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Shock Compression Behavior of Stainless Steel 316L Octet-Truss Lattice Structures

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Abstract

Lattice structures offer desirable mechanical properties for applications of energy absorption and impact mitigation but limited research has been carried out on their shock compression behavior. In this work, the shock compression behavior of stainless steel 316L (SS316L) octet-truss lattice structures was investigated through experimental techniques and numerical simulations. Plate impact experiments with high-speed imaging were conducted at impact velocities of 270 – 390 m/s on lattice specimens with 5x5x10 unit cell geometries additively manufactured (AM) using direct metal laser sintering. High-speed imaging together with digital image correlation was used to extract full-field measurements and define a two-wave structure consisting of an elastic wave and planar compaction (shock) wave which propagated along the impact direction. A linear shock velocity versus particle velocity relation was found to approximate the measurements with a unit slope and a linear fit constant equal to the crushing speed. The shock velocity versus particle velocity relation, full-field measurements, and elastic limit together with the Eulerian form of the Rankine-Hugoniot jump conditions were used to find relations for the stress and internal energy behind the shock. Stress behind the shock increased with relative density and particle velocity, and specific internal energy converged to a single curve similar to that of bulk AM SS316L. Explicit finite element analysis using the Johnson-Cook constitutive model demonstrated similar shock behavior observed in experiments and a linear shock velocity versus particle velocity relation and corresponding Hugoniot calculations were found to be in agreement with experimental results. Numerical simulations confirmed negligible effects of exterior versus interior measurements and further validated the application of one-dimensional shock theory.

Keywords: Lattice structures, Shock, Hugoniot, Digital image correlation, Plate impact

1. Introduction

Lattice structures (or, lattice materials) are a class of architected cellular materials consisting of periodic unit cells composed of rods, plates, or smooth surfaces [1–5]. Advances in additive manufacturing (AM) within the last decade have allowed acceleration of the design space for these materials through increased manufacturing resolution on both the nano- and millimeter scale for metallic and polymeric materials [3, 6]. Lattice structures demonstrate high specific strength and stiffness at low relative densities (volume fractions) [1, 2, 7] and excellent energy absorption properties across various topologies [3, 8–10] which makes them well suited for high strain-rate and impact applications. Deformation of these materials may be considered as both a microscopic unit cell (UC) response and macroscopic bulk material response. Thus, the mechanical behavior of lattice structures inherently depends on both unit cell topology and base material. Strain-rate effects have been observed in two regimes: (1) at moderate to high strain-rates (1000/s), lattice structures

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show strain-rate-dependent mechanical behavior consistent with the base parent material [11–13]; and (2) at very high strain-rates or impact velocities, lattice structures exhibit a compaction shock wave [14, 15] similar to that in cellular materials [16].

Application of shock physics in solid cellular materials, such as metallic foams or periodic structures, serves as motivation for the analysis of lattice structures. Rankine-Hugoniot jump conditions for planar uniaxial shocks following the conservation of mass, momentum, and energy have been extensively applied in analytical models of the shock response in metallic foams [17, 18] but only proposed in lattice structures [19]. For example, Barnes et al. [20] performed experiments and Hugoniot shock analysis on low relative density aluminum foams using a direct impact split-Hopkinson bar with complementary numerical simulations by Gaitanaros and Kyriakides [21]. They observed a shock developing above some critical impact velocity and computed Hugoniot values agreed with experimental and simulation measurements. Additional simulations by Gaitanaros and Kyriakides [22] on aluminum foams showed dependence of the shock velocity-impact velocity relation, stress behind the shock, crushing energy density, and Hugoniot strain on varying relative densities. More recently, plate impact experiments with velocimetry have quantified the compaction shock wave in additively manufactured cellular materials [23], stochastic foams, and periodic materials of layered rods [24]. At very high impact velocities, interesting phenomena such as jetting of the periodic structure has also been observed [25].

The shock response of lattice structures has been studied in a limited manner across various length scales and materials. Hawreliak et al. [14] and Lind et al. [15] have conducted experiments on sub-millimeter UC polymeric materials which clearly show a two-wave structure of an elastic precursor wave followed by compaction shock wave. Branch et al. [26] conducted experiments on stainless steel 316L lattices (2x2x3 UC) using phase contrast imaging and demonstrated significant effects of experimental geometries on the shock response. On a smaller scale, Portela et al. [27] investigated impact behavior of nano-scale brittle lattice structures which showed a compaction shock response with distinct regions of intact and densified material. Few shock experiments on metallic lattice structures on the engineering scale (millimeter UC) have been carried out. The shock regime of lattice structures has been introduced, but not fully explored and requires the development of a consistent theory and analysis across length scales and materials.

In this work, shock definitions and parameters are developed for lattice structures and 1D uniaxial planar shock theory is applied to stainless steel 316L octet-truss lattice structures on the $\mathcal{O}(cm)$ length scale. Normal plate impact experiments are conducted and Rankine-Hugoniot jump conditions are applied for the compaction shock using full-field digital image correlation (DIC) measurements obtained using high-speed imaging. General trends for the Hugoniot field variables are analyzed per particle velocity and relative density. Explicit finite element analysis is also conducted to shed further light on experiments and the observed trends. Section 2 details the experimental methods of this work through specimen design and characterization, description of normal plate impact experiments, and DIC techniques. Section 3 then presents the experimental results and defines and develops Rankine-Hugoniot shock relations for shock

velocity, stress, and internal energy of lattice specimens. Section 4 presents the framework, results, and Rankine-Hugoniot shock analysis for numerical simulations, and discusses these results to validate effects of exterior versus interior measurements, application of one-dimensional shock theory, and comparison to experiments. Finally, Section 5 presents the summary and conclusions of this work.

2. Experimental Methods

2.1. Specimen Design and Characterization

The design and manufacturing process of lattice structure specimens using an octet-truss unit cell geometry [28] with 5x5x10 UC dimensions and one baseplate is illustrated in Fig. 1. Geometries with relative densities, ρ^*/ρ_s , of 10%, 15%, and 30% were designed using SolidWorks (Dassault Systemes, Providence, RI). A characteristic length, L_0 , was maintained in each unit cell geometry while the thickness, t , was varied to modify relative density. Lattice structures were manufactured by Protolabs (Maple Plain, MN, USA) using high-resolution (20 μm layer) direct metal laser sintering (DMLS) AM with stainless steel 316L (SS316L).

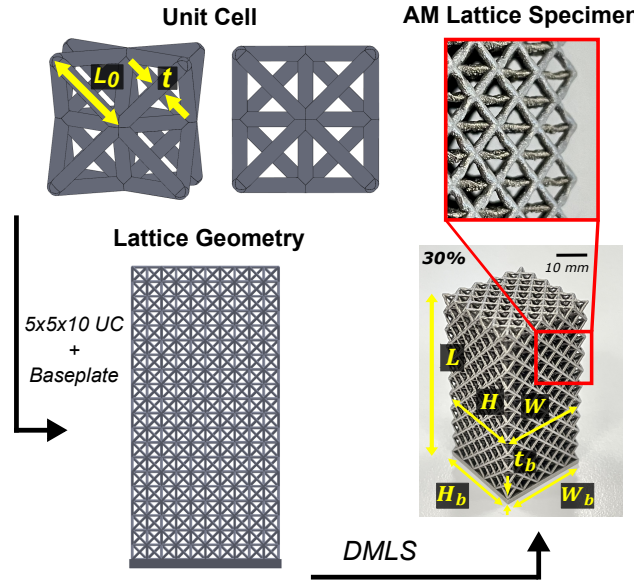


Figure 1: Design and DMLS additive manufacturing process of 5x5x10 UC SS316L octet-truss lattice specimens with characteristic length, L_0 , strut thickness, t , lattice specimen length, L , lattice width, W , lattice height, H , baseplate thickness, t_b , baseplate width, W_b , and baseplate height, H_b .

Table 1 shows relevant physical properties and dimensions of all lattice specimens. Relative density was calculated using volume fraction measurements as follows:

$$\rho^*/\rho_s = \frac{V_{exp} - V_b}{V_f} = \frac{V_{exp} - t_b H_b W_b}{(L - t_b) H W} \quad (1)$$

where V_{exp} is the experimental volume of the full specimen found by an Archimedes' principle suspension technique [29], V_b is the volume of the rectangular baseplate, and V_f is the space-filling box volume of the lattice. V_b was calculated from thickness, t_b , width, W_b , and height, H_b , of the baseplate and V_f

was calculated using the specimen total length, L , width, W , and height, H . Experimental error was computed using measurement uncertainties and error propagation. Mass density values were computed to verify repeatability of measurements and showed good agreement around $7700 \pm 100 \text{ kg/m}^3$, slightly lower than the expected value of 7960 kg/m^3 for AM SS316L [30]. Relative density for computer-aided-design (CAD) models was calculated using Eq. (1) and the corresponding measurements. Due to difficulties in printing small features, experimental relative densities showed significant deviation from CAD values ranging from 7% for $OT10$, 10% for $OT15$, and up to 15% for $OT30$.

#	CAD			Specimen			V_f			V_b			$\rho^*/\rho_s[\%]$
	$L_0[mm]$	$t[\mu m]$	$\rho^*/\rho_s[\%]$	$m[g]$	$V_{exp}[mL]$	$\rho[kg/m^3]$	$L[mm]$	$W[mm]$	$H[mm]$	$t_b[mm]$	$W_b[mm]$	$H_b[mm]$	
$OT10_1$	3.39	440	9.89	27.64	3.71	7450	49.84	24.44	24.44	1.59	24.99	24.97	9.43 ± 0.27
$OT10_2$	3.39	440	9.89	29.72	3.85	7746	49.89	24.43	24.44	1.65	25.02	25.02	9.74 ± 0.10
$OT10_3$	3.39	440	9.89	29.62	3.90	7617	49.91	24.45	24.44	1.60	25.01	25.05	9.99 ± 0.12
$OT10_4$	3.39	440	9.89	28.30	3.68	7710	49.84	24.46	24.49	1.57	25.03	25.04	9.30 ± 0.21
$OT15_1$	3.39	550	14.99	34.90	4.57	7656	49.81	24.54	24.51	1.53	25.00	24.98	12.40 ± 0.26
$OT15_2$	3.39	550	14.99	36.93	4.85	7630	49.62	24.47	24.53	1.28	24.93	25.00	13.92 ± 0.24
$OT15_3$	3.39	550	14.99	37.48	4.89	7677	49.79	24.55	24.56	1.50	25.02	25.02	13.55 ± 0.28
$OT15_4$	3.39	550	14.99	34.87	4.53	7706	49.79	24.55	24.73	1.48	25.00	25.00	12.28 ± 0.21
$OT30_1$	3.39	813	29.98	65.85	8.50	7764	50.01	24.77	24.79	1.61	24.97	25.00	25.15 ± 0.42
$OT30_2$	3.39	813	29.98	66.20	8.81	7535	50.03	24.77	24.77	1.63	24.98	24.99	26.17 ± 0.24

Table 1: Design and experimental parameters for lattice specimen characterization. Measured dimensions of each specimen are used to compute the relative density.

2.2. Normal Plate Impact Experiments

Normal plate impact experiments with high-speed imaging were conducted at varying impact velocities for specimens with nominal relative densities $\rho^*/\rho_s = 10\%, 15\%$, and 30% . A slotted 38.7 mm bore diameter powder gun was used with 76.2 mm long and 38.7 mm diameter aluminum 6061 projectiles (flyers) to impact lattice specimens at velocities of 270-390 m/s in a chamber with a transparent polycarbonate (PC) window. Figure 2 illustrates the experimental set-up. Lattice specimens were epoxied to an aluminum target holder and attached to a mounting gimbal by alumina rods.

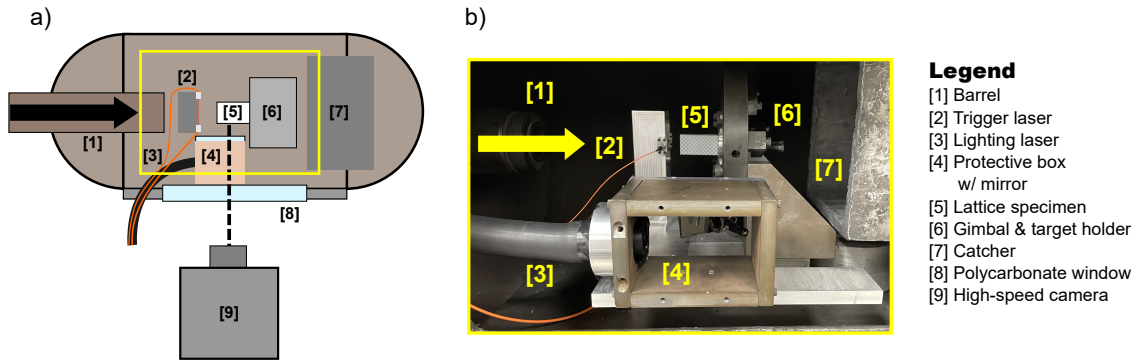


Figure 2: (a) Schematic of the experimental set-up for normal plate impact powder gun experiments on lattice specimens and (b) photograph of the laboratory set-up. One side of the protective box is removed for visibility.

Experimental images were captured using a Hyper Vision HPV-X2 high-speed camera (Shimadzu, Kyoto, Japan) with a Nikon 70-300mm f/4.5-5.6G lens and lighting from a non-coherent CAVILUX Smart laser (Cavitar, Tampere, Finland) with 40 ns pulse lengths which provided sufficient lighting and prevented

motion blur. A protective box with an interior mirror was used to protect the Cavitar laser and redirect light onto the test specimen through a small PC window. 128 images (400x250 pixels (px)) were captured at a constant framing rate (500,000 to 800,000 frames per second) set to maximize the number of images taken normal to the side of the lattice through the PC window during shock compression of the lattice specimen. The framing rate of the high-speed camera was determined by setting the framing interval, Δt , based on target impact velocities of 270, 300, 330, and 380 m/s. Experimental image capture was triggered using an Agilent MSO9404A oscilloscope (20 GSa/s sample rate), Wilcom F6230A visual fault locator, and Thorlabs PDA10A2 photodiode. Interruption of the laser (placed ahead of the specimen) by the flyer caused a measured voltage drop which triggered after a 1 μs time duration.

2.3. Digital Image Correlation

An example of experimental images taken during impact is shown in Fig. 3 and full deformation can be visualized in Supplemental Video S1. 2D digital image correlation (DIC) was performed on the high-speed images using Vic2D (Correlated Solutions, Columbia, SC, USA). Lattice specimens were painted white to improve contrast in experimental images and a random speckle pattern was applied to each flyer. There were two areas of interest (AOI) for these experiments; AOI1: over the flyer used to track rigid flyer displacements, and AOI2: over the lattice specimen to track specimen particle displacements. A random speckle pattern was applied to the flyer using a Sharpie pen (Fine Point) and produced unique subsets for correlation analysis of AOI1, while the lattice geometry itself was used for correlation of AOI2. A subset size of 29 px (4.35 mm) was used for AOI1 and 23 px (3.45 mm) for AOI2 (corresponding to $\approx 3/4$ of a unit cell) and step size of 1 px with incremental correlation was used in both analyses. Due to the inherent periodicity of the lattice specimen, DIC was able to correlate AOI2 subsets that were smaller than a unit cell with seed points placed away from the impact surface. A maximum error of 0.15 px was set to remove highly decorrelated regions.

Some image distortion through the polycarbonate (PC) window was expected. Therefore, calibration images were taken without the PC window and used to measure actual specimen dimensions using ImageJ [31]. These values were then used as calibration lengths in the DIC analyses. The particle velocities of both AOI are then numerically computed by applying the three-point central difference method on the measured displacements at the constant framing rate. Figure 3(b)-(d) shows the DIC results for particle velocity for experiment #OT15₂ on a $\rho^*/\rho_s \approx 15\%$ specimen at $V_{impact} \approx 300$ m/s. A clear compression shock front is visible in the images represented by the discontinuity of crushed and non-crushed regions of the specimen. This discontinuity can also be realized through the overlaid particle velocities which show near zero values ahead of the front and high constant values behind the front. Due to the large amount of deformation, some regions lose correlation and values are removed from the results illustrated by the ‘holes’ in later images.

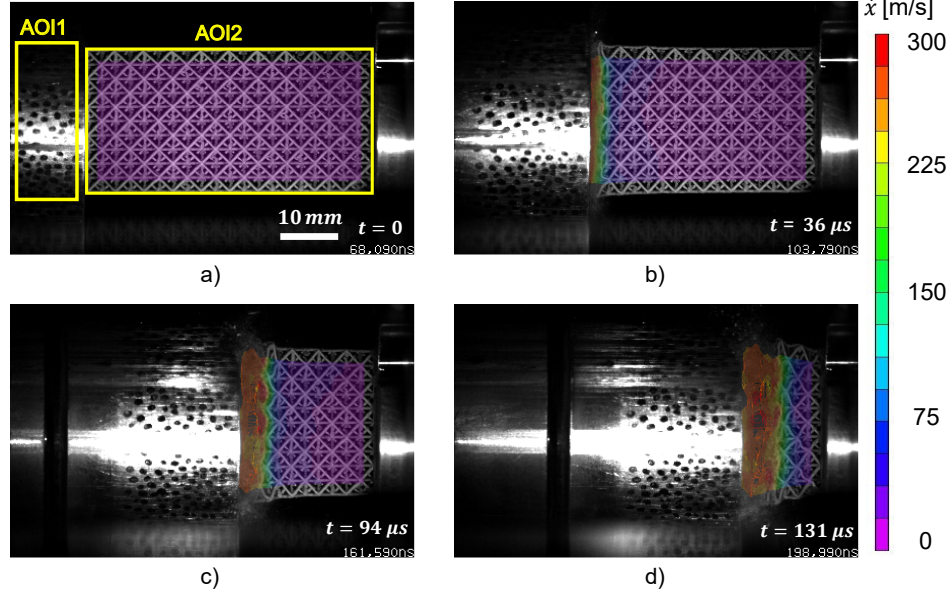


Figure 3: Experimental images for #OT152 at $V_{\text{impact}} \approx 300 \text{ m/s}$ and $\rho^*/\rho_s \approx 15\%$ with (a) DIC areas of interest and (b)-(d) full-field measurements for particle velocity at time instances, $t = 0$, $36 \mu\text{s}$, $94 \mu\text{s}$, and $131 \mu\text{s}$ after impact.

3. Experimental Results and Discussion

The experimental matrix for this study is shown in Table 2. Four impact velocities are tested for 10% and 15% specimens, and two for 30% specimens. The velocities of the flyers were extracted from each experiment using DIC displacements, and the set Δt and corresponding FPS are shown.

#	$\Delta t [\text{ns}]$	FPS	$V_{\text{impact}} [\text{m/s}]$
OT10 ₁	1900	526,320	270
OT10 ₂	1700	588,240	303
OT10 ₃	1500	666,670	331
OT10 ₄	1300	769,230	381
OT15 ₁	1900	526,320	265
OT15 ₂	1700	588,240	295
OT15 ₃	1500	666,670	331
OT15 ₄	1400	714,290	391
OT30 ₁	1700	588,240	290
OT30 ₂	1400	714,290	381

Table 2: Experimental matrix with frames per second (FPS) determined by setting the framing interval (Δt) of the high-speed camera to maximize images captured during deformation by a flyer traveling at the impact velocity, V_{impact} . The specimens are designated as $OTPP_Q$ where PP and Q are the nominal relative density (%) and experiment number for that density, respectively.

3.1. Wave Definitions and Extraction

Similar to shock behavior in bulk materials [32], a two-wave structure consisting of (1) an elastic wave, and (2) a compaction (shock) wave was observed during experiments. Similar observation has been well documented in polymeric lattice structures on the sub-millimeter scale impacted around 200-600 m/s. [14, 15]. The elastic wave and shock wave may be defined and the corresponding location can be extracted

using full-field displacement and velocity measurements. Ravindran et al. have used this technique to accurately define elastic and shock fronts in aluminum foams [33]. Full-field measurements were averaged over the center (middle) row of unit cells to approximate a uniaxial bulk material response. Displacement and velocity measurements were taken as the average over 30 y-pixel (vertical) positions corresponding to the center UC, for each x-pixel (horizontal) position.

The elastic wave front was defined using a $15\ \mu\text{m}$ displacement criterion based on the maximum observed DIC error (confidence interval) of 0.1 px and average image scale of 0.15 mm/px. The confidence interval for all experiments remained under 0.1 px during elastic deformation and therefore the criterion is larger than the experimental resolution. Wave definition and extraction for experiment #OT15₂ is shown in Fig. 4. Figure 4(a) shows particle displacement plotted against horizontal position (Lagrangian/undeformed coordinate) defined using the impact surface as $X = 0$ and time of impact as $t = 0$. Each line represents a single time-instance taken from one experimental image and the marker represents the position of the elastic front. A stricter or looser displacement criterion may slightly alter results, however the steepness and consistent spacing of displacement profiles imply similar results for criteria of 10-20 μm .

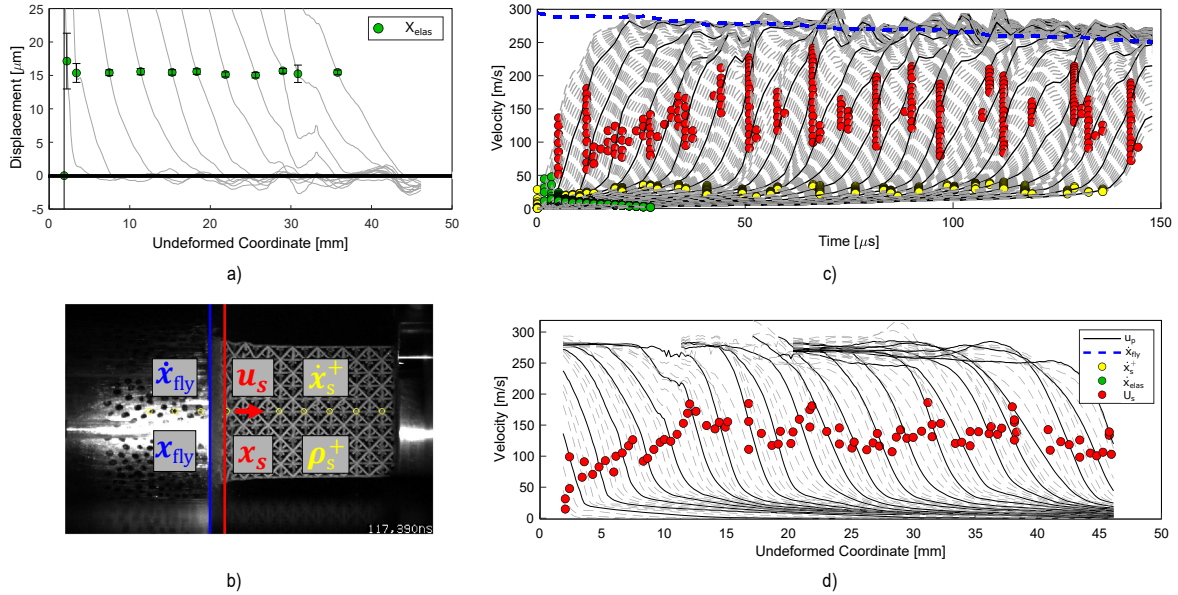


Figure 4: Wave definition and extraction for experiment #OT15₂: (a) displacement position profiles with $15\ \mu\text{m}$ displacement criterion marking the location of the elastic wave front (X_{elas}); (b) visualization of the shock definition and relevant parameters; and DIC particle velocity (u_p) profiles with respect to (c) time and (d) undeformed (Lagrangian) coordinate, with the elastic wave ($\dot{x}_{elas}$), flyer ($\dot{x}_{fly}$), velocity ahead of the shock ($\dot{x}_s^+$), and shock front (U_s) marked. Every 10th line is highlighted to improve data visualization.

The shock front and relevant parameters are defined in both position and time and visualization is shown in Fig. 4(b). Figure 4(c) shows the particle velocity (u_p) - time (t) history of all material points (x-pixel positions). Each material point of the specimen shows an initial low-amplitude particle velocity from the elastic wave followed by steep acceleration to a nearly constant particle velocity similar to the flyer speed. The shock front was defined for each material point as the point of largest change in velocity—equal to infinity

in the case of a mathematically sharp shock. Based upon this definition, the shock front may be defined: with respect to time for one material point (pixel), or with respect to position for one time instance (image). Figure 4(c) shows particle velocities with respect to time (i curves of j data points) and Fig. 4(d) shows particle velocities with respect to position (j curves of i data points) where i is the number of horizontal pixels in experimental images ≈ 200 and j is the number of images analyzed ≈ 90 . Every 10^{th} line is highlighted to improve data visualization. Determination of the shock front using du_p/dt caused ‘lines’ of front positions to emerge in Fig. 4(c) due to the low temporal resolution from the limited number of experimental images. Determination of the shock front using du_p/dX improved extraction of the shock wave as shown in Fig. 4(d) due to an increased spatial resolution ($i > j$). The particle velocity ahead of the shock, $\dot{x}_s^+$, was determined from the particle velocity-time history as the point of maximum change in curvature before the shock front.

The wave fronts were extracted and plotted against time as shown in Fig. 5(a) and visualized using experimental images in Supplemental Video S2. The elastic wave front (X_{elas}) was determined using the displacement criterion, the flyer front (x_{fly}) was determined using displacement measurements of the flyer, and the Lagrangian shock front (X_s) was determined using the spatial derivative technique in Fig. 4(d). The term ‘‘Lagrangian’’ is used to describe parameters defined using the undeformed coordinate system while the term ‘‘Eulerian’’ is used to describe parameters defined using the deformed coordinate system. The Eulerian shock front (x_s) can be determined by mapping the Lagrangian shock front to the deformed coordinate system using full-field displacement (z) measurements: $x_s = X_s + z$. Flyer speed and shock velocities were computed from the front-time histories using a three-point central difference method. Figure 5(b) shows the calculated flyer speed, Lagrangian shock velocity (U_s), and Eulerian shock velocity (u_s) as a function of time with the steady shock defined from time t_1 to t_2 to avoid edge or smoothing effects.

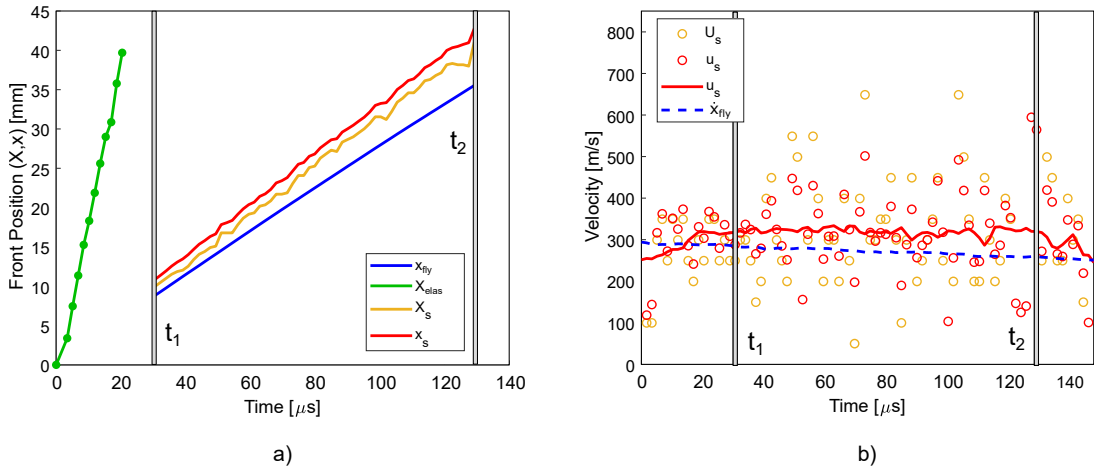


Figure 5: (a) Elastic (X_{elas}), flyer (x_{fly}), Lagrangian shock (X_s), and Eulerian shock (x_s) front-time histories and (b) corresponding flyer speed ($\dot{x}_{fly}$) and shock velocities in the Lagrangian (U_s) and Eulerian (u_s) configurations computed using a three-point central difference method. A 20% moving mean window was applied to smooth experimental scatter of Eulerian shock velocities (u_s) indicated by the red solid line.

Deceleration of the flyer was observed in all experiments and scatter plots of the shock velocities showed large variations in values. These variations may largely be attributed to noise in experimental front locations

combined with small timesteps used in numerical derivative calculations. Eulerian shock velocities were smoothed with a 20% moving mean window to eliminate scatter and resemble the smoothness of the flyer speed. Front positions appear mostly linear in time which supports the formation of a steady shock in the lattice specimens and justifies smoothing of data.

3.1.1. Experimental Elastic Wave Speeds

The elastic wave speeds were computed using a linear fit of the elastic front time history and error is reported as the 95% confidence interval of the fit. Results for all experiments are shown in Fig. 6. There is a clear trend showing positive correlation between elastic wave speed and relative density which agrees with the positive stiffness correlation with relative density found in lattice structures [1]. However, this trend inherently depends on manufacturing defects and geometry which control the stiffness and density of the specimen. Impact velocity does not appear to influence the wave speeds which agrees with results from other studies [15].

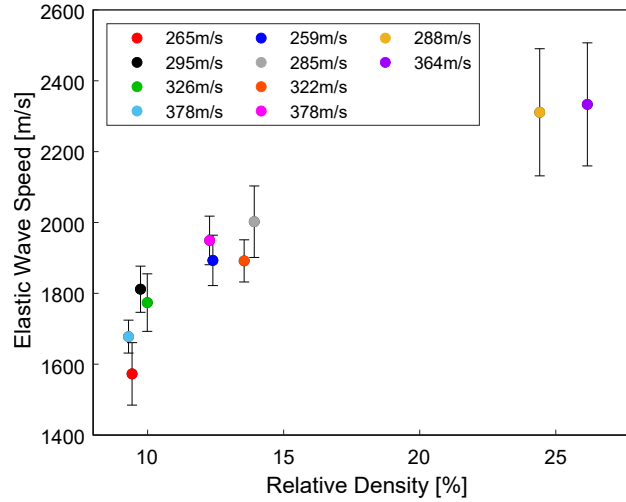


Figure 6: Elastic wave speed as a function of relative density for all experiments. Elastic wave speeds were computed using a linear fit of the elastic front time-history and error is reported as the 95% confidence interval of the fit.

3.2. Rankine-Hugoniot Shock Analysis

Rankine-Hugoniot jump conditions were used to analyze the shock compression behavior of lattice specimens. Full-field displacement measurements were used to map between the undeformed and deformed coordinate systems and allowed for extraction of field variables ahead and behind of the shock. These measurements allow for application of the Eulerian forms of the jump conditions following the conservation of mass, momentum, and energy [34]:

$$[[\rho]]u_s = [[\rho\dot{x}]] \quad (2)$$

$$[[\rho\dot{x}]]u_s = [[\rho\dot{x}^2 - \sigma]] \quad (3)$$

187

188

$$\llbracket \rho \left(\mathcal{E} + \frac{1}{2} \dot{x}^2 \right) \rrbracket u_s = \llbracket \rho \left(\mathcal{E} + \frac{1}{2} \dot{x}^2 \right) \dot{x} - \sigma \dot{x}^2 \rrbracket \quad (4)$$

189 where bracket quantities $\llbracket q \rrbracket$ represent the jump value ($q^+ - q^-$) across the shock, ρ is the density, $\dot{x}$ is the
 190 particle velocity, σ is the stress, and $\mathcal{E}$ is the specific internal energy. The Eulerian form of these equations
 191 was considered to account for the effect of the elastic wave on shock behavior, namely the propagation of
 192 the shock into non-quiescent material. Significant deviation of the Eulerian shock front and Lagrangian
 193 shock front in Fig. 5(a) illustrates a non-negligible effect. The difference in shock fronts is visualized in
 194 Supplemental Video S2 showing the shock and flyer fronts overlaid on experimental images. The Eulerian
 195 (deformed) coordinates show a true tracing of the shock that accounts for elastic deformation.

196 3.2.1. Shock Velocity (u_s) - Particle Velocity (u_p) Relation

197 Full Hugoniot characterization was carried out using known or approximated states ahead of the shock,
 198 conservation equations Eqs. (2)-(4), and an additional relation between shock velocity, u_s , and particle
 199 velocity, u_p . u_s is the Eulerian shock velocity shown in Fig. 5(b) and the particle velocity behind the shock,
 200 u_p , was approximated by the flyer speed, $\dot{x}_{fly}$, at the same time instant. This approximation is based on
 201 the convergence of particle velocities to the flyer speed as seen in Fig. 4(c). The $u_s - u_p$ relation is shown
 202 in Fig. 7 for all experiments. Each data point represents a single time instance at which particle velocities
 203 were computed from one experimental image. Smoothed data with a 20% moving mean window was used
 204 in the final results.

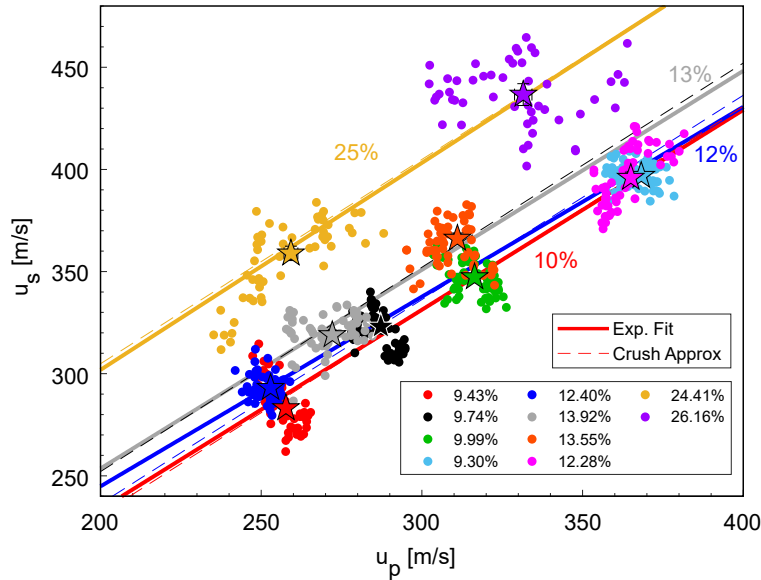


Figure 7: Eulerian shock velocity (u_s) - particle velocity (u_p) relation for all specimens and linear fits for $\rho^*/\rho_s = 10\%, 12\%, 13\%$, and 25% . Each data point corresponds to a measurement from one experimental image and stars represent linearized approximations. Dotted lines show the ‘crushing speed approximation’ discussed in Section 3.2.2.

205 Significant deceleration of the flyer induced a range of u_p during experiments. Linearized approximations
 206 were found by taking a linear fit of the flyer and shock fronts in Fig. 5(a) and are shown as stars in Fig.

7. A linear fit was also applied to scatter data to define the $u_s - u_p$ relation. While there was significant scatter in the results, the linear fit shows good agreement with linearized values. This agreement suggests experimental noise as the major cause of variation and will be further discussed and validated in Section 4.3 of the finite element analysis results.

The linear fit of the data was computed for four groups of relative density, 10% ($OT10_1 - OT10_4$), 12% ($OT15_1, OT15_4$), 13% ($OT15_2, OT15_3$), and 25% ($OT30_1, OT30_2$). Two groups (12%, 13%) were considered from the CAD 15% specimens due to a significant 10% difference in measured relative density values. The linear fit is of the form:

$$u_s = m + Su_p \quad (5)$$

where m is the constant coefficient and S is the slope of the relation. Experimental results for these fits are shown in Table 3 with error found using 95% confidence bounds. The slope, $S \approx 1$ for all relative densities and m increased with relative density for Eulerian coordinate parameters. A consistent S value and positive correlation of m with relative density matches trends observed in simulation of aluminum foams [22]. The Lagrangian shock relations are shown for comparison and demonstrate similar values for S but no trend for m can be determined due to high errors. Lagrangian and Eulerian fit parameters are further compared and discussed in simulation results in Section 4.2.

ρ^* / ρ_s [%]	m_x	S_x	m_X	S_X
10	37.98 ± 12.71	0.978 ± 0.041	50.02 ± 25.83	0.923 ± 0.084
12	59.09 ± 9.52	0.929 ± 0.030	59.89 ± 17.00	0.894 ± 0.054
13	58.99 ± 30.43	0.973 ± 0.105	58.97 ± 54.6	0.940 ± 0.188
25	99.15 ± 28.37	1.014 ± 0.096	50.83 ± 39.98	1.107 ± 0.136

Table 3: Linear fit parameters for experimental linear shock velocity (u_s) – particle velocity (u_p) relations of Eulerian, x , and Lagrangian, X , coordinate systems for $\rho^* / \rho_s = 10\%, 12\%, 13\%$, and 25% .

3.2.2. Crushing Speed Approximation

Most bulk materials typically follow a linear $u_s - u_p$ relation with $m \approx c_0$, where c_0 is the bulk sound speed of the material [34]. Elastic wave speeds $c_{elas} \approx 1600\text{-}2400$ m/s in the lattice specimens (Fig. 6) are much higher than the fitted $m \approx 30\text{-}100$ m/s values in Table 3. This constant, m , is therefore not equivalent to the bulk sound speed of the material, but may be physically interpreted by assuming $S \approx 1$. An S value of approximately one suggests a constant compaction strain in the shocked region and modeling the lattice as an elastic-plastic rigid-locking crushable material [18] where the particle velocity behind the shock is equal to the flyer speed. Taking $S = 1$ in Eq. (5), m can be rewritten:

$$m = u_s - u_p = \frac{d}{dt}(x_s(t) - x_{fly}(t)) = \frac{d}{dt}x_c(t) \quad (6)$$

where the shock velocity (u_s) and particle velocity (u_p) can be rewritten as time derivatives of the front locations, $x_s(t)$ and $x_{fly}(t)$, and the distance $x_s(t) - x_{fly}(t) = x_c(t)$ is recognized as the crushed width of the lattice specimen computed as the distance between the shock front and flyer front. The constant m can

then be interpreted as the slope of a linear fit to the crush width-time history, or, a constant crushing speed. Figure 8(a) shows the crushing width versus time plots for all experiments. Experimental curves show some noise, but a linear fit can be applied. The linear fit is shown through dotted lines in Fig. 8(a) and the slope of the plots are extracted in Fig. 8(b) and plotted against relative density. The crushing speed shows a strong positive correlation to relative density and values show good agreement with the calculated fit values for m . Using $S = 1$ and m as the crushing speed may serve as a general approximation for the linear $u_s - u_p$ relation of lattice materials. These approximations are applied for $\rho^*/\rho_s = 10\%, 12\%, 13\%$, and 25% by taking m as the average value for each group and are plotted as dotted lines alongside experimental fits in Fig. 7. These approximations show good agreement with experimental fit lines within the range of particle velocities.

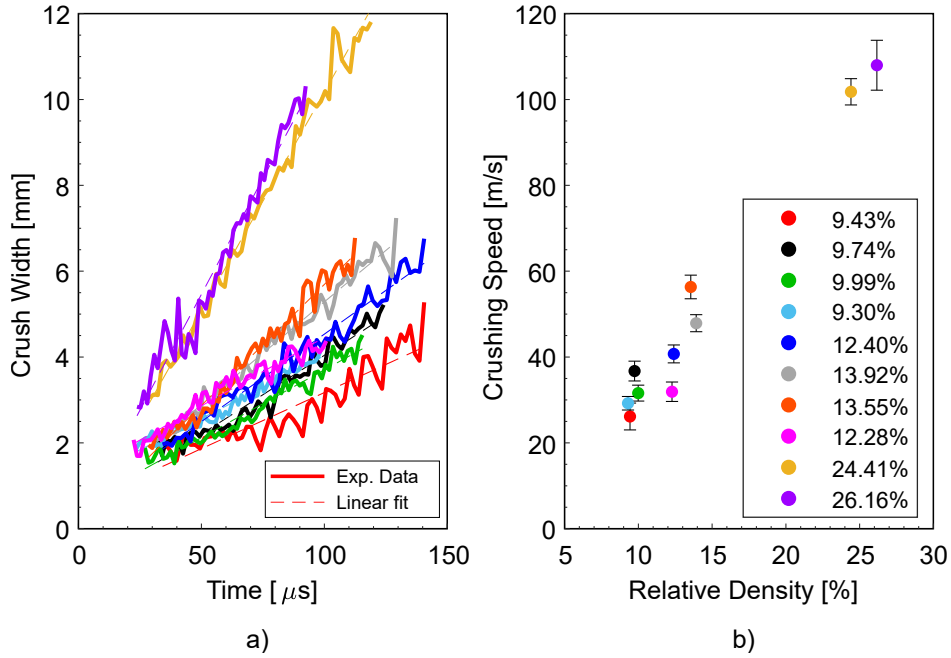


Figure 8: (a) The crush width (distance between flyer and shock front) of shocked specimens as a function of time with linear fit lines; and (b) the crushing speed (slope of linear fit of crush width time history) plotted against relative density.

3.2.3. Hugoniot Relations for Stress and Internal Energy

Stress and internal energy Hugoniot relations can be developed for both elastic and shock waves with the elastic wave treated as a weak shock into a quiescent material and the shock wave treated as a strong shock into a non-quiescent material. In the elastic case, $u_s = c_{elas}$ where c_{elas} is the measured elastic wave speed and independent of impact velocity.

The density ratio ahead of the shock, ρ^+/ρ_0 , was approximated by the ratio of the length of the uncrushed region in the deformed coordinate to the length of the uncrushed region in the undeformed coordinate. These lengths were found as the distance between the shock fronts and non-impacted face of the lattice such that: $\rho^+/\rho_0 = (L - t_b - X_s)/(L - t_b - x_s)$ with specimen length, L , and baseplate thickness, t_b . Using full-field

measurements of $\dot{x}^+$, $\dot{x}^-$, u_s , ρ^+ , and conservation of mass (Eq. (2)), the density behind the shock, ρ^- , may be determined:

$$\rho^- = \frac{\rho^+ u_s - \rho^+ \dot{x}^+}{u_s - \dot{x}^-} \quad (7)$$

It is assumed the elastic wave propagates into a quiescent material, or, $\dot{x}_{el}^+ = 0$. The particle velocity behind the elastic wave, $\dot{x}_{el}^-$, was defined as the corresponding velocity at which the material point satisfies the displacement criterion. Divergence of velocity values was observed for initial data points and so a constant value was approximated for $\dot{x}_{el}^-$ from the fourth image of each experiment (approximately deformation of half a unit cell). The velocity ahead of the shock, $\dot{x}_s^+$, was determined from the particle velocity-time history as the point of maximum change in curvature before the shock front and was chosen to most accurately depict the velocity. Particle velocities ahead of the shock in Fig. 4(c) appeared to gradually increase over time. This could be related to material properties such as plasticity or experimental conditions such as lateral movement of the specimen but regardless must be accounted for in the analysis.

Assumption or measurement of the stress ahead of the shock, σ^+ , and conservation of momentum (Eq. (3)) may be used to find the stress behind the shock, σ^- :

$$\sigma^- = \sigma^+ + u_s [\rho \dot{x}] - [\rho \dot{x}^2] \quad (8)$$

The stress behind the elastic wave, σ_{el}^- , was calculated assuming quiescent initial conditions and used to approximate the stress ahead of the compaction shock, σ_s^+ . Equation (8) was solved for each data point for the $\sigma_s^- - u_p$ relation shown in Fig. 9 and σ_s^- showed a positive correlation with relative density and particle velocity. Increasing the relative density at a constant particle velocity caused a larger increase in stress than increasing particle velocity at a constant relative density.

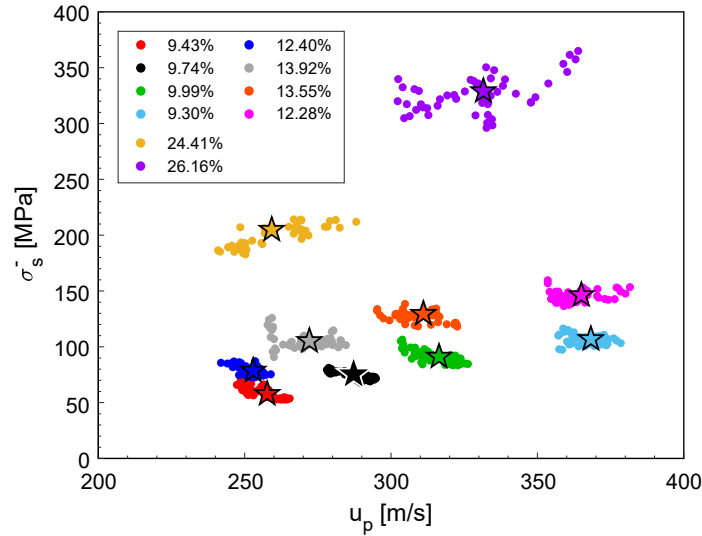


Figure 9: Experimental stress, σ_s^- , vs. particle velocity, u_p , Hugoniot relation. Stars represent linearized values by assuming a constant shock velocity.

In addition to the stress, the internal energy behind the shock, $\mathcal{E}^-$, may be computed in a similar way using conservation of energy (Eq. (4)):

$$\mathcal{E}^- = \frac{u_s[\rho\dot{x}^2] - [\rho\dot{x}^3] + 2[\sigma\dot{x}] + 2\mathcal{E}^+(\rho^+u_s - \rho^+\dot{x}^+)}{2\rho^-(u_s - \dot{x}^-)} \quad (9)$$

Elastic values for $\mathcal{E}_{el}^-$ were solved assuming quiescent initial conditions and used to approximate the internal energy ahead of the shock, $\mathcal{E}_s^+$. Specific internal energy is defined per unit mass and was converted to per unit volume by multiplying by the experimental density (ρ^*). Figure 10 shows the $\mathcal{E}_s^- - u_p$ Hugoniot relation for all experiments (a) per unit mass and (b) per unit volume.

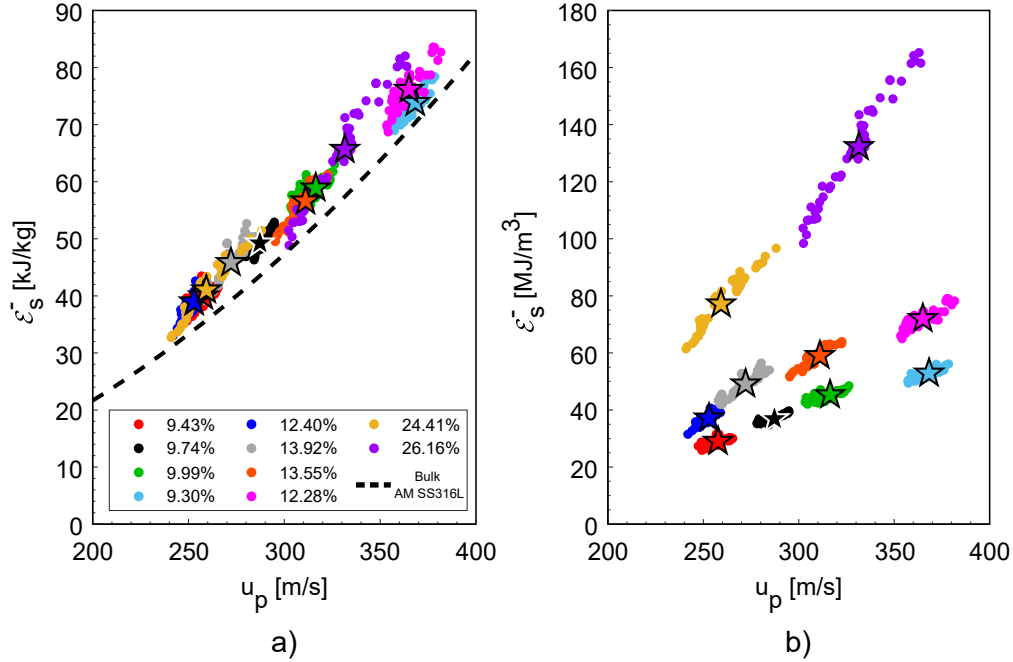


Figure 10: Experimental internal energy, $\mathcal{E}_s^-$ vs. particle velocity, u_p , Hugoniot relation normalized per (a) unit mass and (b) unit volume. Stars represent linearized values by assuming a constant shock velocity. The black dashed line shows the specific internal energy for bulk AM SS316L [30].

The specific internal energy converged to a single curve for all particle velocities and relative densities. The dotted black line in Fig. 10 represents values for bulk Laser Engineering Net Shaping (LENS) additively manufactured stainless steel 316L from shock experiments [30]. The bulk curve was calculated using measured experimental parameters of LENS AM SS316L: bulk sound speed, $c_0 = 4474$ m/s, longitudinal sound speed, $c_L = 5730$ m/s, density, $\rho = 7960$ kg/m³, velocity at the Hugoniot Elastic Limit (HEL), $v_{HEL} = 60$ m/s (approximated from photonic doppler velocimetry wave profiles), and linear coefficient in the $U_s - u_p$ shock relationship, $s = 1.54$. The stress and specific internal energy ahead of the shock were approximated using the conservation of momentum (Eq. 3) and conservation of energy (Eq. 4) and used with a linear $U_s - u_p$ relation for LENS AM SS316L to calculate the specific internal energy behind the shock, $\mathcal{E}_s^-$. The lattice structure curves overall show excellent agreement with the results for the bulk material. Mechanical properties of

Selective Laser Melting (SLM) AM SS316L have been shown to vary based upon crystallographic textures [35, 36] and therefore slight deviations between curves may occur due to differences in mechanical properties from differing AM techniques (LENS vs DMLS). Specific internal energy (per unit mass) of lattice specimens is therefore solely dependent on the parent material properties and particle velocity. Internal energy per unit volume showed a positive correlation with particle velocity and relative density—this is due to an increase of mass in higher relative densities, but little to no change in space-filling volume of the specimens.

4. Numerical Simulations

Numerical simulations were used to investigate the validity of the application of shock definitions and parameters and to explore additional trends in lattice structure shock behavior. A finite element framework with the Johnson-Cook constitutive model was used to model the AM SS316L lattice specimens. The elastic, flyer, and shock front locations and speeds were extracted for each unit cell position and Rankine-Hugoniot shock analysis was applied similar to experiments. Simulation results were used to evaluate the effects of exterior node measurements and non-constant impact velocities, and were also compared to experimental values.

4.1. Finite Element Model Framework

Numerical simulations were carried out using Abaqus/Explicit (Dassault Systemes, Providence, RI) finite element analysis (FEA) software with 5x5x10 CAD geometries used for AM of specimens. The baseplate of each specimen was fixed to an analytically rigid plate and a second rigid plate was used to impact the lattice with displacement boundary conditions corresponding to experimental flyer displacements. A general contact algorithm was defined for self-contact of the specimen and rigid plate-lattice interactions. Tetrahedral free meshing with quadratic C3D10M elements was used with a local mesh size of 1.5 mm corresponding to 4-5 million nodes and 2-3 million elements. Mass scaling was used to increase the computational timestep to 1 ns which corresponded to < 0.1% percent change in mass for all specimens. Physical relevance of simulations was evaluated by comparing the total energy to external work, internal energy, kinetic energy, and plastic dissipation at each timestep. Total energy appeared constant with respect to these energies and reached a maximum of 3.5% of the kinetic energy across all simulations, with an average value approximately equal to 0.5%. Figure 11(a) shows the side-view of the FEA 3D framework with lattice specimen and rigid plates and Fig. 11(b) shows nodes of interest for the numerical analysis. This framework allowed measurement and analysis of both interior and exterior nodes of the lattice specimen. Exterior nodes provided a direct comparison to experimental measurements while interior nodes provided a more appropriate uniaxial strain approximation over the innermost unit cells. Displacements and velocities for interior points were taken as the average over four nodes located at the middle vertices of the unit cell. Validation and comparison of these two techniques is discussed in Section 4.4.

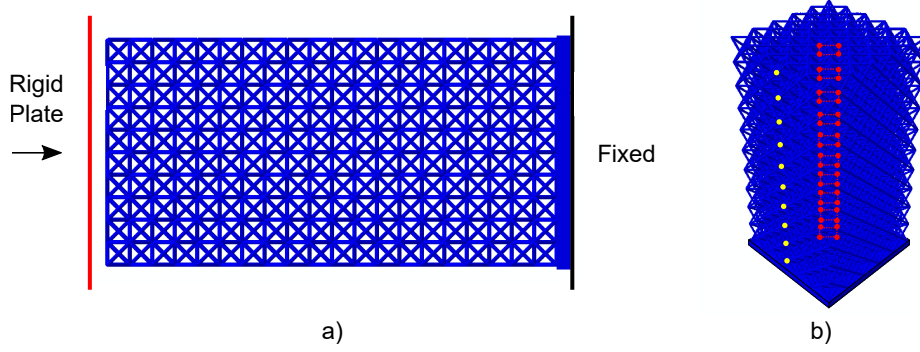


Figure 11: (a) Front view of the 3D finite element model of a 5x5x10 lattice specimen compressed between one fixed and one moving rigid plate and (b) interior and exterior nodes of interest in the 3D model of the lattice specimen.

To model the bulk material, the Johnson-Cook (JC) constitutive model [37] was used for the AM SS316L material:

$$\sigma = [A + B(\epsilon^p)^n] \left[1 + C \ln \left(\frac{\dot{\epsilon}^p}{\dot{\epsilon}_0} \right) \right] (1 - \hat{\theta}^m) \quad (10)$$

where ϵ^p is the equivalent plastic strain, $\dot{\epsilon}^p$ is the equivalent plastic strain-rate, and $\dot{\epsilon}_0$ is the reference strain-rate. The non-dimensional temperature, $\hat{\theta}$, is piecewise defined by: $(T - T_{ref})/(T_{melt} - T_{ref})$ for $T_{ref} < T < T_{melt}$; 0 for $T < T_{ref}$; and 1 for $T > T_{melt}$. The melting temperature, T_{melt} , was taken as the melting temperature of conventional SS316L [38] and a high reference temperature, T_{ref} , was chosen to limit temperature effects.

AM techniques of metals induce different material properties due to grain elongation and partial melting [35, 36] and constitutive modeling using conventional SS316L properties may lead to less accurate results. The parameters for the Johnson-Cook model ($A, B, n, C, \dot{\epsilon}_0$) were taken from experimental data collected by Platek et al. [39] on SLM AM SS316L while elastic parameters, density, and m were chosen from typical values for conventional SS316L [38, 40].

$\rho[kg/m^3]$	E [GPa]	ν [-]	A [MPa]	B [MPa]	n [-]	C [-]	m [-]	T_{ref} [K]	T_{melt} [K]	$\dot{\epsilon}_0$ [-]
8000	193	0.27	542	303	0.293	0.028	0.6	1573.15	1673.15	0.001

Table 4: Material parameters used for the Johnson-Cook (JC) constitutive model in numerical simulations. Elastic parameters, physical parameters, and m are taken from typical values for SS316L [38, 40] and the constitutive parameters are taken from experiments on AM SS316L by Platek et al. [39].

4.2. Simulation Results

Using the defined 3D model, numerical simulations were performed for each corresponding experiment in Table 2. Deformation images from simulation results clearly show the two-wave structure seen in experiments. Figure 12(a) shows the visualization of the shock front defined by particle velocities for $\rho^*/\rho_s = 15\%$ and $V_{impact} \approx 300$ m/s (corresponding to experiment #OT15₂) and full deformation can be visualized in Supplemental Video S1. A clear discontinuity in particle velocity was observed between regions of near zero velocity ahead of the shock and high constant velocity behind the shock.

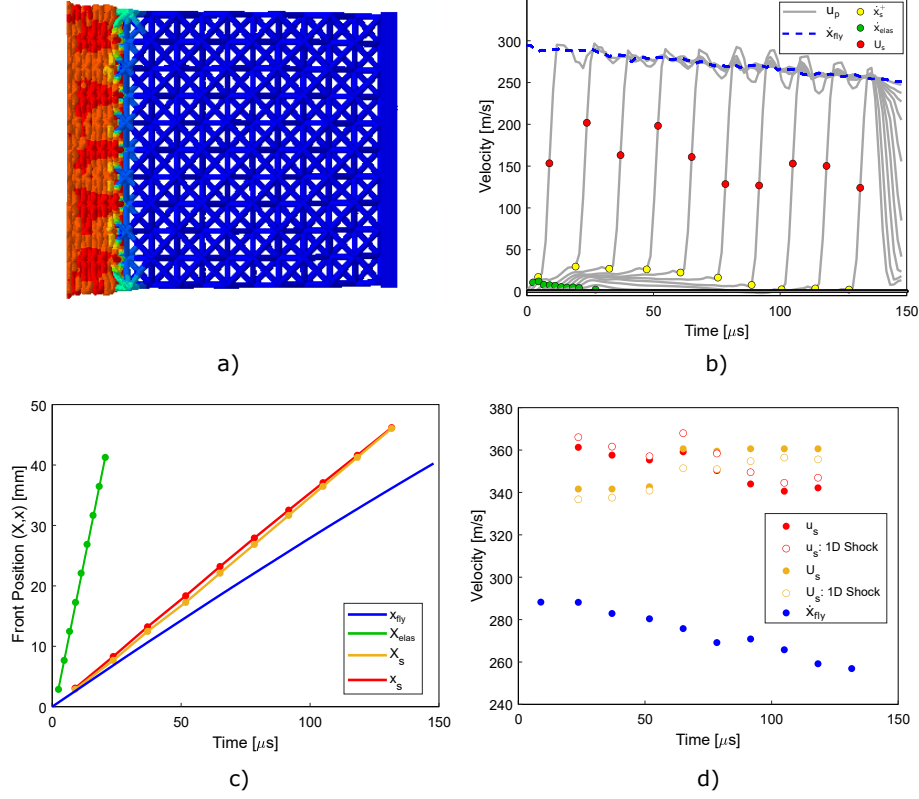


Figure 12: Wave definition and extraction in numerical simulation corresponding to experiment #OT152: (a) deformation images from numerical simulations showed similar shock structure to experimental images; (b) particle velocity profiles for each unit cell position used to determine shock parameters and front locations; (c) elastic (X_{elas}), flyer (x_{fly}), and shock (X_s, x_s) front-time histories; and (d) corresponding shock velocity and flyer speed with 1D shock theory calculations.

Figure 12(b) shows the particle velocities extracted from the interior nodes shown in Fig. 11(b). Particle velocities were extracted from 10 positions along the lattice each corresponding to one unit cell. The shock front was defined using a maximum du_p/dt (change in velocity with respect to time) criterion. This technique was chosen over the maximum du_p/dX technique used in experiments due to the higher temporal resolution (number of frames) than spatial resolution (number of unit cell positions) and therefore provided a more accurate calculation. The elastic wave and velocity ahead of the shock were defined as in experiments: the elastic wave front was defined using a $15 \mu m$ displacement criterion and the velocity ahead of the shock was defined from the velocity-time profile as the point of maximum change in curvature before the shock front. Small oscillations of particle velocity behind the shock were observed but clearly follow the deceleration of the flyer.

The computed elastic, flyer, and shock fronts are shown in Fig. 12(c) and closely resemble trends from experiments. The position of the propagating elastic wave front appears to be progressing linearly in time and elastic wave speed was extracted by applying a linear fit. Differences between the shock fronts in undeformed and deformed configurations were also observed, but were smaller than those in experiments.

Flyer speed was set by experimental displacement boundary conditions and shock velocities were com-

puted using a three-point central difference method and are shown in Fig. 12(d). The lower noise in the simulation results revealed trends within the velocities: Eulerian shock velocity decreased with time while Lagrangian shock velocity increased with time. These opposite trends may be explained by considering definitions of the shock velocity in Eulerian and Lagrangian coordinates derived by Davison [34]:

$$U_s = \frac{\rho^+}{\rho_0}(u_s - \dot{x}^+) \quad (11)$$

Equation (11) quantifies the effect of the elastic wave and density ahead of the shock on the Lagrangian shock velocity. This relation allows the computation of shock velocity in either coordinate given known values for the density and velocity ahead of the shock. The ratio density ahead of the shock, ρ^+/ρ_0 , was again defined as the ratio of distances from the non-compacted lattice face to the shock front: $\rho^+/\rho_0 = (L - t_b - X_s)/(L - t_b - x_s)$. Values of both coordinate shock velocities were computed using u_s , U_s , and Eq. (11) and are represented by the open data points in Fig. 12(d). These calculated values confirm the increasing and decreasing trends of U_s and u_s , respectively. Analysis in either coordinate system is valid, but only the deformed coordinate showed a positive correlation with the flyer speed with both decreasing with time. This correlation serves as the justification for defining the $u_s - u_p$ relation in the deformed coordinate for the Rankine-Hugoniot shock analysis.

4.3. Rankine-Hugoniot Shock Analysis

The Rankine-Hugoniot shock analysis of numerical simulations is shown in Fig. 13. Figure 13(a) shows the Eulerian shock velocity, u_s , flyer speed, u_p relation for $\rho^*/\rho_s = 10\%$, 15% , and 30% . The linear relation, $u_s = m + Su_p$, found in experiments was more evident in simulation results and illustrated by the fit lines along the individual scatter points. Linearized approximations for each simulation are shown by diamond markers. The slope $S \approx 1$ for all simulations and appeared to slightly increase with relative density while m was similar to experimental values and ranged from 30-100 m/s. Fit parameters are shown in Table 5 and are further discussed in Section 4.4.

Figure 13(b) shows the crush width-time history and the corresponding crushing speeds. Crush width ($x_s(t) - x_{fly}(t)$) profiles appeared linear and speeds were found by applying a linear fit. Crushing speed showed a positive correlation with relative density and a slight dependence on particle velocity with higher crushing speeds occurring at higher impact velocities. The $u_s - u_p$ relation was again approximated with $S = 1$ and m as the crushing speed taken as the average for each relative density. Crushing speed $u_s - u_p$ approximations are shown as dotted lines in Fig. 13(a) and show good agreement with data.

Rankine-Hugoniot jump conditions were applied for the elastic wave assuming quiescent initial conditions and wave speed, c_{elas} , which was measured using a linear fit of the front positions as seen in Fig. 12(c). The velocity behind the elastic wave, $\dot{x}_{el}^-$, was taken as the maximum velocity value of the elastic wave and the stress behind the elastic wave, σ_{el}^- was approximated using Eq. (8). Each data point represents results from one unit cell position (taken over four interior nodes about the central UC) and used to calculate the

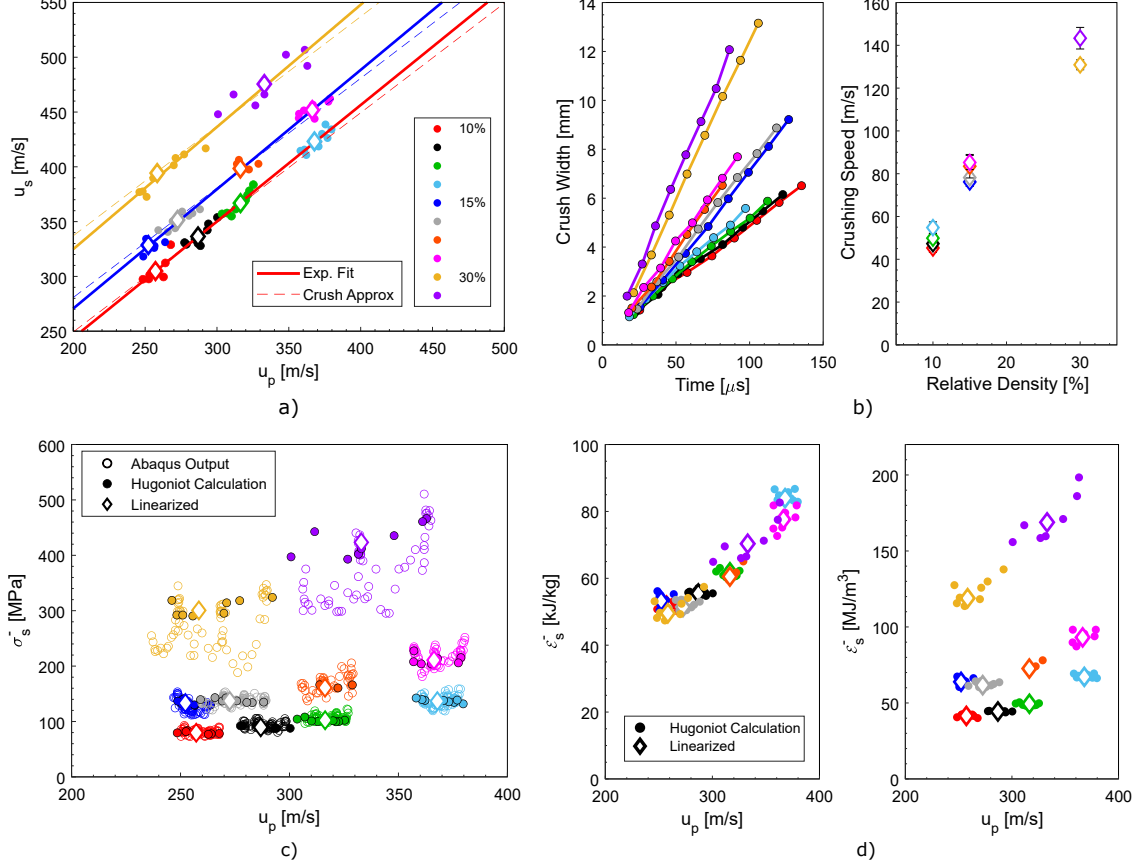


Figure 13: Shock Hugoniot relations for numerical simulations: (a) shock velocity (u_s) – particle velocity (u_p) relation, (b) crushing width-time history and crushing speed versus relative density, (c) stress behind the shock (σ_s^-) – particle velocity (u_p) relation, and (d) internal energy behind the shock ($\mathcal{E}_s^-$) – particle velocity (u_s) relation per unit mass and per unit volume.

Hugoniot response, similar to full-field measurements of experiments. The density behind the shock, ρ_s^- , was calculated using Eq. (7) and the stress behind the shock, σ_s^- , was evaluated using Eq. (8). Additionally, the stress ahead of the shock is approximated as $\sigma_s^+ = \sigma_{el}^-$ as done in experiments. It is noted that calculation of ρ^+/ρ_0 and subsequent parameters is improved over the experimental approach due to the true fixed position (no lateral motion) of the baseplate.

Reaction force measurements on the rigid impactor plate were extracted and directly compared to calculated Hugoniot values. Figure 13(c) shows the $\sigma_s^- - u_p$ relation with the rigid impactor reaction force outputs, Hugoniot calculations, and linearized approximations. Excellent agreement between Hugoniot calculations and reaction force outputs validates application of one-dimensional Rankine-Hugoniot shock analysis.

Figure 13(d) shows the internal energy ($\mathcal{E}_s^-$) – particle velocity (u_p) relation calculated using Eq. (9) and approximation of the state ahead of the shock as $\mathcal{E}_s^+ = \mathcal{E}_{el}^-$. Convergence of curves for internal energy per unit mass and divergence of curves for internal energy per unit volume was observed as in experiments.

4.4. Validation of Node Measurements

Exterior node measurements on the three-dimensional lattice structure may not match internal node measurements due to lateral movement of exterior material during loading. This difference in values is examined through the $u_s - u_p$ relations for these nodes. Figure 14(a) shows the numerical $u_s - u_p$ relation for exterior and interior nodes (shown in Fig. 11(b)) for $\rho^*/\rho_s = 10\%$ specimens with experimental displacements as applied boundary conditions. Exterior nodes showed a lower intercept than interior nodes which may be interpreted as a lower crushing speed. This decrease in crushing speed measured in the axial direction is consistent with the presence of lateral velocities. Overall, differences between shock velocities were small and exterior node measurements adequately approximate interior measurements. $S \approx 1$ with exterior and interior values differing around 2%. While m values varied more, the corresponding shock velocities showed 4-6% difference within relevant particle velocity ranges.

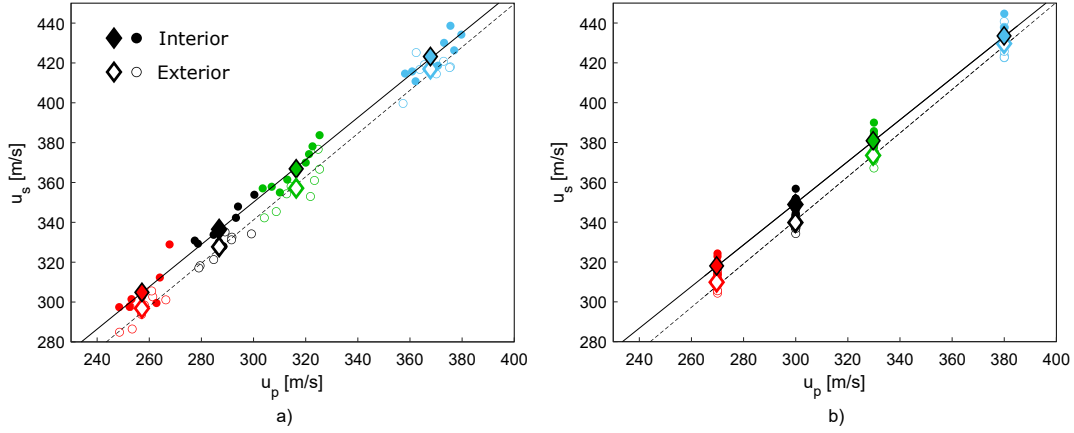


Figure 14: Comparison of shock velocity (u_s) - particle velocity (u_p) relation for interior and exterior nodes for applied boundary conditions of a) experimental displacements and b) constant impact velocities for $\rho^*/\rho_s = 10\%$ specimens. The circles ($\circ$) correspond to the various nodal values and the diamonds ($\diamond$) correspond to linearized values by assuming a constant shock velocity.

Figure 14(b) show the $u_s - u_p$ relation for exterior and interior nodes of $\rho^*/\rho_s = 10\%$ specimens impacted at constant velocities of 270, 300, 330, and 380 m/s. Exterior nodes again showed a lower intercept than interior nodes consistent with a lower axial-measured crushing speed due to lateral velocities. Scatter of 4 - 5% was observed in shock velocities and can be attributed to measurement techniques and resolution. Excellent agreement between linear $u_s - u_p$ relations and linearized shock velocity approximations justify the linearized approximations as relevant quantities.

Table 5 shows computed values for S , m_{fit} , and m_{crush} where S and m_{fit} were found using a linear fit of all data points and m_{crush} was computed as the average crushing speed. Errors for S and m_{fit} were found using 95% confidence bounds and error for m_{crush} was calculated as the standard error. These results provide insights into the sensitivity of the $u_s - u_p$ calculations. $S \approx 1$ within error for all conditions while m_{fit} and m_{crush} varied with significant errors. The difference in fit values between experimental displacements and constant velocities was small (1-2% for S , 10-20% for m_{fit} , and 2% for m_{crush}) and

verifies the use of experimental displacements for $u_s - u_p$ relations. Crushing speed was higher in simulations than experiments for all relative densities which may be attributed to perfect geometries and higher relative densities in simulations.

$\rho^*/\rho_s[\%]$		Node	Loading	S	m_{fit}	m_{crush}
10	Simulation	Exterior	u_{const}	1.094 ± 0.045	12.95 ± 14.47	44.47 ± 2.33
10	Simulation	Interior	u_{const}	1.047 ± 0.044	35.68 ± 14.04	49.41 ± 1.11
10	Experiment	Exterior	-	0.978 ± 0.041	37.98 ± 12.71	30.93 ± 2.23
10	Simulation	Exterior	z_{exp}	1.085 ± 0.050	15.88 ± 15.39	43.56 ± 2.60
10	Simulation	Interior	z_{exp}	1.063 ± 0.045	31.54 ± 13.83	49.34 ± 2.06
12	Experiment	Exterior	-	0.929 ± 0.030	59.09 ± 9.52	36.32 ± 4.41
13	Experiment	Exterior	-	0.973 ± 0.105	58.99 ± 30.43	52.10 ± 4.22
15	Simulation	Interior	z_{exp}	1.089 ± 0.051	53.02 ± 15.30	80.79 ± 2.15
25	Experiment	Exterior	-	1.014 ± 0.096	99.15 ± 28.37	104.89 ± 3.09
30	Simulation	Interior	z_{exp}	1.115 ± 0.123	101.8 ± 37.07	137.12 ± 6.21

Table 5: The linear shock velocity (u_s) – particle velocity (u_p) parameters (S, m_{fit}, m_{crush}) for exterior and interior nodes of experiments and simulations with applied experimental displacements (z_{exp}) for $\rho^*/\rho_s = 10\%, 15\%$, and 30% specimens and constant impact velocities (u_{const}) for $\rho^*/\rho_s = 10\%$ specimens.

4.5. Comparison to Experiments

Numerical simulation results support the Hugoniot equation of state (EOS) in the form of a linear $u_s - u_p$ relation as a reasonable approximation of the shock response of SS316L octet-truss lattices. The linearized results for all experiments and simulations are shown in Fig. 15. Differences between experiment and simulation may be attributed to the constitutive model, additive manufacturing technique [35, 36], stiffness effects from geometric imperfections [41], and lower relative densities from manufacturing defects. Similar trends were observed in both experiment and simulation: a linear $u_s - u_p$ relation (Fig. 15(a)); linear crushing width-time histories and a positive correlation between crushing speed and relative density (Fig. 15(b)); increasing stress behind the shock with particle velocity and relative density (Fig. 15(c)); and converged internal energy per unit mass behind the shock and diverged internal energy per unit volume behind the shock (Fig. 15(d)). Higher values were observed for all quantities in simulations which may be dependent on geometric defects and distributions [26].

Average elastic wave speeds are compared for all relative densities and reported in Table 6. Infinite lattice approximations were found using effective elastic properties from Bloch-wave homogenization of the octet-truss unit cell calculated by Patil and Matlack [42]. Normalized effective uniaxial modulus was found as a function of relative density with scaling $E^*/E_s \propto (\rho^*/\rho_s)^{1.2}$ and values were extracted for $\rho^*/\rho_s = 10\%, 15\%, 30\%$. Wave speed was approximated as $c = \sqrt{E^*/\rho^*}$ using these values. Experiment and simulation values were calculated using linear fits of front profiles as previously discussed.

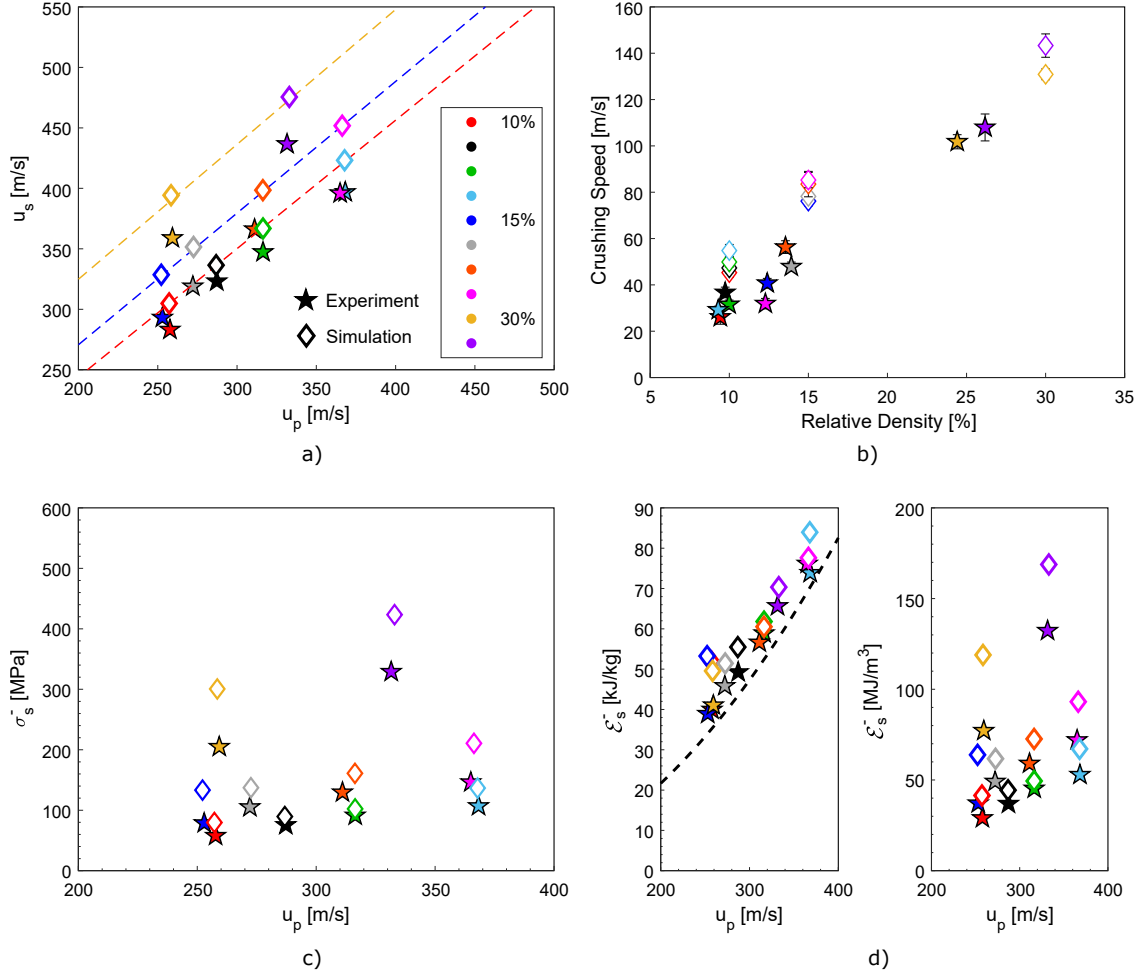


Figure 15: Comparison between simulation and experiment for a) shock velocity (u_s) - particle velocity (u_p) relation with simulation fit lines, b) crushing speed - relative density relation, c) stress (σ_s^-) - particle velocity (u_p) relation, and d) internal energy ($\mathcal{E}_s^-$) - particle velocity (u_p) relation defined per unit mass and per unit volume. The dotted black line represents experimental values for bulk AM SS316L [30].

ρ^*/ρ_s	10%	15%	30%
Experiment	1709 ± 54	1934 ± 26	2322 ± 11.18
Simulation	2028 ± 5	2129 ± 10	2438 ± 6
Bloch-wave	2009	2142	2299

Table 6: Average elastic wave speeds from experiments and simulations for $\rho^*/\rho_s = 10\%$, 15% , and 30% specimens. Error was calculated as the standard error of the measurements. Bloch-wave approximations of wave speeds were found using effective stiffness calculations [42].

In experiment, simulation, and Bloch-wave analysis, elastic wave speed increased with relative density. Wave speeds were lower in experiments and showed better agreement with both simulation and Bloch-wave values at higher relative densities. Bloch-wave approximations of wave speeds showed excellent agreement with simulation values with 1% difference at $\rho^*/\rho_s = 10\%$ and 15% , and 6% difference at $\rho^*/\rho_s = 30\%$. Experimental wave speed at $\rho^*/\rho_s = 10\%$ showed 18.6% difference from simulation values, $\rho^*/\rho_s = 15\%$ showed 10.1% difference, and $\rho^*/\rho_s = 30\%$ showed 4.9% difference.

A major reason for this discrepancy in values could be manufacturing defects. Manufacturing defects were

observed frequently in the horizontal struts of lower density specimens as seen in Fig. 16. High quantities of manufacturing imperfections in $\rho^*/\rho_s = 10\%$ experimental specimens suggest a loss of stiffness from geometric defects and support a larger observed difference in speeds. These defects may be attributed to the manufacturing limits of DMLS technology for features at this length scale such as poor heat conduction due to small feature dimensions or powder bed rollers disrupting weakened struts during printing. The effects of geometric defects are accounted for in the shock analysis by using the experimental elastic wave speed and relative density values.

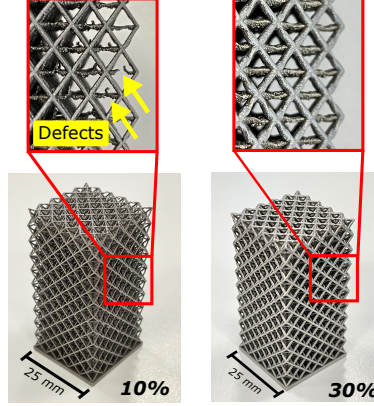


Figure 16: Manufacturing defects of $\rho^*/\rho_s = 10\%$ specimens present on horizontal struts relative to print direction.

5. Summary and Conclusions

The shock compression behavior of stainless steel 316L octet-truss lattice structures was explored and shock definitions and Rankine-Hugoniot shock theory were applied to both experiments and simulations. SS316L octet-truss lattice structures with $5 \times 5 \times 10$ unit cell (5 mm/UC) geometries were additively manufactured using DMLS. Normal plate impact experiments were conducted on specimens with nominal relative densities (volume fraction) of 10%, 15%, and 30% at impact velocities ranging from 270 m/s to 390 m/s. High-speed experimental images were taken at 500,000-800,000 fps and digital image correlation (DIC) was performed on the lattice specimen and flyer to extract full-field particle velocities. Images showed a distinct discontinuity between regions of highly densified and intact material with a densification front propagating in the direction of impact. Particle velocities of the densified material were approximately equal to the flyer speed while the velocities of the intact region were near zero. A two-wave structure consisting of an elastic wave and compaction shock wave was observed in agreement with prior work [14, 15]. The elastic wave was defined using a displacement criterion and elastic wave speeds appear constant for all specimens at a given relative density. Experimental and simulation elastic wave speeds were found to be around 30%-50% of the bulk sound speed of the parent material.

The observed ‘structural shock’ can be modelled using Rankine-Hugoniot jump conditions for one-dimensional planar shocks. The shock front was defined for each material point at the point of maximum

change of particle velocity and described through position and time using full-field measurements of the lattice specimen. Full-field measurements allow determination of density and velocity field variables during shock propagation and permit application of the Rankine-Hugoniot jump conditions in the Eulerian form. The elastic wave was treated as a weak shock and jump conditions were used to approximate field variables of the material behind the elastic wave, but ahead of the shock. Stress and internal energy behind the shock were also estimated using the jump conditions and the experimental measurements.

In addition to experiments, numerical simulations using finite element analysis (Abaqus/Explicit) with the Johnson-Cook constitutive model also exhibit a highly densified region with a steady front separated from the intact material. Numerical results validate experimental techniques by verifying the negligible effects of exterior node measurements and non-constant impact velocities, matching calculated Hugoniot stress values to numerical outputs, and accurately computing shock velocities using 1D shock theory. Rankine-Hugoniot analysis of numerical simulations matches experimental results leading to the following conclusions:

- A linear shock velocity, u_s , particle velocity, u_p , relation was found to describe the equation of state for octet-truss lattice materials: $u_s = m + Su_p$. Linearized approximations of velocities were found by taking a linear fit of the front position-time history and show excellent agreement with fits of the full data.
- A slope of $S \approx 1$ was found in both experiment and simulation and suggests modeling lattice structures as elastic-plastic rigid-locking [18] crushable materials.
- The constant term, m , is correlated with the crushing front speed with values $\mathcal{O}(10 - 100)$ m/s significantly lower than the elastic wave speeds of SS316L lattices with values $\mathcal{O}(2000)$ m/s.
- Stress behind the shock showed a positive correlation with relative density and particle velocity, while specific internal energy calculations showed a correlation only with particle velocity.
- Specific internal energy of lattice specimens closely resembled values obtained from bulk AM SS316L at similar particle velocities.
- 1D shock theory may be applied to numerical simulation results without resorting to a $U_s - u_p$ relation for the bulk material. Qualitative agreement with experimental results shows the shock behavior of lattice structures is dependent on the structural response.

In both experiments and numerical simulations, consistent definitions for shock parameters such as front locations and field variables (density, particle velocity) behind and ahead of the shock have been developed, and 1D Rankine-Hugoniot analysis may be identically applied. With these definitions and theoretical framework, future work entails the exploration of shock behavior of lattice structures with varying topology and parent material. Homogenization and extrapolation of shock behavior based on relative density and unit cell topology would allow for optimization of these materials at a highly decreased computational cost. Extension

to other topologies would allow engineering design of impact-dissipative structures for applications of energy absorption, and strong lightweight materials for dynamic applications.

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References

- [1] Ashby MF. The properties of foams and lattices. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 2006;364(1838):15–30. doi:10.1098/rsta.2005.1678.
- [2] Schwaiger R, Meza LR, Li X. The extreme mechanics of micro- and nanoarchitected materials. *MRS Bulletin* 2019;44(10):758–65. doi:10.1557/mrs.2019.230.
- [3] Zhang X, Wang Y, Ding B, Li X. Design, Fabrication, and Mechanics of 3D Micro-/Nanolattices. *Small* 2020;16(15):1–19. doi:10.1002/smll.201902842.
- [4] Tancogne-Dejean T, Diamantopoulou M, Gorji MB, Bonatti C, Mohr D. 3D Plate-Lattices: An Emerging Class of Low-Density Metamaterial Exhibiting Optimal Isotropic Stiffness. *Advanced Materials* 2018;30(45):1–6. doi:10.1002/adma.201803334.
- [5] Bonatti C, Mohr D. Mechanical performance of additively-manufactured anisotropic and isotropic smooth shell-lattice materials: Simulations & experiments. *Journal of the Mechanics and Physics of Solids* 2019;122:1–26. doi:10.1016/j.jmps.2018.08.022.
- [6] Tofail SA, Koumoulos EP, Bandyopadhyay A, Bose S, O'Donoghue L, Charitidis C. Additive manufacturing: scientific and technological challenges, market uptake and opportunities. *Materials Today* 2018;21(1):22–37. doi:10.1016/j.mattod.2017.07.001.
- [7] Schaedler TA, Jacobsen AJ, Torrents A, Sorensen AE, Lian J, Greer JR, et al. Ultralight metallic microlattices. *Science* 2011;334(6058):962–5. doi:10.1126/science.1211649.
- [8] Ozdemir Z, Tyas A, Goodall R, Askes H. Energy absorption in lattice structures in dynamics: Nonlinear FE simulations. *International Journal of Impact Engineering* 2017;102:1–15. doi:10.1016/j.ijimpeng.2016.11.016.

- [9] Mueller J, Matlack KH, Shea K, Daraio C. Energy Absorption Properties of Periodic and Stochastic 3D Lattice Materials. *Advanced Theory and Simulations* 2019;2(10):1–11. doi:10.1002/adts.201900081.
- [10] Jin N, Wang F, Wang Y, Zhang B, Cheng H, Zhang H. Failure and energy absorption characteristics of four lattice structures under dynamic loading. *Materials and Design* 2019;169:107655. doi:10.1016/j.matdes.2019.107655.
- [11] Weeks JS, Ravichandran G. High strain-rate compression behavior of polymeric rod and plate Kelvin lattice structures. *Mechanics of Materials* 2022;166:104216. doi:10.1016/j.mechmat.2022.104216.
- [12] Tancogne-Dejean T, Spierings AB, Mohr D. Additively-manufactured metallic micro-lattice materials for high specific energy absorption under static and dynamic loading. *Acta Materialia* 2016;116:14–28. doi:10.1016/j.actamat.2016.05.054.
- [13] Hazeli K, Babamiri BB, Indeek J, Minor A, Askari H. Microstructure-topology relationship effects on the quasi-static and dynamic behavior of additively manufactured lattice structures. *Materials & Design* 2019;176:107826. doi:10.1016/j.matdes.2019.107826.
- [14] Hawreliak JA, Lind J, Maddox B, Barham M, Messner MC, Barton NR, et al. Dynamic Behavior of Engineered Lattice Materials. *Scientific Reports* 2016;6:1–7. doi:10.1038/srep28094.
- [15] Lind J, Jensen BJ, Barham MI, Kumar M. In situ dynamic compression wave behavior in additively manufactured lattice materials. *Journal of Materials Research* 2019;34(1):2–19. doi:10.1557/jmr.2018.351.
- [16] Sun Y, Li QM. Dynamic compressive behaviour of cellular materials: A review of phenomenon, mechanism and modelling. *International Journal of Impact Engineering* 2018;112:74–115. doi:10.1016/j.ijimpeng.2017.10.006.
- [17] Harrigan JJ, Reid SR, Seyed Yaghoubi A. The correct analysis of shocks in a cellular material. *International Journal of Impact Engineering* 2010;37(8):918–27. doi:10.1016/j.ijimpeng.2009.03.011.
- [18] Lopatnikov SL, Gama BA, Haque MJ, Krauthauser C, Gillespie JW. High-velocity plate impact of metal foams. *International Journal of Impact Engineering* 2004;30(4):421–45. doi:10.1016/S0734-743X(03)00066-6.
- [19] Messner MC, Barham MI, Kumar M, Barton NR. Modeling shocks in periodic lattice materials. *AIP Conference Proceedings* 2017;1793(January). doi:10.1063/1.4971618.
- [20] Barnes AT, Ravi-Chandar K, Kyriakides S, Gaitanaros S. Dynamic crushing of aluminum foams: Part i - Experiments. *International Journal of Solids and Structures* 2014;51(9):1631–45. doi:10.1016/j.ijsolstr.2013.11.019.

- [21] Gaitanaros S, Kyriakides S. Dynamic crushing of aluminum foams: Part II - Analysis. *International Journal of Solids and Structures* 2014;51(9):1646–61. doi:10.1016/j.ijsolstr.2013.11.020.
- [22] Gaitanaros S, Kyriakides S. On the effect of relative density on the crushing and energy absorption of open-cell foams under impact. *International Journal of Impact Engineering* 2015;82:3–13. doi:10.1016/j.ijimpeng.2015.03.011.
- [23] Winter RE, Cotton M, Harris EJ, Maw JR, Chapman DJ, Eakins DE, et al. Plate-impact loading of cellular structures formed by selective laser melting. *Modelling and Simulation in Materials Science and Engineering* 2014;22(2). doi:10.1088/0965-0393/22/2/025021.
- [24] Branch BA, Ionita A, Patterson BM, Schmalzer A, Clements B, Mueller A, et al. A comparison of shockwave dynamics in stochastic and periodic porous polymer architectures. *Polymer* 2019;160:325–37. doi:10.1016/j.polymer.2018.10.074.
- [25] Lind J, Robinson AK, Kumar M. Insight into the coordinated jetting behavior in periodic lattice structures under dynamic compression. *Journal of Applied Physics* 2020;128(1). doi:10.1063/5.0003776.
- [26] Branch BA, Specht PE, Jensen S, Jared B. Detailed meso-scale simulations of the transient deformation in additively manufactured 316 L stainless steel lattices characterized by phase contrast imaging. *International Journal of Impact Engineering* 2022;161:104112. doi:10.1016/j.ijimpeng.2021.104112.
- [27] Portela CM, Edwards BW, Veysset D, Sun Y, Nelson KA, Kochmann DM, et al. Supersonic impact resilience of nanoarchitected carbon. *Nature Materials* 2021;doi:10.1038/s41563-021-01033-z.
- [28] Deshpande V, Fleck N, Ashby M. Effective properties of the octet-truss lattice material. *Journal of the Mechanics and Physics of Solids* 2001;49(8):1747–69. doi:10.1016/S0022-5096(01)00010-2.
- [29] Hughes SW. Archimedes revisited: A faster, better, cheaper method of accurately measuring the volume of small objects. *Physics Education* 2005;40(5):468–74. doi:10.1088/0031-9120/40/5/008.
- [30] Gray GT, Livescu V, Rigg PA, Trujillo CP, Cady CM, Chen SR, et al. Structure/property (constitutive and spallation response) of additively manufactured 316L stainless steel. *Acta Materialia* 2017;138:140–9. doi:10.1016/j.actamat.2017.07.045.
- [31] Schneider CA, Rasband WS, Eliceiri KW. NIH Image to ImageJ: 25 years of image analysis. *Nature Methods* 2012;9(7):671–5. doi:10.1038/nmeth.2089.
- [32] Meyers MA. *Dynamic Behavior of Materials*. John Wiley and Sons, Ltd; 1994. ISBN 9780470172278. doi:10.1002/9780470172278.
- [33] Ravindran S, Koohbor B, Malchow P, Kidane A. Experimental characterization of compaction wave propagation in cellular polymers. *International Journal of Solids and Structures* 2018;139-140:270–82. doi:10.1016/j.ijsolstr.2018.02.003.

- [34] Davison L. Fundamentals of Shock Wave Propagation in Solids. Shock Wave and High Pressure Phenomena; Berlin, Heidelberg: Springer Berlin Heidelberg; 2008. ISBN 978-3-540-74568-6. doi:10.1007/978-3-540-74569-3.
- [35] Sun Z, Tan X, Tor SB, Chua CK. Simultaneously enhanced strength and ductility for 3D-printed stainless steel 316L by selective laser melting. NPG Asia Materials 2018;10(4):127–36. doi:10.1038/s41427-018-0018-5.
- [36] Maconachie T, Leary M, Lozanovski B, Zhang X, Qian M, Faruque O, et al. SLM lattice structures: Properties, performance, applications and challenges. Materials and Design 2019;183:108137. doi:10.1016/j.matdes.2019.108137.
- [37] Johnson GR, Cook WH. Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures. Engineering Fracture Mechanics 1985;21(1):31–48. doi:10.1016/0013-7944(85)90052-9.
- [38] Harvey PD. Engineering properties of steel. American Society for Metals (ASM); 1982. ISBN 9780871701442.
- [39] Platek P, Sienkiewicz J, Janiszewski J, Jiang F. Investigations on mechanical properties of lattice structures with different values of relative density made from 316L by selective laser melting (SLM). Materials 2020;13(9). doi:10.3390/ma13092204.
- [40] Umbrello D, M'Saoubi R, Outeiro JC. The influence of Johnson-Cook material constants on finite element simulation of machining of AISI 316L steel. International Journal of Machine Tools and Manufacture 2007;47(3-4):462–70. doi:10.1016/j.ijmachtools.2006.06.006.
- [41] Liu L, Kamm P, García-Moreno F, Banhart J, Pasini D. Elastic and failure response of imperfect three-dimensional metallic lattices: the role of geometric defects induced by Selective Laser Melting. Journal of the Mechanics and Physics of Solids 2017;107:160–84. doi:10.1016/j.jmps.2017.07.003.
- [42] Patil GU, Matlack KH. Effective property evaluation and analysis of three-dimensional periodic lattices and composites through Bloch-wave homogenization. The Journal of the Acoustical Society of America 2019;145(3):1259–69. doi:10.1121/1.5091690.