_e^{(0)} \right\} \right\} \\ &= n_e Tr_1 \left\{ L_s(a1) \frac{1}{\Delta\omega - L(1)} L_s(a1) f_o(1) \right\} + n_e^2 Tr_1 \left\{ L_s(a1) f_o(1) \right\} R_a Tr_2 \left\{ L_s(a2) f_o(2) \right\} \end{aligned} \tag{C.2.3}$$

Similar manipulations yield for the second term

$$Tr_e\left\{ L_I \rho_e^{(0)} \right\} R_a Tr_e \left\{ L_I \rho_e^{(0)} \right\} = n_e^2 Tr_1 \left\{ L_s(a1) f_o(1) \right\} R_a Tr_2 \left\{ L_s(a2) f_o(2) \right\} \tag{C.2.4}$$

For the terms containing first order corrections to the density matrix

$$Tr_e\left\{ L_I R_o \rho_{ae}^{(1)} \right\} = n_e Tr_1 \left\{ L_s(a1) \frac{1}{\Delta\omega - L(1)} f_1(a1) \right\} + n_e^2 Tr_1 \left\{ L_s(a1) f_o(1) \right\} R_a Tr_2 \left\{ f_1(a2) \right\} \tag{C.2.5}$$

and

$$Tr_e\left\{ L_I R_o \rho_e^{(0)} \right\} Tr_e \left\{ \rho_{ae}^{(1)} \right\} = n_e^2 Tr_1 \left\{ L_1(a1) f_o(1) \right\} R_a Tr_2 \left\{ f_1(a2) \right\} \tag{C.2.6}$$

Collecting results get

$$M^{(2)}(a,\omega) = n_e Tr_1 \left\{ L_s(a1) \frac{1}{\Delta\omega - L(1)} \left[L_s(a1) f_o(1) + f_1(a1) f_o^{-1}(a) \Delta\omega \right] \right\} \tag{C.2.7}$$

and all terms of $O\left(n_e^2\right)$ cancel.

Acknowledgments

Thanks to James W. Dufty for reading the manuscript. The work by CAI performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

References

M. Baranger, Phys. Rev. 111, 494(1958)

D.B. Boercker & C.A. Iglesias, Phys. Rev. A30, 2771(1984)

D.B. Boercker, F.J. Rogers & R.W. Lee, J. Phys. B16, 3279(1983)

A. Calisti, F. Khelfaoui, R. Stamm et al., Phys. Rev. A9, 5433(1990)

R.D. Cowan, *The Theory of Atomic Structure and Spectra* (University of California Press, Berkeley, 1981)

M.A. Gigosos, J. Phys. D47, 343001(2014)

T.A. Gomez, T. Nagayama, P.B. Cho et al., Phys. Rev. Letts. 127, 235001(2021)

T.A. Gomez, T. Nagayama, P.B. Cho et al., J. Phys. B55, 034022(2022)

T.A. Gomez, T. Nagayama, D.P. Kilcrease et al., Phys. Rev. A98, 012505(2018)

H.R. Griem, *Spectral Line Broadening in Plasmas* (Academic Press, New York, London, (1974)

J.W. Dufty. Phys. Rev. 187, 305(1969)

T.W. Hussey, J.W. Dufty & C.F. Hooper, Phys. Rev. A12, 1084(1975)

T.W. Hussey, J.W. Dufty & C.F. Hooper, Phys. Rev. A16, 1248(1977)

C.A. Iglesias, HEDP 18, 14(2016)

C.A. Iglesias, HEDP 38, 100921(2021)

C.A. Iglesias & J.W. Dufty, *Spectral Line Shapes* (edited by K. Burnett, W. de Guyter, Berlin, 1983, Vol. II, p. 55)

R.R. Johnston, JQSRT 7, 815(1967)

G.C. Junkel, M.A. Gunderson, C.F. Hooper & D.A. Haynes, Phys. Rev. E62, 5584(2000)

R.C. Mancini, D.P. Kilcrease, L.A. Woltz & C.F. Hooper, CPC 63, 314(1991)

J.T. O'Brien & C.F. Hooper, JQSRT 14, 479(1974)

E.W. Smith & C.F. Hooper, Phys. Rev. 157, 126(1967)