

Modeling the Contribution of Precipitation Loss to a Radiation Belt Electron Dropout Observed by Van Allen Probes

Zhi Gu Li¹, Weichao Tu¹, and Richard S. Selesnick²

¹West Virginia University

²Air Force Research Laboratory

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Abstract

A drift-diffusion model is used to simulate the low-altitude electron distribution, accounting for azimuthal drift, pitch angle diffusion, and atmospheric backscattering effects during a rapid electron dropout event on August 21st, 2013, at L=4.5. Additional external loss effects are introduced during times when the low-altitude electron distribution cannot be reproduced by diffusion alone. The model utilizes low-altitude electron count rate data from five POES/MetOp satellites to quantify pitch angle diffusion rates. Low-altitude data provides critical constraint on the model because it includes the drift loss cone region where the electron distribution in longitude is highly dependent on the balance between azimuthal drift and pitch angle diffusion. Furthermore, a newly derived angular response function for the detectors onboard POES/MetOp is employed to accurately incorporate the bounce loss cone measurements, which have been previously contaminated by electrons from outside the nominal field-of-view. While constrained by low-altitude data, the model also shows reasonable agreement with high-altitude data. Pitch angle diffusion rates during the event are quantified and are faster at lower energies. Precipitation is determined to account for all of the total loss observed for 350 keV electrons, 76% for 600 keV and 45% for 900 keV. Predictions made in the MeV range are deemed unreliable as the integral energy channels E3 and P6 fail to provide the necessary constraint at relativistic energies.

1 **Modeling the Contribution of Precipitation Loss to a Radiation Belt Electron**
2 **Dropout Observed by Van Allen Probes**

3 **Zhi Gu Li¹, Weichao Tu¹, and Richard Selesnick²**

4 ¹Department of Physics and Astronomy, West Virginia University, Morgantown, WV, USA

5 ²Space Vehicles Directorate, Air Force Research Laboratory, Kirtland AFB, NM, USA

6 Corresponding author: Zhi Gu Li (zhigu.li@mail.wvu.edu)

7 **Key Points:**

- 8 • The model well reproduces low-altitude electron data by considering azimuthal drift,
9 pitch angle diffusion, and atmospheric backscatter.
- 10 • 5 POES/MetOp satellites are used to constrain model parameter evolution and quantify
11 pitch angle diffusion rates.
- 12 • Precipitation is identified as the dominant loss mechanism for electrons in the energy
13 range 300-850 keV during the event of our study.

14 Abstract

15 A drift-diffusion model is used to simulate the low-altitude electron distribution,
16 accounting for azimuthal drift, pitch angle diffusion, and atmospheric backscattering effects during
17 a rapid electron dropout event on August 21st, 2013, at $L = 4.5$. Additional external loss effects
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19 diffusion alone. The model utilizes low-altitude electron count rate data from five POES/MetOp
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22 is highly dependent on the balance between azimuthal drift and pitch angle diffusion. Furthermore,
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30 energy channels E3 and P6 fail to provide the necessary constraint at relativistic energies.

31 1. Introduction

32 The dynamics of the outer terrestrial radiation belt can be exceedingly variable and is
33 instigated ultimately by solar activity. The population of the outer belt is dominated by energetic
34 (> 0.5 MeV) electrons and extends from approximately 3 to 7 Earth radii (R_E) (Van Allen et al.,
35 1958; Ganushkina et al., 2011). The presence of these MeV electrons heavily influences the
36 operations of near-Earth satellites and can cause catastrophic damage as they penetrate deep into
37 sensitive electronics (Baker, 2000). Furthermore, energetic electrons which precipitate from the
38 radiation belts into the Earth's atmosphere have been found to increase production of nitrogen
39 oxides (NO_x), leading to rapid depletion of the ozone layer (e.g., Fytterer et al., 2015; Meraner &
40 Schmidt, 2018), and enhanced ionospheric conductance (e.g., Robinson et al., 1987; Yu et al.,
41 2018).

42 It has been well known, especially recently with data from the Van Allen Probes (VAP),
43 that relativistic electron flux within the radiation belts can decrease rapidly on orders of
44 magnitudes within only a few hours (see e.g., Shprits et al., 2012; Baker et al., 2016). However,
45 the mechanisms responsible for these so-called rapid electron dropout events are not well
46 understood. Magnetopause shadowing in combination with radial diffusion provides an efficient
47 mechanism responsible for fast electron loss, as they funnel previously trapped electrons towards
48 the magnetopause where they are subsequently lost traversing open field lines (see e.g., Li et al.,
49 1997; Shprits et al., 2006a; Turner et al., 2012; Tu et al., 2019). However, at the heart of outer
50 radiation belts ($L \approx 4$), it is believed that precipitation of electrons onto the terrestrial atmosphere
51 via pitch angle diffusion serves as the dominant mechanism responsible for electron dropouts
52 (Selesnick, 2006; Millan et al., 2010; Turner et al., 2014). At high altitudes, where Coulomb
53 collision is insignificant, stochastic pitch angle scattering of relativistic electron is due to resonant
54 wave-particle interactions with the electromagnetic waves excited by plasma instabilities such as
55 chorus, plasmaspheric hiss, and electromagnetic ion cyclotron (see e.g., Thorne, 2010 and
56 references therein). The pitch angles of trapped electrons can eventually be scattered into the loss

57 cone, a range of pitch angles with mirror points within the dense atmosphere, where they then
58 subsequently precipitate.

59 Early attempts to quantify pitch angle diffusion rates were made by Kennel & Petschek
60 (1966), Thorne & Kennel (1971), and Lyons et al., (1972). However, without comprehensive
61 information about the wave's spectral properties, cold plasma densities, or ion composition, these
62 estimates were hampered by uncertainties and broad assumptions. Currently, estimated electron
63 loss rates from pitch angle scattering still diverge greatly. For example, Shprits et al. (2005)
64 employed electron lifetimes in their model of 10 days within the plasmasphere and 0.7 to 4 days
65 outside the plasmasphere, depending on the K_p index. Comparatively, Barker et al. (2005) used a
66 L dependent electron lifetime ranging from 3 days at $L = 6$ to 29 days at $L = 4$. Modeling work
67 done by Tu et al. (2010) estimated relativistic electron lifetimes to vary from 0.01 days to 10 days
68 during magnetic storm events. These conflicting estimates underscore the persisting ambiguity
69 surrounding pitch angle diffusion rates and their driving mechanisms. Accurate quantification of
70 electron pitch angle diffusion and loss rates would be of value as they are ubiquitous in
71 comprehensive radiation belt models and are also needed to reliably assess electron acceleration
72 processes.

73 Theoretically, the non-dipolar nature of the terrestrial magnetic field causes nonuniform
74 electron loss in magnetic longitude, where the resulting redistribution due to azimuthal drift
75 provides the necessary low-altitude observational constraint to infer pitch angle diffusion rates.
76 This approach was validated by Imhof (1968) where an approximate analytical solution to the
77 diffusion equation showed reasonable agreement with data from low-altitude satellites. Detailed
78 simulations were done by Abel & Throne (1999) further confirming the significant role of both
79 pitch angle diffusion and azimuthal drift in the loss of radiation belt electrons, but however, was
80 not compared to observational data. Subsequently, Selesnick et al., (2003) developed a parametric
81 model, accounting for azimuthal drift and pitch angle diffusion, and used low-altitude SAMPEX
82 satellite data to constrain the rate of pitch angle diffusion within the model. The model was capable
83 of predicting temporal and spatial variation of pitch angle diffusion without the need for global
84 wave measurements, which led to multiple subsequent works (including this one, see also e.g.,
85 Selesnick, 2006; Tu et al., 2010). However, the temporal resolution provided by the model is
86 significantly reduced by the limited availability of data from just one satellite, enabling a cadence
87 as low as only half a day. Magnetospheric conditions during storm time can vary much more
88 rapidly, leading to misleading results, or missing storm-time dynamics. This issue was addressed
89 by Pham et al. (2017) by using an extensive set of low-altitude data obtained from a more recent
90 satellite constellation composed of Polar Orbiting Environmental Satellites (POES) operated by
91 the National Oceanic and Atmospheric Administration (NOAA) and the Meteorological
92 Operational Satellites (MetOp) operated by the European Organization for the Exploitation of
93 Meteorological Satellites (EUMETSAT). Five POES/MetOp satellites were online during their
94 event study, which provided their model with an improved 3-hour cadence.

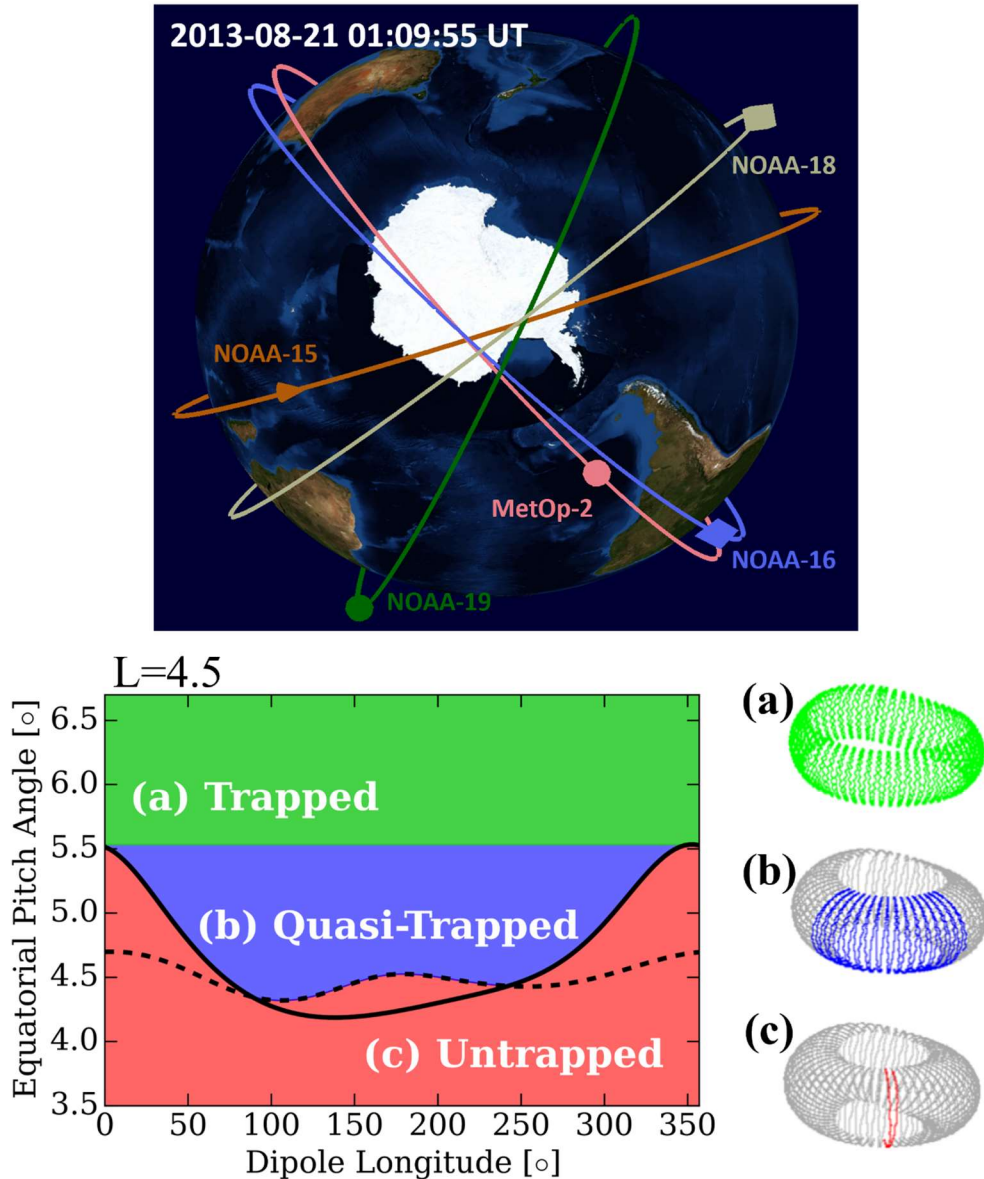
95 An accurate angular response function for detectors is required under weak pitch angle
96 diffusion, where the loss cone is relatively empty. This is because trapped electron flux outside of
97 the loss cone are typically several orders of magnitude higher, where any measurements taken
98 inside the loss cone are dominated by the trapped electrons outside of the field-of-view (FOV).
99 While the bounce-averaged models used in Pham et al. (2017) and preceding works are capable in
100 approximating the loss cone flux under both weak and strong diffusion scenarios, there did not
101 exist any reliable loss cone measurements to constrain the model evolution in such regions.

102 Consequently, their models assumed an empty loss cone, and so their results were valid in only
103 the weak diffusion limit. Selesnick et al. (2020) derived a more accurate angular response function
104 for the detectors onboard POES/MetOp, accounting for the response to particles from all incident
105 angles. By incorporating this new angular response function, POES/MetOp's loss cone
106 measurements can provide a more accurate constrain for model flux inside the loss cone region.

107 In this work, we use an updated drift-diffusion model to dynamically simulate the
108 distribution of low-altitude electrons and quantify the pitch angle diffusion rates during a rapid
109 electron dropout event at $L = 4.5$ on August 21st, 2013. The newly-derived angular response
110 function is used in combination with the previously established energy response function (Yando
111 et al., 2011) to convert the simulated electron distribution function to count rates which are then
112 compared to the observed count rates from POES/MetOp. The model includes the dominant effects
113 of pitch angle diffusion and azimuthal drift as experienced by low-altitude electrons and, in
114 addition, atmospheric backscattering effects of electrons within the loss cone. The model
115 parameters, including pitch angle diffusion rates, are determined by fitting simulated POES/MetOp
116 count rate data to observations. Notably, this approach does not rely on wave measurements. Its
117 results could potentially be used to constrain the distribution of waves that are responsible for the
118 precipitation loss, though we do not attempt that here. Furthermore, with the availability of VAP
119 data, additional insight on the model results can be obtained by comparisons with high-altitude
120 data, which was not done in previous works related to this model. Section 2 provides an overview
121 of low-altitude electron data and their significance in determining pitch angle diffusion rates.
122 Section 3 details the model, including the initial conditions obtained from VAP, while section 4
123 describes the dropout event. Section 5 presents the model results followed finally by comparisons
124 with high-altitude data, conclusion and further discussions in sections 6, and 7 respectively.

125 **2. Low-altitude Electron and Trapping Regions**

126 The distribution of electrons mirroring near and within the upper boundary of the terrestrial
127 atmosphere can be categorized into three distinct groups. Aside from electrons that mirror well
128 above or deeply within the atmosphere, which are categorized as the *trapped* and *untrapped*
129 electron populations, due to non-dipolar nature of the terrestrial magnetic field, there arises an
130 intermediate category called the *quasi-trapped*, where particles can mirror above or within the
131 atmosphere depending on their location in magnetic longitude. This is because, near the surface,
132 the higher-order magnetic multipoles asymmetrically determine the elevation of the particle's
133 mirror point in relation to the 100 km altitude threshold – upper boundary layer of the Earth's
134 atmosphere. So hence, the strength of magnetic field at 100 km altitude divides the low-altitude
135 electrons into these three categories.



136
 137 **Figure 1:** Top panel shows the orbits of 5 POES/MetOp satellites on August 21, 2013, viewed
 138 from above Antarctica. Bottom left panel shows the partitioned regions of trapped, quasi-trapped,
 139 and untrapped electrons based on their equatorial pitch angles at $L = 4.5$. The dashed and solid
 140 black line represents the northern and southern hemisphere bounce loss cones respectively. The
 141 highlighted path on the bottom right tori exemplifies the trajectories of a particle in each category.
 142 Bottom panels are adapted from Tu et al., (2010).

143 Figure 1, bottom left panel, shows the variation of equatorial north (dashed curve) and
 144 south (solid curve) loss cones in magnetic longitude (hereinafter referred to as just longitude) at a
 145 constant McIlwain L-shell value (hereinafter referred to as L , see e.g., McIlwain, 1961). The
 146 particle will attempt to mirror across both hemispheres, so therefore, the larger of the two angles
 147 at each longitude governs whether a particle will mirror below the atmosphere within a single
 148 bounce period. This is known as the bounce loss cone (BLC, red region in Figure 1) and the

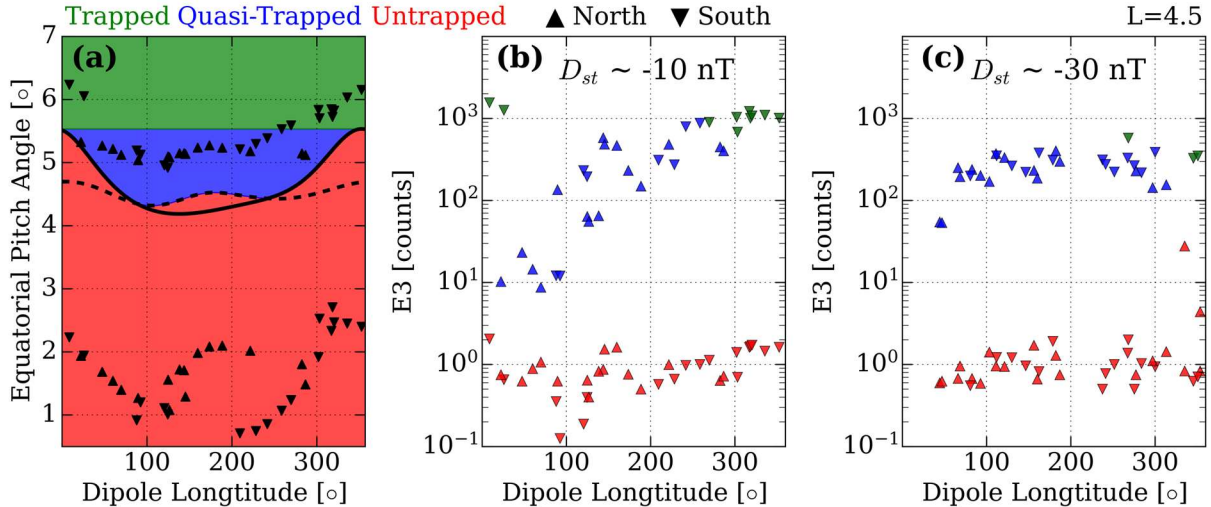
149 untrapped electron population pertains equatorial pitch angles below the BLC curve, since it is
150 likely that they precipitate within a singular bounce.

151 Extending this idea, the quasi-trapped population of electrons exists with certain pitch
152 angles who mirror above the atmosphere at their local longitude but eventually drift into the BLC
153 at another longitude and precipitate. Consequently, most of these electrons are lost near $\sim 300^\circ$
154 longitude to the southern loss cone. This results in enhanced electron precipitation over the south
155 Atlantic and is known as the South Atlantic Anomaly (SAA). The region in momentum space
156 populated by the quasi-trapped electrons constitutes the drift loss cone (DLC, blue region in Figure
157 1), where the boundaries are defined by the maximum value of the BLC in longitude and the BLC
158 itself. Lastly for completeness, the previously mentioned trapped population (green region in
159 Figure 1) are those outside of the DLC and therefore remain trapped within the radiation belts
160 indefinitely.

161 The distribution of electrons within these three categories serves as a good constraint on
162 the rate of pitch angle scattering and electron loss rate within the radiation belts. Count rate data
163 of the low-altitude electrons are obtained from the Medium Energy Proton and Electron Detector
164 (MEPED) which is a part of the Space Environment Monitor 2 (SEM-2) instrument suite (Evans
165 & Greer, 2000). SEM-2/MEPED is flown onboard POES/MetOp. These satellites are in a Sun-
166 synchronous polar orbit at an approximate altitude and period of 800 km and 1.7 hours
167 respectively. Figure 1, top panel, shows the orbits of the 5 POES/MetOp satellites operational
168 during the August 21, 2013, event, with level 2 (16 seconds averaged count rates) data available.
169 The combination of Earth's rotation with the spread in local-time of these 5 satellites provides
170 good coverage of all geographic longitudes within ~ 3 hours.

171 The MEPED detector is described in detail by Evans and Greer (2000). Its entrance
172 aperture has a 15° half-angle nominal field of view (FOV) to incoming particles. Local pitch angles
173 are derived from the IGRF magnetic field model since POES/MetOp does not carry a
174 magnetometer onboard (Finlay et al., 2010; Green, 2013). The electron telescope measuring count
175 rate contains a single silicon detector and three integral energy channels labeled E1, E2, E3
176 corresponding to energies > 30 , > 100 , and > 300 keV respectively. The proton telescope
177 contains two silicon detectors and six integral energy channels P1 to P6 with defined energy
178 ranges. Additionally, P6 can serve as an electron channel, responding to electrons ≥ 700 keV
179 when proton intensities are not too high (Yando et al. 2011). For our interests in energetic electron
180 dropouts, E3 and P6 will be of interest to us for this study as they correspond more closely to
181 radiation belt electron energies. Contamination from protons when using P6 as an electron channel
182 can be an issue, however proton intensities are typically low in the outer belt. Furthermore, proton
183 intensities are verified to be low during this particular event via P5 which is insensitive to electrons
184 (Green, 2013).

185 The nominal values of energy response and FOV can serve as a good indicator of which
186 electron population POES/MetOp are surveying. However, Selesnick et al. (2020) demonstrated
187 that E3 and, in particular, P6 data can be dominated by electrons from outside the nominal FOV.
188 Our model thus uses the more comprehensive energy and angular response functions provided by
189 Yando et al. (2011) and Selesnick et al., (2020) respectively, and the details can be found therein.



190
 191 **Figure 2:** Panels (a) and (b) show equatorial pitch angle and E3 channel count rate measurements
 192 taken by POES/MetOp over a 3-hour period starting from 2013-08-21/00:00:00 UT. The upper
 193 and lower cluster of points on panel (a) are taken by the 90° and 0° telescope respectively. Panel
 194 (c) is taken over a 3-hour period starting from 2013-08-21/06:00:00 UT at a lower D_{st} level.

195 The MEPED detector onboard POES/MetOp has two telescopes oriented in orthogonal
 196 directions. At mid to higher latitudes (or equivalently $L \gtrsim 1.5$, since orbital altitude is constant),
 197 one of the telescopes points in a more magnetic field-aligned direction, while the other points
 198 perpendicular to the magnetic field. These have been appropriately named as the 0° and 90°
 199 telescope respectively. At these altitudes, the local BLC and DLC drastically widen, giving
 200 POES/MetOp an increased ability to distinguish between trapped, quasi-trapped and untrapped
 201 electron populations. Generally, the 90° telescope measures quasi-trapped electrons inside the
 202 DLC, except for near the SAA where it occasionally measures trapped electrons, while the
 203 0° measures exclusively untrapped electrons deep inside the BLC. As an example, Figure 2a shows
 204 the pitch angle measurements made by 5 POES/MetOp satellites interpolated to $L = 4.5$ over a 3-
 205 hour interval needed for sufficient longitudinal coverage. The local pitch angle measured is
 206 assumed to be from the center of the FOV and is mapped to the equator using IGRF-11. As stated
 207 earlier and seen in Figure 2a, the 90° telescope has excellent coverage of the quasi-trapped
 208 population in longitude with the occasional measurement of the trapped population, while on the
 209 other hand, the 0° telescope exclusively measures the untrapped electrons.

210 Figure 2a and 2b are correlated plots in the sense that they visualize the same 3-dimensional
 211 data but on 2 separate y-axes. Notably, each count rate point taken from the E3 channel presented
 212 on Figure 2b corresponds to a unique point on Figure 2a, indicating the associated pitch angle at
 213 the same longitude of the measurement. The minimum value of the Disturbance storm-time (D_{st})
 214 index over the 3-hour interval is -10 nT, suggesting low levels of pitch angle scattering, and this
 215 is reflected in the distribution of quasi-trapped count rates in longitude. The quasi-trapped
 216 population is derived from trapped electrons whose pitch angles have been scattered into the DLC.
 217 Hence, if the pitch angle scattering rate is comparable to the azimuthal drift speed of the electrons,
 218 the quasi-trapped flux should increase gradually as they drift towards larger longitudes.
 219 Subsequently, the increased quasi-trapped flux decreases as the electrons precipitate over the SAA
 220 which means the quasi-trapped count rates should attain its minimum near the region immediately

221 after. This pattern of steady increase and sudden decrease within the DLC is manifested by a strong
 222 gradient of increase in the quasi-trapped count rates (blue triangles) in longitude as seen in Figure
 223 2b. Trapped count rates (green triangles) are generally higher because they do not undergo periodic
 224 losses every drift orbit, while the untrapped count rates (red triangles) are nominally the lowest.

225 Figure 2c panel is in the same style as Figure 2b except its equatorial pitch angle
 226 measurements are not shown, and the data is taken at a different 3-hour interval with a minimum
 227 D_{st} value of -30 nT. Therefore, in this case and with reference to Figure 2b, the scattering rate is
 228 increased while azimuthal drift speed remains the same. When scattering rates dominate the drift
 229 speed, the shape of the quasi-trapped count rate distribution loses its longitudinal dependence as
 230 shown in Figure 2c and the quasi-trapped count rate values become comparable to the trapped over
 231 all longitudes. Moreover, the shape of the quasi-trapped distribution becomes relatively insensitive
 232 to further increases in pitch angle scattering rates. However, even higher rates of pitch angle
 233 scattering can still be inferred by the precipitation models provided the additional constraints from
 234 the untrapped measurements.

235 3. Model Description

236 3.1 Drift-Diffusion Model

237 Selesnick et al. (2003) demonstrated that the primary characteristics of the electron's phase
 238 space density (f) at low altitudes are governed by the azimuthal drift motion and pitch angle
 239 diffusion due to wave-particle interactions. Additionally, the effects of scattering and loss due to
 240 atmospheric interactions for electrons mirroring within the atmosphere can be approximated by
 241 strong pitch angle diffusion and rapid decay respectively which was found to work well in
 242 reproducing the loss cone electron measurements made by POES/MetOp's 0° telescope (Selesnick
 243 et al., 2020). All together, these main contributing effects to the electron dynamics at a particular
 244 L and energy can be described by the bounce-averaged model

$$245 \quad \frac{\partial f}{\partial t} + \omega_d \frac{\partial f}{\partial \phi} = \frac{1}{xT(y)} \frac{\partial}{\partial x} \left(xT(y) D_{xx} \frac{\partial f}{\partial x} \right) - \frac{4}{\tau_b} (1 - F_b) \Theta(x_c - x) f + \left. \frac{\partial f}{\partial t} \right|_{ext.}, \quad (1)$$

246 where $f = f(x, \phi, t)$ is the electron phase space density as a function of magnetic longitude ϕ ,
 247 time t , and $x = \cos \alpha_0$; α_0 is the equatorial pitch angle of the electron. ω_d and τ_b are the azimuthal
 248 drift frequency and bounce period of the electron evaluated under a dipole approximation. The
 249 $T(y)$ function is proportional to the electron bounce period and is approximated in a dipole field
 250 by $T(y) = 1.380173 - 0.639693y^{0.737}$, where $y = \sin(\alpha_0)$ (Davidson, 1976). x_c is the local
 251 (in ϕ) x value of the BLC. The BLC angle is determined by the maximum value between the loss
 252 cones in each hemisphere and is calculated using the IGRF-11 geomagnetic field model. Θ is the
 253 Heavyside step function which ensures that only electrons mirroring inside the atmosphere
 254 experiences atmospheric effects.

255 The advection term accounts for the eastwards azimuthal motion of the electrons due to
 256 gradient-curvature drift in the positive longitude direction (Roederer, 1970, p.27). The x diffusion
 257 term accounts for the wave-particle interactions that stochastically scatters the particles from their
 258 original pitch angles (Schultz & Lanzerotti, 1974, p.77). The decay term (equation (1) second term
 259 on the right) captures the loss of electrons through momentum-degrading collisions with

260 atmospheric particles where the decay rate is controlled by F_b , the ratio of electrons that are not
 261 lost but scattered back out of the atmosphere. Lastly, the third term on the right of equation (1)
 262 accounts for source or loss due to any other external mechanisms (e.g., local heating or
 263 magnetopause shadowing). It is assumed to be the form

$$264 \quad \left. \frac{\partial f}{\partial t} \right|_{ext.} = (S_0 \tilde{E}^{-\nu} - L_0 \tilde{E}^{-\eta}) \frac{g_1(x, \phi)}{p^2}, \quad (2)$$

265 where S_0 and L_0 (not to be confused with the McIlwain L shell parameter) are the external electron
 266 source and loss rates and are strictly positive. ν and η controls the energy dependance of these
 267 source and loss rates. S_0 , L_0 , ν , and η are free parameters to be determined by model fits to the
 268 low-altitude electron data. \tilde{E} is the particle energy normalized by 1 MeV and the electron's
 269 momentum p^2 is included in the denominator so that S_0 and L_0 have dimensions of intensity per
 270 unit time. g_1 is approximately the normalized first order eigenfunction of the drift-diffusion
 271 operator, derived numerically for when equation (1) exhibited steady decay in absence of external
 272 source/loss. This was chosen because all higher order terms of source/loss decay more rapidly and
 273 therefore are not well constrained by data. Furthermore, the functional form of g_1 helps reduce
 274 any artificial diffusion for when source/loss is introduced. Since the source and loss function is not
 275 well constrained, it is only turned on as a last resort if the model cannot reproduce the observed
 276 data via pitch angle diffusion alone.

277 Selesnick (2006), Tu et al. (2010), and Pham et al. (2017) all used a functional dependence
 278 of $D_{xx} \sim x^{-\sigma}$ with a similar drift-diffusion model and reported reasonably good agreements
 279 between model and low-altitude data. We chose a D_{xx} of the form

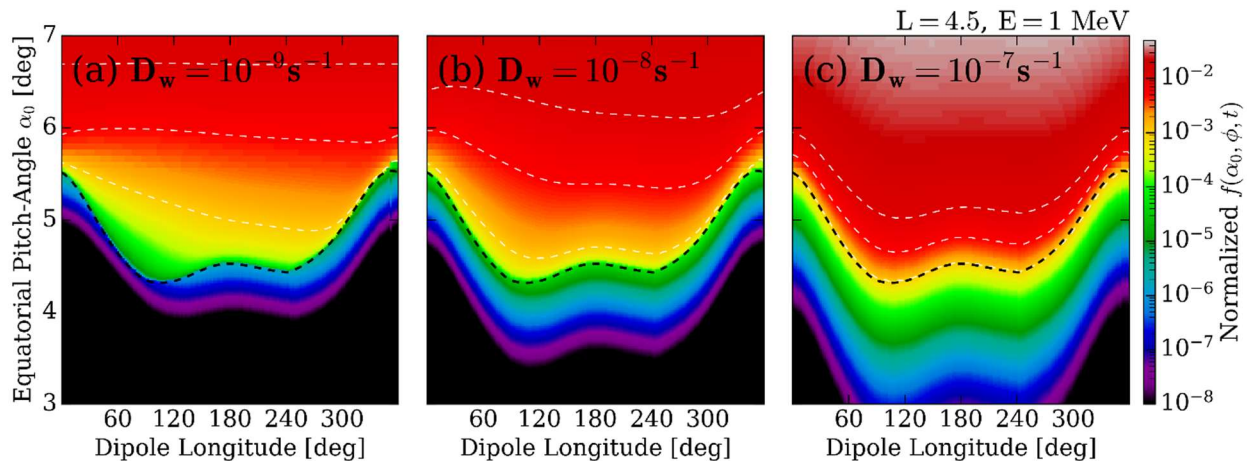
$$280 \quad D_{xx} = \frac{\tilde{E}^{-\mu} D_w}{10^{-4} + x^{20}} + D_b \Theta(x_c - x), \quad (3)$$

281 where we have found $\sigma = 20$ to work well with our event study. D_w includes all pitch angle
 282 diffusion through various wave-particle interactions and μ serves as the energy dependance of the
 283 diffusion activity. Both D_w and μ are free parameters determined by model fits to the low-altitude
 284 data. The factor of 10^{-4} solely serves to maintain numerical stability at low x (higher α_0) values.
 285 The form of this first term in equation (3) results in high diffusion for equatorial particles thus
 286 evolves f into a “flat-top” distribution (consistent with the VAP observations during storm time
 287 as will be discussed in Section 4). At low pitch angles, equation (3) approaches $D_w \tilde{E}^{-\mu}$ which will
 288 largely determine the electron loss rate (Shprits et al., 2006b).

289 F_b is a simplified description of the backscattered electrons that undergo mirroring below
 290 100 km altitude and subsequently re-emerge from the atmosphere. The random scattering in pitch-
 291 angle due to atmospheric collisions is capture in D_b of equation (3) as a diffusive process. The Θ
 292 Heavyside function ensures that this diffusive mechanism is limited to only the electrons within
 293 the BLC. F_b and D_b together dictates that the BLC electrons will undergo scattering of their pitch
 294 angle, and rapid decay due to momentum degrading collisions. The energy loss of the
 295 backscattering process is neglected since Selesnick et al. (2004) found the rate energy change per
 296 emergence out of the atmosphere is much less than the rate of decay itself. In our model, $F_b =$
 297 $0.1/\tilde{E}$ is assumed which dictates that only 10% of 1 MeV BLC electrons are backscattered by the

298 atmosphere, where the energy dependence is a simplified approximation based on the findings in
 299 Selesnick et al. (2004). The combination of F_b with $D_b = 10^{-7} \text{ s}^{-1}$ for all energies is found to
 300 match well with BLC electron measurements.

301 Equation (1) is solved numerically at energies from 300 to 1800 keV, spaced by 50 keV,
 302 using operator splitting to combine Crank-Nicholson method in x with first-order upwind scheme
 303 in ϕ (Press et al., 2007, p.1048). The boundary conditions in x are $\partial f / \partial x = 0$ at $x = 0$, and $x =$
 304 1. This would dictate no flow of electron currents in or out of the unphysical regions in x (or α_0).
 305 Periodic boundary conditions are applied in ϕ such that $f(\phi) = f(\phi + 2\pi)$.



306
 307 **Figure 3:** Example model solutions for a select value of energy and L with varying levels of pitch-
 308 angle diffusion. The color intensity plot is a normalized solution, and the dashed black line is the
 309 angle of the BLC. The dashed white lines are contours of the normalized f . The external
 310 source/loss term is turned off here.

311 Figure 3 shows sample solutions to equation (1) with increasing levels of pitch angle
 312 diffusion. They all have $\mu = 0$ so the diffusion coefficient becomes $D_{xx} = D_w$ at the low pitch
 313 angles near the DLC. From Figure 3, it is evident that the distributions of electrons at low altitudes
 314 are strongly dependent on the diffusion rates or, more specifically, the ratio D_{xx}/ω_d . In the case
 315 of Figure 3a, the distribution of electrons at the lower altitudes is azimuthal drift dominated where
 316 the slow rate of pitch angle diffusion that does not fill in the DLC at all longitudes. Furthermore,
 317 the solution above the DLC is largely unaffected by the shape of the BLC (shown by the lack of
 318 ϕ dependence in the white contour lines). Conversely, Figure 3c demonstrates a case where
 319 diffusion dominates. Here, the effect of azimuthal drift becomes largely irrelevant and the local
 320 solution at each longitude is primarily dictated by its pitch angle relative to the local BLC.
 321 Consequently, lines of constant f tightly hugs the BLC at all longitudes. The solutions of f above
 322 the DLC becomes more longitudinally dependent. Lastly, Figure 3b shows an intermediate case.

323 3.2 Initial Condition

324 The initial conditions for the model are derived from Van Allen Probes data. MagEIS
 325 (Magnetic Electron Ion Spectrometer, Blake et al., 2013) onboard VAP can provide measurements
 326 of electron energies from 30 keV up to 4 MeV, however, large uncertainties are associated with
 327 energies > 1 MeV due to the small geometric factor in its design (Blake et al., 2013). Therefore,
 328 only energy channels ranging from $\sim 240 - 904$ keV are utilized from MagEIS (energies below 240

329 keV are irrelevant since E3 channel of the MEPED detector responds to > 300 keV). Only two
 330 energy channels are of use to us from the REPT (Relativistic Electron-Proton Telescope, Baker et
 331 al., 2012) instrument, 1.8 and 2.4 MeV. This is because higher energies typically go undetected in
 332 integral energy channels like E3 and P6 due to their substantially lower intensities.

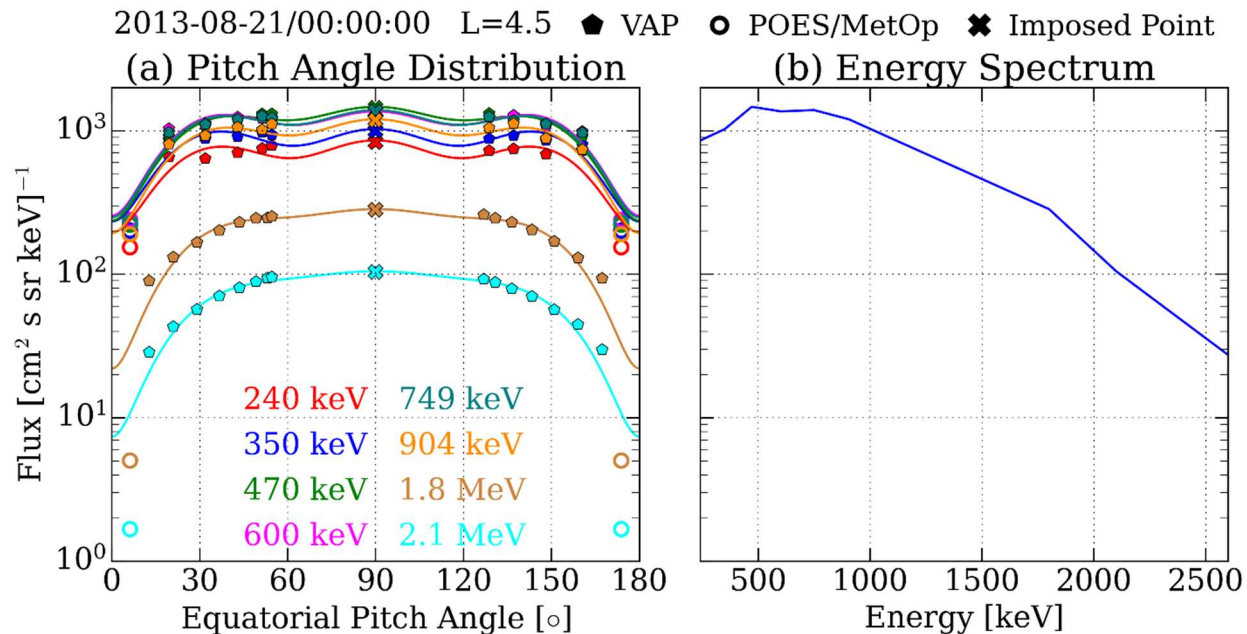
333 Since the VAP measurements cannot cover low pitch angles near the loss cone, we turned
 334 to POES/MetOp to supplement an additional data point around $\alpha_0 \approx 6^\circ$. This additional data point
 335 corresponds to the nearest trapped electron measurement in temporal proximity to the initial time,
 336 acquired from any of the 5 POES/MetOp satellites. The E3, and E4 integral flux data from
 337 POES/MetOp (Green, 2013) are used for this supplementary point, which correspond to energies
 338 > 287 and > 612 keV respectively. The initial condition requires us to find the differential
 339 (energy) flux from the integral flux provided by POES/MetOp, for which we use the energy
 340 spectrum obtained from VAP at the lowest available pitch angle measurement.

341 To obtain the pitch angle distribution (PAD) for all angles, the VAP's local pitch angle
 342 measurements are first mapped to the equator using the TS04D magnetic field model (Tsyganenko
 343 & Sitnov, 2005). Zhao et al., (2018) found using 6th order even Legendre polynomials to fit
 344 energetic electron PADs works well for $L > 4$, so a similar extrapolation technique is applied here:

$$345 \quad j_E(\alpha_0) = \sum_{n=0}^3 c_{2n} P_{2n}(\cos(\alpha_0)), \quad (4)$$

346 where P_{2n} are the even Legendre Polynomials, and c_{2n} are coefficients to be determined via best
 347 fit with data. Note that this is done for each available energy channel and j_E is the PAD for a single
 348 energy.

349 Lastly, during the initial time of this event, VAP was unable to make any measurements
 350 near $\alpha_0 = 90^\circ$. Considering that prior to the initial time high equatorial pitch angle measurements
 351 from VAP were available and show largely a flat PAD near 90° , we introduce an artificial point
 352 at $\alpha_0 = 90^\circ$ with a value 15% larger than the mean of the two closest available measurement to
 353 $\alpha_0 = 90^\circ$. In essence, we manually imposed a flat-top distribution for this initial time, as justified
 354 by observations. Equation (4) is then used to fit over the VAP pitch angle data, supplementary
 355 point provided by POES/MetOp, and the artificially imposed point at $\alpha_0 = 90^\circ$.



356
357 **Figure 4:** Panel (a) shows the initial PAD for the model obtained from VAP with supplementary
358 points from POES/MetOp at the lowest equatorial pitch angles. An additional point is imposed at
359 $\alpha_0 = 90^\circ$ to enforce a flat-top shape. Panel (b) shows the energy spectrum of the initial condition.

360 Figure 4a depicts the initial PAD obtained at each energy for the electron dropout event.
361 Figure 4b shows the energy spectrum, obtained from the flux values at $\alpha_0 = 90^\circ$. At lower energies
362 the Legendre polynomials exhibit multiple changes in concavity and is likely an artifact arising
363 from the limited MagEIS data available at high pitch angles. However, this does not pose a problem
364 because our diffusion coefficient effectively smooths out any gradients at high pitch angles. The
365 energy spectrum in Figure 4b exhibits a phenomenon known as the “bump-on-tail” distribution,
366 which is commonly caused by resonant wave-particle interactions with plasmaspheric hiss waves
367 for lower energy electrons (e.g., Zhao et al., 2019). This initial condition is simulated at a low
368 diffusion case ($D_w = 10^{-9} \text{ s}^{-1}$) for one drift period to initialize the DLC and BLC electron
369 distribution while largely leaving the initial trapped distribution unmodified. The one-drift-period
370 simulated distribution is then used as the initial condition for the event. The model simulates
371 f which is related to the differential flux by $j = p^2 f$.

372 3.3 MEPED Energy and Angular Response

373 The output of our model is the electron phase space density f for a particular L and energy
374 as a function of pitch angle cosine, longitude, and time. To provide constraint from low-altitude
375 electron data obtained from POES/MetOp (e.g., Figure 2b and 2c), the simulated f needs to be
376 converted into count rate. This is achieved by using the MEPED angular response functions
377 (Selesnick et al., 2020) in combination with the previously developed energy response function
378 (Yando et al., 2011). By virtually flying satellites through the simulation, we can replicate the
379 count rates that the detectors would have recorded based on the dynamic state of the electron
380 distribution.

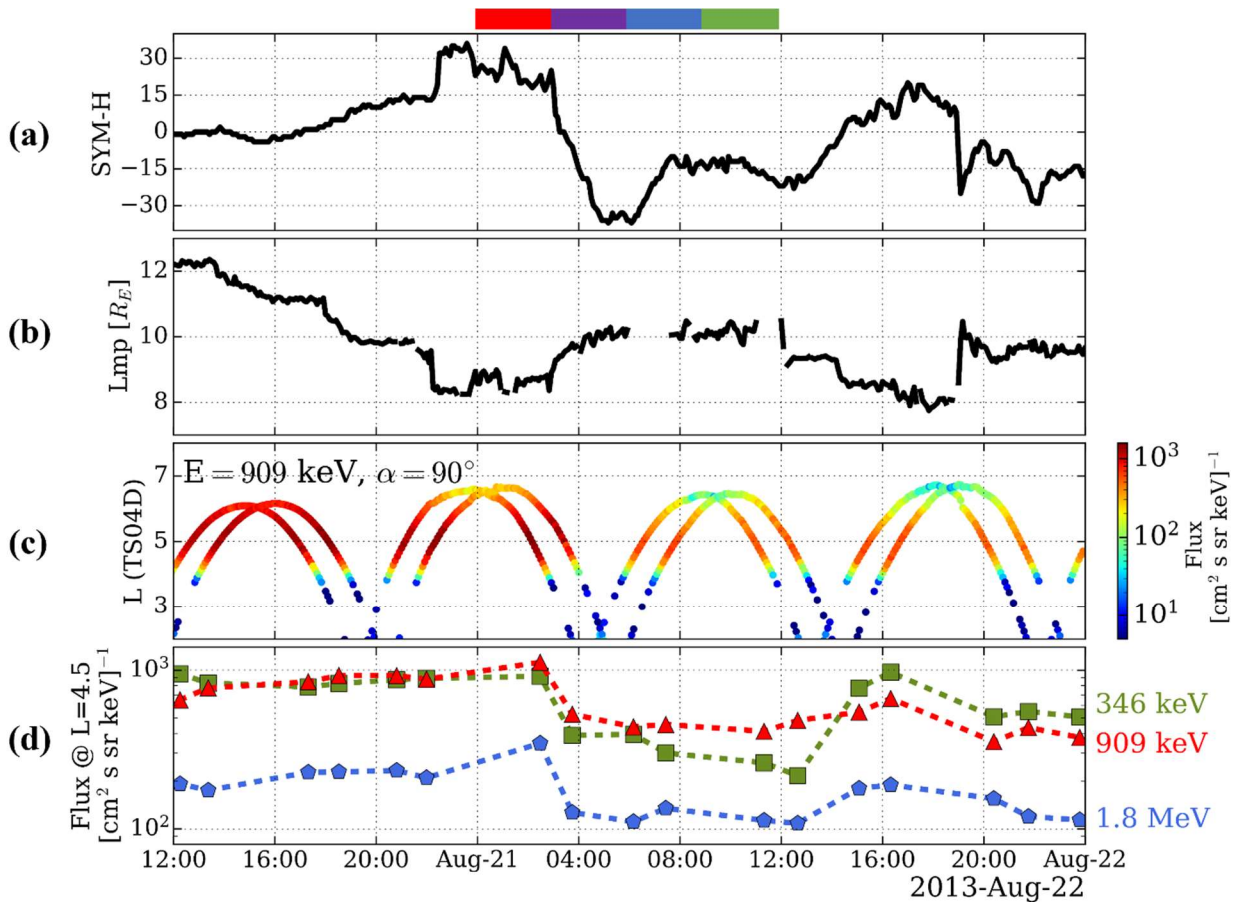
381 **3.4 Parametric Tuning**

382 The 2 (or 6 if external source/loss is turned on) free parameters per simulation interval are
 383 determined by fitting the simulated count rate to the observed count rate. For reasons detailed in
 384 section 2.2, a comprehensive longitudinal coverage of the count rate data is required to provide
 385 the necessary constraint on the pitch angle diffusion rates, and thus, the time duration of this
 386 dropout event is separated into four 3-hour intervals. The parameters are assumed to be constant
 387 within each interval, which can be invalid if magnetospheric conditions change rapidly. To
 388 mitigate this, the interval boundaries are picked by an orderly distribution of quasi-trapped count
 389 rates from the POES/MetOp data such that they tightly follow a general trend (see e.g., Selesnick
 390 et al., 2020, Figures 6, 7, and 8).

391 The parameters are iteratively tuned to minimize the metric:

$$392 \qquad K^2 = \sum_i^N [\log_{10}(d_i) - \log_{10}(p_i)]^2, \qquad (5)$$

393 where, N is the total number of data points, d_i are the observed count rates, and p_i are the simulated
 394 count rates. The Quasi-Newton method is used to iteratively find the set of parameters which
 395 achieves the minimum K^2 starting from an initial set of seed parameters (Tarantola, 2005, p.79).
 396 Constant background rates are estimated for each channel and is determined by the count rate floor
 397 reached by the 0° telescope during quiet times (excluding a few outliers which occasionally fall
 398 below). Count rates below the background rate are included in the model but excluded from the
 399 fit. Lastly, temporal continuity in the model is achieved by using the evolved f of a previous
 400 interval as the initial condition for the next interval, where the parameters are again obtained by
 401 recursive fit.

402 **4. August 2013 Dropout Event**

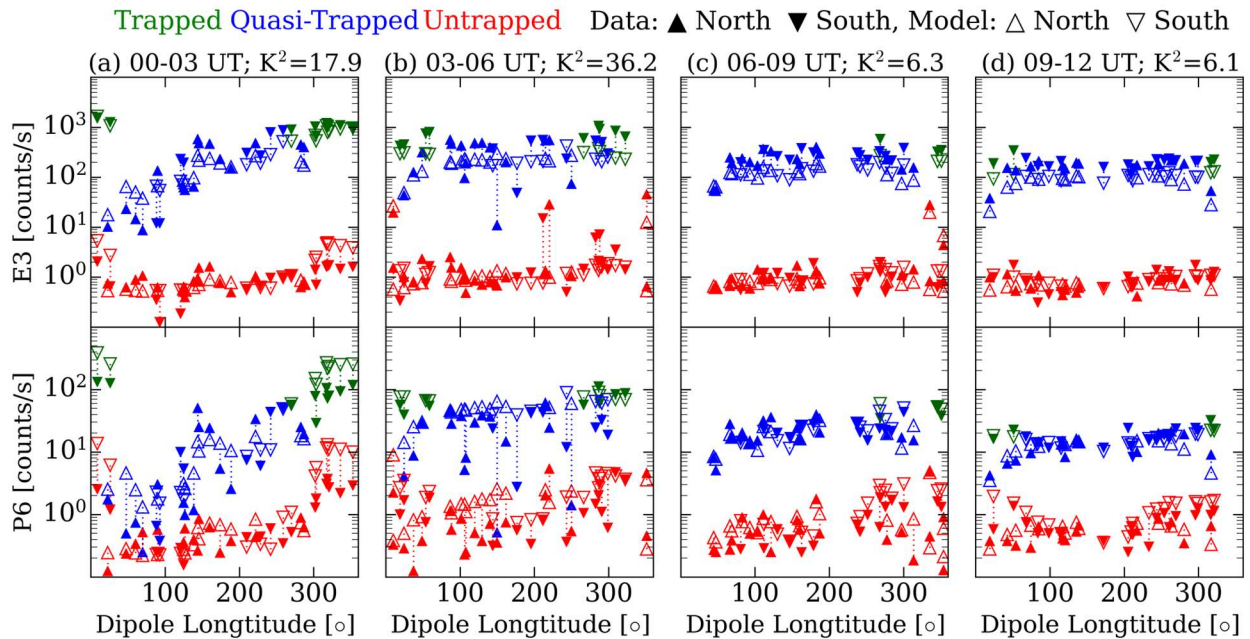
403 **Figure 5:** Panel (a) shows the SYM-H and solar wind pressure variation during the event. Panel
 404 (b) shows the predicted dayside magnetopause location. Panel (c) shows the flux measurement for
 405 a selected energy and local pitch angle as measured by the VAPs along their orbit trajectory, and
 406 (d) shows the flux variation taken at only $L = 4.5$ for a few selected energies. The colored bars on
 407 top of panel (a) correspond to the interval periods of the four 3-hour intervals, starting at 00, 03,
 408 06, 09 UT of August 21st, 2013.

410 A rapid electron dropout was observed by VAP across a broad range of energies at $L = 4.5$
 411 on August 21st, 2013. Figure 5a shows the SYM-H index variation during this event, which attains
 412 a minimum of -37 nT. Despite this being a small storm, it has triggered a significant response in
 413 the radiation belt. Figure 5c shows 909 keV electron flux measurements from the MagEIS 90°
 414 local pitch angle bin along the VAP orbits. To better illustrate the flux drop, Figure 5d shows the
 415 90° observations interpolated to $L = 4.5$, for three selected energies, where the fluxes all decrease
 416 by approximately half an order of magnitude. The precise timing of this dropout is not well
 417 resolved, but it is evident that the dropout occurs within ~76 minutes, between the two inbound
 418 passes of VAP A & B just prior to 04 hours UT. Figure 5b shows that the dayside magnetopause
 419 location predicted by the Shue et al. (1998) model stays above 8 Earth radii during the entirety of
 420 the event. This suggests that the dropout is likely driven by precipitation resulting from pitch angle
 421 diffusion rather than magnetopause shadowing loss in combination with radial diffusion.

422 In this work, we are primarily interested in what happens during the dropout, so our
 423 simulation covers the quiet time just shortly before the dropout and continues until just before
 424 fluxes begin recovering to their pre-storm levels. The simulated time range for this event is from
 425 00 to 12 hours UT on August 21st, 2013. Between approximately 04-12 hours UT, the electron flux
 426 at $L = 4.5$ remain in a depleted state for a broad range of energies and there appears to be no
 427 obvious energy dependence in this dropout. This 12-hour simulation time domain is divided into
 428 four equal 3-hour intervals as marked by the colored bars above Figure 5a. Interval 1 (red) will
 429 quantify model parameters during a quiet time state of the radiation belts, while intervals 2, 3, and
 430 4 will quantify storm-time conditions. This dropout event is chosen for this study since it is a small
 431 storm during which the adiabatic effects on electron flux is insignificant (see e.g., Selesnick, 2006).

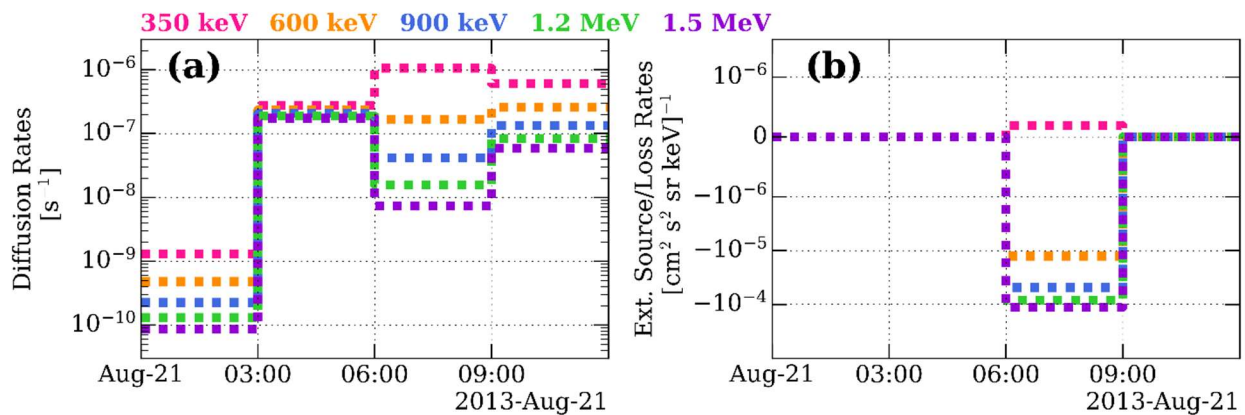
432 5 Model Results

433 Solid markers in Figure 6 show the observed count rates as a function of longitude during
 434 this event taken from the E3 and P6 channels for each of the four 3-hour intervals. Plotted on top
 435 with hollow points are the simulated count rates obtained by flying virtual satellites through the
 436 model. Both the model and data have been categorized based on their measurement taken outside
 437 of the DLC (trapped in green), within the DLC (quasi-trapped in blue) and within the BLC
 438 (untrapped in red) and the hemisphere which the measurement was taken at (upward vs. downward
 439 triangles). The 4 columns all together depict the temporal evolution of the observed and modeled
 440 electron dynamics for this event.



441
 442 **Figure 6:** Electron count rate data (solid triangles) from 5 POES/MetOp satellites interpolated at
 443 $L = 4.5$ on August 21st, 2013. Model count rate (hollow triangles) is shown with dashed lines
 444 connecting each simulated count rate to its corresponding data point. Column (a) is taken during a
 445 3-hour quiet pre-storm interval. Column (b) is during the initial storm main phase. Columns (c)
 446 and (d) are the subsequent hours during the main phase. The goodness of fit per each interval is
 447 provided in the title of each column.

448 Figure 6a depicts the quiet time 3-hour interval before the storm main phase. The quasi-
 449 trapped count rate data increase with longitude, indicating a slow rate of pitch angle diffusion, and
 450 was successfully captured by the model. However, our model slightly overestimates trapped count
 451 rates in the P6 channel during this interval for several possible reasons. First, the diffusion
 452 coefficient's functional form (Equation (3)), derived via observations at a different time, may not
 453 accurately represent the diffusion coefficient during the initial interval. Second, P6 is more
 454 sensitive to > 1 MeV electrons than E3, leading to greater manifestations of uncertainties
 455 associated with interpolating the initial energy spectrum across the 1-1.8 MeV gap, and the
 456 assumed energy spectrum at low pitch angles (as detailed in section 3.2). Third, cross-calibration
 457 between POES/MetOp and VAP is likely required for the initial condition but is not performed for
 458 this work. Finally, the lone data point from POES/MetOp at low pitch angles, especially at higher
 459 energies, for the initial condition was not captured by the fit (see Figure 4a). These inaccuracies
 460 during this interval are associated with higher order effects which are inconsequential and decay
 461 rapidly in later intervals when the diffusion rate increases. This is supported by the following
 462 interval (Figure 6b) when trapped count rates in P6 become reasonably well matched.



463 **Figure 7:** Evolution of the model parameters over the four 3-hour intervals for select energies.
 464 Panel (a) shows the pitch angle diffusion rates ($D_w \tilde{E}^{-\mu}$) and Panel (b) shows the source and loss
 465 rates ($S_0 \tilde{E}^{-\nu} - L_0 \tilde{E}^{-\eta}$) which are only non-zero during the third interval. Linear threshold for the
 466 symmetric logarithmic y-axes is set at 10^{-6} .
 467

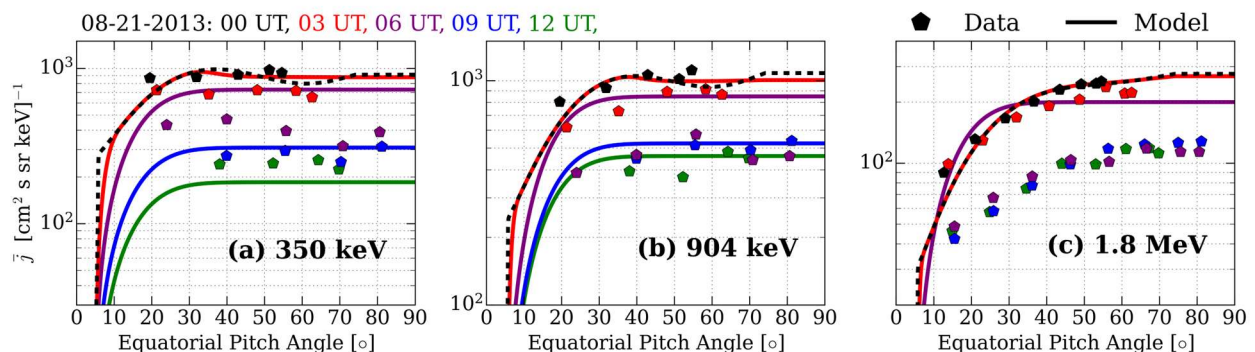
468 Figure 6b shows simulated and observed count rates for the second interval during the
 469 storm's initial main phase. The SYM-H index drops quickly during this interval, and the rapid
 470 changes in magnetospheric conditions could violate the constant parameter assumption imposed
 471 on our model. The scattered count rates in the quasi-trapped population show further indications
 472 of such violation. Despite that, the model still reproduced a substantial portion of the observed
 473 electron distribution, implying that the assumption of constant parameters is only weakly violated,
 474 and its impact remains minor. The scatter explains the relatively higher K^2 during this interval
 475 (provided in the title of each column in Figure 6). Furthermore, during this interval, the flat
 476 distribution exhibited by the quasi-trapped population indicates fast pitch angle diffusion which is
 477 reflected in both the model count rate, and the estimated rates of pitch angle diffusion (shown in
 478 Figure 7a). Figure 7a shows an increase in pitch angle diffusion for all energies by 2 to 3 orders of
 479 magnitude, reaching similar levels for all energies. Since the main trend of the data distribution is
 480 largely replicated by the model during the first two intervals, the external source/loss factor
 481 remains off up to the end of the second interval.

482 Figure 6c shows the model and observed count rates for the third interval. Here, the external
 483 source/loss function was enabled, because without it, the simulated untrapped count rates in the
 484 P6 channel were uniformly overestimated by an order of magnitude (not shown). Energy
 485 dependence of pitch angle diffusion can be identified in the data where the quasi-trapped count
 486 rate distribution is flatter as a function of longitude in the lower energy channel, E3, than the higher
 487 energy channel, P6. This feature was recognized by the model and higher pitch angle diffusion
 488 rates were obtained at lower energies as shown in Figure 7a. The energy dependence of the external
 489 loss during this interval is shown in Figure 7b, which indicates faster external loss at higher
 490 energies. For electron energies < 350 keV, the rates of external source/loss are negligible. This
 491 plays an important role in decreasing the energetic trapped and quasi-trapped electron count rates
 492 in the P6 channel without scattering electrons into the BLC, as required by the data.

493 Figure 6d depicts the final storm-time interval, and the data are again well reproduced by
 494 the model. The pitch angle diffusion rates determined by the model, Figure 7a, show a decrease in
 495 lower energies, but an increase at higher energy compared to the previous interval. The resulting
 496 pitch angle diffusion becomes nearly uniform again in energy, though higher at lower energy, and
 497 is still significantly faster than in quiet times. No external source/loss was required during this
 498 interval, and it was therefore turned off.

499 Notably, slight under prediction of the simulated count rates is present across all intervals
 500 compared to observed E3 count rates, however, this discrepancy is not significant enough to
 501 warrant the introduction of external source/loss mechanisms. Finally, the reasonable match
 502 between the model and observed trapped count rates for all four intervals, in both E3 and P6, shows
 503 that the model has successfully captured the electron flux decrease throughout the event.

504 6 Comparison with High-Altitude Data



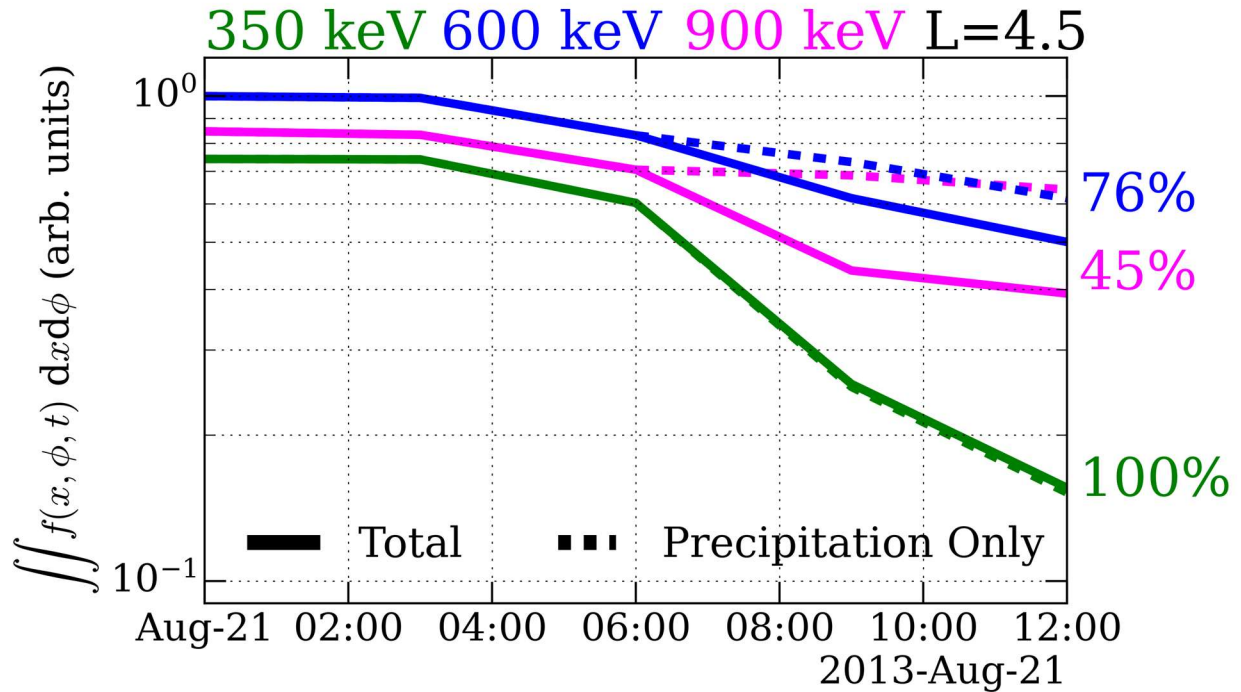
505 **Figure 8:** Drift averaged model solutions (lines) compared with VAP observations (pentagons) at
 506 $L = 4.5$ for different intervals (in different colors) for select energies. High-altitude data on panels
 507 (a) and (b) are taken from the MagEIS instrument, while panel (c) is from REPT. Dashed line
 508 corresponds to the model's initial condition.
 509

510 The successful reproduction of low-altitude electron distributions made by the model may
 511 not imply the same for high altitudes. The model's initial conditions were derived mainly using
 512 high-altitude data from VAP, but its evolution was exclusively guided by the low-altitude electron
 513 distribution. This raises the question: can a model constrained by low-altitude data effectively
 514 replicate the changes at higher altitudes?

515 Figure 8 answers this question, shown with VAP data taken at the end times of each 3-hour
516 interval (in pentagon symbols). The model solutions at the end of each interval are shown by the
517 colored solid curves, while the initial condition is reiterated by the black dashed line. The best fits
518 with low-altitude data, as quantified by K^2 , occurred in intervals 3 and 4. Correspondingly, Figures
519 8a and 8b show the best agreement between observations and the model at 09 UT (blue) and 12
520 UT (green), the end times of the 3rd and 4th interval, for electron energies below 1 MeV (Figures
521 8a and 8b). Interval 2 at low altitudes showed the worst fit between the model and observed count
522 rates which is likely due to the weak violation of the constant parameters assumption. As detailed
523 in Section 5, a few of the quasi-trapped data points during this interval (Figure 6b) deviate from
524 the general trend observed in the other quasi-trapped data points, which serves as an indicator of
525 fluctuating pitch angle diffusion rates. As a result, the match between model and observations
526 during this interval (in purple) is the poorest at high altitudes as demonstrated in Figures 8a and
527 8b. These transient dynamics are challenging for the model to fully capture. For this interval, our
528 quantifications of the model parameters can be interpreted as a 3-hour average.

529 Figure 8c compares model results with VAP observation for 1.8 MeV electrons. Notably,
530 the model predicts a complete extinction of electrons in the 3rd and 4th intervals (thus not appearing
531 in Figure 8c), contradicting observations. This discrepancy is attributed to the limited constraint at
532 relativistic energies provided by the POES/MetOp data. Both E3 and P6 are integral energy
533 channels responding primarily to the sub-1 MeV population and their sensitivity to higher energy
534 electrons is limited by the more abundant population at lower energy.

535 The model was successful in reproducing low-altitude electron distributions as observed
536 by POES/MetOp. Further comparisons with high-altitude data observed by VAP exclusively
537 reveal the discrepancies at relativistic energies. Nonetheless, our model agreed well at low and
538 high altitudes for energies below ~ 1 MeV, and hence, in this range, we can conclude that the
539 parameters quantified by our model are representative of actual magnetospheric conditions.



540 **Figure 9:** The variation of total flux (presented in arbitrary units) in the system for each energy as
 541 a function of time. The percentages on the right represent the change in flux due to precipitation
 542 only.
 543

544 Having established the model's domain of validity, we can now address our primary
 545 question: what is the contribution of precipitation to the observed electron dropout during this
 546 event? The change in total flux within the system (integrated over x or α_0 and ϕ) can be attributed
 547 either to atmospheric loss (precipitation) or to external source/loss mechanisms. Therefore, to
 548 isolate the loss due to precipitation, the total contribution of the external source/loss (Equation (2))
 549 is determined and then removed from the total flux to obtain the flux loss due to precipitation only.
 550 Figure 9 depicts the variation in total flux within the system in comparison to the variation of flux
 551 due to precipitation alone. The contribution of precipitation to the electron dropout progressively
 552 decreases with energy; it is $\sim 100\%$ at 350 keV, 76% at 600 keV, and 45% at 900 keV. This suggests
 553 that for energies below ~ 850 keV precipitation is the dominant cause for the observed rapid
 554 electron dropout, while another mechanism is the primary cause at higher energy.

555 7 Discussion and Conclusions

556 In this study, we used a drift-diffusion model, that includes azimuthal drift, pitch angle
 557 diffusion and atmospheric backscatter, to simulate a rapid electron dropout event at $L = 4.5$ on
 558 August 21st, 2013. Additionally, we used angular and energy response functions for the MEPED
 559 detector onboard POES/MetOp to accurately simulate electron data for comparison with observed
 560 count rates (Selesnick et al., 2020; Yando et al., 2011). Low-altitude data obtained from a
 561 constellation of five POES/MetOp satellites is used to constrain the evolution of the electron
 562 distribution and pitch angle diffusion rates during this event and the contribution of precipitation
 563 to the total loss is determined. We found that during storm-time, pitch angle diffusion was
 564 significantly faster for lower electrons energies. Additional, or external, loss effects were
 565 introduced, which were significant especially at higher energy from 06-09 UT, since without it,

566 low-altitude electron data could not be reproduced by the model from pitch angle diffusion alone.
567 Ultimately, the model determined that precipitation contributed 100% to the total observed loss of
568 350 keV electrons, as observed from high altitudes, 76% for 600 keV, and 45% for 900 keV, so
569 that precipitation was the dominant loss mechanism for electron energies below ~ 850 keV. For the
570 electron population with energies exceeding ~ 1 MeV, the E3 and P6 data provided insufficient
571 constraint, resulting in discrepancies between the model and observed high-altitude data.

572 The energy dependence and the rate of pitch angle diffusion as quantified by the model
573 appears to be consistent with plasmaspheric hiss waves (e.g., Ni et al., 2014) which could serve as
574 the underlying mechanism responsible for the precipitation loss. Statistical studies show that
575 plasmaspheric hiss waves are often observed within and near the plasmopause location (see e.g.,
576 Thorne, 2010 and references therein). An empirical plasmopause model (Liu et al., 2015) shows
577 that, during this event, the plasmopause partially encloses the $L = 4.5$ region, favoring the
578 plasmaspheric hiss interpretation. Chorus waves can also share a similar energy dependence and
579 scattering rate with plasmaspheric hiss and could serve as another possible mechanism for rapid
580 pitch angle scattering of low energy electrons (e.g., Li et al., 2014). Other contenders such as
581 electromagnetic ion cyclotron waves are generally efficient at scattering electrons at highly
582 relativistic energies, which is inconsistent with our model predictions (e.g., Millan et al., 2007;
583 Miyoshi et al., 2008). However, the modeling approach used in this work has quantified pitch
584 angle diffusion rates without relying on accurate wave measurements.

585 External loss rates determined by the model showed faster loss for higher energy. While
586 the precise mechanism is not specified in the model, the energy dependence as quantified by the
587 model is consistent with loss due to magnetopause shadowing in combination with outward radial
588 diffusion (see e.g., Fei et al., 2006; Tu et al., 2019).

589 While the model provides good agreement with low-altitude data and moderate agreement
590 with high-altitude data, there are several areas of possible improvement. As discussed in section
591 6, the energy constraint provided by E3 and P6 are insufficiently sensitive to highly relativistic
592 electrons. Additionally, the energy dependence of the diffusion coefficient, external source/loss,
593 and atmospheric backscatter ratio (F_b) were crudely modeled. Here, we use a reasonable
594 approximation to backscatter given that we also approximate the loss cone distribution with a
595 bounce-averaged model. To correctly model the loss cone distribution and their interactions with
596 the atmosphere would require a more sophisticated Monte-Carlo approach and a non-bounce
597 averaged model, as done in Selesnick et al. (2004). Nonetheless, this was not performed here as it
598 adds significant complexity to the model.

599 Although our simulation focused on a single dropout event, we have demonstrated the
600 effectiveness of a data-drive model for estimating pitch angle diffusion rates without the need for
601 plasma wave measurements. Further model improvements could be made by the inclusion of other
602 low-altitude satellites to improve longitudinal coverage and model cadence. The inclusion of high-
603 altitude data from multiple satellites, as that used for the initial condition, could provide more
604 comprehensive constraints on the model across all pitch angles. However, the model already has
605 distinguished precipitation loss from the total loss observed during an electron dropout event,
606 addressing an unresolved question in radiation belt dynamics.

607 **Acknowledgments**

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609 80NSSC21K2008, DOE grant DE-SC0020294, and NSF Grant AGS 1752736.

610 **Open Research**

611 POES and MetOp SEM-2 data used in this are available online
612 (<https://www.ncei.noaa.gov/data/poes-metop-space-environment-monitor/>). Van Allen Probes
613 MagEIS and REPT data are available online
614 (<https://spdf.gsfc.nasa.gov/pub/data/rbsp/rbspa/13/ect/>). NASA's OMNIWeb data is available at
615 (<https://omniweb.gsfc.nasa.gov/>). The model results of the electron distribution function and
616 quantified model parameters per each simulation interval used in this study are also available
617 online (<https://doi.org/10.5281/zenodo.8436768>).

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14 Abstract

15 A drift-diffusion model is used to simulate the low-altitude electron distribution,
16 accounting for azimuthal drift, pitch angle diffusion, and atmospheric backscattering effects during
17 a rapid electron dropout event on August 21st, 2013, at $L = 4.5$. Additional external loss effects
18 are introduced during times when the low-altitude electron distribution cannot be reproduced by
19 diffusion alone. The model utilizes low-altitude electron count rate data from five POES/MetOp
20 satellites to quantify pitch angle diffusion rates. Low-altitude data provides critical constraint on
21 the model because it includes the drift loss cone region where the electron distribution in longitude
22 is highly dependent on the balance between azimuthal drift and pitch angle diffusion. Furthermore,
23 a newly derived angular response function for the detectors onboard POES/MetOp is employed to
24 accurately incorporate the bounce loss cone measurements, which have been previously
25 contaminated by electrons from outside the nominal field-of-view. While constrained by low-
26 altitude data, the model also shows reasonable agreement with high-altitude data. Pitch angle
27 diffusion rates during the event are quantified and are faster at lower energies. Precipitation is
28 determined to account for all of the total loss observed for 350 keV electrons, 76% for 600 keV
29 and 45% for 900 keV. Predictions made in the MeV range are deemed unreliable as the integral
30 energy channels E3 and P6 fail to provide the necessary constraint at relativistic energies.

31 1. Introduction

32 The dynamics of the outer terrestrial radiation belt can be exceedingly variable and is
33 instigated ultimately by solar activity. The population of the outer belt is dominated by energetic
34 (> 0.5 MeV) electrons and extends from approximately 3 to 7 Earth radii (R_E) (Van Allen et al.,
35 1958; Ganushkina et al., 2011). The presence of these MeV electrons heavily influences the
36 operations of near-Earth satellites and can cause catastrophic damage as they penetrate deep into
37 sensitive electronics (Baker, 2000). Furthermore, energetic electrons which precipitate from the
38 radiation belts into the Earth's atmosphere have been found to increase production of nitrogen
39 oxides (NO_x), leading to rapid depletion of the ozone layer (e.g., Fytterer et al., 2015; Meraner &
40 Schmidt, 2018), and enhanced ionospheric conductance (e.g., Robinson et al., 1987; Yu et al.,
41 2018).

42 It has been well known, especially recently with data from the Van Allen Probes (VAP),
43 that relativistic electron flux within the radiation belts can decrease rapidly on orders of
44 magnitudes within only a few hours (see e.g., Shprits et al., 2012; Baker et al., 2016). However,
45 the mechanisms responsible for these so-called rapid electron dropout events are not well
46 understood. Magnetopause shadowing in combination with radial diffusion provides an efficient
47 mechanism responsible for fast electron loss, as they funnel previously trapped electrons towards
48 the magnetopause where they are subsequently lost traversing open field lines (see e.g., Li et al.,
49 1997; Shprits et al., 2006a; Turner et al., 2012; Tu et al., 2019). However, at the heart of outer
50 radiation belts ($L \approx 4$), it is believed that precipitation of electrons onto the terrestrial atmosphere
51 via pitch angle diffusion serves as the dominant mechanism responsible for electron dropouts
52 (Selesnick, 2006; Millan et al., 2010; Turner et al., 2014). At high altitudes, where Coulomb
53 collision is insignificant, stochastic pitch angle scattering of relativistic electron is due to resonant
54 wave-particle interactions with the electromagnetic waves excited by plasma instabilities such as
55 chorus, plasmaspheric hiss, and electromagnetic ion cyclotron (see e.g., Thorne, 2010 and
56 references therein). The pitch angles of trapped electrons can eventually be scattered into the loss

57 cone, a range of pitch angles with mirror points within the dense atmosphere, where they then
58 subsequently precipitate.

59 Early attempts to quantify pitch angle diffusion rates were made by Kennel & Petschek
60 (1966), Thorne & Kennel (1971), and Lyons et al., (1972). However, without comprehensive
61 information about the wave's spectral properties, cold plasma densities, or ion composition, these
62 estimates were hampered by uncertainties and broad assumptions. Currently, estimated electron
63 loss rates from pitch angle scattering still diverge greatly. For example, Shprits et al. (2005)
64 employed electron lifetimes in their model of 10 days within the plasmasphere and 0.7 to 4 days
65 outside the plasmasphere, depending on the K_p index. Comparatively, Barker et al. (2005) used a
66 L dependent electron lifetime ranging from 3 days at $L = 6$ to 29 days at $L = 4$. Modeling work
67 done by Tu et al. (2010) estimated relativistic electron lifetimes to vary from 0.01 days to 10 days
68 during magnetic storm events. These conflicting estimates underscore the persisting ambiguity
69 surrounding pitch angle diffusion rates and their driving mechanisms. Accurate quantification of
70 electron pitch angle diffusion and loss rates would be of value as they are ubiquitous in
71 comprehensive radiation belt models and are also needed to reliably assess electron acceleration
72 processes.

73 Theoretically, the non-dipolar nature of the terrestrial magnetic field causes nonuniform
74 electron loss in magnetic longitude, where the resulting redistribution due to azimuthal drift
75 provides the necessary low-altitude observational constraint to infer pitch angle diffusion rates.
76 This approach was validated by Imhof (1968) where an approximate analytical solution to the
77 diffusion equation showed reasonable agreement with data from low-altitude satellites. Detailed
78 simulations were done by Abel & Throne (1999) further confirming the significant role of both
79 pitch angle diffusion and azimuthal drift in the loss of radiation belt electrons, but however, was
80 not compared to observational data. Subsequently, Selesnick et al., (2003) developed a parametric
81 model, accounting for azimuthal drift and pitch angle diffusion, and used low-altitude SAMPEX
82 satellite data to constrain the rate of pitch angle diffusion within the model. The model was capable
83 of predicting temporal and spatial variation of pitch angle diffusion without the need for global
84 wave measurements, which led to multiple subsequent works (including this one, see also e.g.,
85 Selesnick, 2006; Tu et al., 2010). However, the temporal resolution provided by the model is
86 significantly reduced by the limited availability of data from just one satellite, enabling a cadence
87 as low as only half a day. Magnetospheric conditions during storm time can vary much more
88 rapidly, leading to misleading results, or missing storm-time dynamics. This issue was addressed
89 by Pham et al. (2017) by using an extensive set of low-altitude data obtained from a more recent
90 satellite constellation composed of Polar Orbiting Environmental Satellites (POES) operated by
91 the National Oceanic and Atmospheric Administration (NOAA) and the Meteorological
92 Operational Satellites (MetOp) operated by the European Organization for the Exploitation of
93 Meteorological Satellites (EUMETSAT). Five POES/MetOp satellites were online during their
94 event study, which provided their model with an improved 3-hour cadence.

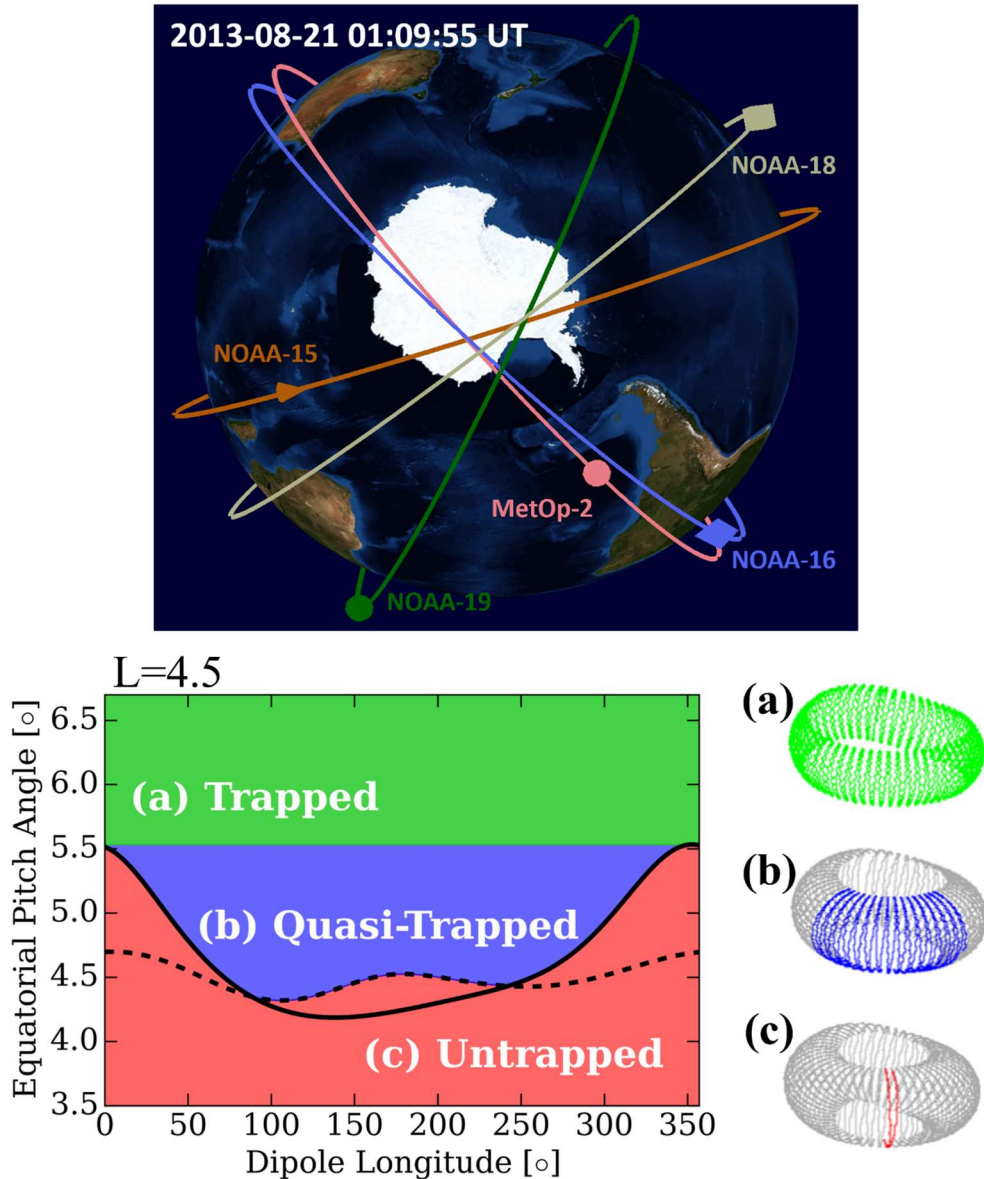
95 An accurate angular response function for detectors is required under weak pitch angle
96 diffusion, where the loss cone is relatively empty. This is because trapped electron flux outside of
97 the loss cone are typically several orders of magnitude higher, where any measurements taken
98 inside the loss cone are dominated by the trapped electrons outside of the field-of-view (FOV).
99 While the bounce-averaged models used in Pham et al. (2017) and preceding works are capable in
100 approximating the loss cone flux under both weak and strong diffusion scenarios, there did not
101 exist any reliable loss cone measurements to constrain the model evolution in such regions.

102 Consequently, their models assumed an empty loss cone, and so their results were valid in only
103 the weak diffusion limit. Selesnick et al. (2020) derived a more accurate angular response function
104 for the detectors onboard POES/MetOp, accounting for the response to particles from all incident
105 angles. By incorporating this new angular response function, POES/MetOp's loss cone
106 measurements can provide a more accurate constrain for model flux inside the loss cone region.

107 In this work, we use an updated drift-diffusion model to dynamically simulate the
108 distribution of low-altitude electrons and quantify the pitch angle diffusion rates during a rapid
109 electron dropout event at $L = 4.5$ on August 21st, 2013. The newly-derived angular response
110 function is used in combination with the previously established energy response function (Yando
111 et al., 2011) to convert the simulated electron distribution function to count rates which are then
112 compared to the observed count rates from POES/MetOp. The model includes the dominant effects
113 of pitch angle diffusion and azimuthal drift as experienced by low-altitude electrons and, in
114 addition, atmospheric backscattering effects of electrons within the loss cone. The model
115 parameters, including pitch angle diffusion rates, are determined by fitting simulated POES/MetOp
116 count rate data to observations. Notably, this approach does not rely on wave measurements. Its
117 results could potentially be used to constrain the distribution of waves that are responsible for the
118 precipitation loss, though we do not attempt that here. Furthermore, with the availability of VAP
119 data, additional insight on the model results can be obtained by comparisons with high-altitude
120 data, which was not done in previous works related to this model. Section 2 provides an overview
121 of low-altitude electron data and their significance in determining pitch angle diffusion rates.
122 Section 3 details the model, including the initial conditions obtained from VAP, while section 4
123 describes the dropout event. Section 5 presents the model results followed finally by comparisons
124 with high-altitude data, conclusion and further discussions in sections 6, and 7 respectively.

125 **2. Low-altitude Electron and Trapping Regions**

126 The distribution of electrons mirroring near and within the upper boundary of the terrestrial
127 atmosphere can be categorized into three distinct groups. Aside from electrons that mirror well
128 above or deeply within the atmosphere, which are categorized as the *trapped* and *untrapped*
129 electron populations, due to non-dipolar nature of the terrestrial magnetic field, there arises an
130 intermediate category called the *quasi-trapped*, where particles can mirror above or within the
131 atmosphere depending on their location in magnetic longitude. This is because, near the surface,
132 the higher-order magnetic multipoles asymmetrically determine the elevation of the particle's
133 mirror point in relation to the 100 km altitude threshold – upper boundary layer of the Earth's
134 atmosphere. So hence, the strength of magnetic field at 100 km altitude divides the low-altitude
135 electrons into these three categories.



136
 137 **Figure 1:** Top panel shows the orbits of 5 POES/MetOp satellites on August 21, 2013, viewed
 138 from above Antarctica. Bottom left panel shows the partitioned regions of trapped, quasi-trapped,
 139 and untrapped electrons based on their equatorial pitch angles at $L = 4.5$. The dashed and solid
 140 black line represents the northern and southern hemisphere bounce loss cones respectively. The
 141 highlighted path on the bottom right tori exemplifies the trajectories of a particle in each category.
 142 Bottom panels are adapted from Tu et al., (2010).

143 Figure 1, bottom left panel, shows the variation of equatorial north (dashed curve) and
 144 south (solid curve) loss cones in magnetic longitude (hereinafter referred to as just longitude) at a
 145 constant McIlwain L-shell value (hereinafter referred to as L , see e.g., McIlwain, 1961). The
 146 particle will attempt to mirror across both hemispheres, so therefore, the larger of the two angles
 147 at each longitude governs whether a particle will mirror below the atmosphere within a single
 148 bounce period. This is known as the bounce loss cone (BLC, red region in Figure 1) and the

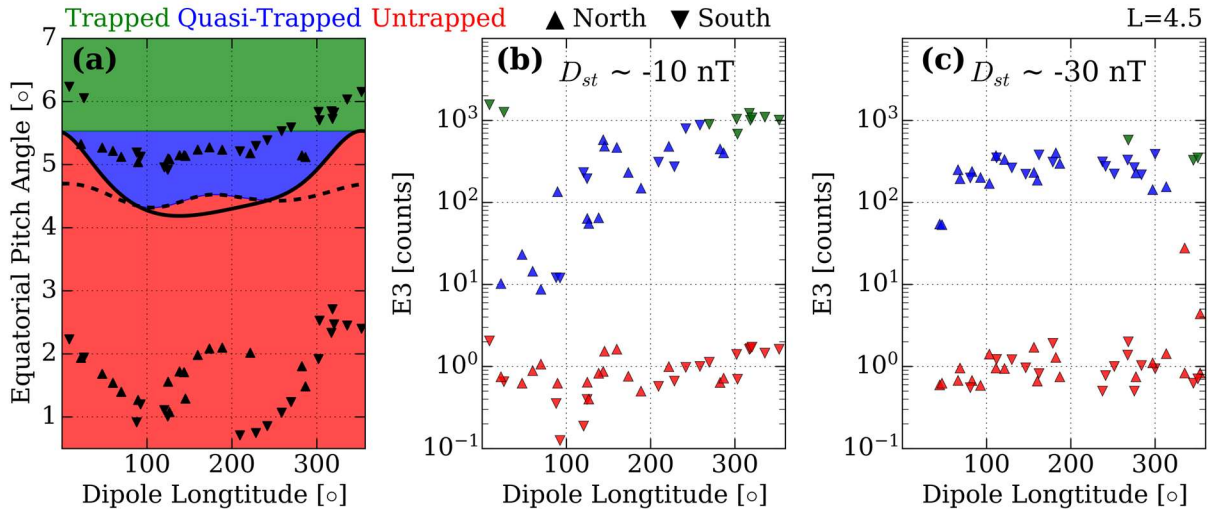
149 untrapped electron population pertains equatorial pitch angles below the BLC curve, since it is
150 likely that they precipitate within a singular bounce.

151 Extending this idea, the quasi-trapped population of electrons exists with certain pitch
152 angles who mirror above the atmosphere at their local longitude but eventually drift into the BLC
153 at another longitude and precipitate. Consequently, most of these electrons are lost near $\sim 300^\circ$
154 longitude to the southern loss cone. This results in enhanced electron precipitation over the south
155 Atlantic and is known as the South Atlantic Anomaly (SAA). The region in momentum space
156 populated by the quasi-trapped electrons constitutes the drift loss cone (DLC, blue region in Figure
157 1), where the boundaries are defined by the maximum value of the BLC in longitude and the BLC
158 itself. Lastly for completeness, the previously mentioned trapped population (green region in
159 Figure 1) are those outside of the DLC and therefore remain trapped within the radiation belts
160 indefinitely.

161 The distribution of electrons within these three categories serves as a good constraint on
162 the rate of pitch angle scattering and electron loss rate within the radiation belts. Count rate data
163 of the low-altitude electrons are obtained from the Medium Energy Proton and Electron Detector
164 (MEPED) which is a part of the Space Environment Monitor 2 (SEM-2) instrument suite (Evans
165 & Greer, 2000). SEM-2/MEPED is flown onboard POES/MetOp. These satellites are in a Sun-
166 synchronous polar orbit at an approximate altitude and period of 800 km and 1.7 hours
167 respectively. Figure 1, top panel, shows the orbits of the 5 POES/MetOp satellites operational
168 during the August 21, 2013, event, with level 2 (16 seconds averaged count rates) data available.
169 The combination of Earth's rotation with the spread in local-time of these 5 satellites provides
170 good coverage of all geographic longitudes within ~ 3 hours.

171 The MEPED detector is described in detail by Evans and Greer (2000). Its entrance
172 aperture has a 15° half-angle nominal field of view (FOV) to incoming particles. Local pitch angles
173 are derived from the IGRF magnetic field model since POES/MetOp does not carry a
174 magnetometer onboard (Finlay et al., 2010; Green, 2013). The electron telescope measuring count
175 rate contains a single silicon detector and three integral energy channels labeled E1, E2, E3
176 corresponding to energies > 30 , > 100 , and > 300 keV respectively. The proton telescope
177 contains two silicon detectors and six integral energy channels P1 to P6 with defined energy
178 ranges. Additionally, P6 can serve as an electron channel, responding to electrons ≥ 700 keV
179 when proton intensities are not too high (Yando et al. 2011). For our interests in energetic electron
180 dropouts, E3 and P6 will be of interest to us for this study as they correspond more closely to
181 radiation belt electron energies. Contamination from protons when using P6 as an electron channel
182 can be an issue, however proton intensities are typically low in the outer belt. Furthermore, proton
183 intensities are verified to be low during this particular event via P5 which is insensitive to electrons
184 (Green, 2013).

185 The nominal values of energy response and FOV can serve as a good indicator of which
186 electron population POES/MetOp are surveying. However, Selesnick et al. (2020) demonstrated
187 that E3 and, in particular, P6 data can be dominated by electrons from outside the nominal FOV.
188 Our model thus uses the more comprehensive energy and angular response functions provided by
189 Yando et al. (2011) and Selesnick et al., (2020) respectively, and the details can be found therein.



190
 191 **Figure 2:** Panels (a) and (b) show equatorial pitch angle and E3 channel count rate measurements
 192 taken by POES/MetOp over a 3-hour period starting from 2013-08-21/00:00:00 UT. The upper
 193 and lower cluster of points on panel (a) are taken by the 90° and 0° telescope respectively. Panel
 194 (c) is taken over a 3-hour period starting from 2013-08-21/06:00:00 UT at a lower D_{st} level.

195 The MEPED detector onboard POES/MetOp has two telescopes oriented in orthogonal
 196 directions. At mid to higher latitudes (or equivalently $L \gtrsim 1.5$, since orbital altitude is constant),
 197 one of the telescopes points in a more magnetic field-aligned direction, while the other points
 198 perpendicular to the magnetic field. These have been appropriately named as the 0° and 90°
 199 telescope respectively. At these altitudes, the local BLC and DLC drastically widen, giving
 200 POES/MetOp an increased ability to distinguish between trapped, quasi-trapped and untrapped
 201 electron populations. Generally, the 90° telescope measures quasi-trapped electrons inside the
 202 DLC, except for near the SAA where it occasionally measures trapped electrons, while the
 203 0° measures exclusively untrapped electrons deep inside the BLC. As an example, Figure 2a shows
 204 the pitch angle measurements made by 5 POES/MetOp satellites interpolated to $L = 4.5$ over a 3-
 205 hour interval needed for sufficient longitudinal coverage. The local pitch angle measured is
 206 assumed to be from the center of the FOV and is mapped to the equator using IGRF-11. As stated
 207 earlier and seen in Figure 2a, the 90° telescope has excellent coverage of the quasi-trapped
 208 population in longitude with the occasional measurement of the trapped population, while on the
 209 other hand, the 0° telescope exclusively measures the untrapped electrons.

210 Figure 2a and 2b are correlated plots in the sense that they visualize the same 3-dimensional
 211 data but on 2 separate y-axes. Notably, each count rate point taken from the E3 channel presented
 212 on Figure 2b corresponds to a unique point on Figure 2a, indicating the associated pitch angle at
 213 the same longitude of the measurement. The minimum value of the Disturbance storm-time (D_{st})
 214 index over the 3-hour interval is -10 nT, suggesting low levels of pitch angle scattering, and this
 215 is reflected in the distribution of quasi-trapped count rates in longitude. The quasi-trapped
 216 population is derived from trapped electrons whose pitch angles have been scattered into the DLC.
 217 Hence, if the pitch angle scattering rate is comparable to the azimuthal drift speed of the electrons,
 218 the quasi-trapped flux should increase gradually as they drift towards larger longitudes.
 219 Subsequently, the increased quasi-trapped flux decreases as the electrons precipitate over the SAA
 220 which means the quasi-trapped count rates should attain its minimum near the region immediately

221 after. This pattern of steady increase and sudden decrease within the DLC is manifested by a strong
 222 gradient of increase in the quasi-trapped count rates (blue triangles) in longitude as seen in Figure
 223 2b. Trapped count rates (green triangles) are generally higher because they do not undergo periodic
 224 losses every drift orbit, while the untrapped count rates (red triangles) are nominally the lowest.

225 Figure 2c panel is in the same style as Figure 2b except its equatorial pitch angle
 226 measurements are not shown, and the data is taken at a different 3-hour interval with a minimum
 227 D_{st} value of -30 nT. Therefore, in this case and with reference to Figure 2b, the scattering rate is
 228 increased while azimuthal drift speed remains the same. When scattering rates dominate the drift
 229 speed, the shape of the quasi-trapped count rate distribution loses its longitudinal dependence as
 230 shown in Figure 2c and the quasi-trapped count rate values become comparable to the trapped over
 231 all longitudes. Moreover, the shape of the quasi-trapped distribution becomes relatively insensitive
 232 to further increases in pitch angle scattering rates. However, even higher rates of pitch angle
 233 scattering can still be inferred by the precipitation models provided the additional constraints from
 234 the untrapped measurements.

235 3. Model Description

236 3.1 Drift-Diffusion Model

237 Selesnick et al. (2003) demonstrated that the primary characteristics of the electron's phase
 238 space density (f) at low altitudes are governed by the azimuthal drift motion and pitch angle
 239 diffusion due to wave-particle interactions. Additionally, the effects of scattering and loss due to
 240 atmospheric interactions for electrons mirroring within the atmosphere can be approximated by
 241 strong pitch angle diffusion and rapid decay respectively which was found to work well in
 242 reproducing the loss cone electron measurements made by POES/MetOp's 0° telescope (Selesnick
 243 et al., 2020). All together, these main contributing effects to the electron dynamics at a particular
 244 L and energy can be described by the bounce-averaged model

$$245 \quad \frac{\partial f}{\partial t} + \omega_a \frac{\partial f}{\partial \phi} = \frac{1}{xT(y)} \frac{\partial}{\partial x} \left(xT(y) D_{xx} \frac{\partial f}{\partial x} \right) - \frac{4}{\tau_b} (1 - F_b) \Theta(x_c - x) f + \left. \frac{\partial f}{\partial t} \right|_{ext.}, \quad (1)$$

246 where $f = f(x, \phi, t)$ is the electron phase space density as a function of magnetic longitude ϕ ,
 247 time t , and $x = \cos \alpha_0$; α_0 is the equatorial pitch angle of the electron. ω_a and τ_b are the azimuthal
 248 drift frequency and bounce period of the electron evaluated under a dipole approximation. The
 249 $T(y)$ function is proportional to the electron bounce period and is approximated in a dipole field
 250 by $T(y) = 1.380173 - 0.639693y^{0.737}$, where $y = \sin(\alpha_0)$ (Davidson, 1976). x_c is the local
 251 (in ϕ) x value of the BLC. The BLC angle is determined by the maximum value between the loss
 252 cones in each hemisphere and is calculated using the IGRF-11 geomagnetic field model. Θ is the
 253 Heavyside step function which ensures that only electrons mirroring inside the atmosphere
 254 experiences atmospheric effects.

255 The advection term accounts for the eastwards azimuthal motion of the electrons due to
 256 gradient-curvature drift in the positive longitude direction (Roederer, 1970, p.27). The x diffusion
 257 term accounts for the wave-particle interactions that stochastically scatters the particles from their
 258 original pitch angles (Schultz & Lanzerotti, 1974, p.77). The decay term (equation (1) second term
 259 on the right) captures the loss of electrons through momentum-degrading collisions with

260 atmospheric particles where the decay rate is controlled by F_b , the ratio of electrons that are not
 261 lost but scattered back out of the atmosphere. Lastly, the third term on the right of equation (1)
 262 accounts for source or loss due to any other external mechanisms (e.g., local heating or
 263 magnetopause shadowing). It is assumed to be the form

$$264 \quad \left. \frac{\partial f}{\partial t} \right|_{ext.} = (S_0 \tilde{E}^{-\nu} - L_0 \tilde{E}^{-\eta}) \frac{g_1(x, \phi)}{p^2}, \quad (2)$$

265 where S_0 and L_0 (not to be confused with the McIlwain L shell parameter) are the external electron
 266 source and loss rates and are strictly positive. ν and η controls the energy dependance of these
 267 source and loss rates. S_0 , L_0 , ν , and η are free parameters to be determined by model fits to the
 268 low-altitude electron data. \tilde{E} is the particle energy normalized by 1 MeV and the electron's
 269 momentum p^2 is included in the denominator so that S_0 and L_0 have dimensions of intensity per
 270 unit time. g_1 is approximately the normalized first order eigenfunction of the drift-diffusion
 271 operator, derived numerically for when equation (1) exhibited steady decay in absence of external
 272 source/loss. This was chosen because all higher order terms of source/loss decay more rapidly and
 273 therefore are not well constrained by data. Furthermore, the functional form of g_1 helps reduce
 274 any artificial diffusion for when source/loss is introduced. Since the source and loss function is not
 275 well constrained, it is only turned on as a last resort if the model cannot reproduce the observed
 276 data via pitch angle diffusion alone.

277 Selesnick (2006), Tu et al. (2010), and Pham et al. (2017) all used a functional dependence
 278 of $D_{xx} \sim x^{-\sigma}$ with a similar drift-diffusion model and reported reasonably good agreements
 279 between model and low-altitude data. We chose a D_{xx} of the form

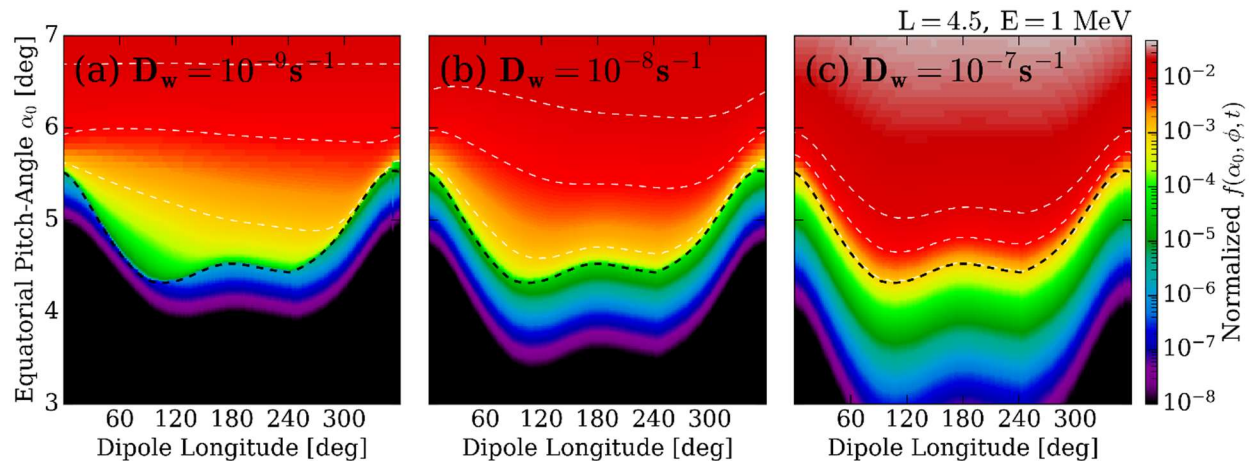
$$280 \quad D_{xx} = \frac{\tilde{E}^{-\mu} D_w}{10^{-4} + x^{20}} + D_b \Theta(x_c - x), \quad (3)$$

281 where we have found $\sigma = 20$ to work well with our event study. D_w includes all pitch angle
 282 diffusion through various wave-particle interactions and μ serves as the energy dependance of the
 283 diffusion activity. Both D_w and μ are free parameters determined by model fits to the low-altitude
 284 data. The factor of 10^{-4} solely serves to maintain numerical stability at low x (higher α_0) values.
 285 The form of this first term in equation (3) results in high diffusion for equatorial particles thus
 286 evolves f into a “flat-top” distribution (consistent with the VAP observations during storm time
 287 as will be discussed in Section 4). At low pitch angles, equation (3) approaches $D_w \tilde{E}^{-\mu}$ which will
 288 largely determine the electron loss rate (Shprits et al., 2006b).

289 F_b is a simplified description of the backscattered electrons that undergo mirroring below
 290 100 km altitude and subsequently re-emerge from the atmosphere. The random scattering in pitch-
 291 angle due to atmospheric collisions is capture in D_b of equation (3) as a diffusive process. The Θ
 292 Heavyside function ensures that this diffusive mechanism is limited to only the electrons within
 293 the BLC. F_b and D_b together dictates that the BLC electrons will undergo scattering of their pitch
 294 angle, and rapid decay due to momentum degrading collisions. The energy loss of the
 295 backscattering process is neglected since Selesnick et al. (2004) found the rate energy change per
 296 emergence out of the atmosphere is much less than the rate of decay itself. In our model, $F_b =$
 297 $0.1/\tilde{E}$ is assumed which dictates that only 10% of 1 MeV BLC electrons are backscattered by the

298 atmosphere, where the energy dependence is a simplified approximation based on the findings in
 299 Selesnick et al. (2004). The combination of F_b with $D_b = 10^{-7} \text{ s}^{-1}$ for all energies is found to
 300 match well with BLC electron measurements.

301 Equation (1) is solved numerically at energies from 300 to 1800 keV, spaced by 50 keV,
 302 using operator splitting to combine Crank-Nicholson method in x with first-order upwind scheme
 303 in ϕ (Press et al., 2007, p.1048). The boundary conditions in x are $\partial f / \partial x = 0$ at $x = 0$, and $x =$
 304 1. This would dictate no flow of electron currents in or out of the unphysical regions in x (or α_0).
 305 Periodic boundary conditions are applied in ϕ such that $f(\phi) = f(\phi + 2\pi)$.



306
 307 **Figure 3:** Example model solutions for a select value of energy and L with varying levels of pitch-
 308 angle diffusion. The color intensity plot is a normalized solution, and the dashed black line is the
 309 angle of the BLC. The dashed white lines are contours of the normalized f . The external
 310 source/loss term is turned off here.

311 Figure 3 shows sample solutions to equation (1) with increasing levels of pitch angle
 312 diffusion. They all have $\mu = 0$ so the diffusion coefficient becomes $D_{xx} = D_w$ at the low pitch
 313 angles near the DLC. From Figure 3, it is evident that the distributions of electrons at low altitudes
 314 are strongly dependent on the diffusion rates or, more specifically, the ratio D_{xx}/ω_d . In the case
 315 of Figure 3a, the distribution of electrons at the lower altitudes is azimuthal drift dominated where
 316 the slow rate of pitch angle diffusion that does not fill in the DLC at all longitudes. Furthermore,
 317 the solution above the DLC is largely unaffected by the shape of the BLC (shown by the lack of
 318 ϕ dependence in the white contour lines). Conversely, Figure 3c demonstrates a case where
 319 diffusion dominates. Here, the effect of azimuthal drift becomes largely irrelevant and the local
 320 solution at each longitude is primarily dictated by its pitch angle relative to the local BLC.
 321 Consequently, lines of constant f tightly hugs the BLC at all longitudes. The solutions of f above
 322 the DLC becomes more longitudinally dependent. Lastly, Figure 3b shows an intermediate case.

323 3.2 Initial Condition

324 The initial conditions for the model are derived from Van Allen Probes data. MagEIS
 325 (Magnetic Electron Ion Spectrometer, Blake et al., 2013) onboard VAP can provide measurements
 326 of electron energies from 30 keV up to 4 MeV, however, large uncertainties are associated with
 327 energies > 1 MeV due to the small geometric factor in its design (Blake et al., 2013). Therefore,
 328 only energy channels ranging from $\sim 240 - 904$ keV are utilized from MagEIS (energies below 240

329 keV are irrelevant since E3 channel of the MEPED detector responds to > 300 keV). Only two
 330 energy channels are of use to us from the REPT (Relativistic Electron-Proton Telescope, Baker et
 331 al., 2012) instrument, 1.8 and 2.4 MeV. This is because higher energies typically go undetected in
 332 integral energy channels like E3 and P6 due to their substantially lower intensities.

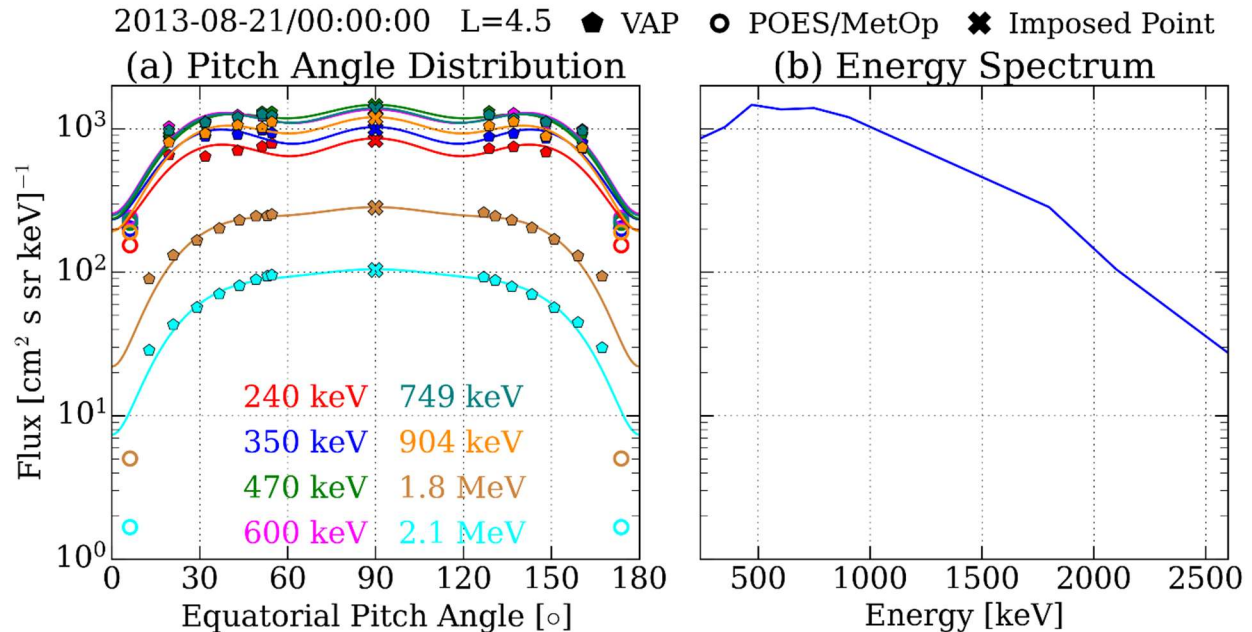
333 Since the VAP measurements cannot cover low pitch angles near the loss cone, we turned
 334 to POES/MetOp to supplement an additional data point around $\alpha_0 \approx 6^\circ$. This additional data point
 335 corresponds to the nearest trapped electron measurement in temporal proximity to the initial time,
 336 acquired from any of the 5 POES/MetOp satellites. The E3, and E4 integral flux data from
 337 POES/MetOp (Green, 2013) are used for this supplementary point, which correspond to energies
 338 > 287 and > 612 keV respectively. The initial condition requires us to find the differential
 339 (energy) flux from the integral flux provided by POES/MetOp, for which we use the energy
 340 spectrum obtained from VAP at the lowest available pitch angle measurement.

341 To obtain the pitch angle distribution (PAD) for all angles, the VAP's local pitch angle
 342 measurements are first mapped to the equator using the TS04D magnetic field model (Tsyganenko
 343 & Sitnov, 2005). Zhao et al., (2018) found using 6th order even Legendre polynomials to fit
 344 energetic electron PADs works well for $L > 4$, so a similar extrapolation technique is applied here:

$$345 \quad j_E(\alpha_0) = \sum_{n=0}^3 c_{2n} P_{2n}(\cos(\alpha_0)), \quad (4)$$

346 where P_{2n} are the even Legendre Polynomials, and c_{2n} are coefficients to be determined via best
 347 fit with data. Note that this is done for each available energy channel and j_E is the PAD for a single
 348 energy.

349 Lastly, during the initial time of this event, VAP was unable to make any measurements
 350 near $\alpha_0 = 90^\circ$. Considering that prior to the initial time high equatorial pitch angle measurements
 351 from VAP were available and show largely a flat PAD near 90° , we introduce an artificial point
 352 at $\alpha_0 = 90^\circ$ with a value 15% larger than the mean of the two closest available measurement to
 353 $\alpha_0 = 90^\circ$. In essence, we manually imposed a flat-top distribution for this initial time, as justified
 354 by observations. Equation (4) is then used to fit over the VAP pitch angle data, supplementary
 355 point provided by POES/MetOp, and the artificially imposed point at $\alpha_0 = 90^\circ$.



356
357 **Figure 4:** Panel (a) shows the initial PAD for the model obtained from VAP with supplementary
358 points from POES/MetOp at the lowest equatorial pitch angles. An additional point is imposed at
359 $\alpha_0 = 90^\circ$ to enforce a flat-top shape. Panel (b) shows the energy spectrum of the initial condition.

360 Figure 4a depicts the initial PAD obtained at each energy for the electron dropout event.
361 Figure 4b shows the energy spectrum, obtained from the flux values at $\alpha_0 = 90^\circ$. At lower energies
362 the Legendre polynomials exhibit multiple changes in concavity and is likely an artifact arising
363 from the limited MagEIS data available at high pitch angles. However, this does not pose a problem
364 because our diffusion coefficient effectively smooths out any gradients at high pitch angles. The
365 energy spectrum in Figure 4b exhibits a phenomenon known as the “bump-on-tail” distribution,
366 which is commonly caused by resonant wave-particle interactions with plasmaspheric hiss waves
367 for lower energy electrons (e.g., Zhao et al., 2019). This initial condition is simulated at a low
368 diffusion case ($D_w = 10^{-9} \text{ s}^{-1}$) for one drift period to initialize the DLC and BLC electron
369 distribution while largely leaving the initial trapped distribution unmodified. The one-drift-period
370 simulated distribution is then used as the initial condition for the event. The model simulates
371 f which is related to the differential flux by $j = p^2 f$.

372 3.3 MEPED Energy and Angular Response

373 The output of our model is the electron phase space density f for a particular L and energy
374 as a function of pitch angle cosine, longitude, and time. To provide constraint from low-altitude
375 electron data obtained from POES/MetOp (e.g., Figure 2b and 2c), the simulated f needs to be
376 converted into count rate. This is achieved by using the MEPED angular response functions
377 (Selesnick et al., 2020) in combination with the previously developed energy response function
378 (Yando et al., 2011). By virtually flying satellites through the simulation, we can replicate the
379 count rates that the detectors would have recorded based on the dynamic state of the electron
380 distribution.

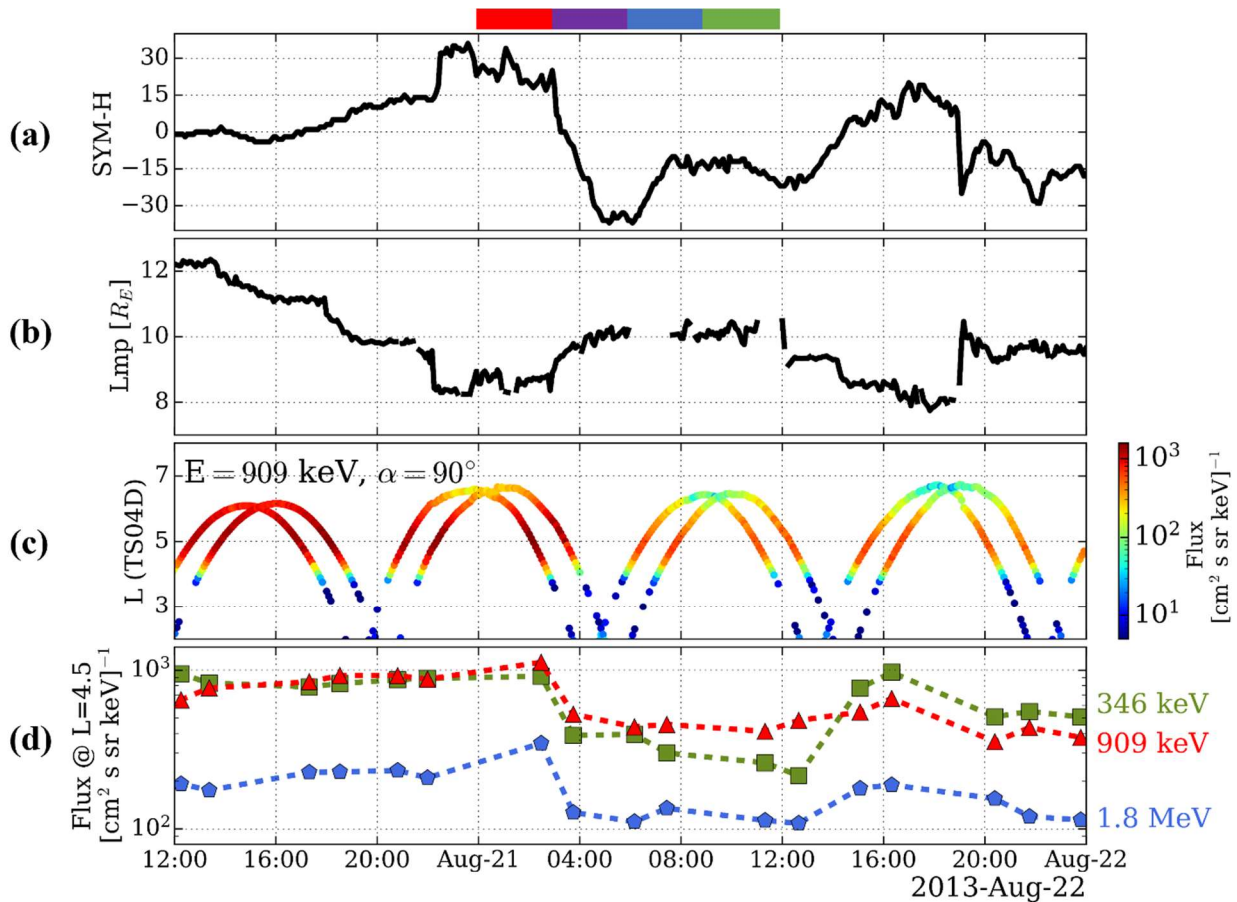
381 **3.4 Parametric Tuning**

382 The 2 (or 6 if external source/loss is turned on) free parameters per simulation interval are
 383 determined by fitting the simulated count rate to the observed count rate. For reasons detailed in
 384 section 2.2, a comprehensive longitudinal coverage of the count rate data is required to provide
 385 the necessary constraint on the pitch angle diffusion rates, and thus, the time duration of this
 386 dropout event is separated into four 3-hour intervals. The parameters are assumed to be constant
 387 within each interval, which can be invalid if magnetospheric conditions change rapidly. To
 388 mitigate this, the interval boundaries are picked by an orderly distribution of quasi-trapped count
 389 rates from the POES/MetOp data such that they tightly follow a general trend (see e.g., Selesnick
 390 et al., 2020, Figures 6, 7, and 8).

391 The parameters are iteratively tuned to minimize the metric:

$$392 \qquad K^2 = \sum_i^N [\log_{10}(d_i) - \log_{10}(p_i)]^2, \qquad (5)$$

393 where, N is the total number of data points, d_i are the observed count rates, and p_i are the simulated
 394 count rates. The Quasi-Newton method is used to iteratively find the set of parameters which
 395 achieves the minimum K^2 starting from an initial set of seed parameters (Tarantola, 2005, p.79).
 396 Constant background rates are estimated for each channel and is determined by the count rate floor
 397 reached by the 0° telescope during quiet times (excluding a few outliers which occasionally fall
 398 below). Count rates below the background rate are included in the model but excluded from the
 399 fit. Lastly, temporal continuity in the model is achieved by using the evolved f of a previous
 400 interval as the initial condition for the next interval, where the parameters are again obtained by
 401 recursive fit.

402 **4. August 2013 Dropout Event**

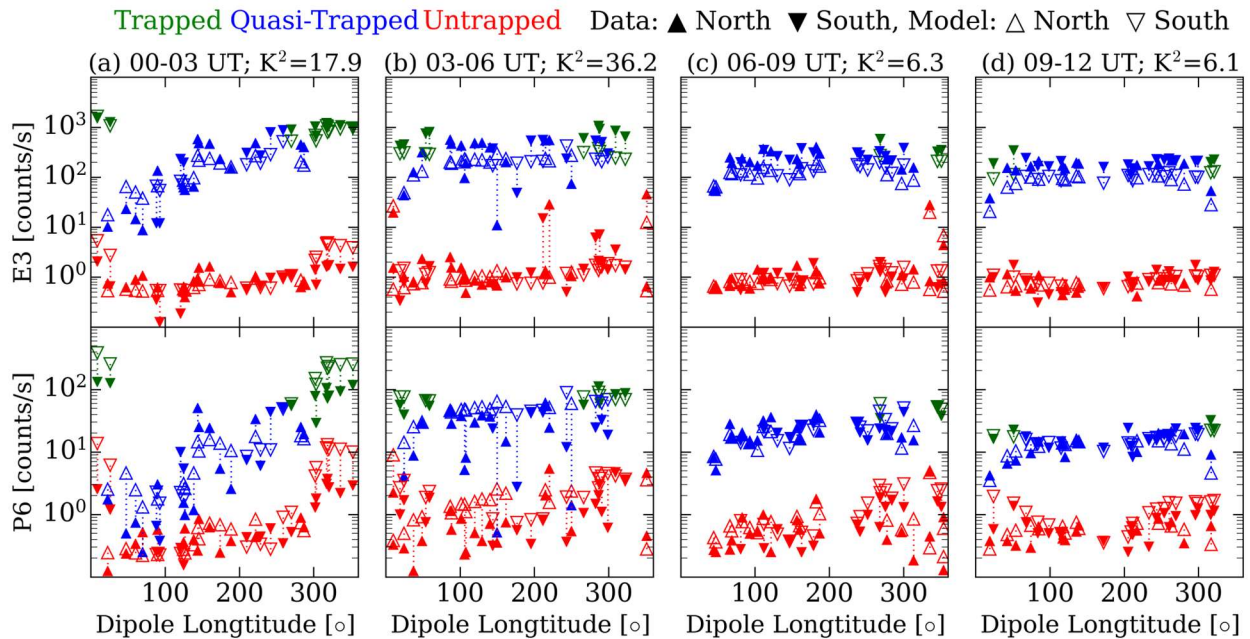
403
 404 **Figure 5:** Panel (a) shows the SYM-H and solar wind pressure variation during the event. Panel
 405 (b) shows the predicted dayside magnetopause location. Panel (c) shows the flux measurement for
 406 a selected energy and local pitch angle as measured by the VAPs along their orbit trajectory, and
 407 (d) shows the flux variation taken at only $L = 4.5$ for a few selected energies. The colored bars on
 408 top of panel (a) correspond to the interval periods of the four 3-hour intervals, starting at 00, 03,
 409 06, 09 UT of August 21st, 2013.

410 A rapid electron dropout was observed by VAP across a broad range of energies at $L = 4.5$
 411 on August 21st, 2013. Figure 5a shows the SYM-H index variation during this event, which attains
 412 a minimum of -37 nT. Despite this being a small storm, it has triggered a significant response in
 413 the radiation belt. Figure 5c shows 909 keV electron flux measurements from the MagEIS 90°
 414 local pitch angle bin along the VAP orbits. To better illustrate the flux drop, Figure 5d shows the
 415 90° observations interpolated to $L = 4.5$, for three selected energies, where the fluxes all decrease
 416 by approximately half an order of magnitude. The precise timing of this dropout is not well
 417 resolved, but it is evident that the dropout occurs within ~76 minutes, between the two inbound
 418 passes of VAP A & B just prior to 04 hours UT. Figure 5b shows that the dayside magnetopause
 419 location predicted by the Shue et al. (1998) model stays above 8 Earth radii during the entirety of
 420 the event. This suggests that the dropout is likely driven by precipitation resulting from pitch angle
 421 diffusion rather than magnetopause shadowing loss in combination with radial diffusion.

422 In this work, we are primarily interested in what happens during the dropout, so our
 423 simulation covers the quiet time just shortly before the dropout and continues until just before
 424 fluxes begin recovering to their pre-storm levels. The simulated time range for this event is from
 425 00 to 12 hours UT on August 21st, 2013. Between approximately 04-12 hours UT, the electron flux
 426 at $L = 4.5$ remain in a depleted state for a broad range of energies and there appears to be no
 427 obvious energy dependence in this dropout. This 12-hour simulation time domain is divided into
 428 four equal 3-hour intervals as marked by the colored bars above Figure 5a. Interval 1 (red) will
 429 quantify model parameters during a quiet time state of the radiation belts, while intervals 2, 3, and
 430 4 will quantify storm-time conditions. This dropout event is chosen for this study since it is a small
 431 storm during which the adiabatic effects on electron flux is insignificant (see e.g., Selesnick, 2006).

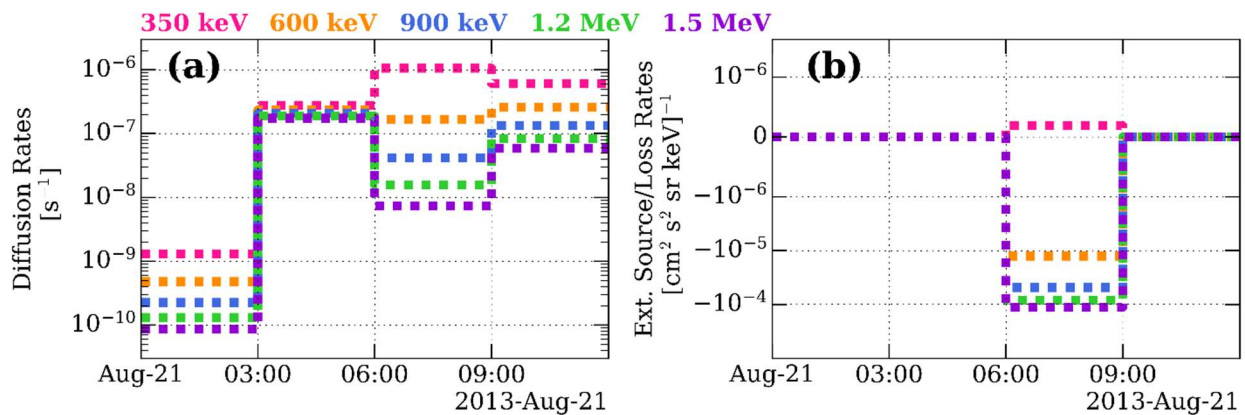
432 5 Model Results

433 Solid markers in Figure 6 show the observed count rates as a function of longitude during
 434 this event taken from the E3 and P6 channels for each of the four 3-hour intervals. Plotted on top
 435 with hollow points are the simulated count rates obtained by flying virtual satellites through the
 436 model. Both the model and data have been categorized based on their measurement taken outside
 437 of the DLC (trapped in green), within the DLC (quasi-trapped in blue) and within the BLC
 438 (untrapped in red) and the hemisphere which the measurement was taken at (upward vs. downward
 439 triangles). The 4 columns all together depict the temporal evolution of the observed and modeled
 440 electron dynamics for this event.



441
 442 **Figure 6:** Electron count rate data (solid triangles) from 5 POES/MetOp satellites interpolated at
 443 $L = 4.5$ on August 21st, 2013. Model count rate (hollow triangles) is shown with dashed lines
 444 connecting each simulated count rate to its corresponding data point. Column (a) is taken during a
 445 3-hour quiet pre-storm interval. Column (b) is during the initial storm main phase. Columns (c)
 446 and (d) are the subsequent hours during the main phase. The goodness of fit per each interval is
 447 provided in the title of each column.

448 Figure 6a depicts the quiet time 3-hour interval before the storm main phase. The quasi-
 449 trapped count rate data increase with longitude, indicating a slow rate of pitch angle diffusion, and
 450 was successfully captured by the model. However, our model slightly overestimates trapped count
 451 rates in the P6 channel during this interval for several possible reasons. First, the diffusion
 452 coefficient's functional form (Equation (3)), derived via observations at a different time, may not
 453 accurately represent the diffusion coefficient during the initial interval. Second, P6 is more
 454 sensitive to > 1 MeV electrons than E3, leading to greater manifestations of uncertainties
 455 associated with interpolating the initial energy spectrum across the 1-1.8 MeV gap, and the
 456 assumed energy spectrum at low pitch angles (as detailed in section 3.2). Third, cross-calibration
 457 between POES/MetOp and VAP is likely required for the initial condition but is not performed for
 458 this work. Finally, the lone data point from POES/MetOp at low pitch angles, especially at higher
 459 energies, for the initial condition was not captured by the fit (see Figure 4a). These inaccuracies
 460 during this interval are associated with higher order effects which are inconsequential and decay
 461 rapidly in later intervals when the diffusion rate increases. This is supported by the following
 462 interval (Figure 6b) when trapped count rates in P6 become reasonably well matched.



463 **Figure 7:** Evolution of the model parameters over the four 3-hour intervals for select energies.
 464 Panel (a) shows the pitch angle diffusion rates ($D_w \tilde{E}^{-\mu}$) and Panel (b) shows the source and loss
 465 rates ($S_0 \tilde{E}^{-\nu} - L_0 \tilde{E}^{-\eta}$) which are only non-zero during the third interval. Linear threshold for the
 466 symmetric logarithmic y-axes is set at 10^{-6} .
 467

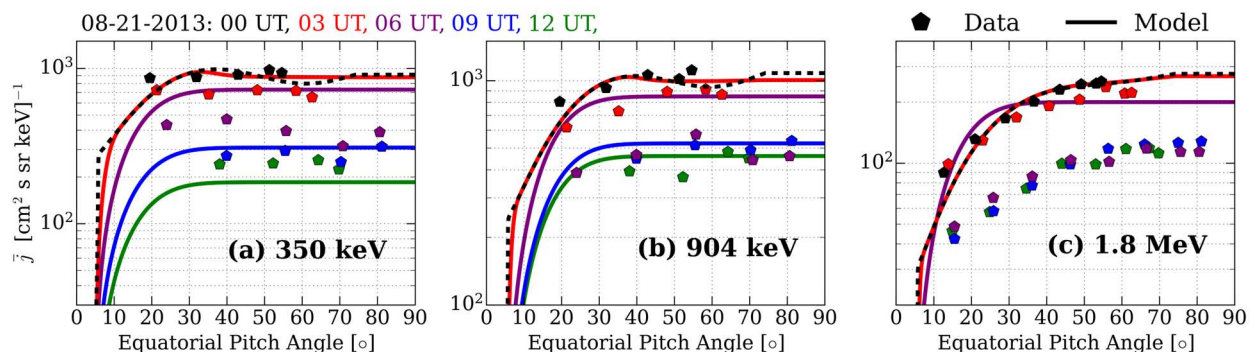
468 Figure 6b shows simulated and observed count rates for the second interval during the
 469 storm's initial main phase. The SYM-H index drops quickly during this interval, and the rapid
 470 changes in magnetospheric conditions could violate the constant parameter assumption imposed
 471 on our model. The scattered count rates in the quasi-trapped population show further indications
 472 of such violation. Despite that, the model still reproduced a substantial portion of the observed
 473 electron distribution, implying that the assumption of constant parameters is only weakly violated,
 474 and its impact remains minor. The scatter explains the relatively higher K^2 during this interval
 475 (provided in the title of each column in Figure 6). Furthermore, during this interval, the flat
 476 distribution exhibited by the quasi-trapped population indicates fast pitch angle diffusion which is
 477 reflected in both the model count rate, and the estimated rates of pitch angle diffusion (shown in
 478 Figure 7a). Figure 7a shows an increase in pitch angle diffusion for all energies by 2 to 3 orders of
 479 magnitude, reaching similar levels for all energies. Since the main trend of the data distribution is
 480 largely replicated by the model during the first two intervals, the external source/loss factor
 481 remains off up to the end of the second interval.

482 Figure 6c shows the model and observed count rates for the third interval. Here, the external
 483 source/loss function was enabled, because without it, the simulated untrapped count rates in the
 484 P6 channel were uniformly overestimated by an order of magnitude (not shown). Energy
 485 dependence of pitch angle diffusion can be identified in the data where the quasi-trapped count
 486 rate distribution is flatter as a function of longitude in the lower energy channel, E3, than the higher
 487 energy channel, P6. This feature was recognized by the model and higher pitch angle diffusion
 488 rates were obtained at lower energies as shown in Figure 7a. The energy dependence of the external
 489 loss during this interval is shown in Figure 7b, which indicates faster external loss at higher
 490 energies. For electron energies < 350 keV, the rates of external source/loss are negligible. This
 491 plays an important role in decreasing the energetic trapped and quasi-trapped electron count rates
 492 in the P6 channel without scattering electrons into the BLC, as required by the data.

493 Figure 6d depicts the final storm-time interval, and the data are again well reproduced by
 494 the model. The pitch angle diffusion rates determined by the model, Figure 7a, show a decrease in
 495 lower energies, but an increase at higher energy compared to the previous interval. The resulting
 496 pitch angle diffusion becomes nearly uniform again in energy, though higher at lower energy, and
 497 is still significantly faster than in quiet times. No external source/loss was required during this
 498 interval, and it was therefore turned off.

499 Notably, slight under prediction of the simulated count rates is present across all intervals
 500 compared to observed E3 count rates, however, this discrepancy is not significant enough to
 501 warrant the introduction of external source/loss mechanisms. Finally, the reasonable match
 502 between the model and observed trapped count rates for all four intervals, in both E3 and P6, shows
 503 that the model has successfully captured the electron flux decrease throughout the event.

504 6 Comparison with High-Altitude Data



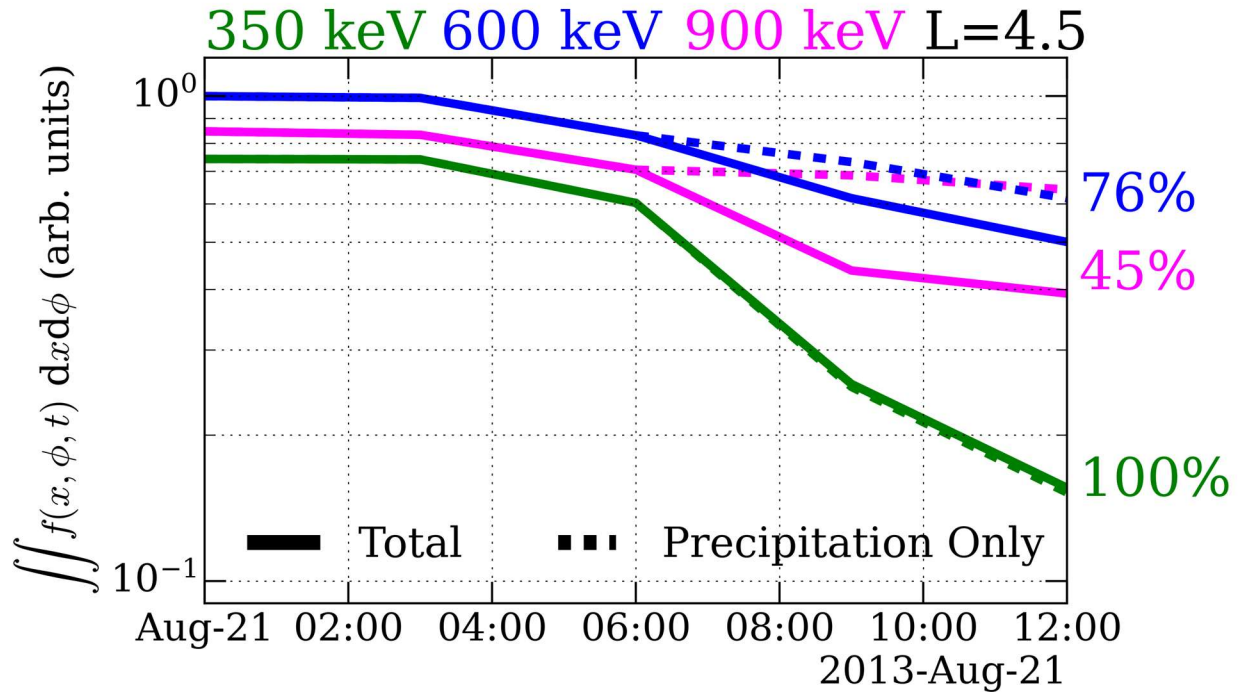
505 **Figure 8:** Drift averaged model solutions (lines) compared with VAP observations (pentagons) at
 506 $L = 4.5$ for different intervals (in different colors) for select energies. High-altitude data on panels
 507 (a) and (b) are taken from the MagEIS instrument, while panel (c) is from REPT. Dashed line
 508 corresponds to the model's initial condition.
 509

510 The successful reproduction of low-altitude electron distributions made by the model may
 511 not imply the same for high altitudes. The model's initial conditions were derived mainly using
 512 high-altitude data from VAP, but its evolution was exclusively guided by the low-altitude electron
 513 distribution. This raises the question: can a model constrained by low-altitude data effectively
 514 replicate the changes at higher altitudes?

515 Figure 8 answers this question, shown with VAP data taken at the end times of each 3-hour
516 interval (in pentagon symbols). The model solutions at the end of each interval are shown by the
517 colored solid curves, while the initial condition is reiterated by the black dashed line. The best fits
518 with low-altitude data, as quantified by K^2 , occurred in intervals 3 and 4. Correspondingly, Figures
519 8a and 8b show the best agreement between observations and the model at 09 UT (blue) and 12
520 UT (green), the end times of the 3rd and 4th interval, for electron energies below 1 MeV (Figures
521 8a and 8b). Interval 2 at low altitudes showed the worst fit between the model and observed count
522 rates which is likely due to the weak violation of the constant parameters assumption. As detailed
523 in Section 5, a few of the quasi-trapped data points during this interval (Figure 6b) deviate from
524 the general trend observed in the other quasi-trapped data points, which serves as an indicator of
525 fluctuating pitch angle diffusion rates. As a result, the match between model and observations
526 during this interval (in purple) is the poorest at high altitudes as demonstrated in Figures 8a and
527 8b. These transient dynamics are challenging for the model to fully capture. For this interval, our
528 quantifications of the model parameters can be interpreted as a 3-hour average.

529 Figure 8c compares model results with VAP observation for 1.8 MeV electrons. Notably,
530 the model predicts a complete extinction of electrons in the 3rd and 4th intervals (thus not appearing
531 in Figure 8c), contradicting observations. This discrepancy is attributed to the limited constraint at
532 relativistic energies provided by the POES/MetOp data. Both E3 and P6 are integral energy
533 channels responding primarily to the sub-1 MeV population and their sensitivity to higher energy
534 electrons is limited by the more abundant population at lower energy.

535 The model was successful in reproducing low-altitude electron distributions as observed
536 by POES/MetOp. Further comparisons with high-altitude data observed by VAP exclusively
537 reveal the discrepancies at relativistic energies. Nonetheless, our model agreed well at low and
538 high altitudes for energies below ~ 1 MeV, and hence, in this range, we can conclude that the
539 parameters quantified by our model are representative of actual magnetospheric conditions.



540
 541 **Figure 9:** The variation of total flux (presented in arbitrary units) in the system for each energy as
 542 a function of time. The percentages on the right represent the change in flux due to precipitation
 543 only.

544 Having established the model's domain of validity, we can now address our primary
 545 question: what is the contribution of precipitation to the observed electron dropout during this
 546 event? The change in total flux within the system (integrated over x or α_0 and ϕ) can be attributed
 547 either to atmospheric loss (precipitation) or to external source/loss mechanisms. Therefore, to
 548 isolate the loss due to precipitation, the total contribution of the external source/loss (Equation (2))
 549 is determined and then removed from the total flux to obtain the flux loss due to precipitation only.
 550 Figure 9 depicts the variation in total flux within the system in comparison to the variation of flux
 551 due to precipitation alone. The contribution of precipitation to the electron dropout progressively
 552 decreases with energy; it is $\sim 100\%$ at 350 keV, 76% at 600 keV, and 45% at 900 keV. This suggests
 553 that for energies below ~ 850 keV precipitation is the dominant cause for the observed rapid
 554 electron dropout, while another mechanism is the primary cause at higher energy.

555 7 Discussion and Conclusions

556 In this study, we used a drift-diffusion model, that includes azimuthal drift, pitch angle
 557 diffusion and atmospheric backscatter, to simulate a rapid electron dropout event at $L = 4.5$ on
 558 August 21st, 2013. Additionally, we used angular and energy response functions for the MEPED
 559 detector onboard POES/MetOp to accurately simulate electron data for comparison with observed
 560 count rates (Selesnick et al., 2020; Yando et al., 2011). Low-altitude data obtained from a
 561 constellation of five POES/MetOp satellites is used to constrain the evolution of the electron
 562 distribution and pitch angle diffusion rates during this event and the contribution of precipitation
 563 to the total loss is determined. We found that during storm-time, pitch angle diffusion was
 564 significantly faster for lower electrons energies. Additional, or external, loss effects were
 565 introduced, which were significant especially at higher energy from 06-09 UT, since without it,

566 low-altitude electron data could not be reproduced by the model from pitch angle diffusion alone.
567 Ultimately, the model determined that precipitation contributed 100% to the total observed loss of
568 350 keV electrons, as observed from high altitudes, 76% for 600 keV, and 45% for 900 keV, so
569 that precipitation was the dominant loss mechanism for electron energies below ~ 850 keV. For the
570 electron population with energies exceeding ~ 1 MeV, the E3 and P6 data provided insufficient
571 constraint, resulting in discrepancies between the model and observed high-altitude data.

572 The energy dependence and the rate of pitch angle diffusion as quantified by the model
573 appears to be consistent with plasmaspheric hiss waves (e.g., Ni et al., 2014) which could serve as
574 the underlying mechanism responsible for the precipitation loss. Statistical studies show that
575 plasmaspheric hiss waves are often observed within and near the plasmopause location (see e.g.,
576 Thorne, 2010 and references therein). An empirical plasmopause model (Liu et al., 2015) shows
577 that, during this event, the plasmopause partially encloses the $L = 4.5$ region, favoring the
578 plasmaspheric hiss interpretation. Chorus waves can also share a similar energy dependence and
579 scattering rate with plasmaspheric hiss and could serve as another possible mechanism for rapid
580 pitch angle scattering of low energy electrons (e.g., Li et al., 2014). Other contenders such as
581 electromagnetic ion cyclotron waves are generally efficient at scattering electrons at highly
582 relativistic energies, which is inconsistent with our model predictions (e.g., Millan et al., 2007;
583 Miyoshi et al., 2008). However, the modeling approach used in this work has quantified pitch
584 angle diffusion rates without relying on accurate wave measurements.

585 External loss rates determined by the model showed faster loss for higher energy. While
586 the precise mechanism is not specified in the model, the energy dependence as quantified by the
587 model is consistent with loss due to magnetopause shadowing in combination with outward radial
588 diffusion (see e.g., Fei et al., 2006; Tu et al., 2019).

589 While the model provides good agreement with low-altitude data and moderate agreement
590 with high-altitude data, there are several areas of possible improvement. As discussed in section
591 6, the energy constraint provided by E3 and P6 are insufficiently sensitive to highly relativistic
592 electrons. Additionally, the energy dependence of the diffusion coefficient, external source/loss,
593 and atmospheric backscatter ratio (F_b) were crudely modeled. Here, we use a reasonable
594 approximation to backscatter given that we also approximate the loss cone distribution with a
595 bounce-averaged model. To correctly model the loss cone distribution and their interactions with
596 the atmosphere would require a more sophisticated Monte-Carlo approach and a non-bounce
597 averaged model, as done in Selesnick et al. (2004). Nonetheless, this was not performed here as it
598 adds significant complexity to the model.

599 Although our simulation focused on a single dropout event, we have demonstrated the
600 effectiveness of a data-drive model for estimating pitch angle diffusion rates without the need for
601 plasma wave measurements. Further model improvements could be made by the inclusion of other
602 low-altitude satellites to improve longitudinal coverage and model cadence. The inclusion of high-
603 altitude data from multiple satellites, as that used for the initial condition, could provide more
604 comprehensive constraints on the model across all pitch angles. However, the model already has
605 distinguished precipitation loss from the total loss observed during an electron dropout event,
606 addressing an unresolved question in radiation belt dynamics.

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610 Open Research

611 POES and MetOp SEM-2 data used in this are available online
612 (<https://www.ncei.noaa.gov/data/poes-metop-space-environment-monitor/>). Van Allen Probes
613 MagEIS and REPT data are available online
614 (<https://spdf.gsfc.nasa.gov/pub/data/rbsp/rbspa/13/ect/>). NASA's OMNIWeb data is available at
615 (<https://omniweb.gsfc.nasa.gov/>). The model results of the electron distribution function and
616 quantified model parameters per each simulation interval used in this study are also available
617 online (<https://doi.org/10.5281/zenodo.8436768>).

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