



Fault isolation and fault-tolerant control for nonlinear stochastic distribution control systems with multiplicative faults

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Abstract

In this paper, a fault isolation, diagnosis and fault tolerant control algorithm is proposed for nonlinear multiple multiplicative faults stochastic distribution control systems employing Takagi–Sugeno fuzzy system. To obtain the detailed fault information, a fault detection algorithm is introduced to discover the fault occurrence time. Then a fault isolation observer is built to produce the residual, and the error system is separated to subsystems affected only by disturbance and multiplicative faults. Moreover, a fault estimation scheme is presented to obtain the fault magnitude information. When faults occur, the system output probability density function will deviate from the desired distribution. So the model predictive control fault tolerant control scheme is needed to minimize the impact of faults as much as possible to make sure that the post fault output probability density function track the desired probability density function. The validity of the designed algorithm is demonstrated through a simulation example, where the fault tolerant control algorithm ensures that the system output probability density function still track the given output probability density function despite the complex case of multiple multiplicative faults occurring simultaneously.

KEYWORDS

fault estimation, fault isolation, fault-tolerant control, model predictive control, multiplicative faults, nonlinear stochastic distribution control systems

1 | INTRODUCTION

Fault and disturbances are inescapable in control systems, there may be serious consequences when faults occur and cannot be resolved in time. When disturbances and faults coexist, there should be an algorithm to isolate the fault. From the perspective of modeling, it can be divided into multiplicative fault and additive fault. Over the past decades, additive fault had gained attention of researchers, but the challenge of multiplicative fault detection and isolation (FDI) had been a difficult one to be overcome.^{1–4} There were many achievements about additive faults. A fault diagnosis method was presented in Reference 5, where the algorithm distinguishes nonisolatable faults by changing the system's architecture. For small fault, a set of neural network-based fault isolation estimators were designed in Reference 6 for nonlinear uncertain systems to determine the location of fault occurrence. In Reference 4, an adaptive observer-based FDI

strategy was designed in active suspension systems of vehicle with faults and uncertainties. Generating residuals through FDI observer, a support vector machine-based evaluation method was introduced in Reference 7 to exhibit robust performance. For multiplicative fault, depending on the different modeling approach, parametric fault was sometimes seen as multiplicative faults too.⁸⁻¹⁰ In Reference 11, by modeling actuator fault as parameter variations, an adaptive Kalman filter was presented such that joint state parameter estimation can be brought together to diagnose fault. In more general cases, researchers considered multiplicative fault directly related to the actuator.^{12,13} In Reference 14, an improved FDI scheme combining the canonical correlation analysis method with the principal component analysis method was proposed to make it sensitive to multiplicative fault. In a situation where multiplicative faults of sensors and actuators occur at the same time, a FDI method based on sliding mode and linear observer was discussed in Reference 15, and an adaptive sliding mode fault diagnosis scheme was presented of Markov systems in Reference 16. However, almost all of the above literatures had only investigated one multiplicative fault and the case of multiple multiplicative faults occurring simultaneously was yet to be addressed.

Receiving inspiration from the fiber distribution in paper-making systems, professor Hong Wang developed the stochastic distribution control (SDC) systems theory, which had been applied to the flame distribution in actual boilers and the molecular weight distribution of polymerization reactions.¹⁷⁻²¹ The ultimate purpose of SDC systems is to enable the shape of system output probability density function (PDF) to track the target shape.^{20,22} The fault information is obtained by FDI and diagnosis algorithm, there needs to be a fault tolerant control (FTC) method alleviates the impact of fault to enable the output PDF to follow the desired PDF.²³ Integrated FDI and FTC algorithms for SDC systems subject to time delay were investigated in Reference 24. In Reference 25, an optimal FTC scheme was discussed for singular SDC systems affected by additive actuator fault. To time-delay collaborative SDC systems, a distributed fault diagnosis method with boundary conditions and a collaborative FTC law were proposed in Reference 26. For cases where the required PDF cannot be determined beforehand, the idea of entropy was brought in Reference 27. FTC strategies for singular SDC systems were performed in Reference 28 using iterative learning estimators. However, these methods can effectively compensate for additional fault, but were not applicable to the case with multiple multiplicative fault.

Nonlinearity is common in actual industrial systems. It is of major importance to research the FDI method for nonlinear systems.²⁹⁻³¹ In Reference 32, a FDI scheme was designed for discrete-time Lipschitz systems, in which an observer with more design parameters was designed to enhance the performance, and the H_2 and H_∞ metrics were adopted to characterize the fault susceptibility and interference robustness, respectively. The problem of active FTC algorithm of Lipschitz nonlinear multi-agent systems with actuator fault was studied based on the graph theory in Reference 33. For the case of simultaneous sensor and actuator faults, FDI strategies had been discussed in the context of uncertain nonlinear systems.³⁴ For systems with stronger nonlinearity, the Takagi–Sugeno (T-S) fuzzy model is the most powerful tool. In Reference 35, a new stability principle adopting the T-S fuzzy model was suggested to solve the system synchronized local stability and fault detection problem. A fault detection observer was constructed of T-S nonlinear systems in Reference 36, where faults and disturbances were considered in two different finite frequency domains and measured by two performance indices, respectively. In Reference 37, a fuzzy control solution was proposed for t-s nonlinear systems with unknown dynamics and actuator saturation. However, little research had been conducted on FDI for SDC systems and systems with multiplicative faults.

In this paper, a FDI and estimation algorithm is presented for nonlinear SDC systems. The main contributions are outlined below.

- (1) To isolate potential multiple multiplicative faults, an adaptive observer is given to gain the observation error system, then the error system is subsequently decomposed into the subsystems affected only by multiplicative faults and the subsystems affected only by nuisances.
- (2) An adaptive fault diagnosis algorithm is designed to obtain the amplitude information of multiplicative faults.
- (3) To alleviate the impact of multiplicative fault, a model predictive control (MPC) FTC scheme is suggested to certify the output PDF to track the expected PDF.

The remaining parts of the paper is arranged as follows. Section 2 demonstrates the system model, including PDF modeling process and T-S fuzzy system model. The fault detection algorithm is designed in Section 3. Section 4 displays the design process of fault isolation observer and fault diagnosis regulation law. The FTC scheme based on the MPC technique is presented in Section 5. A simulation example is displayed in Section 6 to validate the viability of the algorithm, and the conclusions in Section 7.

2 | SYSTEM MODEL DESCRIPTION

The output of SDC systems is define as $y(k) \in [a_1, a_2]$, where a_1 and a_2 are the up-down thresholds. The output PDF can be modeled as

$$\gamma(y, u(k)) = \sum_{s=1}^n v_s(u(k)) \phi_s(y), \quad (1)$$

where $v_s(u(k))$, ϕ_s , $s = 1, 2, \dots, n$ are dynamic weight related to control input and preset primary functions, respectively.

Owing to the restriction factor $\int_{a_1}^{a_2} \gamma(y, u(k)) dy = 1$, Equation (1) could be converted into

$$\gamma(y, u(k)) = \Delta(y)v(k) + R(y), \quad (2)$$

where $\Delta(y) = \left[\phi_1(y) - \frac{\phi_n(y)b_1}{b_n}, \phi_2(y) - \frac{\phi_n(y)b_2}{b_n}, \dots, \phi_{n-1}(y) - \frac{\phi_n(y)b_{n-1}}{b_n} \right]$, $b_i = \int_{a_1}^{a_2} \phi_i(y) dy$, $v(k) = [v_1(y), v_2(y), \dots, v_{n-1}(y)]^T$, $R(y) = \frac{\phi_n(y)}{b_n}$.

The T-S fuzzy model is a strong tool for settling the nonlinear system modeling problem, where the system is linked through if-then rules, and each rule leads to a linear subsystem. The i th rule is displayed below

$$\begin{aligned} & \text{IF } \xi_1(k) \text{ is } \mu_{i1}, \dots \text{ and } \xi_p(k) \text{ is } \mu_{ip}, \text{ Then} \\ & x(k+1) = A_i x(k) + B_i F u(k) + N_i \delta(k) \\ & v(k) = D x(k) \\ & \gamma(y, u(k)) = \Delta(y)v(k) + R(y), \end{aligned} \quad (3)$$

where $x(k) \in R^{n \times 1}$ and $u(k) \in R^{m \times 1}$ are the state and control input. A, B, D, N, G are system parameters matrices with suitable dimension, and $v(k)$ represents the output weight. $F = \text{diag} \{f_1, f_2, \dots, f_m\}$ is the possible actuator multiplicative fault. When no fault occurs, $F = I$, where I is an identity matrix. $\|\delta(k)\| < \omega_0$ is the bounded external disturbance. $\xi_i(k)$ ($i = 1, 2, \dots, p$) is prerequisite variables, μ_{ij} represents the fuzzy set, q is the number of rules.

The global fuzzy system is obtained through fuzzy fusion with the system.

$$\begin{aligned} x(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [A_i x(k) + B_i F u(k) + N_i \delta(k)] \\ v(k) &= D x(k) \\ \gamma(y, u(k)) &= \Delta(y)v(k) + R(y), \end{aligned} \quad (4)$$

where $h_i(\xi(k)) = \frac{\omega_i(\xi(k))}{\sum_{i=1}^q \omega_i(\xi(k))} \geq 0$, $\sum_{i=1}^q h_i(\xi(k)) = 1$, $\omega_i(\xi(k)) = \prod_{k=1}^p \mu_{ik}(\xi_k(k)) \geq 0$.

3 | FAULT DETECTION

To investigate the fault, a detection observer is constructed as follows

$$\begin{aligned} \hat{x}(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [A_i \hat{x}(k) + B_i u(k) + L_{1i} \varepsilon(k)] \\ \hat{v}(k) &= D \hat{x}(k) \\ \hat{\gamma}(y, u(k)) &= \Delta(y) \hat{v}(k) + R(y), \end{aligned} \quad (5)$$

where $\hat{x}(k)$, $\hat{v}(k)$, and $\hat{\gamma}(y, u(k))$ represent the estimation of the state, output weight and out PDF. $\varepsilon(k)$ represents the residual signal, L_1 represents the undetermined gain.

The state error is defined as

$$e(k) = \hat{x}(k) - x(k). \quad (6)$$

The residual signal could be derived as

$$\begin{aligned}\varepsilon(k) &= \int_{a_1}^{a_2} \sigma(y)(\hat{\gamma}(y, u(k)) - \gamma(y, u(k)))dy \\ &= \int_{a_1}^{a_2} \sigma(y)\Delta(y)dy(\hat{v}(k) - v(k)) \\ &= \Sigma D e(k),\end{aligned}\tag{7}$$

where $\Sigma = \int_{a_1}^{a_2} \sigma(y)\Delta(y)dy$.

The detection strategy is designed as

$$\begin{aligned}\|\varepsilon(k)\| &< \bar{\Lambda}, \text{ no fault occur} \\ \|\varepsilon(k)\| &\geq \bar{\Lambda}, \text{ fault occur,}\end{aligned}$$

where $\bar{\Lambda}$ is the detection threshold designed to confirm if a fault occurs.

Furthermore, the fault occurrence moment could be gotten

$$T_f = \inf \bigcup_{j=1}^r \{ \|\varepsilon(k)\| > \Delta_\varepsilon \}.\tag{8}$$

From systems (4) and (5), the error system could be gained as

$$\begin{aligned}e(k+1) &= \hat{x}(k+1) - x(k+1) \\ &= \sum_{i=1}^q h_i(\xi(k)) [A_i e(k) + B_i(I-F)u(k) - N_i \delta(k) + L_{1i} \varepsilon(k)] \\ &= \sum_{i=1}^q h_i(\xi(k)) [(A_i + L_{1i} \Sigma D) e(k) + B_i(I-F)u(k) - N_i \delta(k)].\end{aligned}\tag{9}$$

Theorem 1. For system (4) and the observer (5), the existence of positive definite symmetric (PDS) matrix P_1 and L_1 is required to guarantee the following linear matrix inequality (LMI) holds

$$\begin{bmatrix} \Psi_{11} & (A_i + L_{1i} \Sigma D)^T P_1 N_i \\ * & -N_i^T P_1 N_i \end{bmatrix} \leq 0,\tag{10}$$

afterwards the error system (9) is stable.

where $\Psi_{11} = (A_i + L_{1i} \Sigma D)^T P_1 (A_i + L_{1i} \Sigma D) - P_1$.

Proof. With no fault occurring, there has $F = I$. System (9) can be converted to

$$e(k+1) = \sum_{i=1}^q h_i(\xi(k)) [(A_i + L_{1i} \Sigma D) e(k) - N_i \delta(k)].\tag{11}$$

Select the Lyapunov function as

$$V_1(k) = e^T(k) P_1 e(k).\tag{12}$$

Thus, $V_1(k+1)$ could be obtained as

$$\begin{aligned}V_1(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [e^T(k) (A_i + L_{1i} \Sigma D)^T P_1 (A_i + L_{1i} \Sigma D) e(k) \\ &\quad + 2e^T(k) (A_i + L_{1i} \Sigma D)^T P_1 N_i \delta(k) - \delta^T(k) N_i^T P_1 N_i \delta(k)].\end{aligned}\tag{13}$$

Then the difference between two adjacent sampling points could be derived as

$$\begin{aligned}\Delta V_1 &= V_1(k+1) - V_1(k) \\ &= \sum_{i=1}^q h_i(\xi(k)) \left[e^T(k) \left[(A_i + L_{1i}\Sigma D)^T P_1 (A_i + L_{1i}\Sigma D) - \right. \right. \\ &\quad \left. \left. P_1 \right] e(k) + 2e^T(k)(A_i + L_{1i}\Sigma D)^T P_1 N_i \delta(k) - \delta^T(k) N_i^T P_1 N_i \delta(k) \right],\end{aligned}\quad (14)$$

where $q^T = [e^T(k) \quad \delta^T(k)]$,

$$\begin{aligned}\Psi &= \begin{bmatrix} \Psi_{11} & (A_i + L_{1i}\Sigma D)^T P_1 N_i \\ * & -N_i^T P_1 N_i \end{bmatrix}, \\ \Psi_{11} &= (A_i + L_{1i}\Sigma D)^T P_1 (A_i + L_{1i}\Sigma D) - P_1.\end{aligned}$$

Therefore, when LMI (10) holds, there has $\Delta V_1 < 0$, the error system (9) is stable. ■

4 | FAULT ISOLATION AND ESTIMATION

To isolate the fault, system (4) can be converted to

$$\begin{aligned}x(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [A_i x(k) + B_i u(k) + \sum_{j=1}^m b_j \theta_j^f(k) + N_i \delta(k)] \\ v(k) &= D x(k) \\ \gamma(y, u(k)) &= \Delta(y) v(k) + R(y),\end{aligned}\quad (15)$$

where $\theta_j^f(k) = (f_j - 1)u(k)$, $B = [b_1 \quad b_2 \quad \dots \quad b_m]$.

The fault isolation observer is constructed as

$$\begin{aligned}\hat{x}(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [A_i \hat{x}(k) + B_i u(k) + \sum_{j=1}^m b_j \hat{\theta}_j^f(k) + L_{2i} \varepsilon(k)] \\ \hat{v}(k) &= D \hat{x}(k) \\ \hat{\gamma}(y, u(k)) &= \Delta(y) \hat{v}(k) + R(y),\end{aligned}\quad (16)$$

where $\hat{x}(k)$, $\hat{v}(k)$ and $\hat{\gamma}(y, u(k))$ represent the estimation of the state, output weight and out PDF. $\varepsilon(k)$ represents the residual signal and L_{2i} represents the undetermined gain.

From Equations (15) to (16), the estimation error system can be gained below

$$\begin{aligned}e_1(k+1) &= x(k) - \hat{x}(k) \\ &= \sum_{i=1}^q h_i(\xi(k)) [(A_i - L_{2i}\Sigma D)e(k) + \sum_{j=1}^m b_j \tilde{\theta}_j^f(k) + N_i \delta(k)] \\ \varepsilon_1(k) &= \Sigma D e(k),\end{aligned}\quad (17)$$

where $\tilde{\theta}_j^f(k) = \theta_j^f(k) - \hat{\theta}_j^f(k)$.

To analyse the estimation error, the error system (17) is divided into two subsystems, that is

$$\begin{aligned}e_d(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [(A_i - L_{2i}\Sigma D)e(k) + N_i \delta(k)] \\ \varepsilon_d(k) &= \Sigma D e_d(k),\end{aligned}\quad (18)$$

and

$$\begin{aligned} e_{ff}(k+1) &= \sum_{i=1}^q h_i(\xi(k)) \left[(A_i - L_{2i}\Sigma D)e_{ff}(k) + b_j \tilde{\theta}_j^f(k) \right] \\ \varepsilon_{ff}(k) &= \Sigma D e_{ff}(k) \quad j = 1, 2, \dots, m, \end{aligned} \quad (19)$$

where $e_1(k) = e_d(k) + e_{ff}(k)$, $\varepsilon(k) = \varepsilon_d(k) + \varepsilon_{ff}(k)$.

The adaptive fault regulation law is constructed below

$$\hat{\theta}_j^f(k+1) = \sum_{i=1}^q h_i(\xi(k)) \left[\Gamma_i \varepsilon_{ff}(k) \right], \quad (20)$$

where Γ_i is the undetermined gain of proper dimension. The fault estimation error system could be constructed as

$$\tilde{\theta}_j^f(k+1) = \theta_j^f(k+1) - \sum_{i=1}^q h_i(\xi(k)) \left[\Gamma_i \Sigma D e_{ff}(k) \right]. \quad (21)$$

Theorem 2. *With the fault estimation algorithm (20), if there exist PDS matrix P_2 so that the following LMI holds*

$$\begin{bmatrix} -P_2 + \lambda I & 0 & \Omega_{13} & (\Gamma_i \Sigma D)^T \\ * & -I + \lambda I & b_j^T P_2 & 0 \\ * & * & -P_2 & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (22)$$

then the system (19) is stable.

where $\Omega_{13} = (A_i - L_{2i}\Sigma D)^T P_2$.

Proof. Choose following Lyapunov function

$$V_2(k) = e_{ff}(k)^T P_2 e_{ff}(k) + \tilde{\theta}_j^{fT}(k) \tilde{\theta}_j^f(k). \quad (23)$$

Then $V_2(k+1)$ could be derived as

$$\begin{aligned} V_2(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [e_{ff}(k)^T (A_i - L_{2i}\Sigma D)^T P_2 (A_i - L_{2i}\Sigma D) e_{ff}(k) \\ &\quad + 2e_{ff}(k)^T (A_i - L_{2i}\Sigma D)^T P_2 b_j \tilde{\theta}_j^f(k) + (b_j \tilde{\theta}_j^f(k))^T P_2 b_j \tilde{\theta}_j^f(k) \\ &\quad + \theta_j^{fT}(k+1) \theta_j^f(k+1) - 2e_{ff}(k)^T (\Gamma_i \Sigma D)^T \theta_j^f(k+1) + e_{ff}(k)^T (\Gamma_i \Sigma D)^T \Gamma_i \Sigma D e_{ff}(k)]. \end{aligned} \quad (24)$$

Thus, the difference between two adjacent sampling points can be derived as

$$\begin{aligned} \Delta V_2 &= V_2(k+1) - V_2(k) \\ &= \sum_{i=1}^q h_i(\xi(k)) \left[e_{ff}(k)^T ((A_i - L_{2i}\Sigma D)^T P_2 (A_i - L_{2i}\Sigma D) - P_2 \right. \\ &\quad \left. + (\Gamma_i \Sigma D)^T \Gamma_i \Sigma D) e_{ff}(k) + 2e_{ff}(k)^T (A_i - L_{2i}\Sigma D)^T P_2 b_j \tilde{\theta}_j^f(k) \right. \\ &\quad \left. + \theta_j^{fT}(k+1) \theta_j^f(k+1) + \theta_j^{fT}(k) (b_j^T P_2 b_j - I) \theta_j^f(k) - 2e_{ff}(k)^T (\Gamma_i \Sigma D)^T \theta_j^f(k+1) \right] \\ &= \begin{bmatrix} e_{ff} \\ \tilde{\theta}_i^f \end{bmatrix}^T \Omega \begin{bmatrix} e_{ff} \\ \tilde{\theta}_i^f \end{bmatrix} + \|M_f\|^2, \end{aligned} \quad (25)$$

where $\Omega = \begin{bmatrix} \Omega_{11} & (A_i - L_{2i}\Sigma D)^T P_2 b_j - (\Gamma_i \Sigma D)^T \\ * & b_j^T P_2 b_j - I \end{bmatrix}$, $\Omega_{11} = (A_i - L_{2i}\Sigma D)^T P_2 (A_i - L_{2i}\Sigma D) - P_2 + (\Gamma_i \Sigma D)^T \Gamma_i \Sigma D$.

Combining the famous Schur complement lemma, the follow equation can be obtained as

$$\Phi = \Omega + \lambda I = S_{11} - S_{12}S_{22}^{-1}S_{12}^T,$$

where

$$S_{11} = \begin{bmatrix} -P_2 + \lambda I & 0 \\ 0 & -I + \lambda I \end{bmatrix}, S_{12} = \begin{bmatrix} (A_i - L_{2i}\Sigma D)^T P_2 & (\Gamma_i \Sigma D)^T \\ b_f^T P_2 & 0 \end{bmatrix}, S_{22} = \begin{bmatrix} -P_2 & 0 \\ 0 & -I \end{bmatrix}.$$

It is obvious that Φ and LMI (22) is equivalent. Thus, when condition $\|e_{ff}\| > M_f \sqrt{\frac{1}{\lambda_{\min}(\Omega)}}$ holds, there has $\Delta V_2 \leq 0$, so the system (19) is stable. ■

5 | FAULT-TOLERANT CONTROL

Once the fault information is gathered using the FDI and estimation scheme, a compensating FTC scheme is presented adopting the MPC policy to minimize the impact of faults as much as possible and also to confirm that the output PDF track the expected PDF despite of multiple multiplicative faults occur. The expected PDF is given as

$$\gamma_g = \Delta(y)v_g + R(y), \quad (26)$$

where v_g is the expected weight.

Form system (4), the system at the previous moment can be obtained as

$$\begin{aligned} x(k) &= \sum_{i=1}^q h_i(\xi(k)) [A_i x(k-1) + B_i F u(k-1) + N_i \delta(k-1)] \\ v(k-1) &= D x(k-1) \\ \gamma(y, u(k-1)) &= \Delta(y)v(k-1) + R(y). \end{aligned} \quad (27)$$

The incremental system could be obtained as

$$\begin{aligned} \Delta x(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [A_i \Delta x(k) + B_i F \Delta u(k) + N_i \Delta \delta(k)] \\ v(k) &= D \Delta x(k) + v(k-1) \\ \gamma(y, u(k)) &= \Delta(y)(D \Delta x(k) + v(k-1)) + R(y), \end{aligned} \quad (28)$$

where $\Delta x(k+1) = x(k+1) - x(k)$, $\Delta u(k+1) = u(k+1) - u(k)$.

In MPC, the predictive and control time domain is given as t_1 and t_2 , with $t_2 < t_1$. Since the control time span may be shorter than the predictive one, the control volume remains constant if the control time domain is exceeded.

The t_1 -step predicted output weights and t_2 -step control input increments of the MPC are then converted into

$$\bar{W}(k+1) = \begin{bmatrix} v(k+1) \\ v(k+2) \\ \vdots \\ v(k+t_1) \end{bmatrix}_{t_1 \times 1}, \Delta U(k) = \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+t_2-1|k) \end{bmatrix}_{t_2 \times 1}, \quad (29)$$

where the matrix subscript t_1 and t_2 do not denote the dimensionality of the matrices but rather the number of vectors in the matrices.

Then, the system's future t_1 -step forecast output weight equation is designed as follows

$$V_{t_1}(k+1) = S_{xi} \Delta x(k) + S_n v(k) + S_{ui} F \Delta U(k), \quad (30)$$

where

$$S_{ui} = \begin{bmatrix} DB & 0 & 0 & \cdots & 0 \\ \sum_{j=1}^2 DA_i^{j-1} B & DB & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{j=1}^{t_2} DA_i^{j-1} B & \sum_{j=1}^{t_2-1} DA_i^{j-1} B & \cdots & \cdots & DB \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{j=1}^{t_1} DA_i^{j-1} B & \sum_{j=1}^{t_1-1} DA_i^{j-1} B & \cdots & \cdots & \sum_{j=1}^{t_1-t_2+1} DA_i^{j-1} B \end{bmatrix}_{t_1 \times t_2}, S_n = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix}_{t_1 \times 1}, S_{xi} = \begin{bmatrix} DA_i & \sum_{j=1}^2 DA_i^j & \cdots & \sum_{j=1}^{t_1} DA_i^j \end{bmatrix}_{t_1 \times 1}^T.$$

It is generally hoped that the control action will not vary too much, the objective function is constructed as

$$J = \sum_{i=1}^{t_1} \|\Lambda_{v,i}(v(k+i) - v_g)\|^2 + \sum_{j=1}^{t_2} \|\Lambda_{u,j}(\Delta u(k+j-1))\|^2, \quad (31)$$

where $\Lambda_{v,i}$ and $\Lambda_{u,j}$ is the weighting factor for the output error and the control input respectively, which means that $\Lambda_v = \text{diag}(\Lambda_{v,1}, \Lambda_{v,2}, \dots, \Lambda_{v,t_1})$, $\Lambda_u = \text{diag}(\Lambda_{u,1}, \Lambda_{u,2}, \dots, \Lambda_{u,t_2})$.

Define

$$E(k) = S_v - \sum_{i=1}^q h_i(\xi(k)) [(S_{xi} + S_n D) \Delta x(k)],$$

where $S_v = [v_g, v_g, \dots, v_g]_{t_1 \times 1}^T$ represent the expected weight vector.

Then the objective function (31) can be converted to the following form

$$\begin{aligned} J &= \|\Lambda_v(\bar{W}(k+1) - S_v)\|^2 + \|\Lambda_u \Delta U(k)\|^2 \\ &= \begin{bmatrix} \Lambda_v(\bar{W}(k+1) - S_v) \\ \Lambda_u \Delta U(k) \end{bmatrix}^T \begin{bmatrix} \Lambda_v(\bar{W}(k+1) - S_v) \\ \Lambda_u \Delta U(k) \end{bmatrix} \\ &= \sum_{i=1}^q h_i(\xi(k)) \left(\begin{bmatrix} \Lambda_v S_{ui} F \\ \Lambda_u \end{bmatrix} \Delta U(k) - \begin{bmatrix} \Lambda_v E(k) \\ 0 \end{bmatrix} \right)^T \\ &\quad \times \left(\begin{bmatrix} \Lambda_v S_{ui} F \\ \Lambda_u \end{bmatrix} \Delta U(k) - \begin{bmatrix} \Lambda_v E(k) \\ 0 \end{bmatrix} \right). \end{aligned} \quad (32)$$

The optimal control sequence can be derived from the following equation

$$\begin{aligned} \frac{\partial J}{\partial \Delta U(k)} &= \sum_{i=1}^q h_i(\xi(k)) \begin{bmatrix} \Lambda_v S_{ui} F \\ \Lambda_u \end{bmatrix}^T \left(\begin{bmatrix} \Lambda_v S_{ui} F \\ \Lambda_u \end{bmatrix} \Delta U(k) - \begin{bmatrix} \Lambda_v E(k) \\ 0 \end{bmatrix} \right) \\ &= \sum_{i=1}^q h_i(\xi(k)) (S_{ui}^T \Lambda_v^T F^T F \Lambda_v S_{ui} + \Lambda_u^T \Lambda_u) \Delta U(k) - S_{ui}^T \Lambda_v^T F^T F \Lambda_v E(k). \end{aligned} \quad (33)$$

Thus, the optimal control input increment could be gotten as

$$\Delta U^*(k) = \sum_{i=1}^q h_i(\xi(k)) (S_{ui}^T \Lambda_v^T F^T F \Lambda_v S_{ui} + \Lambda_u^T \Lambda_u)^{-1} S_{ui}^T \Lambda_v^T F^T F \Lambda_v E(k). \quad (34)$$

Since only the first component of the MPC controller affects the system, the control input increments are given below

$$\Delta u(k) = [I \ 0 \ \cdots \ 0]_{1 \times t_2} \Delta U^*(k) = K_{\text{mpc}} E(k). \quad (35)$$

where

$$K_{\text{mpc}} = [I \ 0 \ \dots \ 0]_{1 \times 2} (S_{ui}^T \Lambda_v^T F^T F \Lambda_v S_{ui} + \Lambda_u^T \Lambda_u)^{-1} S_{ui}^T \Lambda_v^T F^T F \Lambda_v, \quad (36)$$

is the gain matrix.

Finally, the FTC controller could be gained

$$u(k) = u(k-1) + K_{\text{mpc}} E(k). \quad (37)$$

The closed-loop incremental system model is gained below

$$\begin{aligned} \Delta x(k+1) &= \sum_{i=1}^q h_i(\xi(k)) [A_i \Delta x(k) + B_i \Delta u(k) + N_i \Delta \delta(k)] \\ &= \sum_{i=1}^q h_i(\xi(k)) [A_i \Delta x(k) + B_i K_{\text{mpc}} E(k) + N_i \Delta \delta(k)] \\ &= \sum_{i=1}^q h_i(\xi(k)) \{ [A_i - B_i K_{\text{mpc}} (S_{xi} + S_n D)] \Delta x(k) + B_i K_{\text{mpc}} S_v + N_i \Delta \delta(k) \}. \end{aligned} \quad (38)$$

The stability criterion of system is that if all the characteristic roots of its system matrix

$$[A_i - B_i K_{\text{mpc}} (S_{xi} + S_n D)],$$

lie within the unit circle, then the system (38) is stable. Hence, when the controller design is completed, the eigenvalues of matrix $[A_i - B_i K_{\text{mpc}} (S_{xi} + S_n D)]$ need to be verified.

6 | SIMULATION RESULTS

To evaluate the introduced FDI and MPC FTC scheme, a simulation experiment is offered, the desirable results are achieved. Considering a typical instance of the wet section of a paper machine, which is illustrated in Figure 1. In the figure, the major parts comprise the headbox approach device, the headbox and the moving wire table. In the headbox, the fibers, supplements and additional chemical additives are blended. After that, as the additives are laid over the moving line table, some water is drained in a white water pit below the table. The flocculation size is random in the pool, but it conforms to a certain distribution. Thus, to control the flocculation distribution, the solids of the effluent water must be controlled and minimized. The distribution is mostly determined by the trapped polymer and shear in headbox. The

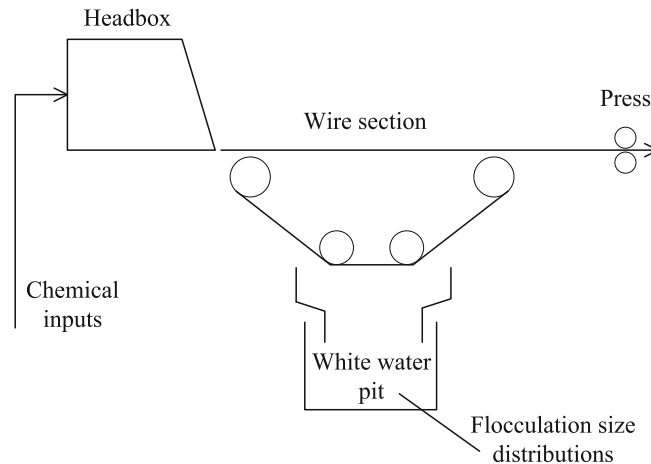


FIGURE 1 Diagram of the wet section with a paper-making machine.

system is inevitably impacted to by extraneous disturbances. If the distribution shifts, it is considered that fault might has occurred. Hence, FDI, fault diagnosis and FTC schemes are necessary to keep the system operational despite the occurrence of multiple faults.

The B-spline neural network is utilized to shape the output PDF. Choose three basis functions as follows

$$\begin{aligned} c_1(y) &= \frac{1}{2}(y-2)^2 I_1 + (-y^2 + 7y - 11.5) I_2 + \frac{1}{2}(y-5)^2 I_3 \\ c_2(y) &= \frac{1}{2}(y-3)^2 I_2 + (-y^2 + 9y - 19.5) I_3 + \frac{1}{2}(y-6)^2 I_4 \\ c_3(y) &= \frac{1}{2}(y-4)^2 I_3 + (-y^2 + 11y - 29.5) I_4 + \frac{1}{2}(y-7)^2 I_5, \end{aligned} \quad (39)$$

where $I_\alpha = \begin{cases} 1 & y \in [\alpha + 1, \alpha + 2] \\ 0 & \text{else} \end{cases}, (\alpha = 1, 2, \dots, 5).$

The identified system parameters are as follows

$$\begin{aligned} A_1 &= \begin{bmatrix} -2 & 0 \\ 0.1 & -3 \end{bmatrix}, A_2 = \begin{bmatrix} -1.8 & 0 \\ 0.3 & -2.4 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1.5 & 0.01 \\ -0.01 & 3.05 \end{bmatrix}, B_2 = \begin{bmatrix} 1.45 & 0.01 \\ -0.02 & 2.64 \end{bmatrix}, \\ D &= \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}, N_1 = [1 \quad 1]^T, N_2 = [0.5 \quad 0.5]^T. \end{aligned}$$

The interference of the system fulfils the uniform distribution at $[-0.5 \quad 0.5]$.

The membership functions are shown below

$$h_1 = \frac{1}{1 + e^{-2x_1}}, h_2 = 1 - h_1.$$

To testify the fault isolation algorithm, there are two potential faults in the system, so two corresponding isolation residual ε_{f1} and ε_{f2} are necessary to isolate the faults. The form of possible faults are supposed as follows

$$f_1(k) = \begin{cases} 0 & k < 30 \\ 3.5 & k \geq 30 \end{cases}, f_2(k) = \begin{cases} 0 & k < 60 \\ 2 & k \geq 60 \end{cases}$$

Through the LMI in Theorem 2, the parameters required for fault isolation and estimation can be calculated as follows

$$\begin{aligned} \Gamma_1 &= [0.007 \quad -0.071], \Gamma_2 = [0.005 \quad -0.065] \\ L_1 &= [0.3129 \quad 0.4962]^T, L_2 = [0.2647 \quad 0.4592]^T. \end{aligned}$$

For the situation that two faults occur at different time, the result of fault isolation is displayed in Figure 2. The result can be gained that fault occurs at $k = 30$ according to the detection residual ε_1 , and through isolation residual ε_{f1} and ε_{f2} , it can be seen more clearly that two faults occur at $k = 30$ and $k = 60$, respectively. In addition, to earn the fault amplitude information, the fault estimation result is given in Figure 3, which indicates that the fault diagnosis algorithm can diagnose the faults in the system with relatively small error. For comparison, the fault diagnosis result presented in Reference 38 is given in Figure 4, which suggests that the algorithm in this paper achieves better results.

Figure 5 displays the FDI results of two faults occurring at the same time. The detection residual indicates that fault occurs at $k = 40$, it can be concluded that two faults occur at $k = 40$ simultaneously from the isolation residual ε_{f1} and ε_{f2} . The fault estimation result is given in Figure 6. From Figures 2–6, it is obvious that the FDI algorithm devised is applicable for a variety of different fault conditions. Similarly, the fault estimation result presented in Reference 38 is given in Figure 7, which clearly shows that the algorithm in this paper works better.

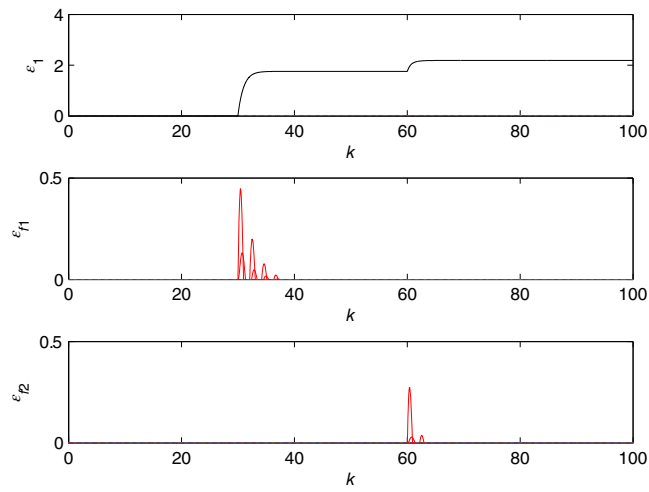


FIGURE 2 Fault detection and isolation results (two faults occurring at different time).

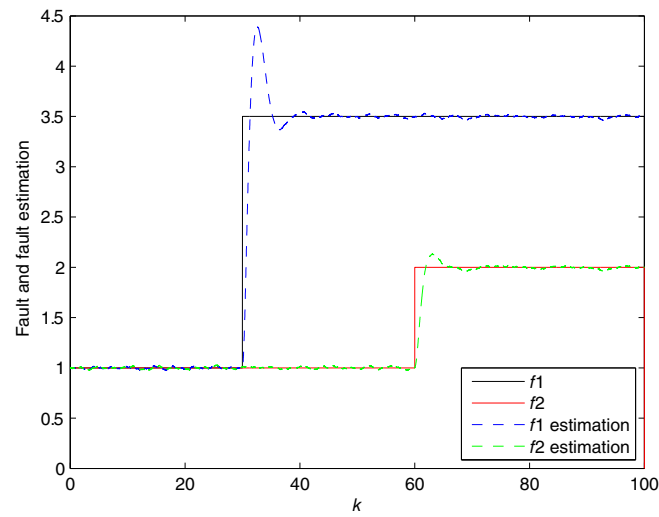


FIGURE 3 Fault diagnosis results (two faults occurring at different time).

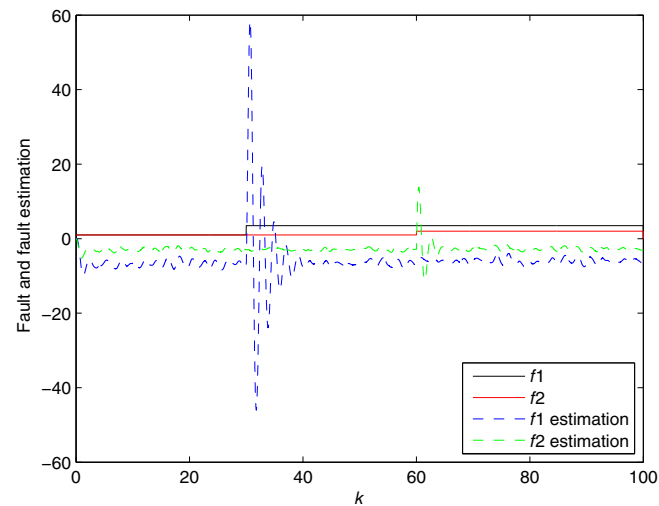


FIGURE 4 Fault diagnosis results (two faults occurring at different time in Reference 38).

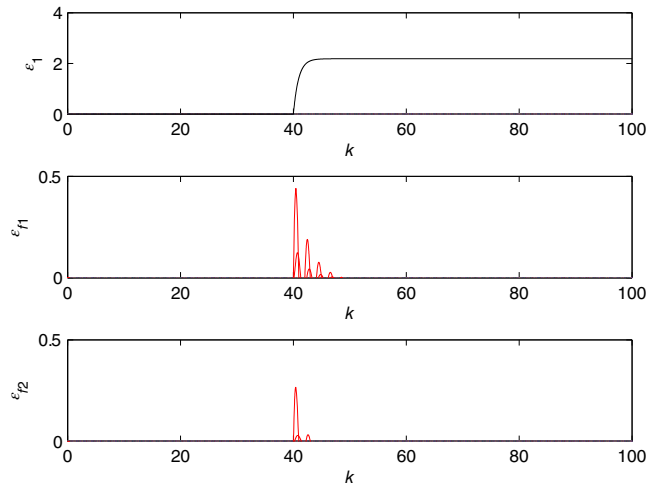


FIGURE 5 Fault detection and isolation results (two faults occurring simultaneously).

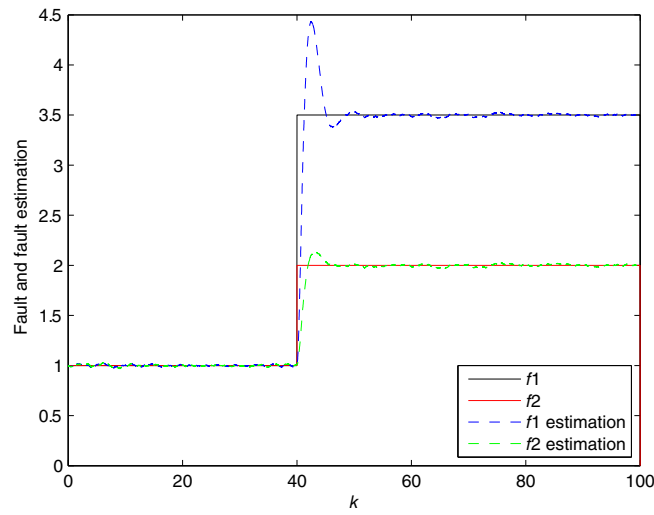


FIGURE 6 Fault diagnosis results (two faults occurring simultaneously).

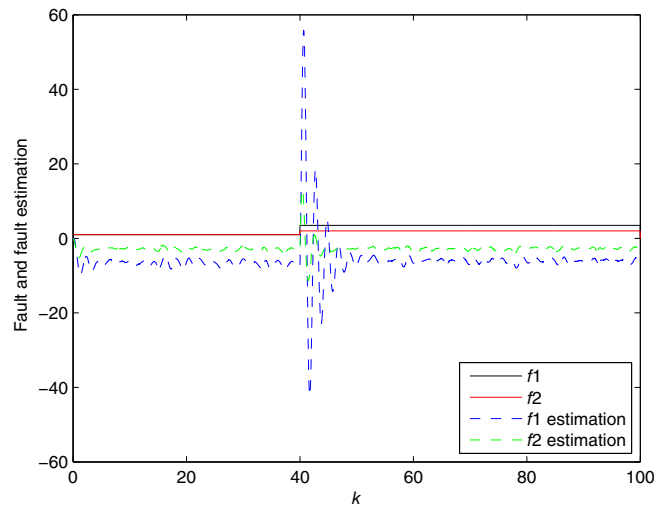


FIGURE 7 Fault diagnosis results (two faults occurring simultaneously in Reference 38).

When the fault information has been obtained through the FDI algorithm, the MPC FTC algorithm is required to reduce the impact of the fault as much as possible. The desired output weight is given as

$$v_g = [0.3, 0.6].$$

The weighting factors in (31) are selected below

$$\Lambda_v = \text{diag} [0.3, 0.5, 0.6, 0.8, 0.5, 0.3, 0.1, 0.8]$$

$$\Lambda_u = \text{diag} [-20.05, 20.58, 0.05, 0.19]$$

The FTC result is given in Figures 8–14. Figure 8 shows the expected PDF. For the situation that two faults occurring at different time, the FTC results are displayed in Figures 9–11, when fault occurs at $k = 30$ and $k = 60$, the shape of the PDF is deformed, but the devised MPC FTC algorithm can eliminate the impact of the fault therefore the output PDF track the expected PDF. However, the PI fault-tolerant controller in Figure 10 cannot eliminate the effects of faults and disturbances, and the effects are very limited. The comparison result of PDF with MPC FTC, the expected PDF, and the PDF with PI FTC is given in Figure 11, where it can be more clearly confirmed that the MPC FTC algorithm enables the output PDF to track the expected PDF with a very small error. Figure 12 shows the FTC result of the two fault occurring

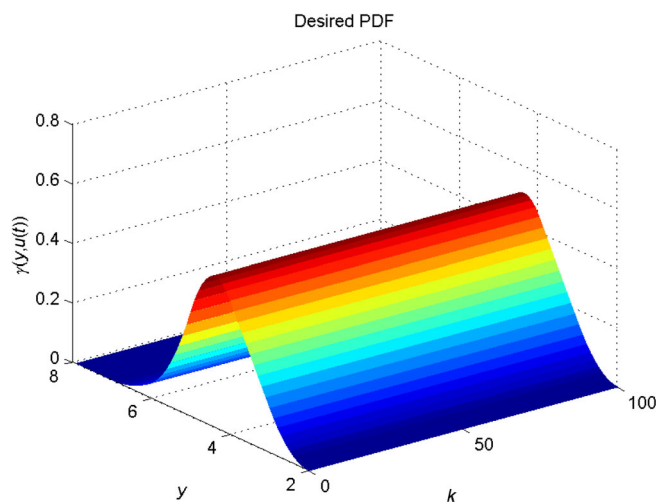


FIGURE 8 The desired probability density function.

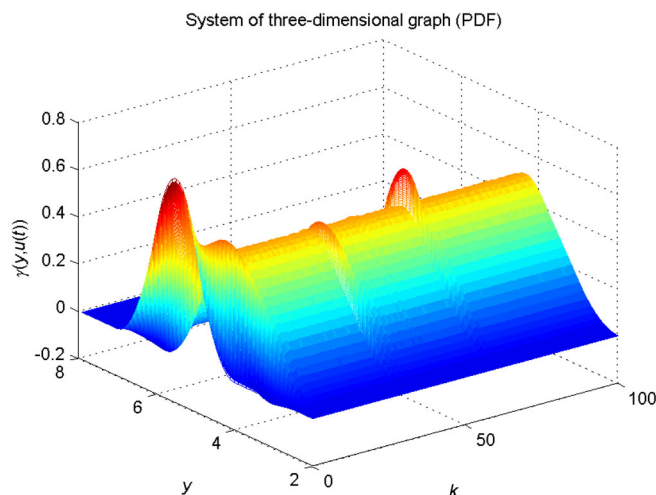


FIGURE 9 Fault tolerant control results (two faults occurring at different time).

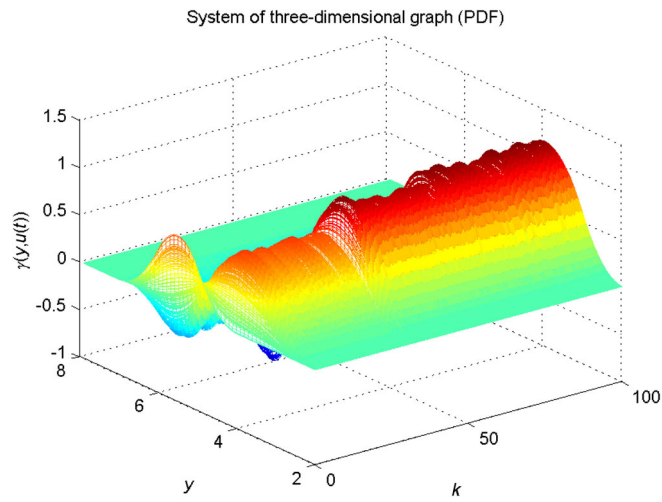


FIGURE 10 PI fault tolerant control results (two faults occurring at different time).

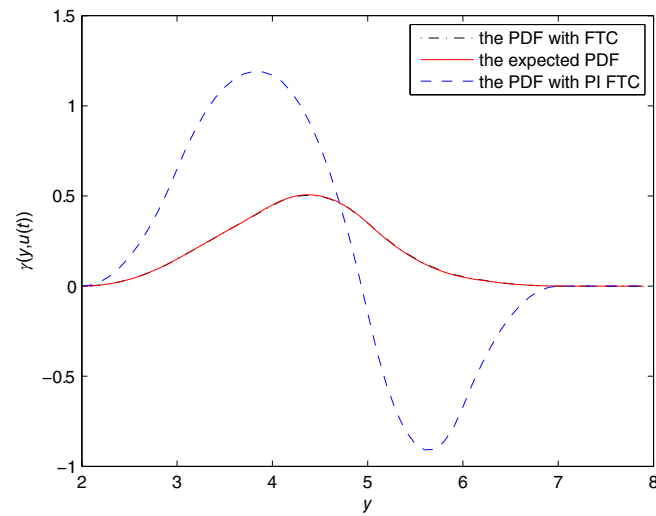


FIGURE 11 The comparison results of probability density function (PDF) with fault tolerant control (FTC) this paper, expected PDF and PDF with PI FTC (two faults occurring at different time).

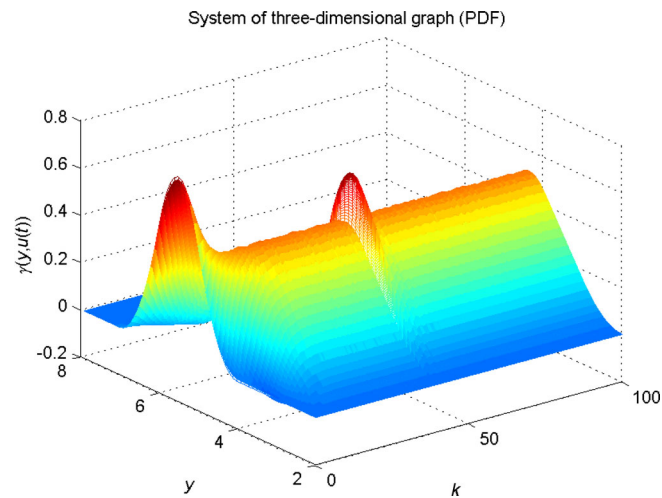


FIGURE 12 Fault tolerant control results (two faults occurring simultaneously).

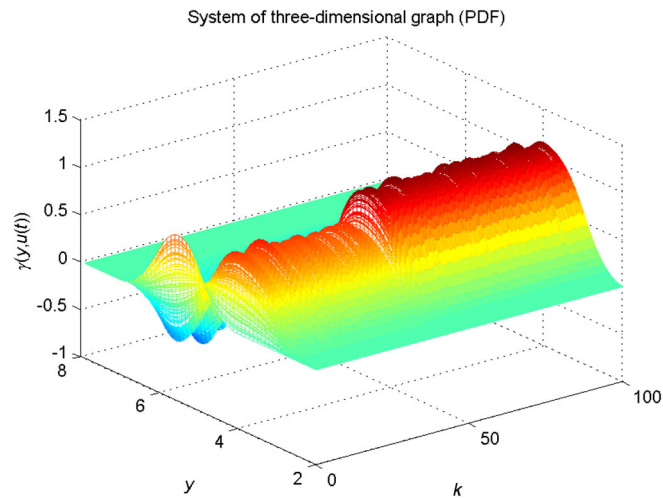


FIGURE 13 PI a fault tolerant control results (two faults occurring simultaneously).

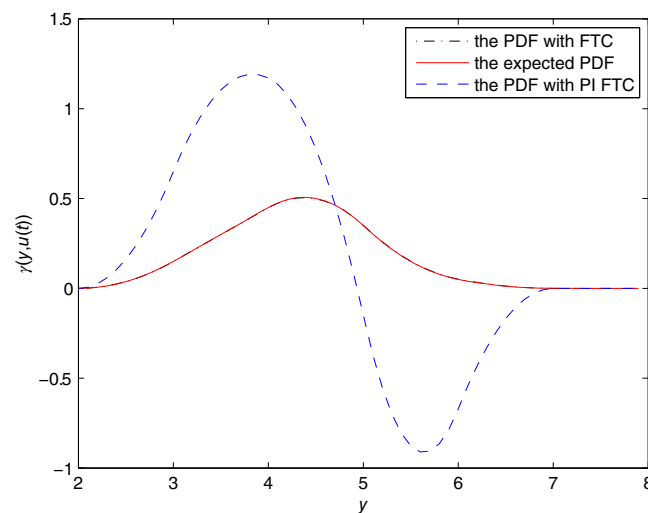


FIGURE 14 The comparison results of probability density function (PDF) with a fault tolerant control (FTC) this paper, expected PDF and PDF with PI FTC (two faults occurring simultaneously).

simultaneously, from which it can be seen that although the PDF deformation is more dramatic when two faults occurred at $k = 40$ simultaneously, but the MPC FTC algorithm can still quickly eliminate the influence of the fault hence the system PDF could track the given PDF. The PI fault-tolerant control result is displayed in Figure 13. In order to highlight the effect of FTC, the PDF comparison results between the PDF with MPC FTC, the desired PDF, and the PDF with PI FTC are presented in Figure 14, from which it is also obvious that output PDF with MPC FTC track the expected PDF with high accuracy. The simulation example proved that the designed FDI and FTC algorithm can be applied in respect of many complex situations.

7 | CONCLUSIONS

In this paper, for SDC systems with multiple multiplicative faults, interference and nonlinearity, FDI, fault estimation and FTC algorithms are introduced. To decouple multiple potential multiplicative faults and interference, an adaptive observer is designed, then the error system is separated into subsystems affected only by multiplicative faults and subsystems affected only by nuisances. A fault estimation algorithm is presented to obtain the detailed fault information like

time, position, magnitude of the multiplicative fault. When the fault information has been gotten, a MPC FTC scheme is put forward to make the output PDF track the desired PDF, mitigating the effect of multiplicative faults on the system as much as possible. The given simulation instance has indicated that the introduced scheme is valid regardless of multiple multiplicative faults occurring simultaneously or at different time.

Additionally, research on FDI and FTC of SDC systems has attracted a lot of attentions, then uncertainties and time-varying faults of SDC systems will be considered in the forthcoming research.

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CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflict of interests.

DATA AVAILABILITY STATEMENT

The data that support the findings will be available in fault diagnosis data at <https://www.baidu.com/> following an embargo from the date of publication to allow for commercialization of research findings.

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