# Local Communication in Populations of Thermostatically Controlled Loads

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**ABSTRACT**

*Thermostatically Controlled Loads (TCLs) have shown great potential for Demand Response (DR) in electric grid operations. However, it has been commonly seen that DR events using TCLs may cause load synchronization and unwanted oscillatory effects, especially in homogeneous populations. In an attempt to mitigate the negative impacts of DR events, a decentralized method is proposed that modifies each thermostat behavior based on the activity of a small number of nearby TCLs. This feedback introduces the possibility of instability in the aggregate behavior. A stability analysis is performed on a linearized model of the aggregate system and the results of that analysis compared to simulation results. The proposed modification of thermostat behavior results in 4-fold reduction in the post-DR peak while suppressing ensuing oscillations at the expense of a modest increase in compressor cycling. The linearized model also provides insight into the aggregate behavior of the population.*

## 1 Introduction

The rapid deployment of intermittent renewable energy generation on the electric grid has presented significant challenges to the entities that are responsible for maintaining the reliability of the electric distribution system. The challenges inherent in this rapid expansion of renewables has been typified by the so-called "duck curve" first reported by the California Independent System Operator (CAISO) [1]. The report highlights two significant challenges of widespread behind-the-meter solar deployment: a steep ramp of demand at sunset, and over generation in mid-day. Some form of energy storage is cited as a solution to these problems, with grid-level batteries dominating the discussion.

But batteries are only one way to tame the duck curve. A growing number of energy observers are pointing out that by enlisting the cooperation of electricity users, and incentivizing changes in their behavior, grid operators can impact the problem by changing consumption at critical times of the day [2]. Such programs fall under the umbrella of demand response (DR) and have been in common use for the past 10 years. Increasing development of the "smart grid" offers unprecedented opportunity for more complex electrical supply and demand interactions in a relationship that has been historically unilateral.

Much of this activity is focused on thermostatically-controlled loads (TCLs) such as those systems used for space heating and cooling, hot water or refrigeration and food storage. Systems that use electricity in this manner are normally designed to maintain temperature within a finite range of values which is defined by the thermostat deadband.

In a typical DR application, residential air conditioning compressors (but not the circulating fan) are temporarily turned off under control of the utility, allowing the inside temperature to rise above the thermostat setting. The grid is relieved of

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the load that the AC compressor would have drawn and the homeowner (if present) may experience a small and possibly noticeable increase in indoor temperature. These programs are nearly universally used to shed load at times of very high demand, but they can also be used to increase consumption at times of energy surplus, resulting in a somewhat cooler home than the set point would imply. A common interpretation of this effect is that energy is being stored in the thermal mass of the home. In this respect, the home acts like a thermal battery, albeit a leaky one with limited storage capacity.

## 2 BACKGROUND

This work builds on a body of literature that spans agent based modeling, demand response, electric markets and modeling of aggregated thermostatically controlled loads.

The use of agent based modeling to understand the aggregate dynamics of the grid and electric markets is becoming mainstream as typified by [3–5]. Much of the focus of these works is on capturing customer and/or market behavior, but they demonstrate the efficacy of this general approach to the current problem.

A key finding of the literature relative to demand response and TCL's is the potential for previously randomized loads to synchronize after a DR event [6, 7]. Specifically, unless great care is taken to avoid this possibility, it is likely that the peak load experienced after a DR event is released (or just prior, in the case of pre-cooling) will exceed the size of the peak avoided by the DR itself. In addition, depending on the heterogeneity of the load, the aggregate load could continue to oscillate through large variations over the next 2-6 hours. This has led many researchers to consider analytical approaches to model the aggregate load of a population of TCL agents.

In related work, Meyn [8–10] has developed a stochastic approach based on mean field theory to controlling deferrable loads to deliver ancillary grid services. In essence, this approach introduces a stochastic element to each deferrable load (varying the time they come on line) and does so in a way that ensure the delivery of certain ancillary services while maintaining quality of service in an average, but not individual sense. The technique is demonstrated on pool pumps and does not incorporate the individual load dynamics.

A priority-stack-based control strategy can be an effective way to control TCLs [11]. Sorting the population of TCLs by temperature into two stacks, one where the TCLs are off and the other where the TCLs are on, allows for the most appropriate selection of the next TCL to turn on or off. Simulations show this approach can be effective, however it requires extensive communication (including information about actual temperature in the home) and computation which may make it difficult to implement at scale.

The line of research that is most pertinent to this paper builds on the seminal work by Malhame and Chong [12] in which the behavior of a large number of identical TCLs is modeled using the a pair of coupled Focker Plank equations. These are 2nd order PDEs that describe the time evolution of the distributions of the TCLs on the temperature deadband of the thermostats. The paper shows how the steady state solutions of these equations can provide insight to the aggregate behavior and can inform methods of controlling them for a demand response event. This work was further advanced by Callaway [2], who developed a control scheme based on the Fokker Plank equations that allowed a collection of TCLs to follow the highly variable signal from a wind farm. This control was brought about by changing the set points of all thermostats in the aggregate population simultaneously. Similarly [13, 14] continued this work through extensive simulations in which they numerically solved the Focker Plank equations on a high resolution grid of the temperature range. The work includes the development of a control framework and demonstrates the ability to control a population of TCLs to follow the production of a large wind farm by simultaneously adjusting the set points of all the thermostats. Perfumo et al. [15] uses the Focker Plank equations and derives a 2nd order ODE that describes the envelope of the expected behavior of the aggregated load. This work provides important insight into how the level of heterogeneity and statistical distribution of the individual agent parameters leads to effective damping of the aggregate load response to DR events or other disturbances. Zhang et al. [7] expanded this approach to a 2nd order model of the individual home and uses a 2D discretization to approximate the distributions. The control scheme suggested is to encourage 'early' switching of the state of the thermostat through a broadcast control signal.

In [16], a centralized controller is used to coordinate many agents without knowing individual agent states. By comparing a reference to a received aggregate output value, the controller determines what signal to broadcast to all agents. Each agent then makes a decision based on the signal and defined probabilities. Similarly, Zhang et al. [7] developed a control scheme where the centralized control signal is broadcast to all agents. The agents then decide how to implement the signal based on their local temperature and power state.

Other researchers have taken a more de-centralized approach. For example, using price signalling and adaptive mechanisms, coupled with smart meters, to prevent loads from synchronizing and creating high peak demand [17].

With the rising number of electric vehicles (EVs), an additional opportunity for demand response has been created. EV charging management allows for the vehicles to charge at times of low demand or to match renewable energy production. Xydas et al. [5] developed a model to demonstrate the effectiveness of "responsive" EV chargers. These chargers determine their charging schedule according to a signal that takes power demand and generation forecasts into account. Their model demonstrated that responsive EVs could reduce the peak charging demand of all EVs, including unresponsive EV chargers,

by shifting demand to a time when the unresponsive chargers were finished charging. They also demonstrated the ability of responsive EVs to charge in response to a real time photovoltaic (PV) generation profile.

## 3 MODELING

The core of this approach is a small amount of information sharing between thermostats that are in close proximity. For this study, thermostats are made aware of the on/off state of the compressors in the four nearest homes. The selection of the connections may be defined by the layout of the neighborhood (e.g. the next door neighbors, the house across the street and over the fence in the backyard) or they may be defined by the topology of the electric distribution system. The thermostats will use that information (the states of neighboring systems) to either delay the transition of the compressor to on, or turn the compressor off prematurely, based on the number of neighbors whose compressor is running.

The logistics of information sharing are not covered in this study, but a large number of options are available covering a spectrum of technologies from internet-based server models and power-line carriers in which the connections can be implemented and programmed centrally, to local communication protocols such as Zigbee and Bluetooth [18]. Similarly, this study does not consider possible bandwidth or latency issues related to this communication. The proposed scheme requires the transmission of one bit of information from each home to 4 nearby homes every minute. Given the fact that the characteristic time constants of the underlying dynamic systems is on the order of hours, this is a reasonable assumption.

Security and privacy are always a concern in information sharing situations, particularly in relation to the energy infrastructure. These privacy concerns are addressed by noting that the information being shared (whether the compressor is on or off) is something a typical next door neighbor can observe by opening the window that is nearest the neighbor's outdoor condenser unit. In some locations, that information could be deduced from a careful observer on the public sidewalk. This information is similar to whether or not inside lights are on. While it is not usually noticed, it is clearly observable from the outside. As for security, securing the national electric grid and all forms of electronic communication from outside interference is a national research priority involving a large number of research groups and will not be addressed in this study.

The model of the individual residence was first proposed by Malhamé and Chong [19]. Callaway furthers this work by contributing equivalent parameters for the first order model and showing populations of thermostatically controlled loads can act as virtual storage devices by collectively managing them [2]. Through this work, Callaway found load populations with greater heterogeneity are better candidates for set point control of thermostatically controlled loads because previous work on homogeneous populations found that their aggregate behavior was more oscillatory, less damped, and in general more difficult to control. The heat transfer dynamics for a single house, are shown in Equation 1.

$$\dot{T} = \frac{1}{RC}\Big(T_\infty - T(t)\Big) + \frac{1}{C}\Big(Q_I - m(t)Q\Big) \tag{1}$$

Where the parameters are defined in Table 1.

Table 1. Model Parameter Definitions

| | | |
|---|---|---|
| $T(t)$ | Indoor Air Temperature | °C |
| $T_\infty$ | Outdoor Air Temperature | °C |
| $C$ | Thermal Capacitance of Building | kWh/°C |
| $R$ | Thermal Resistance of Building | °C/kW |
| $Q_I$ | Internal Heat Gain | kW |
| $Q$ | Load Cooling Capacity | kW |
| $m(t)$ | Discrete State of AC Power | - |

The proceeding analysis is simplified if the equation is transformed using the dimensionless temperature $\theta$, as defined below.

$$\theta_i = \frac{T_i - T_{min,i}}{T_{max,i} - T_{min,i}} = \frac{T_i - T_{sp,i}}{\delta} \tag{2}$$

where the set point temperature, $T_{sp,i}$, is the set point temperature for residence i, defined here as the temperature at the bottom of the thermostat deadband, which is represented by $\delta$.

$$\dot{\theta}_i = -\frac{1}{RC}\theta_i(t) - \frac{Q}{C\delta}m_i(t) + \frac{1}{RC\delta}(T_\infty - T_{sp,i}) \tag{3}$$

Note that the variable $m_i(t)$ is a discrete variable representing the operating state of the air conditioning unit, having a value of one if the unit is operating and a value of zero if the unit is off. This is shown mathematically in Equation 4.

$$m_i(t) = \begin{cases} 0, & \text{if} \quad \theta_i \leq 0 \\ 1, & \text{if} \quad \theta_i \geq 1 \\ m_i(t-\varepsilon), & \text{otherwise} \end{cases} \tag{4}$$

Considering a population containing $N$ TCLs, the total electrical load can be expressed as

$$P_{\text{TCL}}(t) = \sum_{i=1}^{N} \frac{1}{\eta_i} Q_i m_i(t) \tag{5}$$

where $\eta_i$ is the coefficient of performance (COP) of the $i^{th}$ load.

To represent the local communication between individual homes, the conventions of graph theory are used. Graph theory is the mathematical lens through which a network of connected houses can be easily viewed. Networks of connections are often represented as an adjacency matrix, $\mathscr{A}$. For a network containing $N$ nodes, the adjacency matrix has $N$ rows and $N$ columns containing elements that follow the rules:

$$\mathscr{A}_{ij} = \begin{cases} 0, & \text{if nodes } i \text{ and } j \text{ are not connected to each other} \\ 1, & \text{if nodes } i \text{ and } j \text{ are connected to each other} \end{cases} \tag{6}$$

Since all communications are bidirectional, this is an undirected graph and the adjacency matrix is symmetric, $\mathscr{A}_{ij} = \mathscr{A}_{ji}$, and since a house is not connected to itself, the diagonal consists of zeros. The adjacency matrix can be used to find the degree of house $i$ by summing either the column or the row corresponding to that house:

$$d_i = \sum_{j=1}^{N} \mathscr{A}_{ij} = \sum_{j=1}^{N} \mathscr{A}_{ji} \tag{7}$$

Many possible topologies can be implemented in this manner. For the purposes of this study, a square lattice with periodic boundary conditions with a degree of 4 is chosen for the following reasons. The square lattice topology overlays nicely with a typical suburban neighborhood in which the homes are usually on a fairly regular grid. In determining the degree of the graph (i.e. the number of other homes with which each communicates), we wanted to strike a balance between simplicity of implementation and robustness of operation. Previous work has shown that a degree of 2 leaves the network highly vulnerable to complete failure with only a few connections lost while a degree of 4 corresponds well with the four nearest neighbors in the neighborhood [20]. Other topologies can be considered including a ring lattice, random graphs or a star type graph which represents the case in which the grid operator directly controls each individual actor.

Now consider a situation in which the state of the AC unit, $m(t)$, for each house can be communicated from its thermostat to nearby connected thermostats. The variable $\tilde{m}_i$ is introduced to represent the average state of the thermostats communicating with agent $i$. The adjacency matrix representing connected agents can be used to easily calculate all of these values simultaneously:

$$\tilde{\mathbf{m}} = \frac{1}{d}\mathscr{A}\mathbf{m} \tag{8}$$

In Equation 8, $\tilde{\mathbf{m}}$ and $\mathbf{m}$ are vectors of $N$ representing the entire population. $\mathscr{A}$ is the adjacency matrix of $NxN$ size, as defined as in Equation 6 and $d$ is the degree of the graph represented by $\mathscr{A}$.

### 3.1 Multiple Homes in Discrete Time

The goal is to address the behavior of a potentially large number of homes which share information with their closest neighbors. Starting with the first order model in Equation 3, N such homes can be aggregating in state space form as shown

$$\dot{\theta}(t) = \mathbf{A}\theta(t) + \mathbf{B}_{II}\mathbf{m}(t) + \mathbf{B}_{I}(T_{\infty} - T_{sp}) \tag{9}$$

Where the matrices are NxN diagonal matrices in which the diagonal elements are the coefficients in Equation 3. For ease of simulation, and to more faithfully model the manner in which such a system would be implemented, the first order ODE representing the house dynamics is transformed to the discrete domain (sample time = 1 minute).

$$\theta(k+1) = \mathbf{G}\theta(k) + \mathbf{H}_{II}\mathbf{m}(k) + \mathbf{H}_{I}(T_{\infty} - T_{sp}) \tag{10}$$

### 3.2 Modified Thermostat Behavior

As discussed previously, the goal of the shared information is to suppress the activity in any given home if the neighbors' compressors are running. The proposed modified thermostat behavior is shown in Equation 11. The $\tilde{m}$ variable is computed from the state of adjacent thermostats at the previous sampling period.

$$m_i(k+1) = \begin{cases} 0, & \text{if} \quad \theta_i(k) - K_{gain}\tilde{m}_i(k) \leq 0 \\ 1, & \text{if} \quad \theta_i(k) - K_{gain}\tilde{m}_i(k) \geq 1 \\ m_i(k), & \text{otherwise} \end{cases} \tag{11}$$

The variable $K_{gain}$ is a proportional gain for the feedback. While it is possible that this gain could be specific to the individual TCL, for the purposes of this analysis, a uniform value is chosen. In the simplest term, $K_{gain}$ is a knob that lets the grid operator adjust the impact of $\tilde{m}$ thermostat behavior. If $K_{gain}$ is zero, then the thermostats revert to normal behavior.

The addition of the average ON/OFF state of connected neighbors allows agents to reduce overall demand by causing an earlier entry to the OFF state and a later entry to the ON state if many of the neighbors are ON, essentially shifting the deadband up. For example, consider a network with $d = 4$, where two of a house's neighbors are ON, resulting in a $\tilde{m}$ of 0.5. Assuming $K_{gain} = 1$, this house will turn OFF as soon as $\theta = 0.5$, or halfway through the deadband, instead of the standard $\theta = 0$, and won't turn ON until $\theta = 1.5$. With a connection degree of four, each thermostat has the ability to use five upper and five lower bounds depending on the state of neighboring thermostats. In addition, since the upper bounds and lower bounds may move between state transitions, the net effect is also to narrow the deadband in some cases.

### 3.3 System Parameters

Parameter values were adopted from [2] and are displayed in the Table 2. While populations of homogeneous houses are not entirely realistic, they are a fair approximation for tract houses.It is also a very good approximation of a population of more uniform TCLs such as electric water heaters or chest freezers. More importantly, a homogeneous population represents a worst case scenario of the synchronization phenomenon that this approach is designed to mitigate. In heterogeneous populations, the synchronization effect still occurs, although with lower overshoots and shorter duration.

For the purposes of this study, the outdoor air temperature, $T_{\infty}$, is kept at a constant 32°C.

## 4 MODELING AGGREGATE DYNAMICS

Whereas many techniques are proposed for assessing hybrid and nonlinear systems (e.g. [21, 22]), none are well suited to assess the aggregate behavior of a large number of interacting systems.

To model the aggregate dynamics of a large number of residences, it is advantageous to find a linearized model as it allows for compact matrix representation and straightforward stability tests. The graphical representation of a typical thermostat relay is shown in Figure 1. A common linearization of this function is to assume the device controlled by the relay is capable of proportional action, which leads to a approximation as shown in Figure 2. The nonlinear saturation

Table 2. Population Parameter Values

| Parameter | Value |
|---|---|
| $R$, Thermal resistance | 2 °C/kW |
| $C$, Thermal capacitance | 10 kWh/°C |
| $P$, Energy transfer rate | 14 kW |
| $\eta$, Coefficient of Performance | 2.5 |
| $T_{sp0}$, Initial setpoint temperature | 20 °C |
| $T_{\infty}$, Outdoor air temperature | 32 °C |
| $\delta$, Thermostat deadband | 1.0 °C |

behavior remains, but since the purpose of the thermostat is to keep the temperature within the deadband, it is considered inconsequential for this analysis. This particular approximation has the advantage of being an accurate representation of HVAC systems with variable refrigerant flow, which are rapidly gaining market share, particularly in Europe and Oceania [23]. While such linearization eliminates the behavior that leads to the most problematic results in the aggregate, some insight into the aggregate dynamics and stability is still possible. Mathematically, this linearization is equivalent to setting $m$ equal to the position within the thermostat deadband (previously defined at $\theta$), allowing $m$ to take on any value in [0,1].

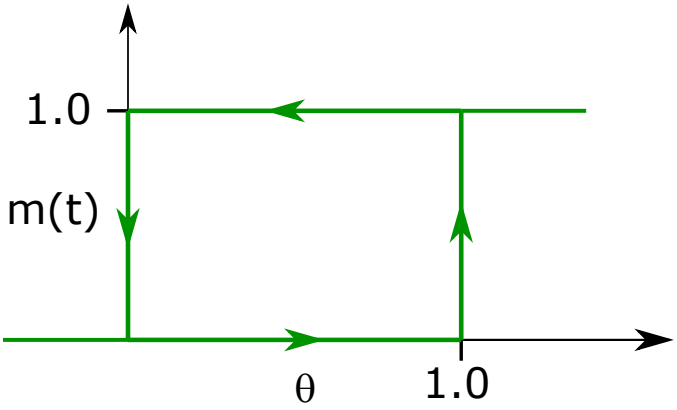


Fig. 1. Graphical representation of a typical thermostat relay

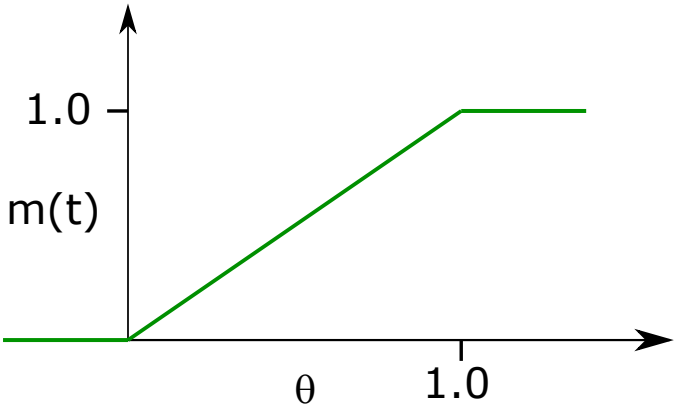


Fig. 2. Graphical representation of a linearized thermostat

In discrete time, the linearized/continuous model of the thermostat behavior is represented as a difference equation seen in Equation 12.

$$m(k+1) = \theta(k) - K_{gain}\tilde{m}(k) \tag{12}$$

Re-write $\tilde{m}$ from Equation 12 in terms of the adjacency matrix.

$$m(k+1) = \theta(k) - \frac{K_{gain}}{d}\mathscr{A}m(k) \tag{13}$$

The system equation described in earlier sections is shown below.

$$\theta(k+1) = G\theta(k) + H_I(T_\infty - T_{sp}) + H_{II}m(k) \tag{14}$$

Note that the thermostat equation in discrete space depends on its previous state and hence the thermostat states become dynamic states of the aggregate system. The aggregate system is now order 2N with the states being $\theta(k)$ and $m(k)$. The new closed loop transition matrix is shown in Equation 15.

$$G_{cl} = \begin{bmatrix} G & H_{II} \\ I & -\left(\frac{K_{gain}}{d}\right)\mathscr{A} \end{bmatrix} \tag{15}$$

By incrementally increasing the gain value, $K_{gain}$, until the magnitude of the largest eigenvalue of the closed-loop state transition matrix falls outside of the unit circle, the gain value at the stability limit is found. This test was repeated for various population sizes and deadbands and the presented in Section 6.

## 5 SIMULATION

To explore the efficacy of the proposed control law, a simulation is conducted. The stimulation is implemented with Matlab (ode45) using the continuous time model, running in one-minute fixed steps. At the end of each step, the thermostat operation is computed and the next step commenced. A population of 10,000 homes is simulated, laid out on a 100x100 periodic square lattice. The parameters are those shown in Table 2. The outdoor air temperature is held constant at 32 $^{\circ}$C and the simulation duration is 5 hours. At the one hour mark, a demand response event is simulated for the entire population which lasts for 15 minutes, turning off all AC units and allowing the temperature in the individual homes to increase according to the dynamics of the model. At the end of the event, the proposed control scheme is implemented for all thermostats.

Three indices of performance were chosen to further investigate the effect of the proportional gain, $K_{gain}$, on the non-linear system, described below.

It has already been pointed out that the proposed feedback law can have the effect of narrowing the thermostat deadband. This in turn increases the frequency of cycling of the AC unit, increasing the wear and tear. To better understand this impact, we monitor the average number of cycles/home/hour, $f_{mean}$.

$$f_{mean} = \frac{1}{NT_{dur}}\sum_{i}^{N}(\#Cycles) \tag{16}$$

In addition to narrowing the deadband, this proposed control law can have the effect of moving the set point up, thus raising the possibility of occupant discomfort. To better understand this impact, we define $M_{high}$ as the number of minutes outside the original deadband per home.

$$M_{high} = \frac{1}{N}\sum_{i}^{N}(\#minutes\,above\,deadband) \tag{17}$$

To capture the peak power following a DR event, we chose the ratio of the peak power to the mean value of the power prior to the DR event, $P_{ratio}$.

$$P_{ratio} = P_{max}/P_{mean} \tag{18}$$

The simulation is initialized by making the initial temperatures uniformly distributed throughout the thermostat deadband and 42.8% of the homes in the "ON" state. This proportion results in constant power demand from the population.

Figure 3 shows the results of this simulation for the nominal case in which a DR event is issued (at t = 1 hr, duration 15 minutes) and no modification of the thermostat behavior. Note that the DR event has the effect of changing the distribution of the homes and they become partially synchronized, resulting in very large peaks and continued oscillations in the hours following. It's worth noting that this result is an idealized case in which all the homes are identical. Previous studies have shown that a more realistically heterogeneous population will result in an oscillation that dies out after one or two cycles, the post DR peak is nevertheless present. The homogeneous population represents a worst-case scenario and any approach to mitigate the problems in this population will be effective in a heterogeneous population as well.

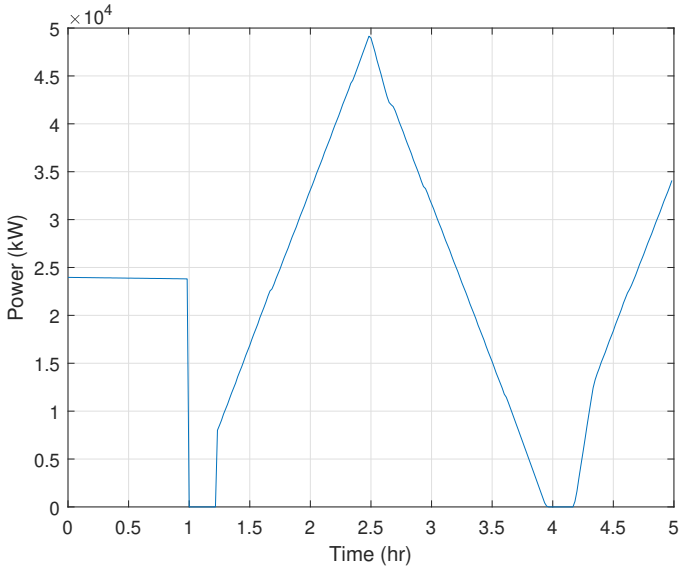


Fig. 3. Aggregate power of population without local communication, N=10,000. At time = 1 hr, a demand response event is implemented and all compressors are shut down for 15 minutes.

As an example of how the local communication can mitigate the response, Figure 4 shows the aggregate power for the same population, but in this case, the information from the locally connected houses is used to suppress the AC compressors as described in Equation 11 with a gain of 0.5. Note that the post DR peak is greatly suppressed and while the aggregate power shows some variation after the peak, there is no appreciable periodic behavior after the first overshoot.

Table 3 shows the indices of performance for the two simulation runs, with and without the local communication. Note that the aggregate power overshoot after the DR event is reduced from a factor of 2 to 1.26. The other indices demonstrate the 'cost' of this reduction in terms of increased compressor cycling (3x more often) and more time the home temperature is outside the deadband. As is shown in the following section, neither of these side-effects lie outside the norm of typical behavior related to DR events.

Table 3. Indices of Performance

| Index | $K_{gain}$=0.0 | $K_{gain}$=0.5 |
|---|---|---|
| $f_{mean}$ (cycle/hr) | 0.60 | 1.86 |
| $M_{high}$(min) | 3.55 | 43.4 |
| $P_{ratio}$ | 2.06 | 1.26 |

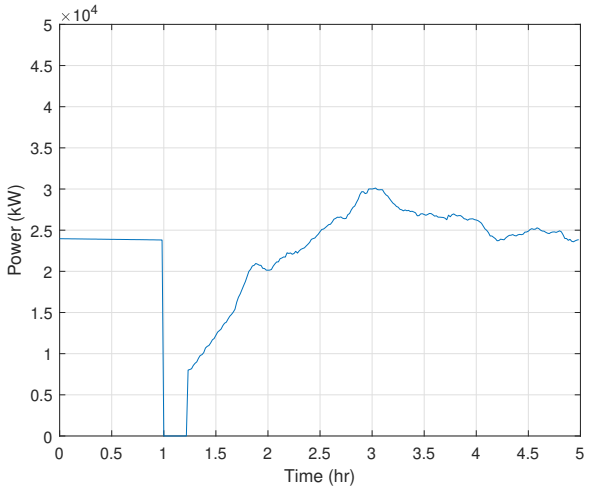


Fig. 4. Aggregate power of population, N=10,000, $K_{gain}$=0.5. At time = 1 hr, a demand response event is implemented and all compressors are shut down for 15 minutes.

## 6 RESULTS

The stability analysis was carried out for various values of $N$, population size, and deadband, $\delta$. Results show that the stability limit (the value of $K_{gain}$ beyond which there are eigenvalues outside the unit circle), was insensitive to $N$ and somewhat dependent on the size of the thermostat deadband. Figure 5 shows that dependency which appears to be asymptotic for both very small and very large deadbands, but is near unity for nearly all cases.

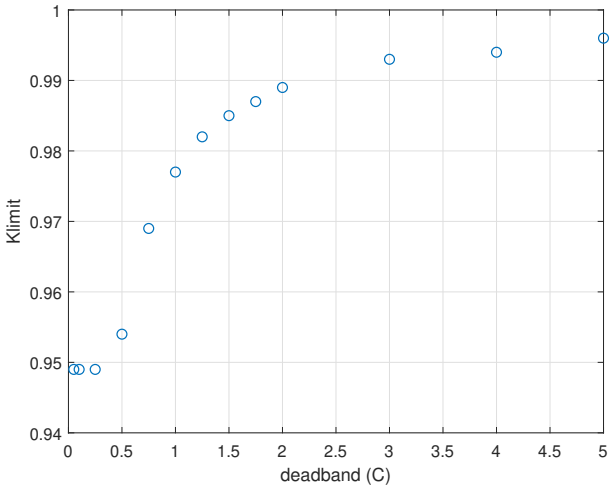


Fig. 5. Stability limit as a function of deadband

Figures 6 and 7 show the aggregate power for gains values on either side of the stability limit for gains of 0.9 and 1.1 respectively. The aggregate power of the response when the gain is above the predicted stability limit shows a few high-frequency oscillations immediately following the DR event, which, while not precisely unstable, certainly indicates undesirable behavior. Those oscillations become much more pronounced and extended as gain increases above 1.1.

At higher gains, the behavior of the aggregate system becomes increasingly erratic. Figure 8 shows the aggregate response for a gain value of 2.0. The oscillations eventually die out, suggesting the system meets the broad definition of stability, but it is clear that such behavior is highly undesirable and must be avoided in any implementation of such a system.

To better understand how the proposed scheme impacts occupant comfort, approximately 1% of the homes are randomly selected and the temperature profiles are plotted in Figure 9. As is the case with all simulations shown herein, the desired set point of 20 $^{\circ}C$ is in the center of a deadband of 1$^{\circ}C$ . As expected, a small portion of the homes exceeded the deadband at the onset of the demand response event. After the event (t = 1.25 hrs), the modified thermostat operation begins. The net effect is

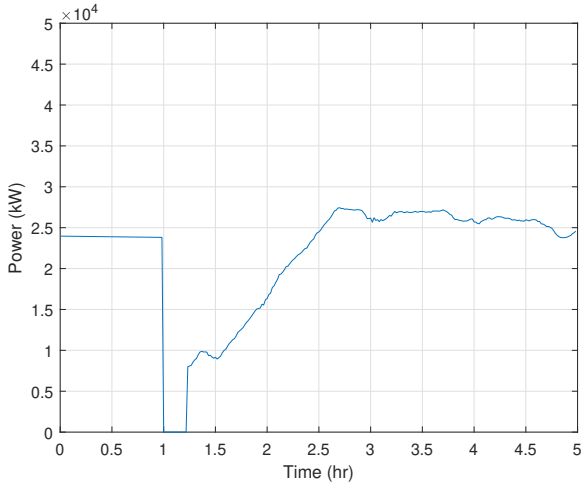


Fig. 6. Aggregate Power of population of 10,000 homes, $\delta$ = 1.0 $^{\circ}C$, $K_{gain}$ = 0.9. At time = 1 hr, a demand response event is implemented and all compressors are shut down for 15 minutes.

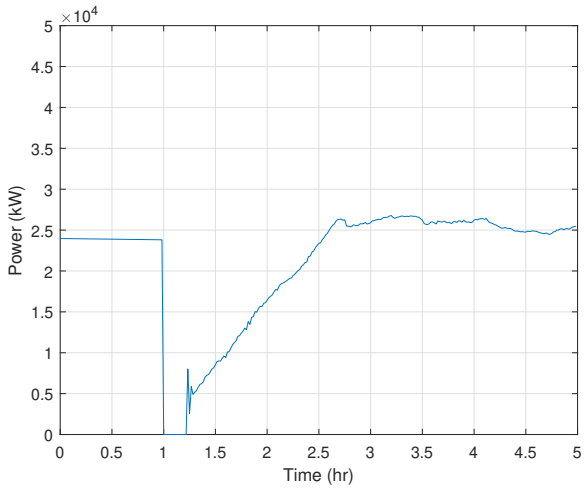


Fig. 7. Aggregate Power of population of 10,000 homes, $\delta$ = 1.0 $^{\circ}C$, $K_{gain}$ = 1.1. At time = 1 hr, a demand response event is implemented and all compressors are shut down for 15 minutes.

to shift the deadbands of the various homes in .25 $^{\circ}C$ increments, depending on the operations of the neighboring homes. Note that this has an effect similar to those schemes which propose to change the setpoints of the individual thermostats [2,13], but without the need for centralized control or universal knowledge of the population. the largest excursion outside the original deadband is 0.5 $^{\circ}C$ for a small portion of the homes.

Figure 10 shows two of the indices of performance, $P_{ratio}$ and $f_{mean}$ as a function of gain for the system of 10,000 homes. Note that $P_{ratio}$ drops quite dramatically for gains values up to about 0.4 while the mean number of oscillations remains low (less than one cycle/hour) for values up to 0.3. This suggests a reasonable trade-off between reducing peak power (from 100% overshoot with gain of zero to only 21%) at $K_{gain}$ = 0.4, while the average number of compressor cycles per hour is only slightly higher than 1 per hour, well within a reasonable range for residential HVAC systems.

Figure 11 shows the $P_{ratio}$ again plotted along with $M_{high}$, the average number of minutes spent above the deadband. Note that $M_{high}$ changes in inverse fashion to the $P_{ratio}$ and the trade-off is clearly indicated, although the 'best' value is not so clearly seen. If a value of 0.4 is chosen (as suggested above), the the average home in the population would spend about 32 minutes (out of 4 hours) outside the deadband of the thermostat.

## 7 Conclusions

A model of an arbitrary number of TCLs was derived and an innovative thermostat modification consisting of a communication mesh among the population was implemented. This modification suppresses the synchronization effects seen following DR events, but at the possible cost of instability. The system model was then linearized to allow a eigenvalue

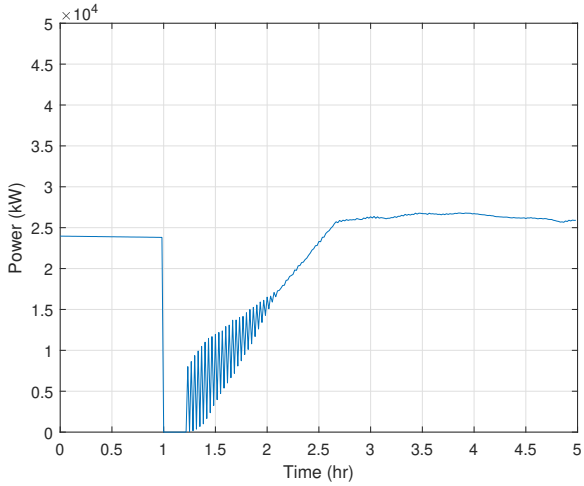


Fig. 8. Aggregate Power of population of 10,000 homes, $\delta$ = 1.0 $^{\circ}C$, $K_{gain}$ = 2.0. At time = 1 hr, a demand response event is implemented and all compressors are shut down for 15 minutes.

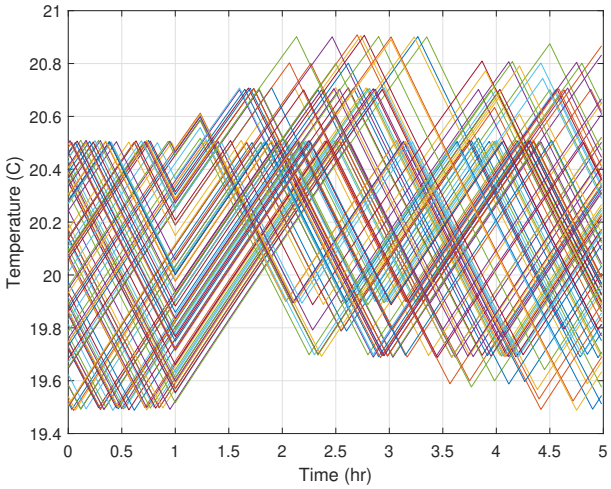


Fig. 9. Temperature profiles of 1% of the homes, $\delta$ = 1.0 $^{\circ}C$, $K_{gain}$ = 0.4. At time = 1 hr, a demand response event is implemented and all compressors are shut down for 15 minutes.

analysis and the stability limit of the gain was shown to be insensitive to the number of TCLs in the population. It was, however, sensitive to the size of the thermostat deadband, with smaller deadbands associated with lower stability limits.

Indices of performance based on peak demand, rate of cycling and time spent outside the original deadband following the DR event are defined and used to examine the results of simulations for a population of 10,000 identical homes, with thermostat deadband of 1 $^{\circ}C$ and various values of the gain incorporated in the thermostat modification.

The eigenvalue analysis indicated that the linearized population would experience unstable behavior at gain values above 0.977. The simulation of the population of nonlinear systems exhibited a stable response for a gain of 0.9 and the beginnings of undesirable oscillatory behavior for a gain value of 1.1 and that these oscillations increased in magnitude and duration with increasing gain.

The approximation of the thermostat deadband allowed for a straightforward stability test at the expense of eliminating the very behavior that can exacerbate the post-DR rebound and oscillation. Nonetheless, the results of the stability analysis provides insight to the dynamics of the population of TCLs.

A comprehensive analysis of the aggregate power dynamics is problematic for several reasons, not the least of which being that the individual TCL's are not themselves stable in the strictest sense of the term. The hysteretic nature of the thermostats leads to limit cycle behavior for all individual agents. At the same time, the aggregate power signal exhibits the tell-tale characteristics of both stable and unstable behavior. It is likely that a comprehensive analysis of the aggregated power dynamics will require some refinement of the notion of stability in general. More work is required before specific guidelines can be stated for the selection of gain for this proposed approach. In addition, this theory should be extended to include other graph topologies and heterogeneous populations of TCLs.

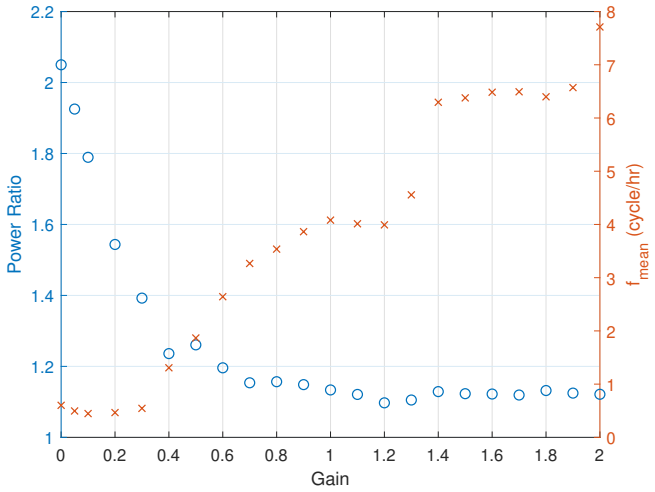


Fig. 10. Power ratio (left axis,'o') and average cycles (right axis,'x') as a function of gain, 10,000 homes, deadband of 1 $^{\circ}C$

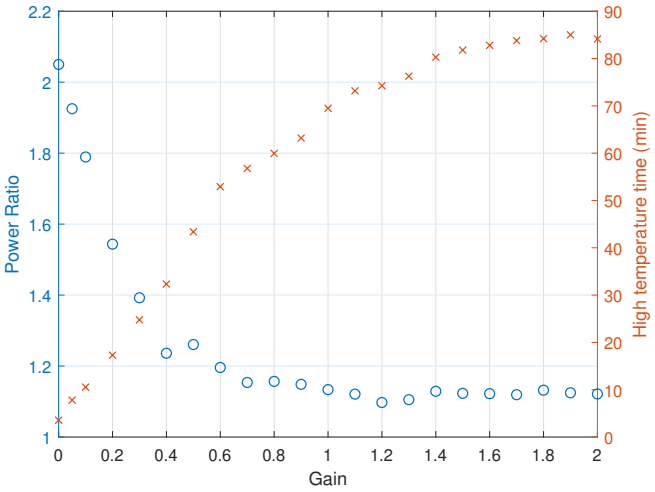


Fig. 11. Power ratio (left axis,'o') and minutes above deadband (right axis,'x') as a function of gain, 10,000 homes, deadband of 1 $^{\circ}C$

Finally, we note that this approach to modifying TCL behavior in a demand response environment can be expanded to any TCL, including heat-pump based heating systems, electric water heaters and refrigerators. In addition, the mesh-like communication scheme proposed herein can be used for other distributed energy resources such as smart inverters and residential batteries. The ability to ensure predictable and stable control of aggregated loads remains an important goal for grid operators.

**Acknowledgements**

This work was supported in part by US Department of Energy, DOE-EE0007726 and the Center for Advanced Energy Studies.

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