The Principle of Minimal Power

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Abstract

This article is devoted to the memory of Yuri P. Raizer, who passed away in 2021. He left a noticeable trace in gas discharge physics. The principle of minimal power (the state that requires minimal power is most probable) is thoroughly used in his books. Although the fundamental laws of physics do not imply this *ad hoc* principle, a detailed analysis of underlying phenomena can often reveal why nature prefers this path. Raizer illustrated this principle for plasma stratification, formation of electrode spots, discharge constriction, the shape of an arc channel, etc. We argue that the nonlinearity of equations describing gas discharges can often justify the realization of a plasma state maintained at minimal electric power. This nonlinearity appears because small groups of energetic electrons often control the ionization processes. The number of these electrons depends strongly on the ratio of the electric field to gas density, *E/N*. Under certain conditions, the ionization rate can also depend nonlinearly on electron density due to stepwise ionization and Coulomb collisions. We use the principle of minimal power to illustrate some of Raizer's contributions to gas discharge physics from a single point of view. We demonstrate that nonlinearity of ionization processes in gas discharges can substantiate this principle for plasma stratification. However, striations of *s*, *p*, and *r* types in Neon could exist with minimal or no ionization enhancement. This reminds us of Raizer's warning that applying the minimal power principle could lead to erroneous predictions, and a proper theory is required in each case to justify its use.

"The phenomenon of striations satisfies the principle of minimal power" – Yuri Raizer

1. Introduction

Prof. Yuri P. Raizer's outstanding scientific merits are reflected in the numerous works of his students and colleagues. His name has taken a worthy place in the Encyclopedia of Low-Temperature Plasma [1] and Wikipedia. He was among the first to receive the title of Soros professor, which allowed him to work fruitfully in the 90s of the twentieth century, the most challenging period for Russian science. In addition to scientific achievements, which were much discussed, the authors would like to say a few words about the role that Professor Raiser played in the formation of generations of undergraduate and graduate students at St. Petersburg University in Russia.

Of all the numerous books written by Raizer, the classical book "Gas Discharge Physics" [2] stands out. This book has become a reference book for all specialists working in the field. Courses of lectures on the fundamentals of plasma physics, the physics of gas discharges, the kinetics of charged particles in plasma, and oscillations and waves in plasma have been taught at the Physics Department of St. Petersburg University using this book.

For several decades, the monthly Seminar held at the Institute of Problems of Mechanics of the Russian Academy of Sciences in Moscow played a unique role in the community. This Seminar bored the official name of Professor Raizer's Seminar. Young people and mature scientists from around the country came to this Seminar to present their work. Many graduate students from the Physics Department of St. Petersburg University attended this Seminar. One of the authors (V.I. Kolobov) presented his Ph.D. dissertations at the Seminar. Discussions of scientific works at the Seminar guaranteed high quality and promoted successful defense of the dissertation. The authors remember numerous personal meetings with Professor Raiser in Moscow, St. Petersburg, and at various international conferences. Communication with this outstanding person left an indelible impression.

As an excellent physicist, Raizer always looked at the essence of phenomena and sought simple explanations of nature. One of the *ad hoc* principles he used to explain the self-organization of gas discharges was the principle of minimal power. Although the fundamental laws of physics do not imply this principle, a detailed analysis of underlying phenomena often reveals why nature prefers this path. Many physical phenomena satisfy the minimum power or energy principle, and nonequilibrium gas-discharge plasma is no exception. Gas discharges are open systems converting electrical power into the creation of charged particles, chemical reactions, light output, etc. The minimal power principle has been used to explain plasma stratification, the formation of cathode and anode spots with a "normal" current density in DC glow discharges and cathode spots in arcs, constriction of the positive column, the shape of an arc channel, etc.

Nonlinear and nonlocal effects are responsible for most self-organization processes in low-temperature plasma. The nonlinear effects appear because a small group of energetic electrons often controls the ionization processes in gas-discharge plasma. The number of these electrons depends strongly on the ratio of the electric field to gas density, E/N. The ionization rate can also depend on electron density nonlinearly due to stepwise ionization and Coulomb collisions. The nonlocal effects appear due to the peculiarities of electron kinetics in collisional plasma.[3,4] The electron mean free path and the electron energy relaxation length can differ by order of magnitude and depend on the gas type and the electron kinetic energy. Understanding specific mechanisms of plasma self-organization in each case and the interplay between the nonlinear and kinetic effects in different gases and discharge types remains a subject of ongoing studies.

Ionization waves (striations) have been studied for more than a century. Their nature was established by the end of the 60th.[5,6,7] Raizer's book describes the basics of plasma stratification and the key properties of the ionization waves. According to Raizer, the positive column of DC discharges prefers a layered state because striated plasma is sustained at lower electric power than homogeneous. The present paper summarizes recent progress in plasma stratification in noble gases. We illustrate that in most cases, striated plasma is indeed maintained at lower power than striation-free plasma. At the same time, our simulations show that striations of *s*, *p*, and *r* types in Neon exist with minimal or no ionization enhancement. So, the minimal power principle cannot predict plasma stratification without nonlinearities of the ionization rate.

Furthermore, we describe Raizer's contributions and recent progress in understanding pattern formation on electrodes, including the effect of the "normal" current density on the cathode and anode spots. The role of nonlinear and nonlocal effects is emphasized. Our analysis confirms

Raizer warnings that applying the minimal power principle could lead to erroneous predictions, and a theory is required in each case to justify its use.

2. The Principle of Minimal Power

The minimal power (energy) principle manifests itself in various fields of physics. A vivid illustration of this principle is the periodic table of elements of Mendeleev. The Pauli principle states that each quantum state characterized by a set of four quantum numbers (principal quantum number n, orbital quantum number l, projection of the orbital m_l, and spin m_s moments) can only have one electron. Per this principle, the number of vacancies on the shell with quantum number n turns out to be equal to $2n^2$. If the filling of electron shells in atoms were regulated only by the Pauli principle, then the number of elements in periods would increase monotonically and would have the values 2, 8, 18, 32, … for $n = 1, 2, 3, 4, \ldots$

The actual Mendeleev's table has the form 2, 8, 8, 18, 18, 32, ... The principle of minimum energy for the electronic configuration operates along with the Pauli principle. The energy of the electronic configuration increases with the growth of the principal and orbital quantum numbers. It may turn out that the energy of a state with quantum numbers, for example, n=3, l=2, will be greater than with the numbers $n = 4$, $l = 0$. Although there are ten vacancies in the three-quantum state at $n = 3$, $l = 2$, it is more energetically advantageous to fill two vacancies in the four-quantum state $n = 4$, $l = 0$. Further, it turns out that filling 10 vacancies in the state $n = 3$, $l = 2$ is energetically more favorable than in 6 states $n = 4$, $l = 1$. Modern methods of calculating the structure of atoms and molecules justify the appearance of actinoids and lanthanides in the periodic table.

Numerous phenomena in molecular physics satisfy the minimum energy principle when the free energy of a system should be minimal in a state of equilibrium. Based on this principle, a drop of liquid in the absence of gravity takes on a spherical shape. When applied to solutions, the principle of minimum free surface energy of a liquid leads to an interesting phenomenon. Adding a surfactant (e.g., soap) to water results in a sharp decrease in the surface tension of the soap solution, which makes it possible to blow large spherical soap bubbles.

The principle of minimal power is relevant to the principle of minimum entropy production in the theory of open (unclosed) systems.[8,9] Indeed, gas discharges are open systems converting electrical power into charged particle creation, chemical reactions, light output, etc. However, a detailed comparison of the two principles is beyond the goals of the present paper.

Many more physical phenomena satisfy the principle of minimum power or energy. Gas discharges are no exception. The best-known example is Steenbeck's minimum principle introduced for a cylindrical arc column.[10] This principle hypothesizes that the temperature of the arc column would be such that the axial electric field was a minimum. Raizer developed a physical model taking into account the heat conduction in the channel to obtain a solution to the arc column problem without using Steenbeck's minimal principle. [11,12]

The minimal power principle left an essential trace in discharge physics.[13] Raizer used this principle to illustrate plasma stratification, the formation of electrode spots, discharge constriction, the shape of an arc channel, etc. However, he emphasized in his book [14] published in the 1970s that applying the minimum power principle may produce erroneous results, as was shown for inductively coupled plasma by M.O. Rozovskii, Raizer's student (see references in chapter 29.2 of that book). The minimum power principle is most valuable until a proper theory is developed. In the book [2], Raizer wrote that the phenomenon of striations satisfies the principle of minimal power. Our studies indicate that the minimal power principle is satisfied for some types of striations but cannot describe the ones observed in the absence of nonlinear dependencies of the ionization rate on electron density or the electric field.

3. Plasma Stratification

DC discharges often have a layered structure. The bright and dark layers (striations) can move along the current direction, invisible to the naked eye but can be visualized by stroboscopic techniques. Striations have been observed in gas discharges long before Langmuir introduced the term plasma. Pekarek first used the term "ionization waves" to distinguish them from ion waves observed at low gas pressures.[15] Figure 1 shows the spontaneous existence of the ionization waves in Neon DC discharges depending on gas pressure p, discharge current i, and tube radius R. No discharge can be maintained in the dark area of the map at low values of pR and i/R. The Debye radius is equal to the tube radius at the line (1). Volume recombination dominates over surface recombination above the line (2). The green area corresponds to a striation-free plasma. In this area, artificial striations have been excited to study the dispersion characteristics of the ionization waves. The ionization waves have interesting dispersion properties in noble gases: their phase and group velocities have similar values but opposite directions. The blue area corresponds to a striation-free, radially constricted arc column. Line (3), which is also called the Pupp boundary, corresponds to the transition from glow to arc discharge. No discharge can be maintained below the line (4) because the ionization cross section has a finite value. The two dashed lines indicate gas pressure p at which the electron mean free path λ and the electron energy relaxation length λ_u equal the tube radius R.

Depending on the gas pressure and discharge current, several striation types have been identified. The most studied are striations in diffuse and constricted discharges near the Pupp boundary. They appear due to a nonlinear dependence of the ionization rate on the electron density. Three types of waves observed at lower currents at pressures below the dashed line $\lambda_u = R$ in Figure 1 are due to nonlocal electron kinetics. Striations in diffuse discharges at medium pressures above the dashed line $\lambda_u = R$ are often irregular and less studied.

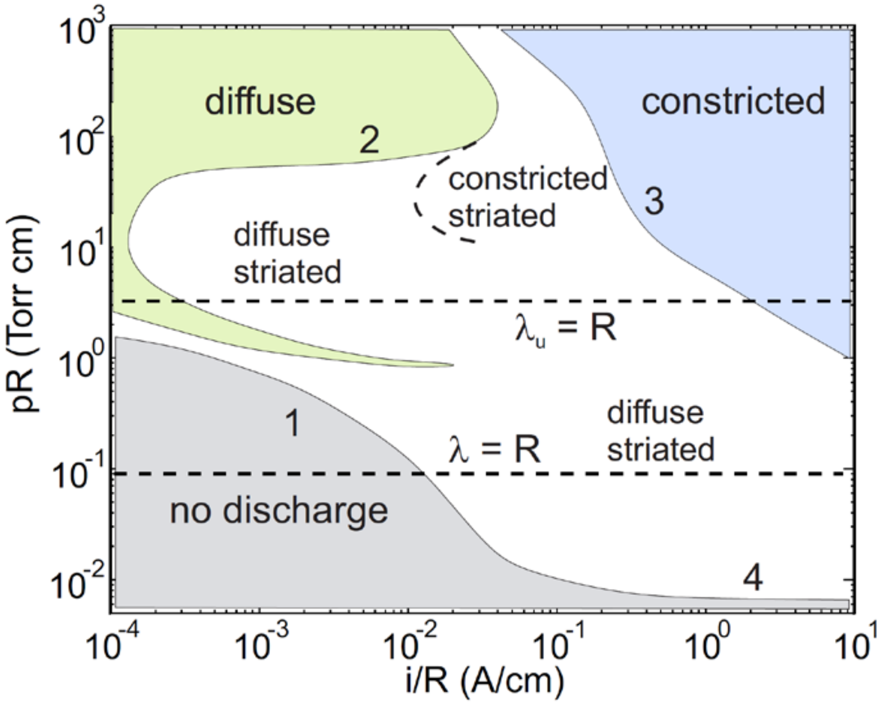

Figure 1: Discharge forms in Neon for different discharge currents *i* and gas pressures *p* (adapted from [28] using data from [16] and [15]).

Ionization waves were described in several reviews and books.[17,18,19] The kinetic nature of the ionization waves in noble gases at low currents was established in the 60th. Novak [20] demonstrated that the potential drop over striation length has near-constant values (19.5. 12.7, and 9.2 V) for *s*, *p*, and *r* striations in Neon. Three types of waves were also observed in Argon. In Helium, only one kind of *s* wave occurs naturally, but the *p* waves were excited in the striation-free region by external perturbations. Novak's potential drop over striation lengths correlated with the excitation threshold ε_1 in noble gases (11.55, 16.62, and 19.32 eV) for Ar, Ne, and He. An analysis of the kinetic equation for electrons in spatially periodic electric fields revealed kinetic resonances for the periods $\Lambda = (p/q)\lambda_\varepsilon$ where $\lambda_\varepsilon = \varepsilon_1/(eE_0)$ and E_0 is an average electric field value, and *p* and *q* are integer numbers. The widths of the resonances are determined by the electron energy loss in elastic collisions, excitations of different states of atoms, the generation of secondary electrons in ionization events, and electron losses to the walls.[21,22,23] Tsendin obtained analytical solutions for Electron Energy Distribution Function (EEDF) to describe the electron bunching process due to the slight energy loss of electrons in the elastic collisions with atoms.

With the advent of computers, studies of the electron relaxation processes have been performed in uniform and spatially modulated electric fields using numerical solutions of the Boltzmann equation and particle-based (PIC) methods.[24,25,26] These studies contributed to understanding the nature of plasma stratification in noble gases and interpreting the famous Frank Hertz experiments. First self-consistent 2d simulations of moving striations in diffuse Argon discharges near the Pupp boundary were reported in Ref. [27] using a fluid model with non-Maxwellian EEDF obtained from a solution of *local* Boltzmann equation for electrons.

These advances were reflected in the reviews.[28,29] However, several questions remained unresolved. What are the key differences between moving and standing striations? What are the dominant mechanisms of EEDF relaxation? How important is electron bunching for plasma stratification in short discharges?

Recent progress has been achieved by computer simulations of plasma stratification in noble gases using fluid [30,31] and hybrid [32] models and Particle in Cell (PIC) simulations.[33] New implicit and coupled fluid plasma solvers enabled studies of ionization waves in diffuse and constricted discharges at relatively high plasma densities. PIC model with periodic boundary conditions [33] allowed self-consistent simulations of moving striations in a positive column of DC discharges in Argon and Neon at low currents without metastable atoms and Coulomb collisions. A hybrid model [32] using a grid-based Fokker-Planck kinetic solver for electrons and a fluid model for ions enabled self-consistent simulations of standing and moving striations in Argon discharges between electrodes at low currents. Standing striations in RF discharges at low plasma densities have been recently obtained in Ref. [34] with the same hybrid solver.

3.1 Fluid simulations of striations at high currents

Since the late 80th, it has been known that the nonlinearities due to EEDF Maxwellization via Coulomb collisions are responsible for plasma stratification in noble gases at high currents (near the Pupp boundary) and for plasma constriction at high gas pressures. However, self-consistent simulations of these phenomena became possible only recently. Moving striations in diffuse Argon discharges were briefly reported[27] using CFD-ACE+ software, but we could not simulate striations in the constricted discharges due to the strong nonlinearity of the system. The recent development of an implicit and coupled fluid plasma solver [35] made it possible to simulate moving striations in diffuse and constricted DC discharges,[30] and standing striations in CCP.[31] Figure 2 shows an example of calculated moving striations in Argon DC discharges at low and high pressures (1 and 400 Torr) in a dielectric tube with a radius of 1 cm and length of 14 cm.

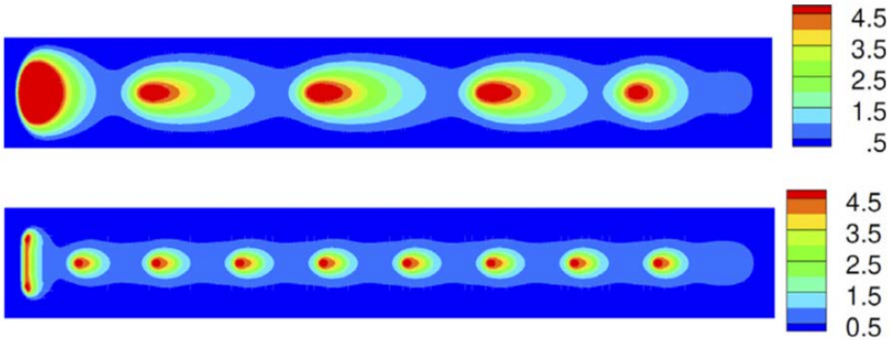

Figure 2: Instantaneous distributions of electron density (in 10^{10} cm^{-3}) in diffuse and constricted DC discharges in a tube of radius 1 cm and length 14 cm at currents near the Pupp boundary (adapted from [35]).

Figure 3 illustrates key differences between 1d striations in diffuse discharges and 2d striations in constricted discharges. In both cases, the electric field has a maximum value near the maximal gradient of plasma density, which corresponds to the dominance of the ambipolar electric field over the conduction component. The maximal value of the electron temperature is shifted towards the cathode relative to plasma density, which corresponds to the propagation of striations towards the cathode. The field is close to zero in the constricted discharge when the plasma density on the axis reaches a maximum value, which agrees with the two-dimensional theory of striations in constricted discharges.[36] The electric field changes the sign between the maximum plasma density and the maximum electron temperature corresponding to highly nonlinear striations in the constructed discharge.

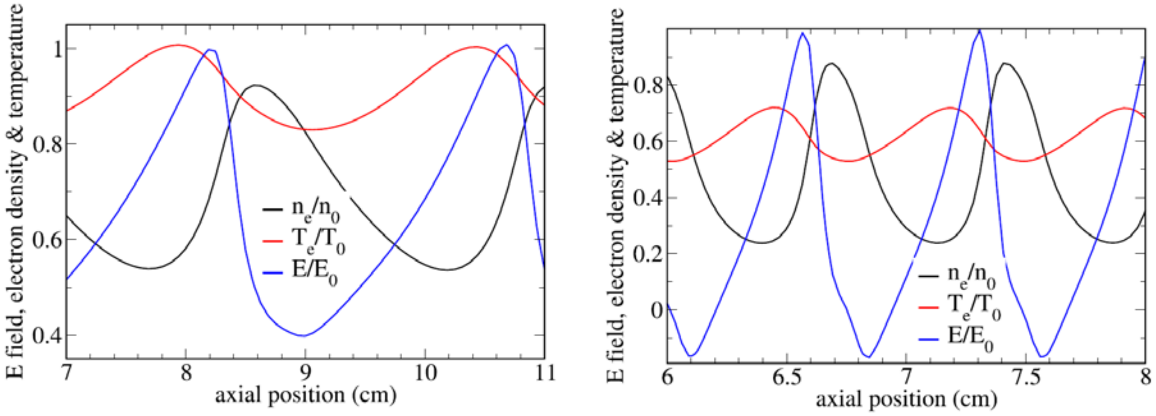

Figure 3: Axial distributions of the normalized electric field, electron density, and temperature for moving striations in the diffuse (left) and constricted (right) discharges at high currents (from Ref. [30])

Standing striations in Argon CCP were experimentally observed [31] at frequencies 3.6, 8.4, and 19 MHz, in a pressure range of 0.05-10 Torr, tube radius 1.1 cm and length 40 cm, for certain discharge currents (see Figure 4 left part). Numerical simulations [31] revealed the common nature of standing striations in CCP and moving striations in DC discharges under similar *pR* and *i/R*. The right part of Figure 4 shows spatial distributions of electron density obtained in 2D simulations of Argon CCP for $R = 1$ cm, $L = 14$ cm, $p = 0.5$ Torr, at a frequency of 20 MHz. The nonlinear dependence of the ionization rate on electron density is the primary reason for plasma stratification at high currents.

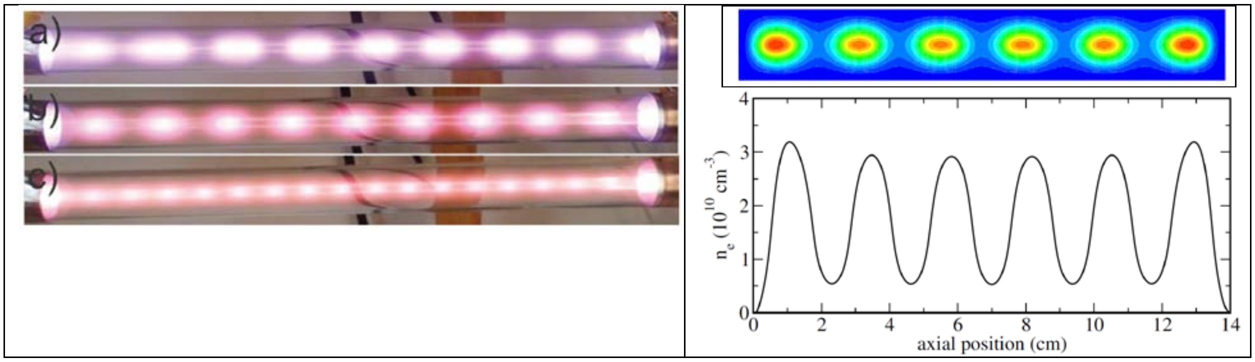

Figure 4: Experimental observation of standing striations in Argon CCP at 3.6 MHz at pressure $p = 0.1$, 1, and 10 Torr (left), and spatial distributions of electron density obtained in 2D simulations at 20 MHz (right) (from Ref. [31]).

3.2 Hybrid models of plasma stratification at low currents

A self-consistent hybrid model of striations in noble gases at low currents has been developed in Ref. [32]. The model consists of a Fokker Planck kinetic solver for electrons, a fluid model for ions, and a Poisson solver for the electric field. The Fokker-Planck equation describes the anisotropic diffusion of electrons in phase space ($\mathbf{r}$,u) (where u denotes the electron kinetic energy) along surfaces of constant total energy. It was demonstrated [32] that standing striations in Townsend discharges began moving with increasing discharge currents above a critical value. Two types of moving striations have been observed with two-level and full-chemistry models, which resemble the ion-guided and metastable-guided *s* and *p* waves.

Figure 5 shows an example of calculated EEDFs and the electric field profiles for the *s* and *p* striations. Two regions in phase space are visible. In the region of strong electric fields, local electron heating (diffusion in energy) dominates. Outside of the heating regions, the spatial diffusion of electrons dominates. Overall, the spatial diffusion and heating (energy diffusion) balance each other over striation length to ensure the potential drop over striation length satisfies Novak's law. This behavior is consistent with the qualitative model of nonlinear kinetic striations previously proposed by Tsendin.[37]

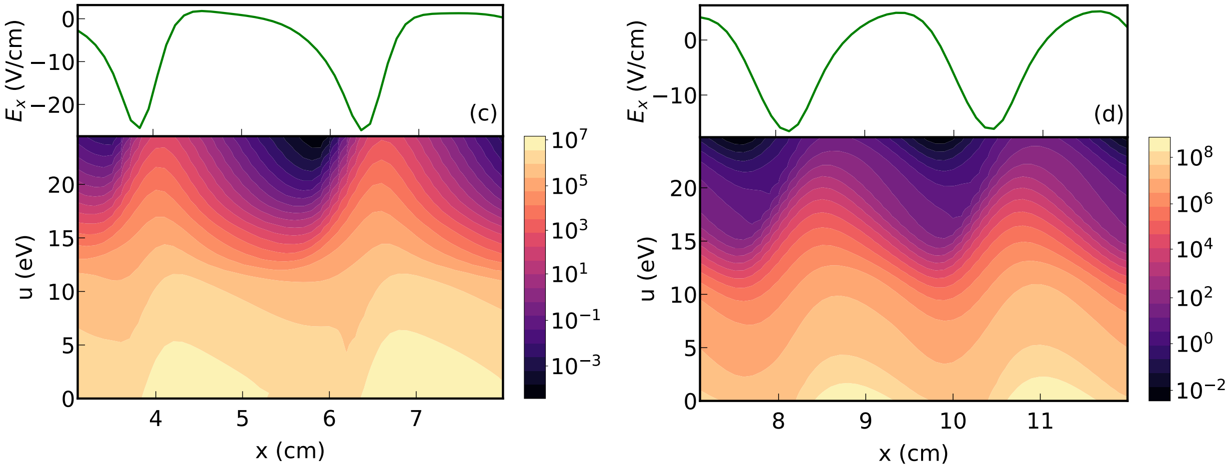

Figure 5: Instantaneous EEDFs for s-striations (left) and p striations (right) and the corresponding electric field profiles (from Ref. [32]).

The hybrid model predicted standing striations in CCP in Argon and Neon over a wide range of driving frequencies from 1 to 50 MHz [34]. The calculated length and amplitude of standing striations depend on the applied voltage. Figure 9 shows an example of simulations for Argon *p* = 0.4 Torr, frequency of 5 MHz, and voltage of 100 V. There are six striations within a 6 cm gap with the potential drop over striation length of about 17 eV. The ion and electron densities remain steady, but the electron temperature and the ionization rate oscillate substantially over the RF period.

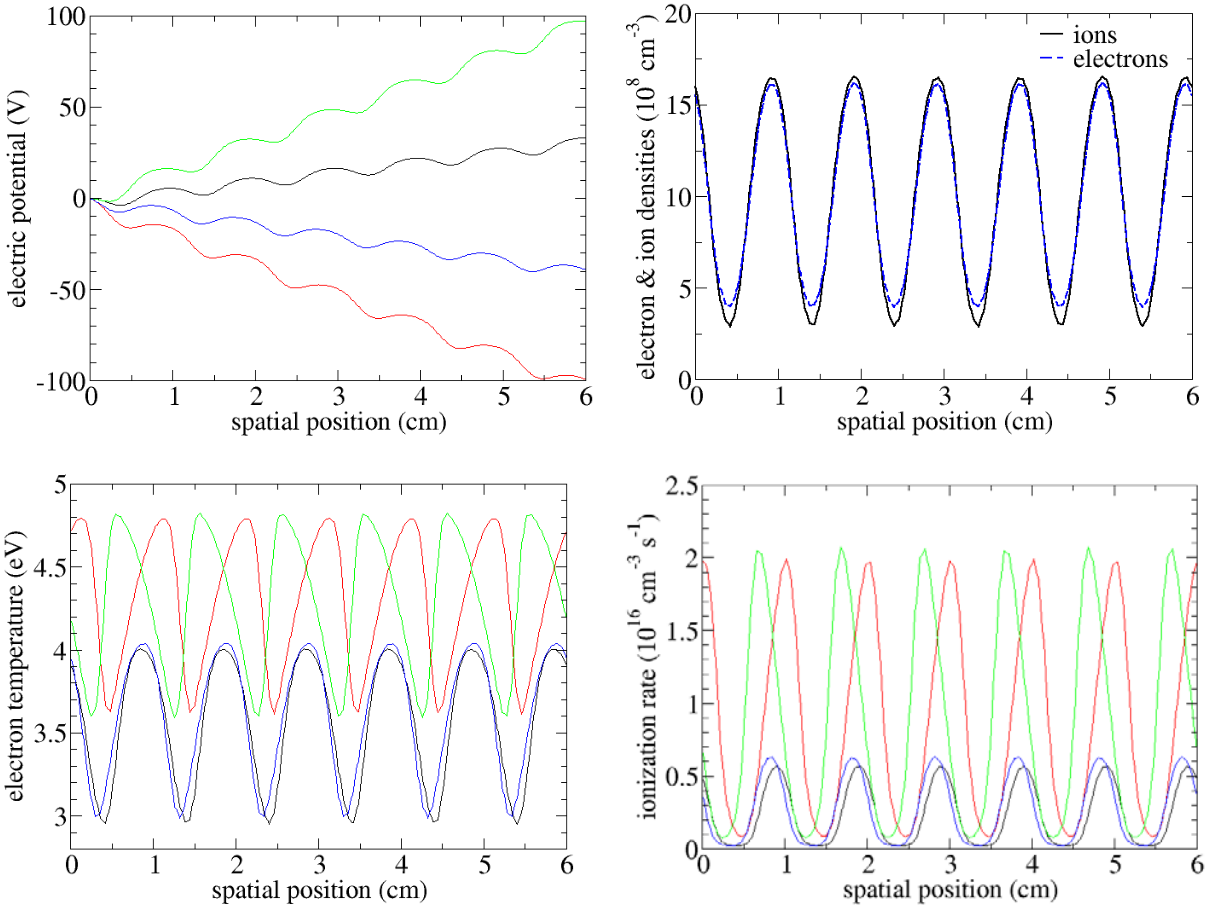

Figure 6: Spatial distributions of the electric potential, electron and ion densities, electron temperature, and the ionization rate at four times (color) during the RF period.

3.3 Minimal power principle for kinetic striations

According to Raizer,[2] ionization waves appear because maintaining a striated plasma state is more efficient than a striation-free state. Figure 7 compares calculated ionization and loss rates in the striated and striation-free positive column in Argon DC discharge at p = 1 Torr, average plasma density ~10^{15} m^{-3}, and the average electric field $\langle E \rangle$ = 17 V/cm. In these simulations, the electric field $\langle E \rangle$ was the same in the striated and striation-free plasma. However, the average value of the ionization frequency in the striated case is 3.3 times higher than in the striation-free case. Therefore, maintaining the striated plasma is 3.3 times more efficient than the striation-free case. Since the ionization must be balanced by loss, the calculated radius of the tube for the striated case (R = 2.5 mm) is much smaller than for the non-striated case (R = 6.8 mm).

The calculated ionization enhancement factor is shown in Figure 7 (on the right) as a function of the electric field $\langle E \rangle$ in Argon and Neon. The ionization enhancement factor is more significant in striated Argon plasma and decreases with increasing $\langle E \rangle$. In Neon, the enhancement factor drops quickly with increasing electric field. At $\langle E \rangle = 5$ V/cm, there is almost no enhancement, which means that the ionization rate is a linear function of the electric field in Neon at these fields.

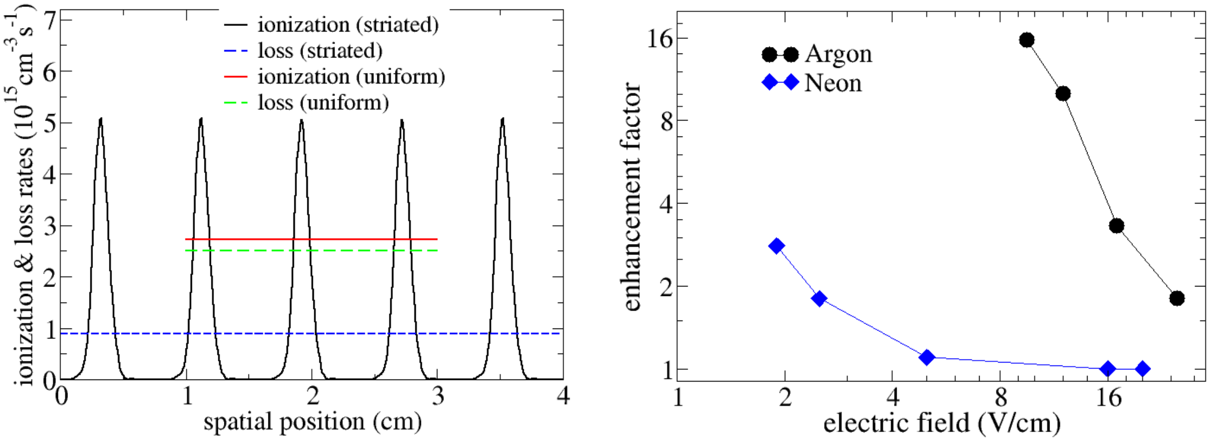

Figure 7: Comparison of the ionization and loss rates in stratified and striation-free DC plasma in Argon (left) and calculated ionization enhancement factor for Argon and Neon as a function of the average electric field $\langle E \rangle$ *(right).*

These simulations confirm that the minimal power principle applies to moving striations in DC discharges of noble gases at low discharge currents. Sustaining stratified plasma is more energy-efficient than striation-free plasma when the ionization rate is a nonlinear function of the electric field. However, the nonlinear dependence of the ionization rate on the electric field appears not necessary for plasma stratification. The case of Neon shows that striations of *s*, *p*, and *r* types could exist with minimal or no ionization enhancement. We expect that the minimal power principle will be applicable for other kinds of striations when the nonlinear dependencies of ionization rate on electron density and the electric field are essential.

4. Plasma self-organization near electrodes

In 1927, Compton and Morsel, in a paper devoted to the cathode fall region, introduced an *ad hoc* principle stating that the potential distribution in the cathode fall region would be such that the ionization of electrons was a maximum.[38] Subsequent work using the complete set of governing equations eliminated the need for such a principle making it a historical curiosity.[39] Yet before this occurred, the Compton-Morse principle may have spawned the Steenbeck's principle of minimum power as a missing piece in theoretical models. This principle does not follow the fundamental laws of physics, and there is no need for it when one can construct a theory in the usual way, as Raizer emphasized.

Raizer analyzed the effect of normal current density in DC discharges based on fundamental principles and explained the operation of the normal cathode spot.[40,41] The methods developed in these works have been further developed to analyze plasma interactions with solid and liquid electrodes. [42,43,44,45]

The initial theory of the normal cathode spot was based on simplifying assumptions, such as a local approximation for the ionization rate on the electric field strength. The local approximation for ionization leads to monotonic profiles of the plasma density and potential, even if diffusion is considered. [46,47] In normal and anomalous discharges, the nonlocality of the ionization is considerable. The plasma density peak is formed in the low field region due to nonlocal ionization, and the field reversal is necessary to retard the electron diffusion in this region. A field configuration that traps the slow electrons is formed, while the ambipolar diffusion describes the ion motion. In sufficiently long discharges, a second field reversal must occur near the Faraday dark space boundary and positive column. The potential and plasma density profiles, the ionization mechanism in the cathode region, the striation formation in this region, the field reversal criterion, can be understood by the kinetic theory of the cathode region. [48]

The kinetic theory of the anode region is not fully developed.[49] Both ion and electron sheaths at the anode have been experimentally observed and appeared in computer simulations of stratified discharges.[32,33] Still, the conditions for the electric field reversals and the formation of trapped electrons remain unclear. The current-voltage characteristic of the anode sheath is non-monotonic, and the current density corresponding to the minimal sheath voltage is called the normal current density at the anode.[50] The presence of normal current density may be the key reason for the formation of anode spots.

The third edition of the book [2] published in 2009 in Russian, contains a new chapter about Dielectric Barrier Discharges (DBD). The high-pressure DBDs have peculiar properties compared to low-pressure CCP. They operate in the Townsend regime at low driving frequencies as a sequence of current pulses corresponding to gas breakdown. At high frequencies, DBD plasma is prone to filamentation. [51,52] The DBDs have been produced in various geometries, gas mixtures, and power regimes and are now actively studied for many applications. [53]

Below, we sketch recent advances in understanding plasma self-organization near electrodes connected to Raizer works.

4.1 Cathode region

In his 1986 review of the cathode region in glow discharges, Raizer focused on nonlocal kinetics of fast electrons, the origin of Faraday dark space, and the nature of normal current density. His follow-up works clarified the role of nonlocal effects in the axial stricture of DC and RF discharges [54,55] and electron diffusion effects in forming cathode and anode spots. [56] The kinetic theory developed in Ref [48] explained the formation of a collisional double layer with three groups of electrons in the cathode region and predicted a possibility of standing striations in the transition from the Faraday dark space to the positive column. Such standing striations were later found in Raizer & Shneider's work.[57]

In this regard, we recall one of the visits of Prof. Raiser to the Laboratory of Plasma Physics of St. Petersburg University in the late 80s. At a seminar, Yury Petrovich spoke about one of the exciting effects theoretically predicted by him and his colleagues. The theory predicted that several spatially damped strata should appear in the cathode region column during the discharge transition from the anomalous to the normal regime. After the seminar, we went to the laboratory, ignited a discharge in Neon, set the necessary pressures and currents, obtained the normal and abnormal discharge maintenance regimes, and, in fact, observed three damped standing striations in the experiment!

The fluid models considering the electron thermal conductivity can qualitatively capture the nonlocal effects responsible for the structure of the cathode region in DC discharges. [58,59] The latest models use implicit and coupled solvers with adaptive Cartesian mesh.[35] The results of such models shown in Figure 8 reproduce the typical structure of DC discharges with the plasma density peak in the negative glow (a), a minimum in the Faraday dark space, an axially constant value in the (striation-free) positive column, and decay near the anode. The two-dimensional electric potential distribution (b) between the equipotential electrodes and a positive column plasma equalizes the electron and ion fluxes to the tube wall. A complicated redistribution of the electric potential forms a collisional double layer in the cathode region. The ionization rate (c) has a sharp peak in the negative glow and decreases sharply in the Faraday dark space before increasing again in the positive column. The ionization rate drops near the anode, forming anode-dark space and producing an off-axis peak (a ring) at the anode surface.

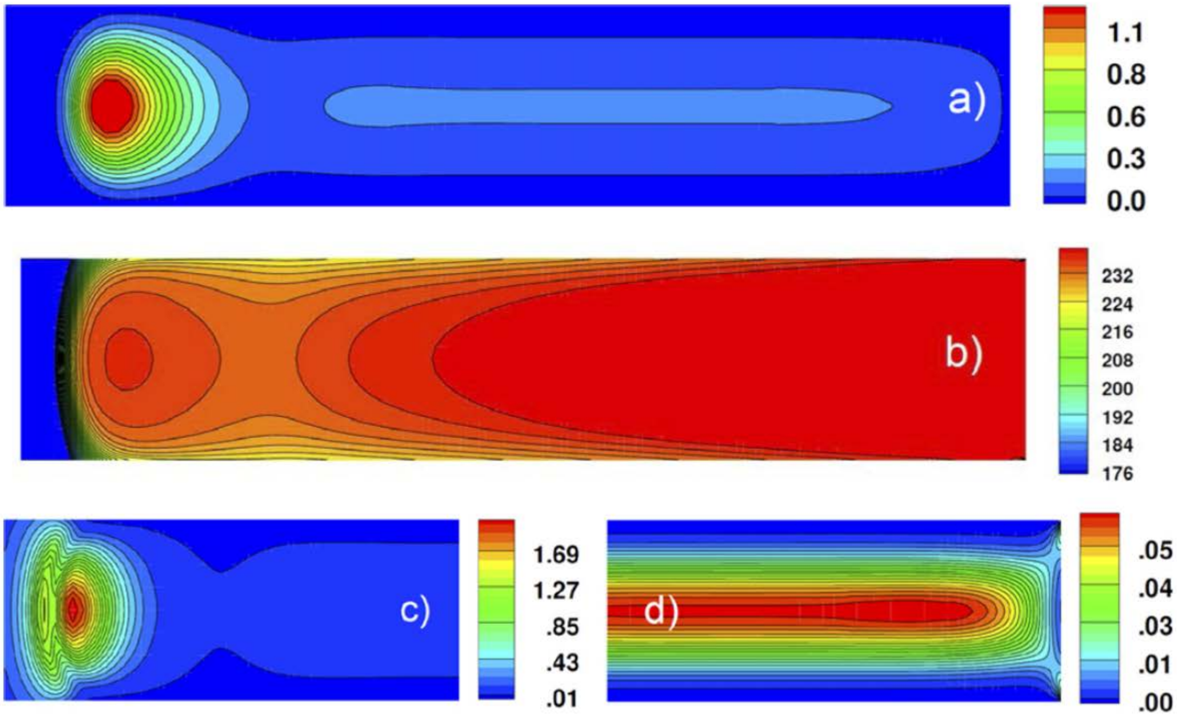

Figure 8: Spatial distributions of electron density (a) in 10^{10} cm^{-3}, electrostatic potential (b) in V, and ionization rate (in 10^{15} cm s^{-1}) near cathode (c) and near anode (d) in a tube of radius R = 1 cm and length d = 7 cm. The cathode is grounded, and a voltage of 235 V is applied to the anode [from Ref. [35]).

The cathode region in DC discharges still actively studied for various applications. It was recently shown [60,61] that the axial structure of the cathode region with hot (thermionic) cathodes is similar to the one with cold cathodes when the electron multiplication in the cathode region exceeds a factor of about 2-3. Raizer's latest papers in this area were devoted to studies of pattern formation on semiconductor cathodes.[62,63]

4.2 Anode Region

One can readily observe spatial patterns on a liquid anode in atmospheric pressure discharges. Figure 9 shows an example of such patterns for different discharge currents.[64] A diffuse spot forms at a low current (5.0 mA), and its diameter increases with increasing discharge current. The spot transforms into a ring with rising currents at a current of 10 mA. With a further increase of discharge current, multiple rings appear gradually. Up to five concentric rings have been observed before the rings break into numerous small spots at a current of 32.0 mA.

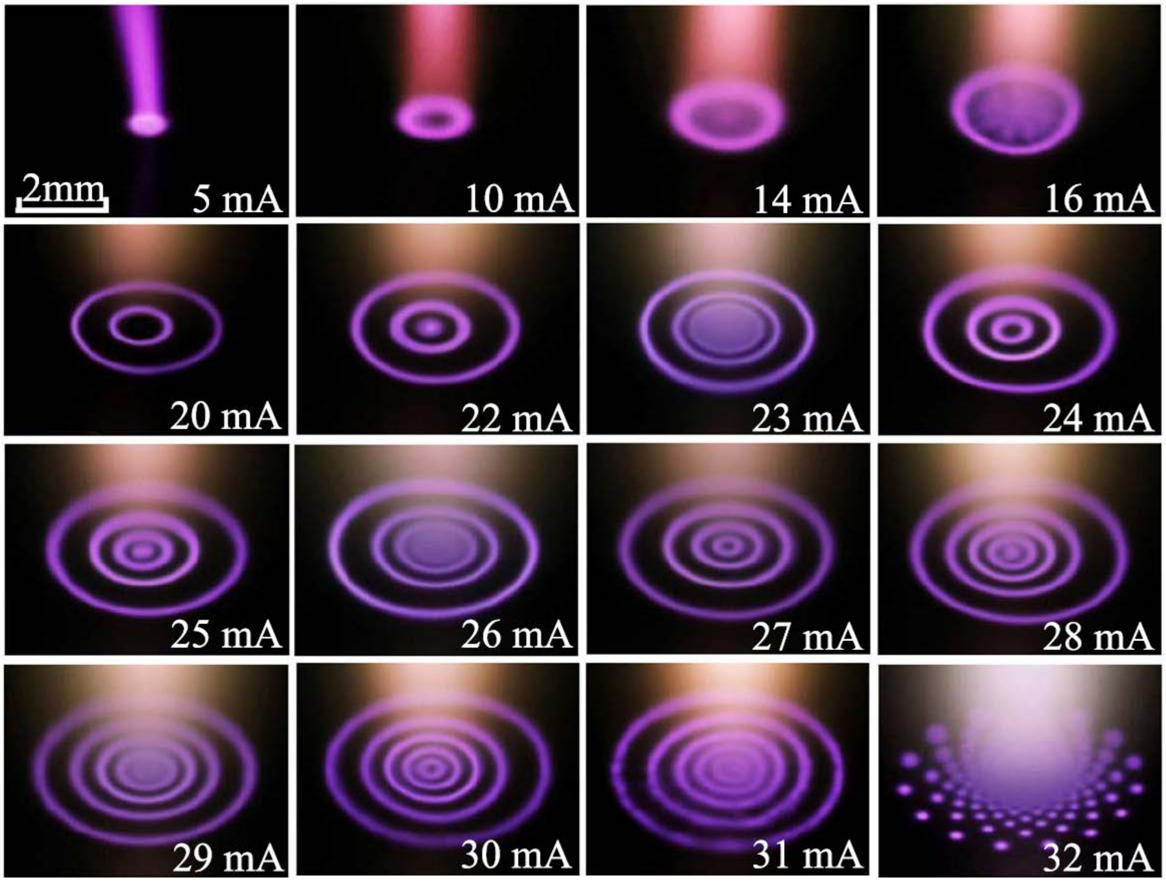

Figure 9: Anode patterns with increasing discharge current. DC discharge in the air with liquid waver anode gap width of 5 mm and exposure time of 100 ms (from Ref [64])

Understanding plasma self-organization in the anode region is far from complete. The kinetic and fluid models predict the appearance of a single luminous ring on a solid anode surface.[65,35,]The recent model [66] relates the anode spots to the change in the sign of the near-anode voltage. However, many phenomena, such as electric field reversals, the appearance of electron groups, and the formation of complicated spatial structures on solid and liquid anodes, remain unclear.

5. Conclusions

We have attempted to grasp the legacy of Professor Raizer using the principle of minimal power as an umbrella to illustrate some of his contributions to gas discharge physics from a single point of view. Raizer used this principle to interpret plasma stratification, pattern formation on electrodes, and other self-organization phenomena in gas discharges. He also warned that applying this *ad hoc* principle may produce erroneous results, and a proper theory could validate its applicability for each case. Our recent studies indicate that the minimal power principle is satisfied for some types of striations but does not hold for other types.

We connected the nonlinearity of equations describing gas discharges with the realization of a plasma state maintained at minimal electric power. Plasma constriction, stratification, and self-organization phenomena near electrodes can be understood from this point of view.

In the present paper, we have illustrated that the minimal power principle applies to moving striations in DC discharges of noble gases at low discharge currents. Sustaining stratified plasma is more energy-efficient than striation-free plasma when the ionization rate is a nonlinear function of the electric field. On the other hand, our simulations have shown that striations of *s*, *p*, and *r* types in Neon could exist with minimal or no ionization enhancement. So, the minimal power principle cannot predict plasma stratification in this case.

Therefore, it is expected that the minimal power principle can be applicable for different types of striations when the nonlinearities of the ionization processes are essential. However, the minimal power principle cannot describe plasma self-organization phenomena in the absence of nonlinearities in plasma production and loss rates. This conclusion agrees with Raizer warnings that applying the minimal power principle could lead to erroneous predictions, and a theory is required in each case to justify its use.

Acknowledgments

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