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Implication of production tax credit on economic dispatch for electricity merchants with storage and wind farms

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ABSTRACT

The production tax credit (PTC) promotes wind energy development, reduces power generation costs, and can affect merchants' joint economic dispatch, particularly for electricity merchants with both energy storage and wind farms. Two common PTC policies are studied – in the first policy, a wind farm receives PTC by selling wind generation to the market and its storage can be used to store energy from the wind generation and energy purchased from the grid but the energy released from the storage cannot receive PTC; in the second policy, the energy released from the storage can also qualify for PTC but purchasing energy from the grid is not allowed. We then employ dynamic programming to study merchants' optimal decision-making while considering PTC and the physical characteristics of storage systems. We analytically show that the state of charge (SOC) range can be segmented into different regions by SOC reference points under two PTC policies. The merchant's optimal action can be conveniently and uniquely determined based on the region within which the current SOC falls. Moreover, this study illustrates that PTC could substantially alter the optimal scheduling policy structures by affecting reference points and their relationships. The results showed that the frequencies for charging and discharging storage decisions decreased with an increase in PTC subsidy. Last, we confirm that, although the first policy allows merchants to buy electricity from the market, the second policy can bring more profits when the PTC is large at the current PTC rates. The findings can provide multistage decision-making guidance to electricity merchants in the wholesale power market.

1. Introduction

The increasing penetration of renewable energy generation brings significant fluctuations and uncertainty to power systems [1,2]. Because the demand and supply of electricity must be matched in real time, it is critical for grid operators to deal with electricity surpluses and insufficiency. Energy storage can rapidly respond to a control command and is expected to play an active role in dealing with renewable generation uncertainty. From a market participant's perspective, storing surpluses for future resale offers an arbitrage opportunity [3].

With the development of renewable sources, wind farm merchants have considered collocating electricity generation and grid-scale storage facilities. Co-optimization of storage and a renewable power plant can be valuable by mitigating intermittent wind energy generation by charge/discharge storage when wind power output mismatches the rated transmission capacity or power demand, time-shifting sales when electricity prices are low, and enabling the purchase of electricity in the future [4,5].

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The value of collocating energy storage with wind generation was reported in DOE (2018) [6] and EPRI (2004) [7]. One example of such a system is the AES Corporation's demonstration project in West Virginia, where a wind farm of 98 MW with a 32 MW lithium-ion battery were collocated [8]. Another example is the Wilmot Energy Center, which includes a 100-MW solar array and a 30-MW battery energy storage system [9].

There are many energy storage facilities, such as pumped storage hydropower (PSH) plants [10-12], battery storage [13,14] and thermal energy storage [15,16]. This paper analyzed battery and PSH storage systems with physical constraints, such as storage capacity, charging/pumping and discharging/generating capacities, and charging/pumping and discharging/generating efficiencies [17].

For modeling energy storage, research on energy inventories traditionally concentrated on optimal bidding strategies or optimal policies. Most existing models [18,19] focus on joint energy arbitrage strategies; however, they neglect the government's economic subsidies from the U.S., such as the production tax credit (PTC) for renewable power plants. These subsidies reduced U.S. wind power costs by 70% [20,21]. Furthermore, when a renewable subsidy—the production tax credit (PTC)—is provided to electricity merchants, the cost of wind power generation decreases, and the profit of selling electricity increases, then such merchants will be encouraged to sell electricity rather than buy and store electricity.

In contrast to previous research, we study how the PTC affects the optimal economic dispatch of merchants and profits. In the traditional study, the profit of the electricity merchant is mainly based on selling power to the market and buying power from the market. In contrast, considering the PTC, the merchant's profit depends not only on the traditional selling and buying operation strategy but also on the government's subsidies by selling renewable power to the market. Thus, the PTC will significantly affect the trading decisions of merchants who have both energy storage and renewable power plants. Therefore, this study aims to develop a better understanding of optimal policy differences in relation to the two assumptions, i.e., PTC vs. no PTC.

Prior research in this field typically assumes that energy remaining in storage is valueless in the final period, which shows that the value of energy (VOE) in storage at the terminal period or residual value of energy equals zero [22]. We interpret this assumption as the merchant's only choice between discharging or remaining idle (doing nothing) during the terminal period. Although Kim and Powell [18] considered the residual value of energy, they did not focus on the energy economics dispatch. This paper incorporated the residual value of energy. In the long term, the residual energy in storage has potential value for the future. When modeling the reward and objective functions, we also consider the residual value of energy in storage.

We are thus motivated to focus on the optimal scheduling of a merchant that has both storage (e.g., PSH, battery) and a renewable power plant (wind farms will be used henceforth to mean power plants). In this configuration, the electricity merchant uses energy storage to manage power transactions with the wholesale power market while considering the physical constraints of storage (e.g., the capacity of the energy storage, charging and discharging limitations/efficiencies), and the wind farm receives PTC by selling the wind-generated electricity to the market in the model. Operational costs and energy value at the end of the optimization horizon were included when calculating the electricity trade on the market.

To capture a wide range of application scenarios, our analyses are carried out in two distinct settings. In the first scenario, a wind farm receives PTC by selling the wind generation to the market. The collocated battery of the merchant can be used to store electricity generated by the wind farm and electricity purchased from the grid , but the stored energy including stored wind generation cannot receive PTC when it is sold to the market (*policy 1*). At this junction, the merchant has four choices: storing wind-generated power and buying power from the market, storing and selling wind-generated power, remaining idle, or discharging storage and selling all wind-generated power to the market. In the second scenario, both wind generation and released energy from the energy storage will qualify for PTC, but purchasing energy from the grid is prohibited. In this scenario, the power stored in the storage must be less than or equal to wind production in each period. Therefore, in this case, an electricity merchant has three choices: storing and selling wind-generated power, remaining idle, or discharging storage and selling all wind-generated power to the market (*policy 2*). We are able to characterize both policies mathematically.

Our analyses aimed to address the following questions: (i) How do electricity merchants with collocated wind farm and storage benefit from considering the PTC? (ii) Under which policy will an individual profit-maximizing merchant achieve more profit? To answer these questions, we first adjust the merchant's traditional reward function by incorporating the PTC subsidy based on the quantity of renewable energy sold to the power market. We then distinguish the optimal decisions of electricity merchant by applying dynamic programming method to maximize his or her profit. We also show how the optimal economic dispatch actions can be modified based on the different PTC credit rates under policies 1 and 2. It should be mentioned that it is nontrivial to achieve analytical results when considering PTC in the problem since it will affect product cost and the traditional structure of reward functions that only consider electricity prices. Such challenges are approached in this paper. We believe these are innovative theoretical conclusions that have never been examined before. This was the first paper to work on the energy storage scheduling/economics dispatch problem via dynamic programming while considering PTC policies to the best of our knowledge.

This research makes three principal contributions. First, for a wind (or renewable) farm merchant with energy storage, this study analytically showed that the state of charge (SOC) reference points at each decision time depended on the currently available energy, the forecasted price, the PTC credit rates, and the available energy of wind generation. On this basis, the storage SOC was divided into different regions by the SOC reference points corresponding to the four actions: storing renewable generation and buying power from the power market, storing and selling renewable generation, remaining idle, and discharging storage and selling all renewable generation to the market. Then, the optimal merchant's actions could be uniquely determined based on the region within the current SOC falls.

Second, compared to the traditional study in which the PTC was ignored, our results show that the PTC influenced the merchant's optimal economic dispatch volume by changing the value of the SOC reference point when the PTC was relatively small. However, when the PTC credit rates were relatively large, the traditional optimal storage policy structure was substantially reconstructed by modifying the relationships among the reference points. The analytical results regarding the boundary conditions (i.e., PTC is large or small) under which the merchants choose the different optimal policies were explained. With increased PTC, the frequencies for pumping and generating decreased.

Third, we also investigate whether stored wind generation will qualify the firm to receive PTC but will make it unable to buy electricity from the grid. Although the optimal SOC reference points in this situation are affected, we can derive valuable insights by employing similar analytical procedures. We find that compared to the policy allowing merchants to buy power from the market (i.e., policy 1), which prohibits purchasing electricity from the grid, and all the energy released from the storage qualifies for the PTC (i.e., policy 2) benefits merchants' profit when the PTC subsidy is large according to the current

PTC credit rates.We find that the merchant must seek to balance perfectly between policy 1 and policy 2. en policy *ne*1 and policy 2.

The rest of this paper is organized as follows. In Section 2, we review the relevant literature. Section 3 modeled a wind farm receiving a PTC or subsidy by selling the wind generation to the market, and it explored battery options to buy electricity from the grid to store. Section 4 examines cases in which storing energy from the grid disqualified the wind farm from receiving PTC. We will verify the proposed method based on a synthesis data case and demonstrate the results by a real electricity price data case from MISO (Midcontinent Independent System Operator, USA) in Section 5. Finally, the conclusions of this paper and research questions for future research are discussed in Section 6.

2. Literature review

This work was primarily related to the co-optimization of energy storage and renewable power plants and the examination of the PTC literature.

2.1. Co-optimization of energy storage and renewable power plants

Researchers have studied the optimal scheduling and energy arbitrage strategy of energy storage with wind farms, considering a series of physical constraints. Renewable electricity generation (e.g., wind power generation) is intermittent and has *high levels of uncertainty*, and the reliability of prediction is low [1,23]. There are many approaches to model the described energy storage problem: an online heuristic method for smoothing the variations of power output to the external grid [24], stochastic optimization in developing energy [25], co-optimizing energy storage for energy arbitrage based on convex relaxations [26], a control procedure for a wind farm, and battery energy storage based on a sequential stochastic decision process [27], dynamic programming for optimal economics dispatch [5,28], and a dynamic programming (DP)-based operation strategy for photovoltaic-battery (PVB) systems [29]. Considering the uncertainties of renewable energy sources and operational flexibility, Mojtaba [30] proposed a multi-period model for the long-term expansion planning of energy storage devices, and power generation technologies.

For the joint operation of a wind farm and energy storage system, Castronuovo and Lopes [31] proposed a discrete hourly optimization algorithm to maximize daily profits and determine the optimal daily operating strategies for wind turbines. Garcia-Gonzalez et al.[32] used two-stage stochastic programming to solve the optimal decision-making scheme of wind generation and pump storage cooperation. Lee [19] solved the short-term power dispatching problem using multi-iteration particle swarm optimization (MIPSO) for the combined pumped storage and wind power system. Huo et al. [33] studied the battery storage and wind energy economic dispatching problem, and they provided an adaptive dynamic programming algorithm to derive the optimal cost strategy for real-time scheduling. Grillo et al.[34] modeled renewable energy as a Markov chain and designed an optimal scheduling strategy for energy storage devices to minimize energy costs and network losses. Kiedanski et al. [27] adopted a sequential stochastic decision-making process to develop a control strategy for a merchant with battery storage and wind farm. Zhou et al. [28] studied the optimal strategy structure of a wind farm with a storage system and examined both positive and negative price scenarios. Oluwaseun et al. [35] studied the optimal scale combination of renewable energy, conventional generation, and energy storage technologies and optimized the corresponding scheduling strategy using an integer-programming optimization model. Liu et al. [36] investigated the optimal operating policy of energy storage without considering renewable power plants and the residual value of energy in storage. Considering the uncertainty of water flow, wind speed, and electricity prices, Avci et al. [4] modeled the storage problem for a pumped hydro energy storage (PHES) facility with a wind farm as a Markov decision process (MDP).

There have been many studies about the joint optimization of energy storage and wind farms, which can effectively smooth loads and improve system operation stability as well as benefit the merchant's profit. However, how the renewable energy subsidy policy for wind generation, such as PTC, impacts the production cost, decision actions, and reward function that result in the merchant's different dispatching decisions has not been studied. Motivated by existing studies, we construct an energy storage model of optimal decisions that reflects the PTC in this paper. It is nontrivial to achieve analytical results when considering PTC in the problem since it will impact the production cost and reward function.

2.2. Production tax credit

Scholars have recognized the importance of PTC in wind power generation. The investing in U.S. wind farms spurred more than $143 billion in private investment over the last ten years because of PTC [37]. Wiser et al. [38] suggested that a long-term extension of the federal PTC would help boost renewable energy growth. Barradale [39] pointed out the negotiation dynamics of power purchase agreements (PPAs) facing the uncertainty of PTC that caused the U.S. wind industry's fluctuating boom-and-bust pattern of investment. Xi et al.[40] applied a spatial/GIS-based financial model to investigate the competitiveness and profitability of onshore wind power. They analyzed the quantified impacts of PTC on the competitive potential of wind power operations. Roach [41] indicated that PTC was more effective at wind energy promotion in deregulated states than in regulated states using a structural supply model of wind power production. Eksioglu et al. [42] developed the mixed-integer nonlinear optimization model to capture the impact of PTC on renewable electricity production. Shrimali et al. [43] showed the effectiveness of the production tax credit that encourages wind energy deployment in the U.S. by empirical examination. Goldfarb et al. [44] pointed out that the public supports the extension of PTC to promote renewable energy development. The PTC plays a significant role in increasing wind energy investments and supporting its growth in the electricity generation sector [45,46]. The influences of PTC and investment tax credit (ITC) on reducing the costs of wind and solar technologies, the installed capacity of renewable generation, and electricity market prices were reported in DOE (2016) [47]. Alizamir [48] studied two types of subsidies for wind energy, investment subsidies (ITCs) and production subsidies (PTCs), and compared their roles.

Although PTC promotes wind energy development, reduces power generation costs and affects reward functions, previous researchers have focused on the macro impact or comparing it with other policies, such as ITC. Unlike some of the above studies, we assess how PTC affects the economic scheduling of electricity merchants with energy storage and wind farms. In addition, two PTC subsidy policies were considered, and the superiority of both policies was compared and studied in the following paper.

3. Consideration of PTC for a merchant with both storage and a wind farm (policy 1)

In this section, the objective functions for electricity merchants were modeled when the merchant had both energy storage and wind farm power plants [18,49]. A wind farm is receiving PTC by selling wind generation to the market and has energy storage that will be able to buy electricity from the grid to store. The merchant's optimal strategies to maximize the profit and to consider the PTC subsidy based on the forecasted price were studied.

3.1. Model setup

This research focused on the co-optimal scheduling of an owner of storage and wind farm plant (i.e., an electricity merchant) using an inventory policy to manage power in the wholesale market and considering the physical constraints of storage. Considering the production tax credit (PTC), a tax policy and subsidy for renewable electricity production, the merchant adopted a storage strategy and managed electricity, generating electricity by the power plant, and buying and selling electricity on the wholesale electricity market. Using discrete-time parameters, the electricity merchant periodically executed trading decisions during a limited horizon for each period $\mathrm{t} \in \{1, 2, 3, \cdots, T\}$.

It was assumed that the storage capacity was finite. The PSH storage had maximum energy capacity $\bar{E}$ and minimum energy level $\underline{E}$, thus $\bar{E} > \underline{E} \geqslant 0$.The PSH had generating/discharging and pumping/charging capacity constraints. $\bar{Q}^{\mathrm{g}}$ and $\underline{Q}^{\mathrm{g}}$ are denoted as the upper and lower limits of generating, respectively, that can be sold to the market in each period. $\bar{Q}^{P}$ and $\underline{Q}^{P}$ express the pumping maximum and minimum limits that can be bought from the power market in one period. For tractability, the study adopted the conventional assumption [5,18,28] that $\underline{Q}^{\mathrm{g}} = \underline{Q}^{P} = 0$.

Three efficiency types with storage were considered. One type of efficiency is denoted by α and β, describing the efficiency of the pumping/charging and generating/discharging modes, where $\alpha, \beta \in (0, 1]$. The other is $\rho \in (0, 1]$, which implies the portion of transmission efficiency: the ratio of power flowing out of a transmission line to the power flowing into another. Losses are incurred at the ends of the transmission lines in both directions. A third of them is a portion $\eta_{\mathrm{t}} \in [0, 1]$, an efficiency of stored energy dissipates during one period because of the evaporation, leakage, and spill rate of the PSH or self-discharge rate of the battery.

The economic dispatch for each period t is defined by q_t^g or $\mathrm{q}_t^{\mathrm{p}}$ to denote the energy change between periods t and t + 1 before considering the efficiency loss. This paper uses $\{\mathrm{c}_p, \mathrm{c}_g\}$($/unit energy) denote the operating cost of PSH storage generating and pumping. Following a previous study [2], it was assumed that energy storage had a linear operating cost. In this study, c_{w} indicates the cost for unit wind generation. For simplicity and brevity of exposition, it was assumed that the renewable power plant followed a linear generation cost. The electricity price in period t is P_t (dollars per unit energy), and the sequential levels of the price are denoted by a vector of $P = (P_1, P_2, \cdots, P_T)$. In this paper, bidding problems were not considered, and we assumed that all electricity submitted to the market was accepted [18,28].

The renewable electricity production tax credit (PTC) is a per-kilowatt-hour (kWh) tax credit for electricity generated using qualified renewable resources. To qualify for this credit, the taxpayer must sell electricity to an unrelated person. In practice, there are two metered technologies for wind farm service [50]. An import meter can measure electricity drawn from the grid, and an export meter is used to measure the renewable power supply to the grid to qualify for the PTC subsidy (https://www.energy.gov/energysaver/grid-connected-renewable-energy-systems). To prevent the merchant from selling all wind-generated electricity to the market to gain high PTC from the government and then buying electricity from the market to store, it was assumed that selling power to the market and purchasing power from the market were not allowed simultaneously. Based on the assumption above, the reward function $R(q_{\mathrm{t}}^g, q_{\mathrm{t}}^{\mathrm{p}}, \mathrm{w}_t, P_t)$ from making decision $(\mathrm{q}_t^g, \mathrm{q}_t^p)$ corresponding to time t, the forecast prices P_t, and the available renewable generation w_t, the PTC subsidy parameter s are defined as follows:

$$R^{(s)}\left(q_{\mathrm{t}}^{g}, q_{\mathrm{t}}^{\mathrm{p}}, w_t, P_t\right) = \begin{cases} -P_{\mathrm{t}} \cdot \left(q_{\mathrm{t}}^{\mathrm{p}}/\alpha - w_t\right)/\rho - \mathrm{c}_p q_{\mathrm{t}}^{\mathrm{p}} - \mathrm{c}_{\mathrm{w}} w_t \ (\mathrm{q}_{\mathrm{t}}^{\mathrm{p}} > \alpha w_t) \\ P_{\mathrm{t}} \cdot \left(w_t - q_{\mathrm{t}}^{\mathrm{p}}/\alpha\right) \cdot \rho - \mathrm{c}_p q_{\mathrm{t}}^{\mathrm{p}} + s\left(w_t - q_{\mathrm{t}}^{\mathrm{p}}/\alpha\right) - \mathrm{c}_w w_t \ (0 \leqslant q_{\mathrm{t}}^{\mathrm{p}} \leqslant \\ P_{\mathrm{t}} \cdot \left(q_{\mathrm{t}}^{g}\beta + w_t\right) \cdot \rho - \mathrm{c}_{\mathrm{g}} q_{\mathrm{t}}^{g} + s w_t - \mathrm{c}_w w_t \ (q_{\mathrm{t}}^{\mathrm{g}} \geqslant 0) \end{cases}$$

Here, the parameter s in superscript (1) indicates the situation considering the subsidy under policy 1. A distinguishing feature of this model is that it incorporates the subsidy s into the reward function for the power generation sold to the market by a wind turbine. This approach rendered the model both novel and practical, as it made the first contribution to the reward function.

The first line in Eq. (1) means the costs when the merchant buys power from the power market, and there is no energy sold to the market in this instance; thus, the merchant does not receive a subsidy. The second line shows the rewards yielded from when the merchant stored part of renewable generation $q_{\mathrm{t}}^{\mathrm{p}}$ to the storage and injected the remaining units $(w_t - q_{\mathrm{t}}^{\mathrm{p}}/\alpha)$ into the transmission line to the market. The additional term $s \cdot (w_t - q_t^p/\alpha)$ represents the PTC from the wind power selling to the market. The third equation, $(q_{\mathrm{t}}^g \beta + w_t)$, shows the reward function when the merchant released the energy from PSH to the market and sold all wind generation. Among all generations sold to the market, additional term $s \cdot w_t$ shows the PTC from the government, which also means that the stored wind generation by time-shifting sales cannot receive the PTC. The terms $\mathrm{c}_p q_{\mathrm{t}}^{\mathrm{p}}$, $\mathrm{c}_{\mathrm{g}} q_{\mathrm{t}}^{g}$, and $\mathrm{c}_{\mathrm{w}} w_t$ denote the pumping operating costs paid by the merchant at time t for $q_{\mathrm{t}}^{\mathrm{p}}$ units of energy (power), generating operating costs paid by the merchant at time t for q_{t}^{g} units of energy (power), and the wind generation cost, respectively.

This paper did not focus on electricity price forecasting methods. Instead, it was assumed that the merchant observed all power prices in advance, based on the forecasted method, so that controlling the storage was deterministic. The target of the study was the situation when wind farms received PTCs or subsidies by selling wind generation to the market and when they had a battery to be able to buy electricity from the grid to store, as shown in $R^{(s)}(q_t^g, q_{\mathrm{t}}^{\mathrm{p}}, \mathrm{w}_t, P_t)$ in Eq. (1).

Denote E_t as the SOC level in the PSH storage at the beginning of period t and the sequential SOC levels of the PSH as $\hat{E} = \left(E_1, E_2, \cdots, E_T\right)$, where $E_i \in [\underline{E}, \bar{E}], \forall i \in \{1, 2, \cdots, T\}$. We defined the feasible trading decisions set based on the current SOC level $E_t \in E$ as:

$$\begin{aligned} \mathrm{Action}(E_t) :& \\ &= \{(\mathrm{q}_t^p, \mathrm{q}_t^g) \\ &\in \mathbb{R} \\ &: 0 \leqslant \mathrm{q}_t^p \leqslant \bar{Q}^{\mathrm{p}} \cdot U_t^p,\ \mathrm{q}_t^p \leqslant \bar{E} \\ &\quad - E_t, 0 \leqslant \mathrm{q}_t^g \leqslant \bar{Q}^{\mathrm{g}} \cdot U_t^{\mathrm{g}},\ \mathrm{q}_t^{\mathrm{g}} \leqslant E_t - \underline{E}\} \end{aligned} \tag{2}$$

Eq. (2) represents the maximum quantity of energy that can be pumped and generated. The first two constraints defined the upper pumping boundary because the maximum limit and PSH storage capacity, respectively. The third and fourth constraints showed the upper generating boundary because of the maximum limit and available energy in the PSH storage. Both binary decision variables U_t^p and U_t^{g} represent the commitment of the unit to pumping and power generation in $[\mathrm{t}, \mathrm{t}+1)$. Without loss of generality, $U_t^{\mathrm{p}} + U_t^g \leqslant 1$, where $U_t^{\mathrm{p}} \in \{0, 1\}$ and $U_t^{\mathrm{g}} \in \{0, 1\}$, means the PSH cannot pump and generate simultaneously, or the battery cannot charge and discharge simultaneously.

At the beginning of period t, the electricity merchant knew the SOC of PSH E_t, the wind generation w_t, the forecasted electricity price P_t, and the PTC subsidy s. The action for each period t is defined by q_t^{p} or q_t^{g} to represent the water/energy changes between periods t and t + 1 before considering the generating and pumping losses. At the end of period t, PSH storage self-discharge occurs, so the SOC at the start of period t + 1 equals $\eta_{\mathrm{t}}(E_t + q_t^{\mathrm{p}} - q_t^g)$. Thus, the following formulation that outlines the state transition or the energy balance from time t to t + 1 for storage is true:

$$E_{t+1} = \eta_t (E_t + q_t^p - q_t^g) \tag{3}$$

3.2. Optimization and analysis

To maximize profit, the merchant periodically produced economic dispatch decisions over a limited horizon in each time $t \in \{1, 2, 3, \cdots, T\}$. The forecasted electricity price in period t is P_t. Following a previous study [28,36], this paper modeled the merchant's economic dispatch strategy as Markov dynamic programming with a finite horizon. The decision variables in each stage t are E_t, w_t, and P_t, and the state at period t is defined by $S(t) = S_t(E_t, w_t, P_t)$. The merchant aimed to find the optimal decision rule π to maximize the expectation of the value function at the initial stage (i.e., stage 1) during the limited period $t \in \{1, 2, 3, \cdots, T\}$.

$$\max_{\pi} \sum_{t=1}^{T} E\left[R^{(s)}(q_t^g, q_t^p, w_t, P_t) |\ S(1)\right]$$
$$s.t. \begin{cases} 0 \leqslant q_t^g \leqslant \bar{Q}^g \cdot U_t^g \\ q_t^g \leqslant E_t - \underline{E} \\ 0 \leqslant q_t^p \leqslant \bar{Q}^p \cdot U_t^p \\ q_t^p \leqslant \bar{E} - E_t \qquad \forall t \in \{1, 2, 3, \cdots, T\} \\ U_t^p \in \{0, 1\}, U_t^g \in \{0, 1\} \\ U_t^p + U_t^g \leqslant 1 \\ E_{t+1} = \eta_t (E_t + q_t^p - q_t^g) \end{cases} \tag{4}$$

This paper neglects the discount factor due to the limited optimization horizon, and E is the expectation concerning (E_t, w_t, P_t). Here, E_1,w_1 and P_1 are the given initial storage levels, the available wind generation, and the advance price, respectively. Let $V_t(S(t))$ symbolize the value function under state $S(t) = S_t(E_t, w_t, P_t) \in \hat{E} \times \widehat{W} \times \hat{P}$ in period t. This function satisfies the Bellman equation:

$$V(S(t)) = \max_{Action(E_t)} [R^{(s)}(q_t^g, q_t^p, w_t, P_t) + E\left(V_{t+1}(S(t+1)|S(t))\right)] \tag{5}$$

Prior research in this field typically expressed the VOE at the end optimization horizon or residual value of energy as $V_{T+1}(S(T+1)) = 0$ [22,28]. Eq. (5), $V_{T+1}(S(T+1)) = VOE_{T+1} \cdot E_{T+1}$, indicates the residual value of energy in storage. As a result, the implications for economic dispatch policy design are quite different when PTC is considered in addition to the value of energy in storage at T. Here, E_{T+1} represents the PSH SOC at the beginning of period T + 1 and the SOC at the end of period T. Next, the optimal policy decision rule of Model (5) was established for the slow storage case [22].

This research substituted the binary decision variables with an equivalent continuous variable and changed Eq. (5) to a traditional Markov Dynamic Programming to obtain the analytical optimal policy rule. Let $E_{t+1} = \eta_t(E_t + q_t^p - q_t^g) = \eta_t(E_t + q_t)$ be the decision variable [22,28,51]. The action (decision) q_t was used for each time t to substitute the decision variables q_t^g, and q_t^p representing the PSH SOC changed between t and t + 1 before accounting for the energy loss.

Similar to Liu *et al.* [36] and Zhou *et al.* [28], q_t<0 represents the SOC decrease due to the action of generating, q_t>0 indicates the SOC increase because of pumping, $q_t = 0$ shows that the SOC did not change, or the electricity merchant did nothing (i.e., idle). Then, there is $q_t = E_{t+1}/\eta_t - E_t$.

The reward function $R^{(s)}(q_t^g, q_t^p, w_t, P_t)$ can be rewritten as $R^{(s)}(q_t, w_t, P_t)$ from decision q_t and is defined as:

$$R^{(s)}(q_t, w_t, P_t) = \begin{cases} -P_t \cdot (q_t/\alpha - w_t)/\rho - c_w w_t - c_p q_t \ \ (q_t > \alpha u \\ -P_t \cdot (q_t/\alpha - w_t) \cdot \rho - s(q_t/\alpha - w_t) - c_w w_t - c_p q_t \ \ (0 \\ -P_t \cdot (q_t \beta - w_t) \cdot \rho - c_w w_t + s w_t + c_g q_t \ \ (q_t < \end{cases}$$

If $s = 0$, a traditional reward function is yielded. The PTC subsidy will change the traditional reward functions, which is an innovation of this study. As a merchant who has both energy storage and a wind farm, the objective function is:

$$\max_{\pi} \sum_{t=1}^{T} E\left[R^{(s)}(q_t, w_t, P_t) |\ S(1)\right] \tag{7}$$

subject to the capacity constraints $\max\{-\bar{Q}^g, \underline{E} - E_t\} \leqslant q_t \leqslant \min\{\bar{Q}^p, \bar{E} - E_t\}$ and the storage energy balance constraints $\eta_t(E_t + q_t) = E_{t+1}$, where $t \in \{1, 2, 3, \cdots, T\}$.

3.3. Profit maximization and optimal decision rule

To obtain the optimal decision rules, the optimization in (7) was first split into three optimizations corresponding to three different actions: *storing renewable generation and buying power from the market, storing and selling renewable generation, and discharging storage and selling all renewable generation to the market.* Then, the optimal solutions for these three optimizations were found:

$$\begin{cases} V_t^{(1-s)*}(S(t)) = \max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \{-\left(P_t/\alpha\rho + c_p\right) q_t - w_t(-P_t/\rho + c_w) + E \\ V_t^{(2-s)*}(S(t)) = \max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \{-\left((P_t\rho + s)/\alpha + c_p\right) q_t - w_t[-P_t\rho + c_w - s \\ V_t^{(3-s)*}(S(t)) = \max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \{(-\left(P_t\beta\rho - c_g\right) q_t - w_t[-P_t\rho + c_w - s] + \end{cases}$$

The original optimization problem can be subdivided into three subproblems $\max V_t^{(1-s)}(S(t))$, $\max V_t^{(2-s)}(S(t))$, and $\max V_t^{(3-s)}(S(t))$ subject to $\max\{-\bar{Q}^g, \underline{E} - E_t\} \leqslant q_t \leqslant \min\{\bar{Q}^p, \bar{E} - E_t\}$, and $\underline{E} \leqslant E_t \leqslant \bar{E}$.

If $V_t(S(t))$ is the value function, it should produce the following conclusions based on the Bellman equation [28,52]. Similarly, maximization (8) can be addressed by removing the given state $S(t)$ (i.e., the given value E_t, w_t, and P_t). The optimal results are:

$$\begin{cases} V_t^{(1-s)*}(S(t)) = \max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left(E[V_{t+1}^*(S(t+1)|S(t)] - \left(P_t/\alpha\rho + c_p\right) E_{t+1} \right. \\ V_t^{(2-s)*}(S(t)) = \max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left(E[V_{t+1}^*(S(t+1)|S(t)] - \left((P_t\rho + s)/\alpha + c_p\right) \right. \\ V_t^{(3-s)*}(S(t)) = \max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left(E[V_{t+1}^*(S(t+1)|S(t) - \left(P_t\beta\rho - c_g\right) E_{t+1} \right. \end{cases}$$

Unlike the previous study [18,28,51], we considered the PTC in the reward function, which yielded for every $t \in \{1, 2, 3, \cdots, T\}$, if the price of electricity at time t, $P_t < +\infty$ holds, in each stage t, the value functions $V_t(S(t))$ and $E[V_{t+1}(S(t+1)|S(t)]$ are concave in $E_t \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, w_t, P_t)$. The optimal solutions for these three optimizations were found. Hence, the different SOC solutions, as in Lemma 1 (see Appendix A), were obtained.

Lemma 1. *When considering the PTC subsidy and electricity merchant, let* $E_{t+1}^{(1-s)*}$, $E_{t+1}^{(2-s)*}$, *and* $E_{t+1}^{(3-s)*}$ *be the optimal results in* (10), *respectively:*

$$\begin{cases} E_{t+1}^{(1-s)*} = \arg\max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left(E\left[V_{t+1}^{*}(S(t+1)\,|\,S(t)\right] - \left(P_t/\alpha\rho + c_p\right) E_{t+1}/\eta_t \right) \\ E_{t+1}^{(2-s)*} = \arg\max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left(E\left[V_{t+1}^{*}(S(t+1)\,|\,S(t)\right] - \left((P_t\rho + s)/\alpha + c_p\right) E_{t+1}/\eta_t \right) \\ E_{t+1}^{(3-s)*} = \arg\max_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left(E\left[V_{t+1}^{*}(S(t+1)\,|\,S(t)\right] - \left(P_t\beta\rho - c_g\right) E_{t+1}/\eta_t \right) \end{cases} \quad (10$$

In Eq. (10), the first finding for the optimal SOC solution (i.e., SOC reference point) $E_{t+1}^{(1-s)*}$ does not relate to the PTC subsidy because the merchant did not receive the PTC when he or she stored all renewable generation. To be qualified for credit, renewable electricity must be sold by the merchant to the market to an unrelated person.

Second, a wind farm can still receive PTC when the merchant performs the action: storing and selling renewable generation and discharging storage. Hence, it was clear that the optimal solution $E_{t+1}^{(2-s)*}$ is related to the wind generation that was sold to the market and the PTC subsidy that was received.

Third, although the reward function changed when the merchant discharged storage and sold all renewable generation to the market when the PTC was considered, neither the PTC nor the wind generation affected the SOC level. Therefore, the optimal solution $E_{t+1}^{(3-s)*}$ was obtained and had nothing to do with wind generation.

In Eq. (10), the following relations were obtained:

(1) For positive electricity prices $P_t \geqslant 0$, the following is true:

$$\begin{cases} 1)\ \text{If } s \leqslant \left(P_t\left(1-\rho^2\right)\right)/\rho, \textit{there is } E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(3-s)*} \\ 2)\ \text{If } s \geqslant \left(P_t\left(1-\rho^2\right)\right)/\rho, \textit{there is } E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*} \end{cases} \quad (11)$$

(2) For negative electricity prices $P_t<0$, the study obtained the following:

(a) When there is $-(c_g + c_p)/(1/\alpha\rho - \beta\rho) \leqslant P_t<0$ holding, for any $s>0$, there is:

$$E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*} \quad (12)$$

(b) When $P_t< -(c_g + c_p)/(1/\alpha\rho - \beta\rho)<0$ hold,

$$\begin{cases} 1)\ \textit{If } 0 < s \leqslant (\alpha\beta - 1)\rho P_t - \alpha\left(c_g + c_p\right), \text{we} \textit{will get } E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(2-s)*} \geqslant E_{t+1}^{(3-s)} \\ 2)\ \textit{If } s \geqslant (\alpha\beta - 1)\rho P_t - \alpha\left(c_g + c_p\right), \text{we } \textit{will get } E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(3-s)*} \geqslant E_{t+1}^{(2-s)*} \end{cases}$$

Without loss of generality, this paper only focused on positive electricity price scenarios. The optimal operation policy was obtained by comparing the current SOC with the SOC reference points. It should be mentioned that when considering PTC and the PTC was large (e.g.,$s \geqslant (P_t(1-\rho^2))/\rho$), the traditional relationship among the optimal SOC reference points was changed. Therefore, the following conclusions and insights are proposed that have never been explored before. Based on Eq. (11), the corresponding optimal results were delivered in the following proposition (proofs are given in Appendix A):

Proposition 1. *For every period* $t \in \{1, 2, 3, \cdots, T\}$ *and forecasted electricity price* $P_t \in P$, *unique optimal SOC reference points exist* $E_{t+1}^{(1-s)*}, E_{t+1}^{(2-s)*}, E_{t+1}^{(3-s)*}$, *which depend on the state* $S(t)$. *When the subsidy per renewable generation sold to the market was small (i.e.,* $s \leqslant P_t(1-\rho^2)/\rho$*), there is* $E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(3-s)*}$, *so the optimal action in each state* $S(t) = S_t(E_t, w_t, P_t) \in \hat{E} \times \hat{G} \times \hat{P}$ *can be specified (see the following).*

Case 1. If $\alpha w_t< \min\{E_{t+1}^{(1-s)*}, \bar{Q}^p\}$ (less wind generation), the SOC can be segmented into four regions: storing renewable generation and buying electricity from the market, storing and selling renewable generation, discharging storage and selling all renewable generation to the market, and doing nothing.

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(1-s)*} - E_t, \overline{Q}^p\right\},\ E_t \in \left[0, E_{t+1}^{(1-s)*} - \alpha w_t\right] \left(\text{store renewable and p}\right. \\ \min\left\{E_{t+1}^{(2-s)*} - E_t, \alpha w_t\right\}, E_t \in \left(E_{t+1}^{(1-s)*} - \alpha w_t, E_{t+1}^{(2-s)*}\right] \left(\text{store renewal}\right. \\ 0, E_t \in \left(E_{t+1}^{(2-s)*}, E_{t+1}^{(3-s)*}\right]\ (\text{keep SOC uncha} \\ \max\left\{E_{t+1}^{(3-s)*} - E_t, -\overline{Q}^g\right\}, E_t \in \left(E_{t+1}^{(3-s)*}, \overline{E}\right] \left(\text{ge}\textit{nerate and } \text{sell}\right. \end{cases}$$

Case 2. If $\alpha w_t \geqslant \min\{E_{t+1}^{(1-s)*}, \bar{Q}^p\}$ (more wind generation), the SOC will be segmented into three regions: storing and selling renewable generation, discharging storage and selling all renewable generation to the market, and doing nothing (i.e., idle/offline).

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(2-s)*} - E_t, \alpha w_t, \overline{Q}^p\right\}, E_t \in \left[0, E_{t+1}^{(2-s)*}\right] \left(\text{stor}\right. \\ 0, E_t \in \left[E_{t+1}^{(2-s)*}, E_{t+1}^{(3-s)*}\right] (\text{kee} \\ \max\left\{E_{t+1}^{(3-s)*} - E_t, -\overline{Q}^g\right\}, E_t \in (E_{t+1}^{(3-s)*}, \overline{E}] \left(\text{gene}\right. \end{cases}$$

The first part of proposition 1 (i.e., Case 1) analytically shows that for a PSH and wind farm merchant that seeks to maximize his or her profit when there is less available wind energy, the storage SOC was divided into four possible regions by three SOC reference points ($E_{t+1}^{(1-s)*}$, $E_{t+1}^{(2-s)*}$, and $E_{t+1}^{(3-s)*}$ *that depend on the currently available energy* E_t, *the forecasted electricity price* P_t, *the PTC* s, *and the wind generation source* w_t) that corresponded to four different actions: 1) storing all wind generation and buying power from the market; 2) storing and selling partial wind generation; 3) remaining idle; and 4) releasing storage and selling all wind generation to the market. If there was less energy in the storage than $E_{t+1}^{(1-s)*} - \alpha w_t$ and less available renewable energy (i.e., $\alpha w_t< \min\{E_{t+1}^{(1-s)*}, \bar{Q}^p\}$), the merchant should (a) store all the renewable sources and (b) purchase electricity from the market, then force the SOC up to $E_{t+1}^{(1-s)*}$.Otherwise, the optimal action is similar with the second part of Proposition 1 as follows.

The second part of Proposition 1 (i.e., Case 2) indicated that if there were more available renewable sources (i.e., $\alpha w_t \geqslant \min\{E_{t+1}^{(1-s)*}, \bar{Q}^p\}$), then the SOC storage was divided into three regions by two optimal reference points ($E_{t+1}^{(2-s)*}$ and $E_{t+1}^{(3-s)*}$) that corresponded to actions 2)–4). 2-–4. If there was less energy in the storage/reservoir than $E_{t+1}^{(2-s)*}$, the merchant did not need to purchase power from the market to increase the SOC, but he or she needed to (a) store renewable sources to increase the SOC to $E_{t+1}^{(2-s)*}$ as close as possible and then (b) sell the reset of the renewable power to the market. Considering the efficiency loss, the income from selling the same power to the market was less than the cost of buying the same energy from the market in the exact time period. Similarly, if the stored energy was within the boundaries of the two optimal reference points (i.e., $E_{t+1}^{(2-s)*} \leqslant E_t \leqslant E_{t+1}^{(3-s)*}$), the merchant should do nothing, and if there was more energy in the PSH than in the optimal reference point $E_{t+1}^{(3-s)*}$, the merchant should release energy from storage and sell all renewable generation to the market, thus reducing the SOC to $E_{t+1}^{(3-s)*}$ to maximize profit.

Proposition 2. *If the subsidy per wind generation sold to the market was* *large (i.e., $s \geqslant P_t(1-\rho^2)/\rho$), it yielded $E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*}$ for the optimal reference points on the SOC, which depended on the current state $S(t)$ and PTC, and the optimal decisions are specified as follows:*

Case 3. If $\alpha w_t < \min\{E_{t+1}^{(2-s)*}, \bar{Q}^p\}$ (less wind generation), the feasible SOC can be classified into four regions: storing and selling renewable generation, storing renewable generation and buying electricity from the market, discharging storage and selling all renewable generation to the market, and doing nothing:

(1) If $\alpha w_t < E_{t+1}^{(1-s)*} - E_{t+1}^{(2-s)*}$

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(1\text{-}s)*} - E_t, \overline{Q}^p\right\}, E_t \in (0, E_{t+1}^{(2\text{-}s)*} - \alpha w_t] \text{ (store re} \\ \min\left\{E_{t+1}^{(2\text{-}s)*} - E_t, \overline{Q}^p\right\}, E_t \in (E_{t+1}^{(2\text{-}s)*} - \alpha w_t, E_{t+1}^{(2\text{-}s)*}] \text{ (} \\ \min\left\{E_{t+1}^{(1\text{-}s)*} - E_t, \overline{Q}^p\right\}, E_t \in (E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(1\text{-}s)*} - \alpha w_t] \text{ (sto} \\ 0, E_t \in (E_{t+1}^{(1\text{-}s)*} - \alpha w_t, E_{t+1}^{(3\text{-}s)*}] \\ \max\left\{E_{t+1}^{(3\text{-}s)*} - E_t, -\overline{Q}^g\right\}, E_t \in (E_{t+1}^{(3\text{-}s)*}, \overline{E}] \text{ (gen} \end{cases}$$

(3) If $\alpha w_t > E_{t+1}^{(1-s)*} - E_{t+1}^{(2-s)*}$

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(1\text{-}s)*} - E_t, \overline{Q}^p\right\}, E_t \in (0, E_{t+1}^{(2\text{-}s)*} - \alpha w_t] \text{ (stc} \\ \min\left\{E_{t+1}^{(2\text{-}s)*} - E_t, \overline{Q}^p\right\}, E_t \in (E_{t+1}^{(2\text{-}s)*} - \alpha w_t, E_{t+1}^{(2\text{-}} \\ 0, E_t \in (E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(3\text{-}s)*}] \\ \max\left\{E_{t+1}^{(3\text{-}s)*} - E_t, -\overline{Q}^g\right\}, E_t \in (E_{t+1}^{(3\text{-}s)*}, \overline{E}] \text{ (} \end{cases}$$

Case 4. If $\min\{E_{t+1}^{(1-s)*}, \bar{Q}^p\} \geqslant \alpha w_t \geqslant \min\{E_{t+1}^{(2-s)*}, \bar{Q}^p\}$ (middle-level wind generation), the feasible SOC can be segmented into three or four regions:

(1) If $E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(1-s)*} - \alpha w_t \geqslant E_{t+1}^{(2-s)*}$, the SOC has four regions: storing renewable generation and buying power from the market, storing and selling renewable generation, discharging/generating storage and selling all renewable generation to the market, and doing nothing (i.e., idle/offline):

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(2\text{-}s)*} - E_t, \overline{Q}^p\right\}, E_t \in (0, E_{t+1}^{(2\text{-}s)*}] \text{ (store r} \\ \min\left\{E_{t+1}^{(1\text{-}s)*} - E_t, \overline{Q}^p\right\}, E_t \in (E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(1\text{-}s)*} - \alpha w_t] \text{ (sto} \\ 0, E_t \in (E_{t+1}^{(1\text{-}s)*} - \alpha w_t, E_{t+1}^{(3\text{-}s)*}] \\ \max\left\{E_{t+1}^{(3\text{-}s)*} - E_t, -\overline{Q}^g\right\}, E_t \in (E_{t+1}^{(3\text{-}s)*}, \overline{E}] \text{ (gen} \end{cases}$$

(2) If $E_{t+1}^{(1-s)*} - \alpha w_t \leqslant E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*}$, the SOC is divided into three regions: storing renewable generation and buying power from the market, discharging/generating storage and selling all renewable generation to the market, and doing nothing (i.e., idle/offline):

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(2\text{-}s)*} - E_t, \alpha w_t, \overline{Q}^p\right\}, E_t \in \left(0, E_{t+1}^{(2\text{-}s)*}\right] \text{ (store renewable and wit} \\ 0, E_t \in \left(E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(3\text{-}s)*}\right] \text{ (keep SOC unchanged)} \\ \max\left\{E_{t+1}^{(3\text{-}s)*} - E_t, -\overline{Q}^g\right\}, E_t \in \left(E_{t+1}^{(3\text{-}s)*}, \overline{E}\right] \text{ (generate and sell renew} \end{cases}$$

Case 5. If $\alpha w_t \geqslant \min\{E_{t+1}^{(1-s)*}, \bar{Q}^p\}$ (more wind generation), the feasible SOC can also be divided into three regions: storing and purchasing electricity, generating PSH storage and selling all renewable generation to the market, and doing nothing. Thus, an optimal action in each state $S(t) = S_t(E_t, g_t, P_t) \in \hat{E} \times G \times P$ can be shown as follows:

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(2\text{-}s)*} - E_t, \alpha w_t, \overline{Q}^p\right\}, E_t \in \left[0, E_{t+1}^{(2\text{-}s)*}\right] \text{ (store renewable} \\ 0, E_t \in \left(E_{t+1}^{(2\text{-}s)*}, E_{t+1}^{(3\text{-}s)*}\right] \text{ (keep SOC unchanged)} \\ \max\left\{E_{t+1}^{(3\text{-}s)*} - E_t, -\overline{Q}^g\right\}, E_t \in \left(E_{t+1}^{(3\text{-}s)*}, \overline{E}\right] \text{ (generate and sell renewal} \end{cases}$$

Compared to Proposition 1, the first part of Proposition 2 (i.e., Case 3) analytically shows that for a PSH and wind farm merchant seeking to maximize his or her profit and with relatively less available wind energy, the storage SOC was divided into four or five regions by three or four SOC reference points ($E_{t+1}^{(2-s)*}$, $E_{t+1}^{(1-s)*}$ *and* $E_{t+1}^{(3-s)*}$, *which depend on the current state* $S(t)$ *and PTC, and the relationship between them with* αw_t) that corresponded to actions 1)–4). I) If the current SOC in the storage/reservoir was more than $E_{t+1}^{(3-s)*}$, the merchant should release energy from storage and generate energy for the market and then decrease the SOC to $E_{t+1}^{(3-s)*}$. II) If there was less energy in the storage/reservoir than $E_{t+1}^{(1-s)*}$ and more energy in the storage/reservoir than $E_{t+1}^{(2-s)*}$, the optimal actions would be as follows: if there were fewer available renewable sources (i.e., $\alpha w_t < E_{t+1}^{(1-s)*} - E_{t+1}^{(2-s)*}$), then the merchant should (a) store all the renewable sources and (b) purchase power from the power market, then increase the SOC up to $E_{t+1}^{(1-s)*}$. If there were more available renewable sources (i.e., $\alpha w_t > E_{t+1}^{(1-s)*} - E_{t+1}^{(2-s)*}$), the merchant should do nothing. III) If there was less energy in the storage/reservoir than $E_{t+1}^{(2-s)*}$ but the SOC could approach $E_{t+1}^{(2-s)*}$ by storing power without buying, the merchant should store renewable sources and increase the SOC as close as possible to $E_{t+1}^{(2-s)*}$. Otherwise, if there was less energy in the storage/reservoir than $E_{t+1}^{(2-s)*} - \alpha w_t$, they should increase SOC up to $E_{t+1}^{(1-s)*}$ by storing all the renewable sources and purchasing power from the market. When the merchant faces two optimal choices: storing renewable generation and buying power from the power market to increase SOC up to $E_{t+1}^{(1-s)*}$ or storing and selling renewable energy to decrease SOC as close as possible to $E_{t+1}^{(2-s)*}$, the second choice was more profitable and should be preferred.

The second part of Proposition 2 (i.e., Case 4) indicated that if there was a middle available wind source (i.e., $\min\{E_{t+1}^{(2-s)*}, \bar{Q}^p\} \leqslant \alpha w_t \leqslant \min\{E_{t+1}^{(1-s)*}, \bar{Q}^p\}$), then the SOC storage was divided into three or four regions by two or three SOC reference points ($E_{t+1}^{(2-s)*}$, $E_{t+1}^{(1-s)*} - \alpha w_t$ *and* $E_{t+1}^{(3-s)*}$) that corresponded to actions 1)–4) or 2)–4). If there was less energy in the storage/reservoir than $E_{t+1}^{(2-s)*}$, it was recommended that the merchant store renewable sources and increase SOC as close as possible to $E_{t+1}^{(2-s)*}$. If there was less energy in the storage than $E_{t+1}^{(1-s)*}$ and more energy in the storage than $E_{t+1}^{(2-s)*}$, similarly, the optimal actions will be as follows: If there was less available wind source (i.e., $E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(1-s)*} - \alpha w_t \geqslant E_{t+1}^{(2-s)*}$), then the merchant should (a) store all the renewable source and (b) purchase electricity from the market, then increase SOC to $E_{t+1}^{(1-s)*}$. If there were more available renewable sources (i.e.,

$E_{t+1}^{(1-s)*} - \alpha w_t \leqslant E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*}$), it is recommended that the merchant do nothing. If there was more energy in the PSH than the reference point $E_{t+1}^{(3-s)*}$, the merchant should release energy from storage and sell all renewable generation, then reduce the SOC to $E_{t+1}^{(3-s)*}$.

The last part of Proposition 2 (i.e., Case 5) indicated that if there was more available renewable source (i.e., $\alpha w_t \geqslant \min\{E_{t+1}^{(1-s)*}, \bar{Q}^p\}$), then the storage SOC should be divided into three regions by two SOC reference points $E_{t+1}^{(2-s)*}$ and $E_{t+1}^{(3-s)*}$ that corresponded to actions 2–4. The merchant only needed to store renewable sources, increase the SOC up to $E_{t+1}^{(2-s)*}$, and then sell partial renewable power to the market.

Special Case A): If $\rho = 1$ (i.e., without transmission efficiency loss), $E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*}$ for any $s \geqslant 0$ is yielded, which means that the traditional results of proposition 1 [28] do not hold.

Special Case B): If $w_t = 0$ (i.e., no wind generation or the forecasted wind generation equals zero), the merchant had storage/PSH only, and the optimal reference point $E_{t+1}^{(2-s)*}$ did not exist.

For wind power, the merchant has two choices: store it in PSH/storage (and sell it at a later time) or sell it to the market at the time the wind power is generated. There are two opportunity costs for the two choices respectively. The opportunity cost of the first choice is the lost PTC subsidies because the wind power is not sold to the market at the time of generation. The opportunity cost of the second choice is the lost arbitrage profit from selling the power at a more preferable price point at a later time. When the opportunity cost of storing, the first opporunity cost, is less than the opporunity cost of selling, the second opportunity cost, (i.e., $s \leqslant P_t(1-\rho^2)/\rho$), the relationship between the reference points remain the same as those in the traditional study which do not consider PTC, but the PTC subsidy will affect the transaction quantity. However, sometimes the opportunity cost of storing is more than selling (i.e., $s > P_t(1-\rho^2)/\rho$). In this case, the PTC subsidy affects the optimal dispatch policy structure by changing the relationships among the reference points. Only when electricity price is low and wind generation is small, the merchant will buy electricity from the market to store for energy arbitrage.

Propositions 1 and 2 yielded the insight and application, as follows.

Insight and Application. For a profit-maximization merchant with both storage and a wind farm, three states of charge (SOC) reference points depend on the current energy level, the forecast price, the available energy of wind source, and the PTC subsidy. The SOC range was split into different regions by SOC reference points. The electricity merchant will obtain identical optimal operations by examining the current storage SOC with the optimal SOC reference points.

3.4. Production tax credit (PTC) analysis

In traditional treatments, electricity merchants do not consider subsidies. The optimal scheduling strategy for merchants owning a wind farm with energy storage when considering PTC was studied. As described in propositions 1 and 2, the subsidy policy can influence trading decisions: when the subsidy was large, electricity merchants tended to sell electricity instead of storing and buying electricity. When subsidies rose, merchants sold more electricity, which is a dynamic that affected merchants' decisions.

Proposition 3. *For any given level of current energy storage E_t, available wind generation w_t, and forecast price P_t ($t \in \{1,2,3,\cdots,T\}$), the production tax credit (PTC) was considered, and then the optimal value functions exhibited the following relations:*

$$\begin{cases} E\left[V_{t+1(s>0)}^{*}(S(\mathrm{t}+1)|\,S(t))\right] \geqslant E\left[V_{t+1(s=0)}^{*}(S(\mathrm{t}+1)|\,S(\\ \max\limits_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t)_{(s>0)}|\;S(1)\right] \geqslant \max\limits_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t \end{cases}$$

Proposition 3 stated that the PTC considerably altered the optimal economic dispatch policy structure and optimal expected profits of electricity merchants under policy one. An electricity merchant who ignores PTC on the power market will overestimate his or her power generation costs. The decisions of an electricity merchant were naturally affected, so when the subsidy was relatively large, the electricity merchant made trading decisions that deviated from optimal scheduling.

To avoid selling all wind-generated electricity to the market to gain high PTC from the government rather than storing electricity for smoothing loads to improve system operation stability. Next, a merchant with a wind farm and an energy storage and could also obtain PTC for stored wind generation, but he or she could not buy electricity from the grid to store. That is, storing energy from the grid disqualified one from receiving PTC.

4. Receiving PTC prohibits buying electricity from the grid to store (policy 2)

The findings presented in Section 3 depended on a wind farm receiving PTC by selling the wind generation to market and that the merchant also had a battery to buy electricity from the grid to store. The implications for the optimal decision changed when the power stored in the energy storage had to be less than or equal to the wind production, and the stored wind generation was also qualifying for PTC. Therefore, in Section 4.1, this paper proposed a model for policy 2. Then, the optimal solutions in Section 4.2 were derived.

4.1. Model setup

It was also assumed that selling power to the market and purchasing power from the market were not allowed simultaneously. Compared to Section 3, wind farm merchants receiving PTC cannot buy electricity from the grid to store; thus, there are only three possible actions: a) storing and selling renewable generation, b) idling, and c) discharging storage and selling all renewable generation to the market.

Thus, we can generate the following reward functions:

$$R^{(PTC)}(q_t, w_t, P_t) = \begin{cases} -P_t \cdot (q_t/\alpha - w_t)\cdot\rho - s(q_t/\alpha - w_t) - c_w w_t - c_p q_t \;(0 \leqslant q_t \leqslant \alpha \\ -P_t \cdot (q_t\beta - w_t)\cdot\rho - c_w w_t - s(q_t\beta - w_t) + c_g q_t \;(q_t < 0) \end{cases}$$

Unlike Eq. (1), if the wind farm merchants receiving PTC cannot buy electricity from the grid to store, then all energy sold to the market is wind generation, so all the energy released from the storage was qualifying for the PTC.

4.2. Optimization and analysis

Similar to Section 3, with the subsidy, the objective function is shown:

$$\max_{\pi} \sum_{t=1}^{T} E\left[R^{(PTC)}(q_t, w_t, P_t)|\;S(1)\right] \tag{23}$$

To obtain the optimal decision rule, we first decompose the optimization in (23) into two optimizations that corresponded to two different actions: a) and c). Meanwhile, the capacity constraints and the balance constraints on stored energy remained unchanged. Then, by optimizing the value function $V_t(E_t, w_t, \hat{P}_t)$), subject to $\underline{E} \leqslant E_{t+1} \leqslant \bar{E}$, the following equations were yielded based on the Bellman equation [28,51,52] :

$$\begin{cases} V_t^{(2\text{ - }PTC)*}(S(t)) = \max\limits_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \{-((P_t\rho + s)/\alpha + c_p)\,q_t - w_t[-P_t\rho + \\ V_t^{(3\text{ - }PTC)*}(S(t)) = \max\limits_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \{-((P_t\rho + s)\,\beta - c_g)\,q_t - w_t[-P_t\rho + c \end{cases}$$

Using the same method in Section 3, the optimal solutions of these two suboptimizations were found.

Lemma 2. *When considering the PTC subsidy and the electricity merchant, let $E_{t+1}^{(2-\mathrm{PTC})*}$ and $E_{t+1}^{(3-\mathrm{PTC})*}$ be the optimal results in (24), respectively:*

$$\begin{cases} E_{t+1}^{(2-PTC)*} = \arg\max\limits_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)] - ((P_t\rho + s)/\right. \\ E_{t+1}^{(3-PTC)*} = \arg\max\limits_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)] - ((P_t\rho + s)\right. \end{cases}$$

In Eq. (25), we obtain the following relations:

(1) For electricity prices $P_t \geqslant -(\alpha(c_g + c_p) + s(1-\beta\alpha))/(\rho - \beta\alpha\rho)$, there is

$$E_{t+1}^{(2-\mathrm{PTC})*} \leqslant E_{t+1}^{(3-\mathrm{PTC})*} \tag{26}$$

(2) For electricity prices $P_t < -(\alpha(c_g + c_p) + s(1-\beta\alpha))/(\rho - \beta\alpha\rho) < 0$, there is

$$E_{t+1}^{(2-PTC)*} \geqslant E_{t+1}^{(3-PTC)*} \tag{27}$$

The optimal analytical results were obtained from the perspective of profit maximization. Similarly, positive electricity prices were still targeted. The conclusions and insights that follow were based on these analytical results (see Appendix B for the derivations).

Proposition 4. *For every stage $t \in \{1,2,3,\cdots,T\}$ and positive forecasted electricity price $\hat{P}_t \in P$, unique optimal storage levels $E_{t+1}^{(2-\mathrm{PTC})*} \leqslant E_{t+1}^{(3-\mathrm{PTC})*}$ were generated, and they depended on the state $S(t)$ and PTC. The optimal decisions were specified as follows:*

The feasible SOC of storage can be divided into three regions: storing and selling renewable generation, discharging/generating storage and selling all renewable generation to the market, and doing nothing (i.e., idle/offline). Thus, an optimal action in each state $S(t) = S_t(E_t, w_t, P_t) \in \hat{E}\times\hat{W}\times\hat{P}$ was specified:

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(2-PTC)*} - E_t, \overline{Q}^p, \alpha w_t\right\}, & E_t \in \left[0, E_{t+1}^{(2-PTC)*}\right] \\ 0, & E_t \in (E_{t+1}^{(2-PTC)*}, E_{t+1}^{(3)*}] \quad (\mathrm{k} \\ \max\left\{E_{t+1}^{(3-PTC)*} - E_t, -\overline{Q}^g\right\}, & E_t \in (E_{t+1}^{(3-PTC)*}, \overline{E}] \end{cases}$$

This proposition indicated that two SOC reference points $E_{t+1}^{(2-\mathrm{PTC})*}$ and $E_{t+1}^{(3-PTC)*}$ were included, and the merchant only needed to store a portion of the renewable source, increase the SOC as close as possible to $E_{t+1}^{(2-\mathrm{PTC})*}$, and then sell the rest of the renewable power to the market. If the current storage level in the storage/reservoir was more than $E_{t+1}^{(3-PTC)*}$, the merchant should generate or discharge energy from PSH and sell all renewable generation to the market, then decrease the SOC to $E_{t+1}^{(3-PTC)*}$. Proposition 4 displays the optimal economic dispatch decision for the merchant when considering PTC under policy two.

Overall, the findings confirmed that when a PTC-subsidized wind farm with storage cannot buy electricity from the grid for storage, it affects the optimal SOC solutions (i.e., SOC reference points), but these insights were qualitatively unchanged by such adjustments.

5. Numerical simulation and case study

We first validated the stated approaches and outcomes using a synthesis data case to show the analysis process under the two policies and compared them with MILP (mixed-integer linear programming) in Section 5.1. Furthermore, Section 5.2 used a real data case from MISO electricity prices, wind generation, and PTC to show the related insights.

5.1. Case study and comparison

For simplicity, a three-period case was used to explain the details of the proposed approaches. In this case, it was assumed that there were three periods ($T = 3$). At each period, the electricity price took one of the values in set $P_t = \{p^M, p^L, p^H\} = \{6, 3, 10\}$. This study assumed that the storage capacity was 10 (i.e., $\underline{E} = 0, \bar{E} = 10$), the maximum generating/discharging capacity was 12 and the maximum pumping/charging capacity was 7. This means that full storage could be emptied in one period, and empty storage could not be filled in one period, but it could be filled in fewer than two periods. It holds that $\underline{E} + \bar{Q}^p \leqslant \bar{E}$ (resp. $\bar{E} - \underline{E} > \bar{Q}^g$) and $\underline{E} + 2\bar{Q}^p \geqslant \bar{E}$. Assume the operating cost is one (i.e., $c_p = c_g = 1$), the wind generation cost is zero (i.e., $c_w = 0$), the charging, discharging and transmission efficiencies are 0.9 (i.e., $\alpha = \beta = 0.9 = \rho$), the self-discharging efficiency is one (i.e., $\eta = 1$), and the wind generation is $w_t = \{3, 5, 0\}$. This research employed the method proposed in Section 3 and the optimal actions proposed in Propositions 1 and 2 to obtain the optimal actions and SOC reference points under three different PTC credit rates (i.e., $s = \{0, 1, 3\}$) with two different initial SOCs, as shown in Table 1 (the proof method is given in Appendix X).

Table 1 indicates that PTC affects the optimal actions. Compared with the current study (i.e., an electricity merchant ignoring the PTC), increasing PTC leads to an increased maximum expectation of profits. When wind farm merchants cannot buy power from the grid to store (i.e., policy 2) to receive PTC subsidies, then, the method proposed in Section 4 is used to obtain the SOC reference points and the optimal actions under three different PTC credit rates (i.e., $s = \{0, 1, 3\}$) with two initial SOCs, as shown in Table 2.

Table 2 shows the optimal actions and profits when the wind farm merchant receives a PTC subsidy, but he or she cannot buy electricity from the grid to store (i.e., policy 2). Compared with Table 1, we find that when the subsidy is large, the profit under policy 2 may be higher, indicating that electricity merchants are willing to give up the opportunity to buy electricity and qualify for the subsidy for all wind generation.

We also employed the traditional MILP model [12,53] to obtain the optimal solutions and compared them with the optimal results in Table 1 and Table 2. The same optimal results were obtained under both dynamic programming and MILP methods for case 1. The above optimal results and optimal profits are verified in AIMMS.

5.2. MISO case study

In this section, 1-hour time units were used for the electricity price series in the day-ahead market $LMP = \{LMP_1, LMP_2, \ldots, LMP_T\}$ (\$/MW). Wind generation with T = 336 stages corresponding to a 2-week period from 1 to 14 May 2021 in MISO (electricity prices and wind generation data are available at https://www.misoenergy.org/)

Table 1
Optimal Results with PTC Credit Rates (Policy 1).

	$s=3, E_1=1$	$s=3, E_1=5$	$s=1, E_1=1$	$s=1, E_1=5$	$s=0, E_1=1$	$s=0, E_1=5$
$\left(E_4^{(1-s)*}, E_4^{(2-s)*}, E_4^{(3-s)*}\right)$	(0,0,0)		(0,0,0)		(0,0,0)	
$\left(E_3^{(1-s)*}, E_3^{(2-s)*}, E_3^{(3-s)*}\right)$	(10,0,10)		(10,0,10)		(10,10,10)	
$\left(E_2^{(1-s)*}, E_2^{(2-s)*}, E_2^{(3-s)*}\right)$	(0,0,10)		(0,0,10)		(0,3,10)	
q_3^*	−8	−10	−8	−10	−10	−10
q_2^*	7	5	7	5	7	5
q_1^*	0	0	0	0	2	0
V_1^*	65.7407	89.3481	59.7407	83.3478	56.9	80.3478

Table 2
Optimal Results with PTC Credit Rates(policy2).

	$s=3, E_1=1$	$s=3, E_1=5$	$s=1, E_1=1$	$s=1, E_1=5$	$s=0, E_1=1$	$s=0, E_1=5$
$\left(E_4^{(2-PTC)*}, E_4^{(3-PTC)*}\right)$	(0,0)		(0,0)		(0,0)	
$\left(E_3^{(2-PTC)*}, E_3^{(3-PTC)*}\right)$	(10,10)		(10,10)		(10,10)	
$\left(E_2^{(2-PTC)*}, E_2^{(3-PTC)*}\right)$	(0,10)		(0,10)		(5.5,10)	
q_3^*	−5.5	−9.5	−5.5	−9.5	−8.2	−10
q_2^*	4.5	4.5	4.5	4.5	4.5	4.5
q_1^*	0	0	0	0	2.7	0.5
V_1^*	74.6	113.8	58.7	90.7	51.02	79.2

was also used. This paper adopted three different PTC credit rates [54] to show the optimal results.

Values 2000 and 20,000 were assigned the minimum and maximum SOC of storage (upper reservoir of PSH) $\underline{E}$ and $\bar{E}$, respectively. Here, $\underline{E}>0$ indicates that the merchant cannot empty the upper reservoir, which is a realistic statement for a PSH in the power market. The maximum generation rate and pumping rate are $\bar{Q}^{p} = 2000$ and $\bar{Q}^{g} = 2000$, respectively. Units of MW hours were used for storage quantities. Units of GW were used to measure both pumping and generating rates, where the pumping and generating efficiencies in this case are $\alpha = \beta = 0.9$. It was assumed that it took $(\bar{E} - \underline{E})/\bar{Q}^{g} = 9$ hours for the PSH to empty the upper reservoir and that it took $(\bar{E} - \underline{E})/\bar{Q}^{p} = 9$ hours to refill the upper reservoir; these values corresponded to the approximate durations exhibited by a large-scale pumped storage hydropower plant in Ludington, Michigan (see https://www.consumersenergy.com/for details). In this case, the pumping and generating operating costs were calculated as $c_p = c_g = 1$ (\$/MW). Self-discharging was ignored, and it was assumed that $\eta = 1$. The wind generation cost did not affect the optimal results; therefore, the wind generation cost was ignored, and the study only focused on the PTC subsidy. In this section, the VOE at the end of time T (i.e., $\mathrm{VOE}_{T+1} = 0$) was not considered. Regularly, two weeks is an optimization cycle for Ludington in the power market. Based on Propositions 1 and 2 in Sections 3 and 4, the transmission efficiency and PTC played a crucial role in determining the optimal actions. Thus, this study considered two different situations: $\rho = 0.9$ and $\rho = 1$.

The renewable electricity PTC was a per kWh credit for electricity generated using qualified energy resources[1]. Under the current law, facilities for which construction began before January 1, 2021, may be qualified for the PTC. However, the credit rates are various for wind facilities that depend on the year when one began construction. Following Sherlock's report [54] and the Renewable Electricity Production Tax Credit (see https://www.epa.gov/lmop/renewable-electricity-production-tax-credit-information/), five different PTC Credit Rates (i.e.,0,1.0,1.5,2.3,2.5¢/kWh; that are 0,10,15,23,25\$/MWh) were considered to yield results.

For a merchant owning both energy storage (such as the Ludington PSH) and a wind farm, the plots in Figs. 1 and 2 show the optimal scheduling under policy 1 and policy 2 with the initial energy in the storage of 2000 MW hours (i.e., 2 GWh) when the transmission efficiency was $\rho = 0.9$. The optimal decisions are made from the perspectives of the PSH owner, considering the four different PTC credit rates and the traditional approach of ignoring the PTC in each figure.

The plots in Figs. 3 and 4 illustrate the optimal SOC curves under policy 1 and policy 2 that relate to the optimal operations in Figs. 1 and 2 with the initial energy in the storage of 2 GWh.

Figs. 1–4 led to the following observations and conclusions: the electricity merchant required different trading strategies because the PTC subsidy changed the objective function. Hence, it affects trading decisions under two different policies. Obviously, under policy 2, the charging electricity amount in each period is less than the optimal decision under policy 1 since the power stored in the battery must be less than or equal to the wind production under policy 2. Therefore, a merchant must strike a perfect trade-off between the PTC credit rates and the power transition quantity. The optimal actions were obtained from Proposition 2 when the merchant ignored the transmission loss

[1] The renewable electricity production credit can be found in §45 of the Internal Revenue Code (IRC).

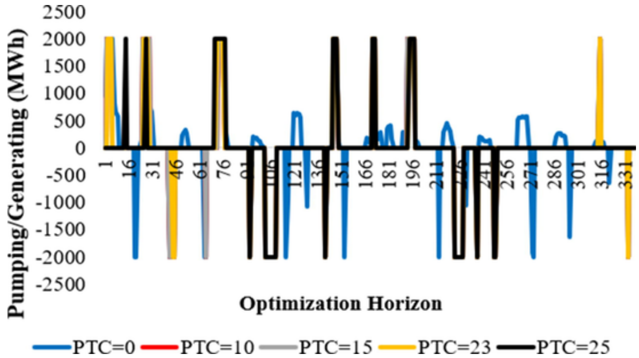

Fig. 1. The optimal actions under policy 1.

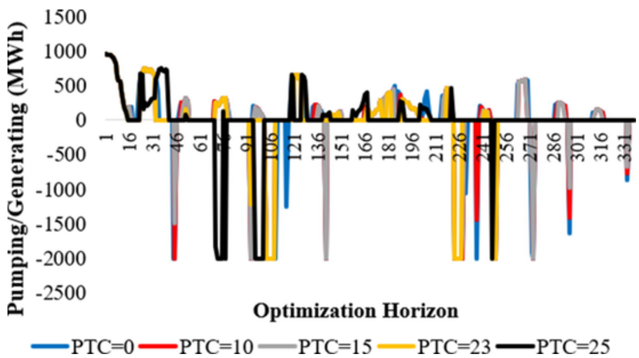

Fig. 2. The optimal actions under policy 2.

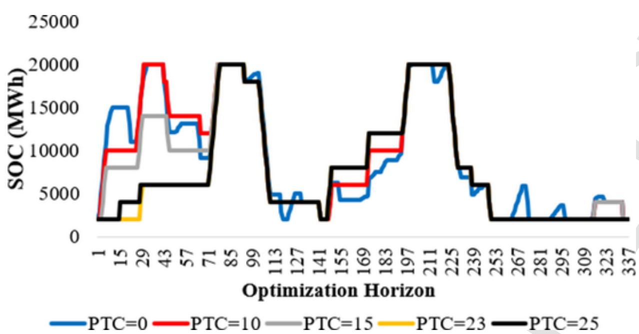

Fig. 3. The optimal SOC curves under policy 1.

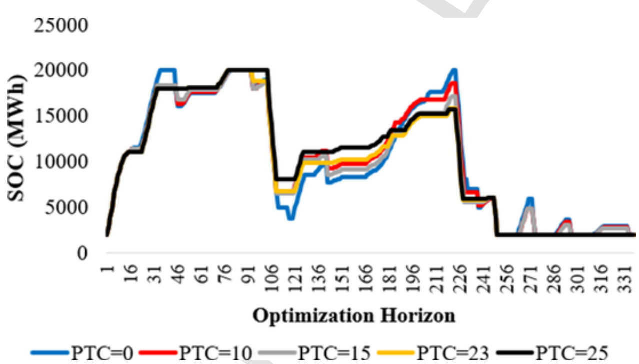

Fig. 4. The optimal SOC curves under policy 2.

(i.e., $\rho = 1$), which is shown in Figs. 5 and 6 with different PTC policies when the initial energy (i.e., E1) in the storage was 2 GWh.

Compared with Figs. 1 and 2, ignoring the loss of transmission efficiency will increase the opportunity to buy and sell electricity, and then the frequency of pumping and generating will be increased. The plots in Figs. 7 and 8 illustrate the optimal SOC curves with initial energy (i.e., E1) in the storage of 2000 MW hours under two PTC policies that corresponded to the optimal actions in Figs. 5 and 6 when the merchant ignored the transmission loss.

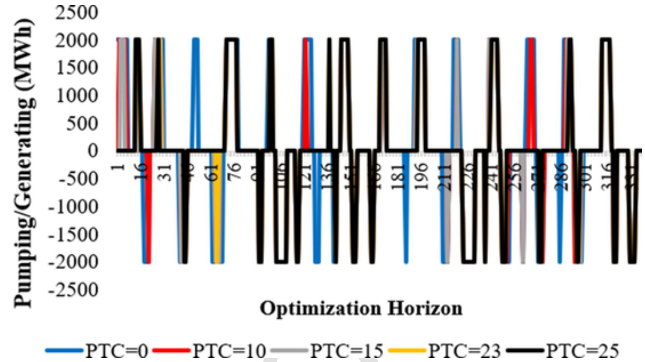

Fig. 5. The optimal actions under policy 1.

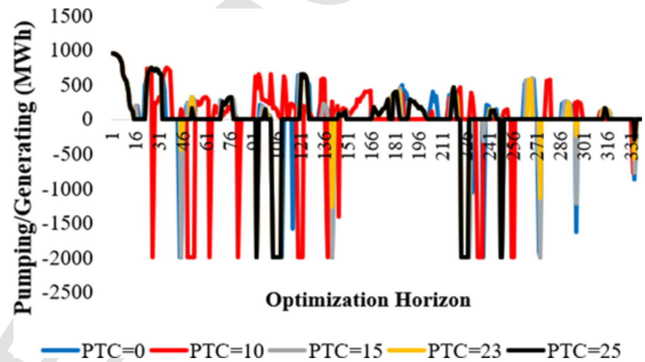

Fig. 6. The optimal actions under policy 2.

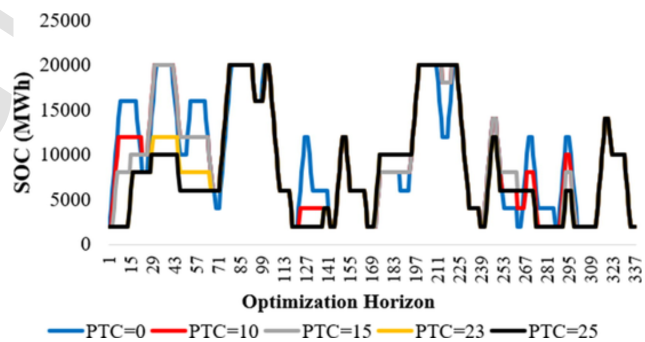

Fig. 7. The optimal SOC curves under policy 1.

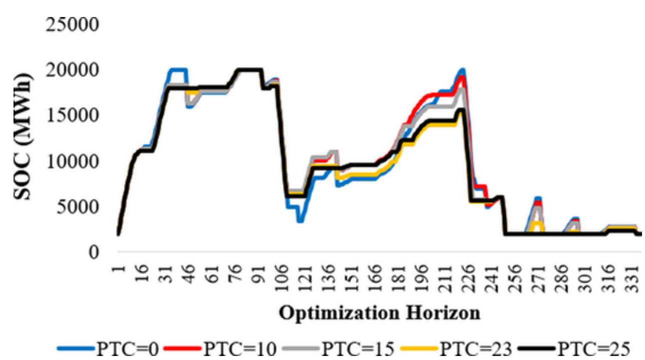

Fig. 8. The optimal SOC curves under policy 2.

Figs. 5–8 show that regardless of whether there was transmission loss, the PTC credit rates impacted the optimal decisions for profit maximization for the merchant with energy storage and wind farms. This part proved the conclusion outlined in Sections 3 and 4 through numerical simulation.

Figs. 1–8 (i.e., considering the transmission loss or not) show that the frequency of pumping and generating (i.e., the number of trades during the optimization horizon) was reduced with the increased PTC. Without considering the PTC, or when the PTC was small, the electricity merchants only made optimal economic dispatching decisions based on the electricity prices. Under policy 1, when the price was very low, the electricity merchant took actions: 1) storing all wind generation and buying power from the market or 2) storing and selling partial wind generation. When the price was fairly high, the electricity merchant chose action 4), releasing storage and selling all wind generation to the market. Otherwise, the merchant took action 3) doing nothing (i.e., selling all the wind generation to market for subsidies). Under policy 2, the merchants have the three actions (2, 3 and 4) to choose from.

Table 3 reports simulation results for the number of idle actions—based on 336 decision periods in two weeks—of an electricity merchant with both storage and a wind farm under different PTC credit rates.

This table reports the number of instances in which the merchant chose to do nothing during the optimization horizon to maximize his or her profit. In the second row, where PTC = 0, we can see that there are 191 (resp., 161) periods when the merchant remains idle in two weeks of optimization when PTC is not considered under policy 1 (resp., policy 2). With improvements in the PTC credits, idle frequencies will increase monotonically. For different PTC credits, the number of idle actions (i.e., the storage had done nothing, and all the wind power generation was sold to the market) under policy 1 is greater than that under policy 2.

According to the current PTC credit rates, when the PTC was large, the merchant was willing to sell wind power generation to obtain large subsidies instead of storing the wind generation, even if the market prices were low. Generally speaking, the increase of PTC and efficiency loss will lead to reduced frequency of the action of storing wind generation for electricity merchants. Therefore, the frequencies of pumping and generation are decreased. Furthermore, compared with policy 1, the number of idle actions for merchants under policy 2 is smaller. This is because under policy 2, merchants chose to store wind generation and sell it when electricity prices were high and could also receive subsidies.

Next, we compare the optimal results for the preceding two PTC policies and then analyze which PTC policy merchants should adopt to maximize profit. Table 4 demonstrates the Ludington PSH optimal profit results with initial energy in the storage of 2000 MW hours under two policies when the merchant considered and ignored the transmission loss.

Table 4 exhibits the optimal profits under two PTC policies when the wind generation and power price correspond to 2 weeks from 1 to 14 May 2021 in the MISO for a merchant having both storage and a wind farm. According to the current PTC credit rates, we found that the electricity merchant will achieve more profit under policy 2 than policy 1 when the PTC subsidy is large. Compared to allowing merchants to buy power from the market at a low price for time-shifting resales, a large PTC subsidy will bring more profit. If the PTC subsidy is small for electricity merchants, they should aim to buy power from the market at a low price and sell power at a high price to increase their profit by energy arbitrage. Therefore, policy 1 benefits an electricity merchant's profit. However, if the PTC subsidy is large, then the increased revenue from energy arbitrage cannot offset any loss because stored wind generation for time-shifting sales cannot receive PTC under policy 1. To obtain subsidies, under policy 1, the merchant can only sell electricity in the current period, while under policy 2, the merchant can store wind generation in storage when the prices are low and sell it when the electricity prices are high and receive subsidies at the same time.

Tables 3 and 4 are consistent with our intuition: the electricity merchant derives more profit from the wholesale power market with the improvement of transmission efficiency in the power market. It is straightforward that the PTC led to high maximum expectations of profit because the PTC policy reduced the wind generation costs and increased the profit. When the transmission efficiency is equal to 1 (i.e., ignoring the transmission loss), the optimal profit under policy 1 is greater than that under policy 2. This is because policy 2 relies on storage for arbitrage, while policy 1 relies more on directly selling wind power to obtain the PTC, so no transmission loss is more beneficial to policy 1.

To verify the robustness of our conclusions, we also conducted a numerical simulation of the cases using hourly market prices and wind generation from MISO with 336 stages corresponding to two weeks from December 3 to 18, 2020 . Similar results and insights to those from our previous simulations were obtained. (See numerical simulation in Appendix)

6. Conclusions and future research

This research is the first to model the joint economic dispatch problem via dynamic programming for an electricity merchant with energy storage and a wind farm receiving PTC. We derived the optimal policy structure from multistage decision-making. The optimal decision structure extends the classic results, and it differed from conventional policies, known as optimal in the literature, without considering the PTC.

For an electricity merchant with energy storage and a wind farm, the optimal scheduling policy that corresponds to the optimal SOC reference points at each decision period depends on (a) the current SOC in the storage, (b) the forecast electricity prices, (c) the availability of wind generation, and (d) the PTC. The study analytically showed that policy 1 (where the wind farm but not the storage can receive PTC by selling the wind generation to market and be able to buy electricity from the grid to store), there were three SOC reference points. The SOC range was segmented into four regions, each of which corresponded to one of four distinct actions. On the other hand, we further find that policy 2 (where a wind farm and storage receive PTC but cannot buy electricity from the grid to store), there are two SOC reference points, such that the feasible SOC

Table 3
Number of Idle Actions with PTC Credit Rates Under Two Policies.

Transmission efficiency $\rho = 0.9$					
Policies/PTC	PTC = 0	PTC = 10$/MWh	PTC = 15$/MWh	PTC = 23$/MWh	PTC = 25$/MWh
Policy 1	196	291	291	295	301
Policy 2	161	192	207	253	259
Transmission efficiency $\rho = 1$					
Policies/PTC	PTC = 0	PTC = 10$/MWh	PTC = 15$/MWh	PTC = 23$/MWh	PTC = 25$/MWh
Policy 1	195	231	239	253	255
Policy 2	160	185	199	231	246

range was split into three regions, each of which corresponded to one of three distinct actions.

The optimal energy storage decision of the electricity merchant was achieved by simply comparing the current SOC of storage with the optimal SOC reference points. However, the PTC credit rates changed the traditional relationships between optimal SOC reference points. Under the two policies, PTC played a significant role in defining optimal policy. Under policy 1, the merchant should perfectly balance the opportunity cost of storing wind generation in storage and that of selling electricity to arbitrage. When the PTC subsidy was small, although similar results as the traditional optimal policy structure were found, the optimal economic dispatch volume was changed. On the other hand, based on the current PTC credit rates, if the PTC was large, the merchants should adopt the corresponding optimal actions that all wind-generated electricity is sold to the market for high PTC subsidies. Only when the electricity price is low and wind generation is small, the merchant should buy electricity from the market to store for arbitrage.

This study showed that with an increase in PTC, the frequency of trading decision (i.e., charging and discharging) was reduced. Our results also confirmed that an electricity merchant should adopt policy two when the PTC subsidy is large, which prohibits the purchase of electricity from the grid and all the energy released from storage qualified for PTC subsidy, resulting in more profit to the merchants. When the PTC subsidy is small, the electricity merchant should take advantage of policy one. These new findings add to the collective knowledge of managing wind generation, PTC subsidy policies and joint economic dispatch of energy storage and wind farm, which constitutes a significant contribution to this research topic.

Toward the end of establishing a reasonably tractable framework and deriving valuable insights, we have made some simplifying assumptions about the linear market impact and the zero lower limits of generation and pumping. Future research could relax these assumptions and thereby extend our understanding of this problem. This paper assumes that the merchant's charging and discharging decisions *do not* affect electricity prices. However, large-scale energy storage will be reflected in price arbitrage, in which case the merchant's operating decisions will affect electricity prices in the market. Another related consideration for future work is examining the market impact of large-scale storage and determining how to model the market impact and build the corresponding reward functions. The extent study identifies individual profit-maximizing decisions of generating companies that align with the welfare-maximizing solution of the ISO when electricity prices are sent to each generating unit. Nevertheless, they consider energy storage and wind farms as well as PTC subsidies in their studies; specifically, whether it is beneficial for the merchant to let ISOs dispatch a merchant's energy storage directly and whether a merchant with energy storage and the wind farm will make less profit if he or she follows the ISO's dispatch instead of seeking his or her own profit maximization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Table 4
Optimal Profit with PTC Credit Rates Under Two Policies..

Transmission efficiency $\rho = 0.9$					
Profit	PTC = 0	PTC = 10$/MWh	PTC = 15$/MWh	PTC = 23$/MWh	PTC = 25$/MWh
Profit on Policy 1($)	2,588,335	3,442,529	3,914,204	4,694,023	4,891,781
Profit on Policy 2($)	2,504,431	3,445,301	3,922,480	4,697,404	4,893,548
Transmission efficiency $\rho = 1$					
Policies	PTC = 0	PTC = 10$/MWh	PTC = 15$/MWh	PTC = 23$/MWh	PTC = 25$/MWh
Profit on Policy 1($)	3,229,613	4,017,197	4,452,313	5,195,041	5,387,410
Profit on Policy 2($)	2,794,687	3,733,006	4,208,833	4,977,473	5,172,282

Appendix A.

Proof of Lemma 1.

(1) The uniqueness of the optimal results:

We show the current payoff rewards as follows for the merchant with the renewable power plant and storage and when the merchant can receive PTC s per renewable generation sold to the market:

$$R^{(s)}(q_t, w_t, P_t) = \begin{cases} -P_t \cdot (q_t/\alpha - w_t)/\rho - c_w w_t - c_p q_t \ (q_t > \alpha w_t) \\ -P_t \cdot (q_t/\alpha - w_t)\cdot \rho - s(q_t/\alpha - w_t) - c_w w_t - c_p q_t \ (0 \leqslant q_t < \alpha w_t) \\ -P_t \cdot (q_t\beta - w_t)\cdot \rho - c_w w_t + s w_t + c_g q_t \ (q_t < 0) \end{cases} \quad (A1)$$

Where, q_t is the energy/inventory change from period t to period t + 1 before accounting for energy loss. By using the method $E_{t+1} = \eta_t \cdot (E_t + q_t)$, we will get the following rewards function at time t.

$$R^{(s)}(q_t, w_t, P_t) = \begin{cases} -(P_t/\alpha\rho)\, q_t - c_p q_t - w_t(-P_t/\rho + c_w)\ (q_t > \alpha w_t) \\ -((P_t\rho + s)/\alpha)\, q_t - c_p q_t - w_t[-P_t\rho + c_w - s]\ (0 \leqslant q_t < \alpha w_t) \\ -P_t\beta\rho q_t + c_g q_t - w_t[-P_t\rho + c_w - s]\ \ (q_t < 0) \end{cases}$$
$$\Leftrightarrow \begin{cases} -(P_t/\alpha\rho)(E_{t+1}/\eta_t - E_t) - c_p(E_{t+1}/\eta_t - E_t) - w_t(-P_t/\rho + c_w)\ (q_t > \alpha w_t) \\ -((P_t\rho + s)/\alpha)(E_{t+1}/\eta_t - E_t) - c_p(E_{t+1}/\eta_t - E_t) - w_t[-P_t\rho + c_w - s]\ (0 \leqslant q_t < \alpha w_t) \\ -P_t\beta\rho(E_{t+1}/\eta_t - E_t) + c_g(E_{t+1}/\eta_t - E_t) - w_t[-P_t\rho + c_w - s]\ \ (q_t < 0) \end{cases} \quad (A2)$$

In the end of the period T or in the beginning of the period T + 1:

$V_T(E_T, P_T) = [R(q_T, w_T, P_T) + VOE_{T+1} \cdot E_{T+1}]$

Where,

$$\begin{cases} V_T^{(1-s)*}(S(T)) = \max\limits_{\underline{E} \leqslant E_{T+1} \leqslant \bar{E}} \{-(P_t/\alpha\rho)\cdot q_T - c_p q_T - w_T(-P_T/\rho + c_w) + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\} \\ V_T^{(2-s)*}(S(T)) = \max\limits_{\underline{E} \leqslant E_{T+1} \leqslant \bar{E}} \{-(P_T\rho + s)/\alpha \cdot q_T - c_p q_T - w_T[-P_T\rho + c_w - s] + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\} \\ V_T^{(3-s)*}(S(T)) = \max\limits_{\underline{E} \leqslant E_{T+1} \leqslant \bar{E}} \{-P_t\beta\rho q_T + c_g q_T - w_T[-P_T\rho + c_w - s] + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\} \end{cases} \quad (A3)$$

Here, we can also get the following relation of equivalence:

$$\begin{cases} V_T^{(1-s)*}(S(T)) = \max\limits_{\underline{E} \leqslant E_{T+1} \leqslant \bar{E}} \{-(P_t/\alpha\rho + c_p)(E_{T+1}/\eta_T - E_T) + VOE_{T+1} \cdot E_{T+1}\} \\ V_T^{(2-s)*}(S(T)) = \max\limits_{\underline{E} \leqslant E_{T+1} \leqslant \bar{E}} \{-((P_T\rho + s)/\alpha + c_p)(E_{T+1}/\eta_T - E_T) + VOE_{T+1} \cdot E_{T+1}\} \\ V_T^{(3-s)*}(S(T)) = \max\limits_{\underline{E} \leqslant E_{T+1} \leqslant \bar{E}} \{-(P_t\beta\rho - c_g)(E_{T+1}/\eta_T - E_T) + VOE_{T+1} \cdot E_{T+1}\} \end{cases} \quad (A4)$$

The best response functions (first-order derivative) of $V_T^*(S(T))$ on E_{T+1} are shown as:

$$\begin{cases} \partial V_T^{(1-s)*}(S(T))/\partial E_{T+1} = -(P_t/\alpha\rho + c_p)/\eta_T + VOE_{T+1} \\ \partial V_T^{(2-s)*}(S(T))/\partial E_{T+1} = -((P_T\rho + s)/\alpha + c_p)/\eta_T + VOE_{T+1} \\ \partial V_T^{(3-s)*}(S(T))/\partial E_{T+1} = -(P_t\beta\rho - c_g)/\eta_T + VOE_{T+1} \end{cases} \quad (A5)$$

We also get the following second-order derivative functions of $V_T^*(S(T))$ on E_{T+1}.

$$\begin{aligned} \partial^2 V_T^{(1-s)*}(S(T))/\partial E_{T+1}^2 &= 0;\ \partial^2 V_T^{(2-s)*}(S(T))/\partial E_{T+1}^2 \\ &= 0;\ \partial^2 V_T^{(3-s)*}(S(T))/\partial E_{T+1}^2 \\ &= 0 \end{aligned} \quad (A6)$$

Since there is $\begin{aligned} & P_T/\alpha\rho + c_p \geqslant (P_T\rho + s)/\alpha + c_p \\ \Leftrightarrow\ & P_T \geqslant (P_T\rho + s)\rho \\ \Leftrightarrow\ & s \leqslant P_T(1-\rho^2)/\rho \end{aligned}$, thus

(1) when $s \leqslant P_T(1-\rho^2)/\rho$, there are $(P_T\beta\rho - c_g)/\eta_T < ((P_T\rho + s)/\alpha + c_p)/\eta_T \leqslant (P_t/\alpha\rho + c_p)/\eta_T$

(a) If $VOE_{T+1} < \left(P_T\beta\rho - c_g\right)/\eta_T$,there are $\partial V_T^{(1-s)*}(S(T))/\partial E_{T+1}<0$,$\partial V_T^{(2-s)*}(S(T))/\partial E_{T+1}<0$, and $\partial V_T^{(3-s)*}(S(T))/\partial E_{T+1}<0$. We will get the following optimal results:

$$\begin{cases} E_{T+1}^{(1-s)*} = \arg\max\limits_{\underline{E}\leqslant E_{T+1}\leqslant \bar{E}} \left\{-\left(P_t/\alpha\rho + c_p\right)(E_{T+1}/\eta_T - E_T) + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} = \underline{E} \\ E_{T+1}^{(2-s)*} = \arg\max\limits_{\underline{E}\leqslant E_{T+1}\leqslant \bar{E}} \left\{-\left((P_T\rho + s)/\alpha + c_p\right)(E_{T+1}/\eta_T - E_T) + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} = \underline{E} \\ E_{T+1}^{(3-s)*} = \arg\max\limits_{\underline{E}\leqslant E_{T+1}\leqslant \bar{E}} \left\{-\left(P_t\beta\rho - c_g\right)(E_{T+1}/\eta_T - E_T) + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} = \underline{E} \end{cases} \tag{A7}$$

(b) If $\left(P_T\beta\rho - c_g\right)/\eta_T < V_{T+1} < \left((P_T\rho + s)/\alpha + c_p\right)/\eta_T$, there are $\partial V_T^{(1-s)*}(S(T))/\partial E_{T+1}<0$, $\partial V_T^{(2-s)*}(S(T))/\partial E_{T+1}<0$, and $\partial V_T^{(3-s)*}(S(T))/\partial E_{T+1}>0$. We will get the following optimal results:

$$\left\{E_{T+1}^{(1-s)*} = \underline{E};\ E_{T+1}^{(2-s)*} = \underline{E};\ E_{T+1}^{(3-s)*} = \bar{E}\right. \tag{A8}$$

(c) If $\left((P_T\rho + s)/\alpha + c_p\right)/\eta_T < VOE_{T+1} < \left(P_T/\alpha\rho + c_p\right)/\eta_T$, there are $\partial V_T^{(1-s)*}(S(T))/\partial E_{T+1}<0$,

$\partial V_T^{(2-s)*}(S(T))/\partial E_{T+1}>0$, and $\partial V_T^{(3-s)*}(S(T))/\partial E_{T+1}>0$, then we will get the following optimal results:

$$\left\{E_{T+1}^{(1-s)*} = \underline{E};\ E_{T+1}^{(2-s)*} = \bar{E};\ E_{T+1}^{(3-s)*} = \bar{E}\right. \tag{A9}$$

(d) If $VOE_{T+1} > \left(P_T/\alpha\rho + c_p\right)/\eta_T$,there are $\partial V_T^{(1-s)*}(S(T))/\partial E_{T+1}>0$, $\partial V_T^{(2-s)*}(S(T))/\partial E_{T+1}>0$, and $\partial V_T^{(3-s)*}(S(T))/\partial E_{T+1}>0$ then we will get the following optimal results:

$$\left\{E_{T+1}^{(1-s)*} = \bar{E};\ E_{T+1}^{(2-s)*} = \bar{E};\ E_{T+1}^{(3-s)*} = \bar{E}\right. \tag{A10}$$

(2) When $s > P_T(1-\rho^2)/\rho$, there are $(P_T\beta\rho - c_g)/\eta_T < (P_T/\alpha\rho + c_p)/\eta_T < \left((P_T\rho + s)/\alpha + c_p\right)/\eta_T$

(e) If $VOE_{T+1} < \left(P_T\beta\rho - c_g\right)/\eta_T$,there are $\partial V_T^{(1-s)*}(S(T))/\partial E_{T+1}<0$, $\partial V_T^{(2-s)*}(S(T))/\partial E_{T+1}<0$, and

$\partial V_T^{(3-s)*}(S(T))/\partial E_{T+1}<0$ we will get the following optimal results:

$$\begin{cases} E_{T+1}^{(1-s)*} = \arg\max\limits_{\underline{E}\leqslant E_{T+1}\leqslant \bar{E}} \left\{-\left(P_t/(\alpha\rho) + c_p\right)(E_{T+1}/\eta_T - E_T) + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} = \underline{E} \\ E_{T+1}^{(2-s)*} = \arg\max\limits_{\underline{E}\leqslant E_{T+1}\leqslant \bar{E}} \left\{-\left((P_T\rho + s)/\alpha + c_p\right)(E_{T+1}/\eta_T - E_T) + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} = \underline{E} \\ E_{T+1}^{(3-s)*} = \arg\max\limits_{\underline{E}\leqslant E_{T+1}\leqslant \bar{E}} \left\{-\left(P_T\beta\rho - c_g\right)(E_{T+1}/\eta_T - E_T) + E[V_{T+1}^*(S(T+1)\,|\,S(T)]\right\} = \underline{E} \end{cases} \tag{A11}$$

(f) If $(P_T\beta\rho - c_g)/\eta_T < VOE_{T+1} < (P_T/\alpha\rho + c_p)/\eta_T$,there are $\partial V_T^{(1-s)*}(S(T))/\partial E_{T+1}<0$,$\partial V_T^{(2-s)*}(S(T))/\partial E_{T+1}<0$, and $\partial V_T^{(3-s)*}(S(T))/\partial E_{T+1}>0$, we get the following optimal results:

$$\left\{E_{T+1}^{(1-s)*} = \underline{E};\ E_{T+1}^{(2-s)*} = \underline{E};\ E_{T+1}^{(3-s)*} = \bar{E}\right. \tag{A12}$$

(g) If $(P_T/\alpha\rho + c_p)/\eta_T < VOE_{T+1} < \left((P_T\rho + s)/\alpha + c_p\right)/\eta_T$, there are $\partial V_T^{(1-s)*}(S(T))/\partial E_{T+1}>0$, $\partial V_T^{(2-s)*}(S(T))/\partial E_{T+1}<0$, and $\partial V_T^{(3-s)*}(S(T))/\partial E_{T+1}>0$ we get the following optimal results:

$$\left\{E_{T+1}^{(1-s)*} = \bar{E};\ E_{T+1}^{(2-s)*} = \underline{E};\ E_{T+1}^{(3-s)*} = \bar{E}\right. \tag{A13}$$

(h) If $VOE_{T+1} > \left((P_T\rho + s)/\alpha + c_p\right)/\eta_T$,there are $\partial V_T^{(1-s)*}(S(T))/\partial E_{T+1}>0$, $\partial V_T^{(2-s)*}(S(T))/\partial E_{T+1}>0$, and

$\partial V_T^{(3-s)*}(S(T))/\partial E_{T+1}>0$ we will get the following optimal results:

$$\left\{E_{T+1}^{(1-s)*} = \bar{E};\ E_{T+1}^{(2-s)*} = \bar{E};\ E_{T+1}^{(3-s)*} = \bar{E}\right. \tag{A14}$$

For the state at $t \in \backslash\{1, 2, \cdots, T\backslash\}$

By optimizing of the value function $V_t(E_t, w_t, P_t)$, subject to $\underline{E} \leqslant E_{t+1} \leqslant \bar{E}$, we will get the following equations based on the Bellman equation.

$$\begin{cases} V_t^{(1-s)*}(S(t)) = \max_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left\{-\left(P_t/\alpha\rho + c_p\right)q_t - w_t(-P_t/\rho + c_w) + E[V_{t+1}^*(S(t+1)|S(t)]\right\} \\ \Leftrightarrow \max_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V_{t+1}^*(S(t+1)|S(t)] - (P_t/\alpha\rho + c_p)\cdot(E_{t+1}/\eta_t)\right) \\ V_t^{(2-s)*}(S(t)) = \max_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left\{-\left((P_t\rho + s)/\alpha + c_p\right)q_t - w_t[-P_t\rho + c_w - s] + E[V_{t+1}^*(S(t+1)|S(t)]\right\} \\ \Leftrightarrow \max_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V_{t+1}^*(S(t+1)|S(t)] - \left((P_t\rho + s)/\alpha + c_p\right)\cdot(E_{t+1}/\eta_t)\right) \\ V_t^{(3-s)*}(S(t)) = \max_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left\{(-\left(P_t\beta\rho - c_g\right)q_t - w_t[-P_t\rho + c_w - s] + E[V_{t+1}^*(S(t+1)|S(t)]\right\} \\ \Leftrightarrow \max_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V_{t+1}^*\left(S(t+1)|S(t) - \left(P_t\beta\rho - c_g\right)\cdot(E_{t+1}/\eta_t)\right)\right) \end{cases} \tag{A15}$$

Recall the previous proof at state T, we know that every $t \in \{1, 2, 3, \cdots, T\}$, in every stage t, the value function $V_t(S(t))$ and $E[V_{t+1}^*(S(t+1)|S(t)]$ are concave in $E_t \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, w_t, P_t)$.Clearly, $E[V_{t+1}^*(S(t+1)|S(t)]$ and functions (A15) are concave in $E_t \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, g_t, P_t)$ by using

$$\begin{aligned}&\partial E[V_{t+1}^*(S(t+1)|S(t)]/\partial E_{t+1}^2\\ &\quad= \left(\partial E[V_{t+1}^*(S(t+1)|S(t)]/\partial E_t^2\right.\\ &\quad\left.\cdot(\partial E_t/\partial E_{t+1})^2\right) \leqslant 0\end{aligned}$$

(1) When $q_t > \alpha w_t$, by optimizing the function (A15), subject to $E_{t+1} \in [\underline{E}, \bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\begin{aligned}&\frac{\partial V_t^{(1-s)*}(S(t))}{\partial E_{t+1}}\\ &= \frac{\partial\left(E[V_{t+1}^*(S(t+1)|S(t)] - \left(P_t/\alpha\rho\right)\cdot\left(E_{t+1}/\eta_t\right)\right)}{\partial E_{t+1}}\\ &= \frac{\partial E[V_{t+1}^*(S(t+1)|S(t)]}{\partial E_{t+1}} - \left(\frac{P_t}{\alpha\rho} + c_p\right)\frac{1}{\eta_t}\end{aligned}$$

The second-order derivative:$\partial V_t^{(1-s)*}(S(t))/\partial E_{t+1}^2 = \partial E[V_{t+1}^*(S(t+1)|S(t)]/\partial E_{t+1}^2 < 0 < 0$

Since the second-order derivative is negative, we can find the unique optimal solutions through the first-order condition. We also will get the following optimal results:

$$\begin{aligned}E_{t+1}^{(1-s)*} = \arg&\max_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V_{t+1}^*(S(t+1)|S(t)]\right.\\ &\left.- \left(P_t/\alpha\rho + c_p\right)(E_{t+1}/\eta_t)\right)\end{aligned} \tag{A16}$$

(2) When $0 \leqslant q_t < \alpha w_t$, by optimizing the function (A15), subject to $E_{t+1} \in [\underline{E}, \bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\frac{\partial\left(E[V_{t+1}^*(S(t+1)|S(t)] - ((P_t\rho + s)/\alpha)\cdot(E_{t+1}/\eta_t)\right)}{\partial E_{t+1}} = \frac{\partial E[V_{t+1}^*(S(t+1)|S(t)]}{\partial E_{t+1}} - \left(\frac{(P_t\rho + s)}{\alpha} + c_p\right)\frac{1}{\eta_t}]$$

The second-order derivative: $\partial E[V_{t+1}^*(S(t+1)|S(t)]/\partial E_{t+1}^2 < 0$.

Since the second-order derivative is negative, we can find the unique optimal solutions through the first-order condition. We also will get the following optimal results:

$$\begin{aligned}E_{t+1}^{(2-s)*} = \arg&\max_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V_{t+1}^*(S(t+1)|S(t)]\right.\\ &\left.- \left((P_t\rho + s)/\alpha + c_p\right)(E_{t+1}/\eta_t)\right)\end{aligned} \tag{A17}$$

(3) When $q_t < 0$, by optimizing the function (A15), subject to $E_{t+1} \in [\underline{E}, \bar{E}]$, we can derive the response function (i.e., first-order derivative) as follows:

$$\begin{aligned}&\frac{\partial\left(E[V_{t+1}^*(S(t+1)|S(t)] - P_t\beta\rho(E_{t+1}/\eta_t)\right)}{\partial E_{t+1}}\\ &\quad= \frac{\partial E[V_{t+1}^*(S(t+1)|S(t)]}{\partial E_{t+1}} - \frac{P_t\beta\rho - c_g}{\eta_t}\end{aligned}$$

The second-order derivative:$\partial E[V_{t+1}^*(S(t+1)|S(t)]/\partial E_{t+1}^2 < 0$.

Similarly, we can find the unique optimal solutions through the first-order condition. We also will get the following optimal results:

$$E_{t+1}^{(3\text{ - s})*} = \arg\max_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)] - \left(P_t\beta\rho - c_g\right)(E_{t+1}/\eta_t)\right) \tag{A18}$$

(2) The relations among three reference points:

(2) Recall the proof 1), for the state at $t \in \{1, 2, \cdots, T\}$,there have the following two equations:

$$\begin{cases} E_{t+1}^{(1\text{ - s})*} = \arg\max_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)] - \left(P_t/\alpha\rho + c_p\right)(E_{t+1}/\eta_t)\right) \\ E_{t+1}^{(2\text{ - s})*} = \arg\max_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)] - \left((P_t\rho + s)/\alpha + c_p\right)(E_{t+1}/\eta_t)\right) \end{cases}$$

Then, we can get the following in equations:

$$\left.\begin{array}{c} (a)\ \left(E\left[V_{t+1}\left(E_{t+1}^{(1\text{ - s})*}, P_{t+1}\right)\,|\,(E_t, P_t)\right] - \left(P_t/\alpha\rho + c_p\right)E_{t+1}^{(1\text{ - s})*}/\eta_t\right) \geqslant \\ (b)\ \left(E\left[V_{t+1}\left(E_{t+1}^{(2\text{ - s})*}, P_{t+1}\right)\,|\,(E_t, P_t)\right] - \left(P_t/\alpha\rho + c_p\right)E_{t+1}^{(2\text{ - s})*}/\eta_t\right) \\ (c)\ \left(E\left[V_{t+1}\left(E_{t+1}^{(2\text{ - s})*}, P_{t+1}\right)\,|\,(E_t, P_t)\right] - \left((P_t\rho + s)/\alpha + c_p\right)E_{t+1}^{(2\text{ - s})*}/\eta_t\right) \geqslant \\ (d)\ \left(E\left[V_{t+1}\left(E_{t+1}^{(1\text{ - s})*}, P_{t+1}\right)\,|\,(E_t, P_t)\right] - \left((P_t\rho + s)/\alpha + c_p\right)E_{t+1}^{(1\text{ - s})*}/\eta_t\right) \end{array}\right\} \tag{A19}$$

Based on the above in equations, we can get the relationship of $(a) - (d) \geqslant (b) - (c)$.
That is, for any given current state $S(t) = S_t(E_t, w_t, P_t) \in \hat{E} \times \hat{W} \times \hat{P}$, we will get:

$$\left((P_t/\alpha\rho + c_p) - \left((P_t\rho + s)/\alpha + c_p\right)\right)[E_{t+1}^{(2-\text{s})*}/\eta_t - E_{t+1}^{(1-\text{s})*}/\eta_t] \geqslant 0$$

Here, we have the following relation:

$$\textbf{(a)}\quad \begin{aligned} P_t/\alpha\rho \geqslant (P_t\rho + s)/\alpha \geqslant 0 &\Leftrightarrow P_t \geqslant (P_t\rho + s)\rho \\ &\Leftrightarrow P_t - P_t\rho^2 \geqslant s\rho \quad ; \\ &\Leftrightarrow s \leqslant P_t(1-\rho^2)/\rho \end{aligned}$$

$$\textbf{(b)}\quad \begin{aligned} 0 \leqslant P_t/\alpha\rho \leqslant (P_t\rho + s)/\alpha &\Leftrightarrow 0 \leqslant P_t \leqslant (P_t + s)\rho^2 \\ &\Leftrightarrow P_t - P_t\rho^2 \leqslant s\rho \\ &\Leftrightarrow s \geqslant P_t(1-\rho^2)/\rho \end{aligned}$$

Therefore, we will obtain the following relationship:

(1) For positive electricity price $P_t \geqslant 0$, if $s \leqslant P_t(1-\rho^2)/\rho$, there is $E_{t+1}^{(1-\text{s})*} \leqslant E_{t+1}^{(2-s)*}$.
(2) For positive electricity price $P_t \geqslant 0$, if $s \geqslant P_t(1-\rho^2)/\rho$, we will get $E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(2-\text{s})*}$.

Obviously, if $P_t<0$, we will get $s \geqslant 0 \geqslant P_t(1-\rho^2)/\rho$.
So, for any $s \geqslant 0$, we will get $E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(2-\text{s})*}$ for all the negative electricity price.

(2) Recall the proof 1), we also have the following two equations:

$$\begin{cases} E_{t+1}^{(2\text{ - s})*} = \arg\max_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)] - \left((P_t\rho + s)/\alpha + c_p\right)\cdot(E_{t+1}/\eta_t)\right) \\ E_{t+1}^{(3\text{ - s})*} = \arg\max_{\underline{E}\leqslant E_{t+1}\leqslant \bar{E}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t)] - \left(P_t\beta\rho - c_g\right)\cdot(E_{t+1}/\eta_t)\right) \end{cases}$$

Then, we can get the following in equations:

$$\left.\begin{array}{c} (e)\ \left(E\left[V_{t+1}^{*}\left(E_{t+1}^{(2\text{ - s})*}, P_{t+1}\right)\,|\,(E_t, P_t)\right] - \left((P_t\rho + s)/\alpha + c_p\right)\cdot\left(E_{t+1}^{(2\text{ - s})*}/\eta_t\right)\right) \geqslant \\ (f)\ \left(E\left[V_{t+1}^{*}\left(E_{t+1}^{(3\text{ - s})*}, P_{t+1}\right)\,|\,(E_t, P_t)\right] - \left((P_t\rho + s)/\alpha + c_p\right)\cdot\left(E_{t+1}^{(3\text{ - s})*}/\eta_t\right)\right) \\ (g)\ E\left[V_{t+1}^{*}\left(E_{t+1}^{(3\text{ - s})*}, P_{t+1}\right)\,|\,(E_t, P_t)\right] - \left(P_t\beta\rho - c_g\right)\cdot\left(E_{t+1}^{(3\text{ - s})*}/\eta_t\right) \geqslant \\ (h)\ E\left[V_{t+1}^{*}\left(E_{t+1}^{(2\text{ - s})*}, P_{t+1}\right)\,|\,(E_t, P_t)\right] - \left(P_t\beta\rho - c_g\right)\cdot\left(E_{t+1}^{(2\text{ - s})*}/\eta_t\right) \end{array}\right\} \tag{A20}$$

Obviously, there is $(e) - (h) \geqslant (f) - (g)$, that is

$$0 \geqslant \left((P_t\beta\rho - c_g) - \left((P_t\rho + s)/\alpha + c_p\right)\right) \cdot \left(E_{t+1}^{(3-s)*}/\eta_t - E_{t+1}^{(2-s)*}/\eta_t\right)$$

For any $P_t \geqslant 0$, Since there is $0<\rho \leqslant 1, 0<\alpha \leqslant 1, 0<\beta \leqslant 1$, $c_g>0$, and $c_p>0$, so, $\left(P_t\beta\rho - c_g\right) < \left((P_t\rho + s)/\alpha + c_p\right)$ hold.

Therefore, we will get the relationship $E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(3-s)*}$.

To sum up, we can draw the relationship for positive prices $P_t \geqslant 0$ and $s \leqslant P_t(1-\rho^2)/\rho$:

$$E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(3-s)*} \tag{A21}$$

Obviously, if $P_t \leqslant 0$, we also have $\left((P_t\beta\rho - c_g) - \left((P_t\rho + s)/\alpha + c_p\right)\right) \cdot \left(E_{t+1}^{(3-s)*}/\eta_t - E_{t+1}^{(2-s)*}/\eta_t\right) \leqslant 0$.

Here, we have the following relation:

(a) $$\begin{aligned}&\left(P_t\beta\rho - c_g\right) - \left((P_t\rho + s)/\alpha + c_p\right) \geqslant 0\\ &\Leftrightarrow \left(P_t\beta\rho - c_g\right)\alpha \geqslant (P_t\rho + s) + \alpha c_p \quad ;\\ &\Leftrightarrow s \leqslant (\alpha\beta - 1)\,\rho P_t - \alpha\left(c_g + c_p\right)\end{aligned}$$

(b) $$\begin{aligned}&\left(P_t\beta\rho - c_g\right) - \left((P_t\rho + s)/\alpha + c_p\right) \leqslant 0\\ &\Leftrightarrow \left(P_t\beta\rho - c_g\right)\alpha \leqslant (P_t\rho + s) + \alpha c_p\\ &\Leftrightarrow s \geqslant (\alpha\beta - 1)\,\rho P_t - \alpha\left(c_g + c_p\right)\end{aligned}$$

For the negative price if $s \geqslant 0$ and $s \leqslant (\alpha\beta - 1)\,\rho P_t - \alpha\left(c_g + c_p\right)$ hold, we can draw the following relation:

$$\begin{aligned}&(\alpha\beta - 1)\,\rho P_t - \alpha\left(c_g + c_p\right) \geqslant 0\\ &\Rightarrow (\alpha\beta - 1)\,\rho P_t \geqslant \alpha\left(c_g + c_p\right)\\ &\Rightarrow P_t \leqslant -\alpha\left(c_g + c_p\right) / ((1 - \alpha\beta)\,\rho)\end{aligned}$$

Therefore, we will obtain the following relationship:

(1) For $P_t < -\alpha(c_g + c_p)/((1-\alpha\beta)\rho)$, if $0<s \leqslant (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p)$, there is $E_{t+1}^{(2-s)*} \geqslant E_{t+1}^{(3-s)*}$.

(2) For $P_t < -\alpha(c_g + c_p)/((1-\alpha\beta)\rho)$, if $s \geqslant (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p)$, we will get $E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(3-s)*}$

To sum up, for negative prices $P_t < -\alpha(c_g + c_p)/(1-\alpha\beta)\,\rho$ and $0<s \leqslant (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p)$:

$$E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(2-s)*} \geqslant E_{t+1}^{(3-s)*} \tag{A22}$$

In this way, if $-\alpha(c_g + c_p)/((1-\alpha\beta)\rho) \leqslant P_t<0$,and $s \geqslant 0$, we also have $E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(3-s)*}$.

(3) Recall the proof 1), we also have the following two equations:

$$\begin{cases} E_{t+1}^{(1-s)*} = \arg\max\limits_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \left(P_t/\alpha\rho + c_p\right)(E_{t+1}/\eta_t)\right)\\ E_{t+1}^{(3-s)*} = \arg\max\limits_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \left(P_t\beta\rho - c_g\right)(E_{t+1}/\eta_t)\right) \end{cases}$$

Then, we can get the following in equations:

$$\left.\begin{aligned} &\text{(i)}\ \left(E\left[V_{t+1}\left(E_{t+1}^{(1-s)*}, P_{t+1}\right)|(E_t, P_t)\right] - \left(P_t/\alpha\rho + c_p\right)\left(E_{t+1}^{(1-s)*}/\eta_t\right)\right) \geqslant\\ &(j)\ \left(E\left[V_{t+1}\left(E_{t+1}^{(3-s)*}, P_{t+1}\right)|(E_t, P_t)\right] - \left(P_t/\alpha\rho + c_p\right)\left(E_{t+1}^{(3-s)*}/\eta_t\right)\right)\\ &(k)\ E\left[V_{t+1}^*\left(E_{t+1}^{(3-s)*}, P_{t+1}\right)|(E_t, P_t)\right] - \left(P_t\beta\rho - c_g\right)\left(E_{t+1}^{(3-s)*}/\eta_t\right) \geqslant\\ &(l)\ E\left[V_{t+1}^*\left(E_{t+1}^{(1-s)*}, P_{t+1}\right)|(E_t, P_t)\right] - \left(P_t\beta\rho - c_g\right)\left(E_{t+1}^{(1-s)*}/\eta_t\right) \end{aligned}\right\} \tag{A23}$$

Obviously, there is $(\mathrm{i}) - (l) \geqslant (j) - (k)$, that is

$$0 \geqslant \left((P_t\beta\rho - c_g) - (P_t/\alpha\rho + c_p + c_p)\right)\left(E_{t+1}^{(3-s)*}/\eta_t - E_{t+1}^{(1-s)*}/\eta_t\right)$$

For positive price $P_t \geqslant 0$, $(P_t/\alpha\rho + c_p) \geqslant (P_t\beta\rho - c_g)$ holds due to $0<\rho \leqslant 1, 0<\alpha \leqslant 1, 0<\beta \leqslant 1, c_g>0$, and $c_p>0$. Therefore, we will get the relationship $E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*}$.

To sum up, we can draw the relationship for positive prices $P_t \geqslant 0$ and $s \geqslant P_t(1-\rho^2)/\rho$:

$$E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*} \tag{A24}$$

Obviously, if $\begin{matrix}(P_t\beta\rho - c_g) - (P_t/\alpha\rho + c_p) \geqslant 0 \\ \Leftrightarrow (\beta\rho - 1/\alpha\rho)P_t \geqslant c_g + c_p \\ \Rightarrow P_t \leqslant -(c_g + c_p)/(1/\alpha\rho - \beta\rho)\end{matrix}$ holds, there is $E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(3-s)*}$. And if $-(c_g+c_p)/(1/\alpha\rho - \beta\rho) \leqslant P_t \leqslant 0$, we still have $E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*}$.

$$s \geqslant (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p)$$

Therefore, we can draw the following results:

For positive electricity prices $P_t \geqslant 0$

$$\begin{cases} \text{If } s \leqslant P_t(1-\rho^2)/\rho, \textit{there is } E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(3-s)*} \\ \text{If } s \geqslant P_t(1-\rho^2)/\rho, \textit{there is } E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*} \end{cases} \tag{A25}$$

For negative electricity prices $-(c_g + c_p)/(1/\alpha\rho - \beta\rho) \leqslant P_t<0$.

$$\left\{ \textit{If } s>0, \text{we } \textit{will get } E_{t+1}^{(2-s)*} \leqslant E_{t+1}^{(1-s)*} \leqslant E_{t+1}^{(3-s)*} \right. \tag{A26}$$

For negative electricity prices $P_t < -(c_g + c_p)/(1/\alpha\rho - \beta\rho)$.

$$\begin{cases} 1)\ \textit{If}\ 0 < s \leqslant (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p), \text{we } \textit{will get } E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(2-s)*} \geqslant E_{t+1}^{(3-s)*} \\ 2)\ \textit{If}\ s \geqslant (\alpha\beta - 1)\rho P_t - \alpha(c_g + c_p), \text{we } \textit{will get } E_{t+1}^{(1-s)*} \geqslant E_{t+1}^{(3-s)*} \geqslant E_{t+1}^{(2-s)*} \end{cases} \quad \blacksquare \tag{A27}$$

Proof of Proposition 3 (Production tax credit (PTC) analysis). For the electricity merchant with storage and wind farm considering production tax credit (PTC), recall the proof the Lemma 1, for any given state $S(t)$, and we can get the following results:

$$\begin{cases} E_{t+1(s>0)}^{(1-s)*} = \arg\max\limits_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left\{ -\left(P_t/\alpha\rho + c_p\right)(E_{t+1}/\eta_t - E_t) + E[V_{t+1}^*(S(t+1)|S(t))] \right\} \\ E_{t+1(s>0)}^{(2-s)*} = \arg\max\limits_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left\{ -\left((P_t\rho + s)/\alpha + c_p\right)(E_{t+1}/\eta_t - E_t) + sw_t + E[V_{t+1}^*(S(t+1)|S(t))] \right\} \\ E_{t+1(s>0)}^{(3-s)*} = \arg\max\limits_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left\{ -\left(P_t\beta\rho - c_g\right)(E_{t+1}/\eta_t - E_t) + sw_t + E[V_{t+1}^*(S(t+1)|S(t))] \right\} \end{cases}$$

If the electricity merchants ignore the production tax credit (PTC) in trading decisions (i.e., traditional study), we have the following results:

$$\begin{cases} E_{t+1(s=0)}^{(1-s)*} = \arg\max\limits_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left\{ -\left(P_t/\alpha\rho + c_p\right)(E_{t+1}/\eta_t - E_t) + E[V_{t+1}^*(S(t+1)|S(t))] \right\} \\ E_{t+1(s=0)}^{(2-s)*} = \arg\max\limits_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left\{ -\left(P_t\rho/\alpha + c_p\right)(E_{t+1}/\eta_t - E_t) + E[V_{t+1}^*(S(t+1)|S(t))] \right\} \\ E_{t+1(s=0)}^{(3-s)*} = \arg\max\limits_{\underline{E} \leqslant E_{t+1} \leqslant \bar{E}} \left\{ -\left(P_t\beta\rho - c_g\right)(E_{t+1}/\eta_t - E_t) + E[V_{t+1}^*(S(t+1)|S(t))] \right\} \end{cases} \tag{A28}$$

By using the payoff rewards function (A1), we know that there have the following relations:

$$\frac{R^{(s)}(q_t, w_t, P_t)}{\partial s} = \begin{cases} 0\ (q_t>\alpha g_t) \\ -(q_t/\alpha - w_t) \geqslant 0\ (0 \leqslant q_t<\alpha g_t) \\ w_t \geqslant 0\ (q_t<0) \end{cases}$$

Then, for positive electricity prices and state $t \in \{1, 2, 3, \cdots, T\}$, we will get

$$\partial R^{(s)}(q_t, w_t, P_t)/\partial s \geqslant 0 \tag{A29}$$

Thus, we will get the following relations: $\sum_{t=1}^{T} R^{(s)}(q_t^*, w_t, P_t)\Big|_{s\geqslant 0} \geqslant \sum_{t=1}^{T} R^{(s)}(q_t^*, w_t, P_t)\Big|_{s=0}$, and the value function of $V_{t+1(s>0)}^*(S(t+1)|S(t))$ increases with the PTC (s), then we will get the following relations:

$$E\left[V^{*}_{t+1(s\geqslant 0)}\left(S(\mathrm{t}+1)|\,S(t)\right)\right] \geqslant E\left[V^{*}_{t+1(s=0)}\left(S(\mathrm{t}+1)|\,S(t)\right)\right] \tag{A20}$$

In this way, we will get

$$\max_{\pi}\sum_{t=1}^{T}E\left[R^{(s)}(q_{\mathrm{t}},w_t,P_t)_{(s\geqslant 0)}|\ S(1)\right] \geqslant \times\max_{\pi}\sum_{t=1}^{T}E\left[R^{(s)}(q_{\mathrm{t}},w_t,P_t)_{(s=0)}|\ S(1)\right] \quad \blacksquare \tag{31}$$

Appendix B.

Proof of Lemma 2.

(1) The uniqueness of the optimal results:

The current payoff rewards are shown as follows for the merchant when the merchant can receive PTC s per wind generation sold to the market:

$$R^{(PTC)}\left(q_{\mathrm{t}},w_{\mathrm{t}},P_t\right)=\begin{cases}-P_{\mathrm{t}}\left(q_{\mathrm{t}}/\alpha-w_{\mathrm{t}}\right)\cdot\rho-s\left(q_{\mathrm{t}}/\alpha-w_{\mathrm{t}}\right)-\mathrm{c_w}w_{\mathrm{t}}-\mathrm{c_p}q_{\mathrm{t}}\ \left(0\leqslant q_{\mathrm{t}}\leqslant\alpha w_{\mathrm{t}}\right)\\ -P_{\mathrm{t}}\cdot\left(q_{\mathrm{t}}\beta-w_{\mathrm{t}}\right)\cdot\rho-\mathrm{c}_w w_{\mathrm{t}}-\mathrm{s}\left(q_{\mathrm{t}}\beta-w_{\mathrm{t}}\right)\ +\mathrm{c_g}q_{\mathrm{t}}\ \left(q_{\mathrm{t}}<0\right)\end{cases} \tag{B1}$$

Where,q_t is the energy/inventory change from period t to period t + 1 before accounting for energy loss. Recall the proof for Lemma 1 in Appendix 1, we will get the following rewards function at time t.

$$\begin{aligned}R^{(PTC)}\left(q_{\mathrm{t}},w_{\mathrm{t}},P_t\right)&=\begin{cases}-P_{\mathrm{t}}\left(q_{\mathrm{t}}/\alpha-w_{\mathrm{t}}\right)\cdot\rho-s\left(q_{\mathrm{t}}/\alpha-w_{\mathrm{t}}\right)-\mathrm{c_w}w_{\mathrm{t}}-\mathrm{c_p}q_{\mathrm{t}}\ \left(0\leqslant q_{\mathrm{t}}\leqslant\alpha w_{\mathrm{t}}\right)\\ -P_{\mathrm{t}}\cdot\left(q_{\mathrm{t}}\beta-w_{\mathrm{t}}\right)\cdot\rho-\mathrm{c}_w w_{\mathrm{t}}-\mathrm{s}\left(q_{\mathrm{t}}\beta-w_{\mathrm{t}}\right)\ +\mathrm{c_g}q_{\mathrm{t}}\ \left(q_{\mathrm{t}}<0\right)\end{cases}\\ &=\begin{cases}-\left(P_{\mathrm{t}}\rho+s\right)q_{\mathrm{t}}/\alpha+\ P_{\mathrm{t}}w_{\mathrm{t}}\cdot\rho+\mathrm{s}w_{\mathrm{t}}-\mathrm{c_w}w_{\mathrm{t}}-\mathrm{c_p}q_{\mathrm{t}}\ \left(0\leqslant q_{\mathrm{t}}\leqslant\alpha w_{\mathrm{t}}\right)\\ -\left(P_{\mathrm{t}}\rho+s\right)q_{\mathrm{t}}\beta+\left(P_{\mathrm{t}}\rho+s\right)w_{\mathrm{t}}-\mathrm{c}_w w_{\mathrm{t}}\ +\mathrm{c_g}q_{\mathrm{t}}\ \left(q_{\mathrm{t}}<0\right)\end{cases}\end{aligned} \tag{B2}$$

Similarly, recall the proof for Lemma 1, we can achieve the following optimal SOC results:

$$\begin{cases}E^{(2\text{ - PTC})*}_{t\ +\ 1}=\arg\max\limits_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V^{*}_{\mathrm{t}+1}\left(S\left(\mathrm{t}+1\right)|S\left(t\right)\right]-\left(P_{\mathrm{t}}\rho+s\right)/\alpha\cdot E_{t+1}/\eta_t-\mathrm{c_p}\cdot E_{t+1}/\eta_t\right)\\ \text{or }\ \partial E[V^{*}_{\mathrm{t}\ +\ 1}\left(S\left(\mathrm{t}+1\right)|S\left(t\right)\right]/\partial E_{t+1}-\left(\left(P_t\rho+s\right)/\alpha+\mathrm{c_p}\right)/\eta_t\Big|_{E_{t+1}=E^{(2\text{ - PTC})*}_{t\ +\ 1}}=0\end{cases} \tag{B3}$$

and

$$\begin{cases}E^{(3\text{ - PTC})*}_{\mathrm{t}+1}=\arg\max\limits_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V^{*}_{\mathrm{t}+1}\left(S\left(\mathrm{t}+1\right)|S\left(t\right)\right]-\left(P_{\mathrm{t}}\rho+s\right)\beta\cdot\left(E_{t+1}/\eta_t\right)\ +\mathrm{c_g}\cdot E_{t+1}/\eta_t\right)\\ \text{or }\ \partial E[V^{*}_{\mathrm{t}\ +\ 1}\left(S\left(\mathrm{t}+1\right)|S\left(t\right)\right]/\partial E_{t+1}-\left(\left(P_{\mathrm{t}}\rho+s\right)\beta-\mathrm{c_g}\right)/\eta_t\Big|_{E_{t+1}\ =\ E^{(3\text{ - PTC})*}_{\mathrm{t}+1}}=0\end{cases} \tag{B4}$$

The relations among two reference points:

(1) Recall the proof 1), we can have the following two equations:

$$\begin{cases}E^{(2-PTC)*}_{t+1}=\arg\max\limits_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V^{*}_{\mathrm{t}+1}\left(S\left(\mathrm{t}+1\right)|S\left(t\right)\right]-\left(\left(P_t\rho+s\right)/\alpha+\mathrm{c_p}\right)\cdot\left(E_{t+1}/\eta_t\right)\right)\\ E^{(3-PTC)*}_{t+1}=\arg\max\limits_{\underline{E}\leqslant E_{t+1}\leqslant\bar{E}}\left(E[V^{*}_{\mathrm{t}+1}\left(S\left(\mathrm{t}+1\right)|S\left(t\right)\right]-\left(\left(P_{\mathrm{t}}\rho+s\right)\beta-\mathrm{c_g}\right)\cdot\left(E_{t+1}/\eta_t\right)\right)\end{cases}$$

Then, we can get the following in equations:

$$\left.\begin{aligned}&(a')\ \left(E\left[V^{*}_{\mathrm{t}+1}\left(E^{(2-PTC)*}_{\mathrm{t}+1},P_{\mathrm{t}\ +\ 1}\right)|\left(E_{\mathrm{t}},P_{\mathrm{t}}\right)\right]-\left(\left(P_t\rho+s\right)/\alpha+\mathrm{c_p}\right)\left(E^{(2-PTC)*}_{\mathrm{t}+1}/\eta_t\right)\right)\geqslant\\ &(b')\ \left(E\left[V^{*}_{\mathrm{t}+1}\left(E^{(3-PTC)*}_{\mathrm{t}+1},P_{\mathrm{t}\ +\ 1}\right)|\left(E_{\mathrm{t}},P_{\mathrm{t}}\right)\right]-\left(\left(P_t\rho+s\right)/\alpha+\mathrm{c_p}\right)\cdot\left(E^{(3-PTC)*}_{\mathrm{t}+1}/\eta_t\right)\right)\\ &(c')\ \left(E\left[V^{*}_{\mathrm{t}+1}\left(E^{(3-PTC)*}_{\mathrm{t}+1},P_{\mathrm{t}\ +\ 1}\right)|\left(E_{\mathrm{t}},P_{\mathrm{t}}\right)\right]-\left(\left(P_{\mathrm{t}}\rho+s\right)\beta-\mathrm{c_g}\right)\cdot\left(E^{(3-PTC)*}_{\mathrm{t}+1}/\eta_t\right)\right)\geqslant\\ &(d')\ \left(E\left[V^{*}_{\mathrm{t}+1}\left(E^{(2\text{ - }PTC)*}_{\mathrm{t}+1},P_{\mathrm{t}\ +\ 1}\right)|\left(E_{\mathrm{t}},P_{\mathrm{t}}\right)\right]-\left(\left(P_{\mathrm{t}}\rho+s\right)\beta-\mathrm{c_g}\right)\left(E^{(2-PTC)*}_{\mathrm{t}+1}/\eta_t\right)\right)\end{aligned}\right\}$$

Obviously, there is $(a') - (d') \geqslant (b') - (c')$, that is

$$\begin{aligned}&\left(((P_t\rho + s)\beta - c_g)\right.\\&\left.- ((P_t\rho + s)/\alpha + c_p)\right)(E_{t+1}^{(2-PTC)*}/\eta_t\\&- E_{t+1}^{(3-PTC)*}/\eta_t) \geqslant 0\end{aligned}$$

For any positive prices $P_t \geqslant 0$, there is $((P_t\rho + s)\beta - c_g) - ((P_t\rho + s)/\alpha + c_p) < 0$, then we will get

$$E_{t+1}^{(2-PTC)*} \leqslant E_{t+1}^{(3-PTC)*}$$

For the negative prices $P_t \leqslant 0$, if $\begin{aligned}&\left(((P_t\rho + s)\beta - c_g)\right.\\&\left.- ((P_t\rho + s)/\alpha + c_p)\right)(E_{t+1}^{(2-PTC)*}/\eta_t\\&- E_{t+1}^{(3-PTC)*}/\eta_t) \geqslant 0\end{aligned}$ holds.

$$\begin{cases} ((P_t\rho + s)\beta - c_g) - \left(\frac{(P_t\rho+s)}{\alpha} + c_p\right) \geqslant 0 \Leftrightarrow s \leqslant \frac{P_t\rho(\alpha\beta-1)-\alpha(c_g+c_p)}{(1-\alpha\beta)} \Leftrightarrow 0 > P_t \geqslant -\frac{s(1-\alpha\beta)+\alpha(c_g+c_p)}{\rho(1-\alpha\beta)} \\ ((P_t\rho + s)\alpha\beta - (P_t\rho + s)) - \alpha(c_g + c_p) \leqslant 0 \Leftrightarrow s \geqslant \frac{P_t\rho(\alpha\beta-1)-\alpha(c_g+c_p)}{(1-\alpha\beta)} \Leftrightarrow P_t \leqslant -\frac{s(1-\alpha\beta)+\alpha(c_g+c_p)}{\rho(1-\alpha\beta)} < 0 \end{cases}$$

Therefore, we will get the following relationship:
If $P_t \geqslant -\left(\alpha(c_g + c_p) + s(1-\beta\alpha)\right)/(\rho(1-\beta\alpha))$,there is

$$E_{t+1}^{(2-PTC)*} \leqslant E_{t+1}^{(3-PTC)*} \tag{B5}$$

If $P_t \leqslant -\left(\alpha\left(c_g + c_p\right) + s(1-\beta\alpha)\right)/(\rho(1-\beta\alpha)) < 0$ there is

$$E_{t+1}^{(2-PTC)*} \geqslant E_{t+1}^{(3-PTC)*}$$

For positive price, we will get the following results:

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(2-PTC)*} - E_t, \overline{Q}^p\right\},\ E_t \in \left[0, E_{t+1}^{(2-PTC)*}\right] \left(\text{store renewable without buying up to } E_{t+1}^{(2-PTC)*}\right) \\ 0, E_t \in \left(E_{t+1}^{(2-PTC)*}, E_{t+1}^{(3-PTC)*}\right] \text{ (keep SOC unchanged)} \\ \max\left\{E_{t+1}^{(3-PTC)*} - E_t, -\overline{Q}^g\right\}, E_t \in \left(E_{t+1}^{(3-PTC)*}, \overline{E}\right] \left(disch\arg e\ \ and \text{ sell renewable down to } E_{t+1}^{(3-PTC)*}\right) \end{cases} \quad \blacksquare \tag{B6}$$

Appendix X. Case study

In this case, we assume there are three time periods ($T = 3$). At each period, the power price takes one of the values in set $P_t = \{p^M, p^L, p^H\} = \{6, 3, 10\}$. We also assume the storage energy capacity cannot refill it fully or sell it empty in one time period, but fewer than two time periods. In detail, when the full (resp. empty) storage can be emptied (resp. filled up) in more than one period but fewer than two periods, it holds that $\underline{E} + \bar{Q}^p \leqslant \bar{E}$ (resp. $\bar{E} - \underline{E} > \bar{Q}^g$) and $\underline{E} + 2\bar{Q}^p \geqslant \bar{E}$ (resp. $\bar{E} - \underline{E} \leqslant 2\bar{Q}^g$). We assume the storage capacity is 10 (i.e., $\underline{E} = 0, \bar{E} = 10$), the generating/discharging max capacity is 12 and the pumping/charging max capacity is 7. Let the operating cost be one(i.e., $c_p = c_g = 1$),wind generation cost (i.e.,$c_w = 0$), the pumping and generating efficiencies be 0.9 (i.e., $\alpha = \beta = 0.9 = \rho$), self-discharging and transmission efficiencies be one (i.e., $\eta = 1$), the wind generation are $w_t = \{3, 5, 0\}$.

Using the above problem and related data, we will show the method to get the optimal solution based on the optimal actions proposed in propositions 1 and 2. In order to compare with the traditional study that without considering the PTC, the optimal results under three different PTC credit rates (i.e., $s = \{0, 1, 3\}$) under two initial SOC (i.e., $E_1 = \{1, 5\}$) .

Case 1(Policy 1): When the PTC $s = 3$.

In stage 4:

$$\begin{aligned} VOE_4 &= (6 + 3 + 10)/3 \\ &= 6.33 \\ &\Rightarrow V_4 = VOE_4 \cdot E_4 \\ &= 6.33E_4 \end{aligned}$$

Plug in the data, we will get the following optimal references points:

$$\begin{cases} E_4^{(1\ -\ s)*} = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}}\left\{-\left(P_3/\alpha\rho + c_p\right)(E_4/\eta_3)\ +\ E\left[V_4^*\left(S(4)|\ S(3)\right)\right]\right\} \\ = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}}\{-(10/0.81\ +\ 1)\ E_4\ + 6.33E_4\} = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}}\{-7.0157E_4\} \\ E_4^{(2\ -\ s)*} = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}}\left\{-\left(\left(P_3\rho + s\right)/\alpha + c_p\right)(E_4/\eta_3) + E\left[V_4^*\left(S(4)|\ S(3)\right)\right]\right\} \\ = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}}\{-((10\times 0.9 + 3)/0.9\ +\ 1)\ E_4 + 6.33E_4\} = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}}\{-8.0033E_4\} \\ E_4^{(3\ -\ s)*} = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}\left\{-\left(P_3\beta\rho - c_g\right)(E_4/\eta_3)\ +\ E\left[V_4^*\left(S(4)|\ S(3)\right)\right]\right\} \\ = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}}\{-(10\times 0.81 - 1)\ E_4\ + 6.33E_4\} = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}}\{-0.77E_4\} \end{cases} \tag{X1}$$

$$\Rightarrow E_4^{(1\ -\ s)*} = E_4^{(2\ -\ s)*} = E_4^{(3\ -\ s)*} = 0$$

In Stage 3:

Action 3: Release power and make the storage level down to $\underline{E} = 0 = E_4^*$, thus, $V_4^* = 0$

$$q_3^*(S_3) = -E_3, E_3 \in (0, \overline{E}\,]\ \text{(sell energy and make SOC down to 0 as close as possible)} \tag{X2}$$

The reward function at stage3 is shown as

$$\begin{aligned} \mathrm{R}_3^{(s)} &= -P_3\cdot(q_3\beta - w_3)\cdot\rho - c_w w_3 + s w_3 + c_g q_3 \\ &= -10\cdot\left(\left(-E_3\right)0.9 - 0\right)\cdot 0.9 + \left(-E_3\right) \\ &= 7.1E_3\ (q_3{<}0) \end{aligned} \tag{X3}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_3^* = \max\left\{\mathrm{R}_3^{(s)} + V_4^*\right\} = 7.1E_3 \tag{X4}$$

In stage 2:

By using the Eqs. (A16), (A17), and (A18), we will get the following results for merchants:

$$\begin{cases} E_3^{(1\ -s)^*} = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}\left(V_3^* - \left(\frac{P_2}{\alpha\rho} + c_p\right)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}\left(7.1E_3 - \left(\frac{3}{0.9\times 0.9} + 1\right)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}(2.3963E_3) \\ E_3^{(2\ -\ s)} = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}\left(V_3^* - \left(\frac{P_2\rho + s}{\alpha} + c_p\right)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}\left(7.1E_3 - \left(\frac{3\times 0.9 + 3}{0.9} + 1\right)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}(-0.23E_3) \\ E_3^{(3\ -\ s)*} = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}\left(V_3^* - \left(P_2\beta\rho - c_g\right)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}\left(7.1E_3 - (3\times 0.9\times 0.9 - 1)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}}(5.67E_3) \end{cases} \tag{X5}$$

$$\Rightarrow E_3^{(1\ -\ s)*} = 10, E_3^{(2\ -\ s)*} = 0, E_3^{(3\ -\ s)*} = 10$$

Thus, we will get the optimal reference points at stage 3 that are:

$$E_3^{(1\ -\ s)*} = 10, E_3^{(2\ -\ s)*} = 0, E_3^{(3\ -\ s)*} = 10 \tag{X6}$$

Since there are $s \geqslant P_2(1-\rho^2)/\rho$, $\alpha w_2 \geqslant \min\{E_{t+1}^{(2\ -\ s)*}, \bar{Q}^p\}$,and $E_{t+1}^{(1)*} \geqslant E_{t+1}^{(1)*} - \alpha w_t \geqslant E_{t+1}^{(2\ -\ s)*}$. The optimal actions at stage 2 are shown as

$$q_t^*(S_t) = \begin{cases} \min\left\{E_{t+1}^{(2\ -s)^*} - E_t, \overline{Q}^p\right\}, E_t \in (0, E_{t+1}^{(2\ -s)^*}]\ \left(\text{store renewable and without buy up to } E_{t+1}^{(2\ -s)^*}\right) \\ \min\left\{E_{t+1}^{(1\ -s)^*} - E_t, \overline{Q}^p\right\}, E_t \in (E_{t+1}^{(2\ -s)^*}, E_{t+1}^{(1\ -s)^*} - \alpha w_t]\ \left(\text{store renewable and purchased electricity up to } E_{t+1}^{(1\ -s)^*}\right) \\ 0, E_t \in (E_{t+1}^{(1\ -s)^*} - \alpha w_t, E_{t+1}^{(3\ -s)^*}]\ \ (\text{keep SOC unchanged}) \\ \max\left\{E_{t+1}^{(3\ -s)^*} - E_t, -\overline{Q}^g\right\}, E_t \in (E_{t+1}^{(3\ -s)^*}, \overline{E}\,]\ \left(\text{generate and sell renewable down to } E_{t+1}^{(3\ -s)^*}\right) \end{cases} \tag{X7}$$

$$\Rightarrow q_2^*(S_2) = \begin{cases} 7,\ \mathrm{E}_2 \in (0, 3] \\ 10 - \mathrm{E}_2,\ \mathrm{E}_2 \in (3, 5.5] \\ 0,\ \mathrm{E}_2 \in (5.5, 10] \end{cases}$$

The reward payoff functions at stage 2 are shown as follows:

$$\begin{aligned} \mathrm{R}_2^{(s)} &= \begin{cases} -P_2\cdot(q_2/\alpha - w_2)/\rho - c_w w_2 - c_p q_2 \\ -P_2\cdot(q_2/\alpha - w_2)\cdot\rho - s\,(q_2/\alpha - w_2) - c_w w_2 - c_p q_2 \end{cases} \\ &= \begin{cases} -3\cdot(7/0.9 - 5)/0.9 - 7,\ \mathrm{E}_2 \in (3, 5.5] \\ -3\cdot((10 - \mathrm{E}_2)/0.9 - 5)/0.9 - (10 - \mathrm{E}_2),\ \mathrm{E}_2 \in (3, 5.5] \\ -3\cdot(0/0.9 - 5)\cdot 0.9 - 3\,(0/0.9 - 5) - 0,\ \mathrm{E}_2 \in (5.5, 10] \end{cases} = \begin{cases} -16.2593,\ \mathrm{E}_2 \in (0, 3] \\ 4.7037\mathrm{E}_2 - 30.3704,\ \mathrm{E}_2 \in (3, 5.5] \\ 28.5,\ \mathrm{E}_2 \in (5.5, 10] \end{cases} \end{aligned} \tag{X8}$$

Therefore, the optimal value function at stage 3 is shown as:

$$V_2^* = \max\{R_2^{(s)} + V_3^*\}$$
$$= \begin{cases} 7.1E_3 - 16.2593|_{E_3=E_2+7}, \ E_2 \in (0,3] \\ 7.1E_3 + 4.7037E_2 - 30.3704|_{E_3=10}, \ E_2 \in (3,5.5] \\ 7.1E_3 + 28.5|_{E_3=E_2}, \ E_2 \in (5.5,10] \end{cases}$$
$$= \begin{cases} 7.1E_2 + 33.4407, \ E_2 \in (0,3] \\ 4.7037E_2 + 40.6296, \ E_2 \in (0,5.5] \\ 7.1E_2 + 28.5, \ E_2 \in (5.5,10] \end{cases} \tag{X9}$$

In stage 1:
By using the Eqs. (A16), (A17), and (A18), we will get the following results for merchants:

$$\begin{cases} E_2^{(1-s)*} = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left(\frac{P_1}{\alpha\rho} + c_p\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left(\frac{6}{0.9*0.9} + 1\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - 8.4074E_2 + 28.5\right) \\ E_2^{(2-s)*} = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left(\frac{P_1\rho+s}{\alpha} + c_p\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left(\frac{6*0.9+3}{0.9} + 1\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - 10.3233E_2 + 28.5\right) \\ E_2^{(3-s)*} = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left(P_1\beta\rho - c_g\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - (6*0.9*0.9-1)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - 3.86E_2 + 28.5\right) \end{cases} \tag{X10}$$

(1) Scenario1: If $E_2 \in (0,3]$, there is $V_2^* = 7.1E_2 + 33.4407$.

$$\begin{cases} E_2^{(1-s)*} = \arg\max_{E_2\in[0,\ 3]} (7.1E_2 + 33.4407 - (6/0.81 + 1)E_2/\eta_1) = \arg\max_{E_2\in[0,\ 3]} (-1.3074E_2 + 33.4407) \\ E_2^{(2-s)*} = \arg\max_{E_2\in[0,\ 3]} (7.1E_2 + 33.4407 - ((6\times 0.9 + 3)/0.9 + 1)E_2/\eta_1) = \arg\max_{E_2\in[0,\ 3]} (-3.2233E_2 + 33.4407) \\ E_2^{(3-s)*} = \arg\max_{E_2\in[0,\ 3]} (7.1E_2 + 33.4407 - (6\times 0.9\times 0.9 - 1)E_2/\eta_1) = \arg\max_{E_2\in[0,\ 3]} (3.24E_2 + 33.4407) \end{cases}$$
$$\Rightarrow E_2^{(1-s)*} = E_2^{(2-s)*} = 0, E_2^{(3-s)*} = 3 \tag{X11}$$

(2) Scenario2: If $E_2 \in (3,5.5]$, there is $V_2^* = 4.7037E_2 + 40.6296$.

$$\begin{cases} E_2^{(1-s)*} = \arg\max_{E_2\in[3,5.5]} (4.7037E_2 + 40.6296 - (6/0.81 + 1)E_2/\eta_1) = \arg\max_{E_2\in[3,5.5]} (-3.7037E_2 + 40.6296) \\ E_2^{(2-s)*} = \arg\max_{E_2\in[3,5.5]} (4.7037E_2 + 40.6296 - ((6\times 0.9 + 3)/0.9 + 1)E_2/\eta_1) = \arg\max_{E_2\in[3,5.5]} (-5.6296E_2 + 40.6296) \\ E_2^{(3-s)*} = \arg\max_{E_2\in[3,5.5]} (4.7037E_2 + 40.6296 - (6\times 0.9\times 0.9 - 1)E_2/\eta_1) = \arg\max_{E_2\in[3,5.5]} (0.8437E_2 + 40.6296) \end{cases}$$
$$\Rightarrow E_2^{(1-s)*} = E_2^{(2-s)*} = 3, E_2^{(3-s)*} = 5.5 \tag{X12}$$

(3) Scenario3: If $E_2 \in (5.5,10]$, there is $V_2^* = 7.1E_2 + 28.5$.

$$\begin{cases} E_2^{(1-s)*} = \arg\max_{E_2\in[5.5,10]} (7.1E_2 + 28.5 - (6/0.81 + 1)E_2/\eta_1) = \arg\max_{E_2\in[5.5,10]} (-1.3074E_2 + 28.5) \\ E_2^{(2-s)*} = \arg\max_{E_2\in[5.5,10]} (7.1E_2 + 28.5 - ((6\times 0.9 + 3)/0.9 + 1)E_2/\eta_1) = \arg\max_{E_2\in[5.5,10]} (-3.2233E_2 + 28.5) \\ E_2^{(3-s)*} = \arg\max_{E_2\in[5.5,10]} (7.1E_2 + 28.5 - (6\times 0.9\times 0.9 - 1)E_2/\eta_1) = \arg\max_{E_2\in[5.5,10]} (3.24E_2 + 28.5) \end{cases}$$
$$\Rightarrow E_2^{(1-s)*} = E_2^{(2-s)*} = 5.5, E_2^{(3-s)*} = 10 \tag{X13}$$

Next, we will choose the optimal references point between the above three scenarios.

(1) Compare $E_2^{(1-s)*}$

$$\left.\begin{cases} 1)\begin{cases} If\ \ E_2 \in [0,\ 3] \Rightarrow E_2^{(1-s)*} = 0 \\ E_2^{(1-s)*} = \arg\max_{E_2\in[0,\ 3]} (-1.3074E_2 + 33.4407) \Rightarrow -1.3074E_2 + 33.4407|_{E_2^{(1-s)*}=0} = 33.4407 \end{cases} \\ 2)\begin{cases} If\ \ E_2 \in [3,\ 5.5] \Rightarrow E_2^{(1-s)*} = 3 \\ E_2^{(1-s)*} = \arg\max_{E_2\in[3,5.5]} (-3.7037E_2 + 40.6296) \Rightarrow -3.7037E_2 + 40.6296|_{E_2^{(1-s)*}=3} = 29.5185 \end{cases} \\ 3)\begin{cases} If\ \ E_2 \in [5.5,\ 10] \Rightarrow E_2^{(1-s)*} = 5.5 \\ E_2^{(1-s)*} = \arg\max_{E_2\in[5.5,10]} (-1.3074E_2 + 28.5) \Rightarrow -1.3074E_2 + 28.5|_{E_2^{(1-s)*}=5.5} = 21.3093 \end{cases} \end{cases}\right\} \Rightarrow E_2^{(1-s)*} = 0 \tag{X14}$$

(2) Compare $E_2^{(2-s)*}$

$$\left\{\begin{array}{l} 1)\left\{\begin{array}{c} If\ \ E_2\in[0,\ \ 3]\Rightarrow E_2^{(2-s)*}=0 \\ E_2^{(2-s)*}=\arg\max\limits_{E_2\in[0,\ 3]}(-3.2233E_2+33.4407)\Rightarrow -3.2233E_2+33.4407|_{E_2^{(2-s)*}=0}=33.4407 \end{array}\right. \\ 2)\left\{\begin{array}{c} If\ \ E_2\in[3,\ \ 5.5]\Rightarrow E_2^{(2-s)*}=3 \\ E_2^{(2-s)*}=\arg\max\limits_{E_2\in[3,5.5]}(-5.6296E_2+40.6296)\Rightarrow -5.6296E_2+40.6296|_{E_2^{(2-s)*}=3}=23.7408 \end{array}\right. \\ 3)\left\{\begin{array}{c} If\ \ E_2\in[5.5,\ \ 10]\Rightarrow E_2^{(2-s)*}=5.5 \\ E_2^{(2-s)*}=\arg\max\limits_{E_2\in[5.5,10]}(-3.2233E_2+28.5)\Rightarrow -3.2233E_2+28.5|_{E_2^{(2-s)*}=5.5}=10.77185 \end{array}\right. \end{array}\right\}\Rightarrow E_2^{(2-s)*}=0 \tag{X15}$$

(3) Compare $E_2^{(3\ -\ s)*}$

$$\left|\begin{array}{l} 1)\left\{\begin{array}{c} If\ \ E_2\in[0,\ \ 3]\Rightarrow E_2^{(3-s)*}=3 \\ E_2^{(3-s)*}=\arg\max\limits_{E_2\in[0,\ 3]}(3.24E_2+33.4407)\Rightarrow 3.24E_2+33.4407|_{E_2^{(3-s)*}=3}=43.1607 \end{array}\right. \\ 3)\left\{\begin{array}{c} If\ \ E_2\in[3,\ \ 5.5]\Rightarrow E_2^{(3-s)*}=5.5 \\ E_2^{(3-s)*}=\arg\max\limits_{E_2\in[3,5.5]}(0.8437E_2+40.6296)\Rightarrow 0.8437E_2+40.6296|_{E_2^{(3-s)*}=5.5}=45.27 \end{array}\right. \\ 3)\left\{\begin{array}{c} If\ \ E_2\in[5.5,\ \ 10]\Rightarrow E_2^{(3-s)*}=10 \\ E_2^{(3-s)*}=\arg\max\limits_{E_2\in[5.5,10]}(3.24E_2+28.5)\Rightarrow 3.24E_2+28.5|_{E_2^{(3-s)*}=10}=60.9 \end{array}\right. \end{array}\right\}\Rightarrow E_2^{(3-s)*}=10 \tag{X16}$$

Thus, we will get the optimal reference points at stage 2 that are shown as:

$$E_2^{(1-s)*}=E_2^{(2-s)*}=0, E_2^{(3-s)*}=10 \tag{X17}$$

Similarly, because there are $s\geqslant P_1(1-\rho^2)/\rho$, $\alpha w_1\geqslant\min\{E_{t+1}^{(2-s)*},\bar{Q}^p\}$, and $E_{t+1}^{(1-s)*}-\alpha w_t\leqslant E_{t+1}^{(2-s)*}\leqslant E_{t+1}^{(1-s)*}$, the optimal actions at stage 1 are shown as

$$q_t^*(S_t)=\left\{\begin{array}{c} \min\left\{E_{t+1}^{(2\ -\ s)*}-E_t,\alpha w_t,\overline{Q}^p\right\}, E_t\in\left(0,E_{t+1}^{(2-s)*}\right]\ \left(\text{store renewable and without buy up to }E_{t+1}^{(2\ -\ s)*}\right) \\ 0, E_t\in\left(E_{t+1}^{(2\ -\ s)*},E_{t+1}^{(3\ -\ s)*}\right]\ \ (\text{keep SOC unchanged}) \\ \max\left\{E_{t+1}^{(3\ -\ s)*}-E_t,-\overline{Q}^g\right\}, E_t\in\left(E_{t+1}^{(3\ -\ s)*},\overline{E}\right]\ \left(generate\ and\ \text{sell renewable down to }E_{t+1}^{(3\ -\ s)*}\right) \end{array}\right.$$
$$\Rightarrow q_1^*(S_1)=0, E_t\in\left(E_{t+1}^{(2\ -\ s)*},E_{t+1}^{(3\ -\ s)*}\right]\ \Leftrightarrow q_1^*(S_1)=0, E_1\in\left(0,\overline{E}\right](keep\ SOC\ unchanged) \tag{X18}$$

The reward payoff functions at stage 1 are shown as follows:

$$\begin{aligned} \mathrm{R}_1^{(s)} &= -P_1\cdot(q_1/\alpha-w_1)\cdot\rho-\mathrm{s}(q_1/\alpha-w_1)-\mathrm{c}_w w_1-\mathrm{c}_p q_1 \\ &= -6\cdot(0/0.9-3)\cdot 0.9-3(0/0.9-3)=25.2,\ E_1\in(0,\overline{E}] \end{aligned} \tag{X19}$$

Therefore, we will get the following optimal value functions at stage 1/initial stage.

$$\begin{aligned} V_1^* &= \max\left\{\mathrm{R}_1^{(s)}+V_2^*\right\} \\ &= \left\{\begin{array}{c} 7.1E_2+33.4407+25.2|_{E_2=E_1},\ \mathrm{E}_1\in\left(0,3\right] \\ 4.7037\mathrm{E}_2+40.6296+25.2|_{E_2=E_1},\ \mathrm{E}_1\in\left(0,5.5\right] \\ 7.1E_2+28.5+25.2|_{E_2=E_1},\ \mathrm{E}_1\in\left(5.5,10\right] \end{array}\right. \\ &= \left\{\begin{array}{c} 7.1E_1+58.6407,\ \mathrm{E}_1\in\left(0,3\right] \\ 4.7037\mathrm{E}_1+65.8296,\ \mathrm{E}_1\in\left(0,5.5\right] \\ 7.1E_1+53.7,\ \mathrm{E}_1\in\left(5.5,10\right] \end{array}\right. \end{aligned} \tag{X20}$$

The corresponding optimal actions are shown:

In stage 1, $q_1^*(S_1)=0,\ E_1\in(0,\bar{E}]$ (*keep SOC unchanged*)

In stage 2, $q_2^*(S_2)=\left\{\begin{array}{c} 7,\ \mathrm{E}_2\in(0,3] \\ 10-\mathrm{E}_2,\ \mathrm{E}_2\in(3,5.5] \\ 0,\ \mathrm{E}_2\in(5.5,10] \end{array}\right.$

In stage 3, $q_3^*(S_3)=-E_3, E_3\in(0,\overline{E}]$ (sell energy and make SOC down to 0 as close as possible)

To sum up, we get the following results:

If $E_1 = 1$ **(The initial SOC in the storage)**
Stage 1: If $E_1 = 1$, (action: sell generation 3 to the market, and keep SOC unchanged), then we will get $E_2 = 1$ (i.e., $q_1^* = 0, R_1 = 25.2$);
Stage 2: If $E_2 = 1$, (action: store renewable and purchased electricity up to $\bar{Q}^p$), then, we will get $E_3 = 8$ (i.e., $q_2^* = 7, R_2 = -16.2593$);
Stage 3: If $E_3 = 8$,(action: generating and selling), then, we have $E_4 = 0$ (i.e., $q_3^* = -8, R_3 = 56.8$).
Based on the forecasted price, total rewards are shown as $R = R_1 + R_2 + R_3 = 65.7407 = V_1^*$.
If $E_1 = 5$ **(The initial SOC in the storage)**
Stage 1: If $E_1 = 5$,(action 1: do nothing), then, the relation of $E_2 = 5$ (i.e., $q_1^* = 0, R_1 = 25.2$) holds;
Stage 2: If $E_2 = 5$,(action 2: store renewable and purchased electricity up to $E_{t+1}^{(1-s)*}$), then there exists $E_3 = 10$ (i.e., $q_2^* = 5, R_2 = -6.8519$);
Stage 3: If $E_3 = 10$,(action 3: generating and selling), then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -10, R_3 = 71$).
Therefore, total rewards in three periods are shown as $R = R_1 + R_2 + R_3 = 89.3481 = V_1^*$ if $E_1 = 5$.
Case 2(Policy 1): When PTC $s = 1$
Recall the proof of Case 1, we will get the following results:

$$\begin{cases} E_4^{(1-s)*} = E_4^{(2-s)*} = E_4^{(3-s)*} = 0 \\ E_3^{(1-s)*} = E_3^{(2-s)*} = E_3^{(3-s)*} = 10 \\ E_2^{(1-s)*} = E_2^{(2-s)*} = 0, E_2^{(3-s)*} = 10 \end{cases} \tag{X21}$$

The corresponding optimal value function at stage 1 are shown:

$$\begin{aligned} &V_1^* \\ &= \max\{R_1^{(s)} + V_2^*\} \\ &= \begin{cases} 7.1E_2 + 33.4407 + 19.2|_{E_2=E_1}, \ E_1 \in [0,3] \\ 4.7037E_2 + 40.6293 + 19.2|_{E_2=E_1}, \ E_1 \in [3,5.5] \\ 5.11E_2 + 38.389 + 19.2|_{E_2=E_1}, \ E_1 \in [5.5,3] \end{cases} \\ &= \begin{cases} 7.1E_1 + 52.6407, \ E_1 \in [0,3] \\ 4.7037E_1 + 59.8293, \ E_1 \in [3,5.5] \\ 5.11E_1 + 57.589, \ E_1 \in [5.5,3] \end{cases} \end{aligned} \tag{X22}$$

In stage 1, $q_1^*(S_1) = 0$, if $E_1 \in (0, \bar{E}]$*(keep SOC unchanged)*
In stage 2, $q_2^*(S_2) = \{7, \text{ if } E_2 \in [0,3]; \ 10 - E_2, \text{ if } E_2 \in [3,10]$
In stage 3, $q_3^*(S_3) = -E_3$, if $E_3 \in (0, \overline{E}]$ (sell energy and make SOC down to 0 as close as possible)
If $E_1 = 1$ **(The initial SOC in the storage)**
Stage 1: If $E_1 = 1$, then we will get $E_2 = 1$ (i.e., $q_1^* = 0, \ R_1 = 19.2$);
Stage 2: If $E_2 = 1$, then, we will get $E_3 = 8$ (i.e., $q_2^* = 7, R_2 = -16.2593$);
Stage 3: If $E_3 = 8$, then, we have $E_4 = 0$ (i.e., $q_3^* = -8, R_3 = 56.8$).
Based on the forecasted price, total rewards are shown as $R = R_1 + R_2 + R_3 = 59.7407 = V_1^*$.
If $E_1 = 5$ **(The initial SOC in the storage)**
Stage 1: If $E_1 = 5$, then, the relation of $E_2 = 5$ (i.e., $q_1^* = 0, R_1 = 19.2$) holds;
Stage 2: If $E_2 = 5$, then there exists $E_3 = 10$ (i.e., $q_2^* = 5, R_2 = -6.8522$);
Stage 3: If $E_3 = 10$, then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -10, R_3 = 71$).
Therefore, total rewards in three periods are shown as $R = R_1 + R_2 + R_3 = 83.3478 = V_1^*$ if $E_1 = 5$.
Case 3 (Policy 1): When PTC$s = 0$
Recall the proof of Case 1, we will get the following results:

$$\begin{cases} E_4^{(1-s)*} = E_4^{(2-s)*} = E_4^{(3-s)*} = 0 \\ E_3^{(1-s)*} = E_3^{(2-s)*} = E_3^{(3-s)*} = 10 \\ E_2^{(1-s)*} = 0, E_2^{(2-s)*} = 3, E_2^{(3-s)*} = 10 \end{cases} \tag{X23}$$

The corresponding optimal value function at stage 1 are shown:

$$V_1^* = \begin{cases} 7.1E_1 + 49.9107, E_1 \in [0,0.3] \\ 7E_1 + 49.9407, E_1 \in [0.3,3] \\ 4.7037E_1 + 56.8293, \ E_1 \in [3,5.5] \\ 4E_1 + 60.7, \ E_1 \in [5.5,10] \end{cases} \tag{X24}$$

In stage 1, $q_1^*(S_1) = \{2.7, \text{ if } E_1 \in [0,0.3); \ 3 - E_1, \ if \ E_1 \in (0,0.3]; \ 0, \text{ if } E_1 \in (3,10]$;
In stage 2, $q_2^*(S_2) = \{7, \text{ if } E_2 \in [0,3]; \ 10 - E_2, \text{ if } E_2 \in [3,10]$
In stage 3, $q_3^*(S_3) = -E_3$, if $E_3 \in (0, \overline{E}]$ (sell energy and make SOC down to 0 as close as possible)
If $E_1 = 1$ **(The initial SOC in the storage)**
Stage 1: If $E_1 = 1$, ($q_1^* = 2, \ R_1 = 2.2$); **Stage 2:** If $E_2 = 3$, ($q_2^* = 7, R_2 = -16.2593$);
Stage 3: If $E_3 = 10$, ($q_3^* = -10, R_3 = 71$).

Based on the forecasted price, total rewards are shown as $R = R_1 + R_2 + R_3 = 56.9407 = V_1^*$.

If $E_1 = 5$ **(The initial SOC in the storage)**

Stage 1: If $E_1 = 5$, $(q_1^* = 0, R_1 = 16.2)$; **Stage 2:** If $E_2 = 5$, $(q_2^* = 5, R_2 = -6.8522)$;

Stage 3: If $E_3 = 10$, then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -10, R_3 = 71$).

Therefore, total rewards in three periods are shown as $R = R_1 + R_2 + R_3 = 80.3478 = V_1^*$ if $E_1 = 5$.

Case 4(Policy 2): When the PTC $s = 3$.

In stage 4:

$$
\begin{aligned}
VOE_4 &= (6+3+10)/3 \\
&= 6.33 \\
&\Rightarrow V_4 = VOE_4 \cdot E_4 \\
&= 6.33E_4
\end{aligned}
$$

Plug in the data, we will get the following optimal references points:

$$
\begin{cases}
E_4^{(2-PTC)*} = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}} \left\{-\left((P_3\rho + s)/\alpha + c_p\right)(E_4/\eta_3) + E\left[V_4^*(S(4)|\, S(3))\right]\right\} \\
= \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}} \{-((10*0.9+3)/0.9+1)\, E_4 + 6.33E_4\} = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}} \{-8.0033E_4\} \Rightarrow E_4^{(2\text{ - PTC})*} = 0 \\
E_4^{(3-PTC)*} = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}} \left\{-\left((P_3\rho + s)\beta - c_g\right)(E_4/\eta_3) + E\left[V_4^*(S(4)|\, S(3))\right]\right\} \\
= \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}} \{-((10*0.9+3)*0.9-1)\, E_4 + 6.33E_4\} = \arg\max\limits_{\underline{E}\leqslant E_4\leqslant \bar{E}} \{-3.47E_4\} \Rightarrow E_4^{(3\text{ - PTC})*} = 0
\end{cases}
\tag{X25}
$$

In Stage 3:

Action 3: Release power and make the storage level down to $\underline{E} = 0 = E_4^*$, thus, $V_4^* = 0$

$$
q_3^*(S_3) = -E_3, E_3 \in (0, \overline{E}] \text{(sell energy and make SOC down to 0 as close as possible)} \tag{X26}
$$

The reward function at stage3 is shown as

$$
\begin{aligned}
&\mathrm{R}_3^{(\mathrm{PTC})} = -P_3 \cdot (q_3\beta - w_3)\cdot\rho - \mathrm{c}_w w_3 - \mathrm{s}(q_3\beta - w_3) + \mathrm{c}_g q_3 \ (q_3 < 0) \\
&= -10 \cdot ((-E_3)0.9 - 0)\cdot 0.9 - 3*((-E_3)0.9 - 0) + (-E_3) = 9.8E_3
\end{aligned}
\tag{X27}
$$

Therefore, the optimal value function at stage 3 is shown as:

$$
V_3^* = \max\{R_3^{(PTC)} + V_4^*\} = 9.8E_3 \tag{X28}
$$

In stage 2:

By using the Eqs. (B3) and (B4), we will get the following results for merchants:

$$
\begin{cases}
E_3^{(2\text{ - PTC})*} = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}} \left(V_3^* - \left(\frac{P_2\rho+s}{\alpha} + \mathrm{c_p}\right)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}} \left(9.8E_3 - \left(\frac{3*0.9+3}{0.9} + 1\right)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}} (2.47E_3) \\
E_3^{(3\text{ - PTC})*} = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}} \left(V_3^* - ((P_2\rho + \mathrm{s})\beta - c_g)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}} \left(9.8E_3 - ((3*0.9+3)*0.9-1)\frac{E_3}{\eta_2}\right) = \arg\max\limits_{\underline{E}\leqslant E_3\leqslant \bar{E}} (5.67E_3)
\end{cases}
\tag{X29}
$$

Thus, we will get the optimal reference points at stage 3 that are:

$$
E_3^{(2\ -\ \mathrm{PTC})*} = E_3^{(3\ -\ \mathrm{PTC})*} = 10 \tag{X30}
$$

The optimal actions at stage 2 are shown as

$$
\begin{aligned}
&q_t^*(S_t) = \begin{cases}
\min\left\{E_{t+1}^{(2-PTC)*} - E_t, \overline{Q}^p, \alpha w_t\right\},\ E_t \in \left[0, E_{t+1}^{(2-PTC)*}\right] \left(\text{store renewable bring SOC up to } E_{t+1}^{(2-PTC)*}\right) \\
0, E_t \in \left(E_{t+1}^{(2-PTC)*}, E_{t+1}^{(3-PTC)*}\right] \ \text{(keep SOC unchanged)} \\
\max\left\{E_{t+1}^{(3-PTC)*} - E_t, -\overline{Q}^g\right\}, E_t \in \left(E_{t+1}^{(3-PTC)*}, \overline{E}\right] \left(disch\arg e \text{ make SOC down to } E_{t+1}^{(3-PTC)*}\right)
\end{cases} \\
&\Rightarrow q_2^*(S_2) = \min\{10 - E_2, 4.5\}\ \mathrm{E}_2 \in (0, 10] \Leftrightarrow q_2^*(S_2) = \begin{cases} 4.5,\ \mathrm{E}_2 \in (0, 5.5] \\ 10 - \mathrm{E}_2,\ \mathrm{E}_2 \in (3, 5.5] \end{cases}
\end{aligned}
\tag{X31}
$$

The reward payoff functions at stage 2 are shown as follows:

$$
\begin{aligned}
&\mathrm{R}_2^{(\mathrm{PTC})} = -P_2 \cdot (q_2/\alpha - w_2)\cdot\rho - \mathrm{s}(q_2/\alpha - w_2) - \mathrm{c}_w w_2 - \mathrm{c}_p q_2 \\
&= \begin{cases} -3\cdot(4.5/0.9 - 5)\cdot 0.9 - 3*(4.5/0.9 - 5) - 4.5,\ \mathrm{E}_2 \in (0, 5.5] \\ -3\cdot((10 - \mathrm{E}_2)/0.9 - 5)\cdot 0.9 - 3*((10 - \mathrm{E}_2)/0.9 - 5) - (10 - \mathrm{E}_2), \mathrm{E}_2 \in (5.5, 10] \end{cases} \\
&= \begin{cases} -4.5,\ \mathrm{E}_2 \in (0, 5.5] \\ 7.33\mathrm{E}_2 - 44.8, \mathrm{E}_2 \in (5.5, 10] \end{cases}
\end{aligned}
\tag{X32}
$$

Therefore, the optimal value function at stage 3 is shown as:

$$
\begin{aligned}
V_2^* &= \max\{R_2^{(PTC)} + V_3^*\} \\
&= \begin{cases} 9.8E_3 - 4.5|_{E_3=E_2+4.5}, & E_2 \in (0, 5.5] \\ 9.8E_3 + 7.33E_2 - 44.8|_{E_3=10}, & E_2 \in (5.5, 10] \end{cases} \\
&= \begin{cases} 9.8E_2 + 39.6, & E_2 \in (0, 5.5] \\ 7.33E_2 + 53.2, & E_2 \in (5.5, 10] \end{cases}
\end{aligned} \tag{X33}
$$

In stage 1:

By using the Eqs. (B3) and (B4), we will get the following results for merchants:

$$
\begin{cases}
E_2^{(2\text{ - PTC})*} = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left(\frac{P_1\rho+s}{\alpha} + c_p\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left(\frac{6*0.9+3}{0.9} + 1\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - 10.33E_2\right) \\
E_2^{(3\text{ - PTC})*} = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left((P_1\rho + s)\beta - c_g\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - \left((6*0.9+3)*0.9 - 1\right)\frac{E_2}{\eta_1}\right) = \arg\max_{\underline{E}\leqslant E_2\leqslant \bar{E}} \left(V_2^* - 6.56E_2\right)
\end{cases} \tag{X34}
$$

(1) scenario1: If $E_2 \in (0, 5.5]$, $V_2^* = 9.8E_2 + 39.6$,

$$
\begin{cases}
E_2^{(2\text{ - PTC})*} = \arg\max_{E_2\in[0,5.5]} \left(V_2^* - 10.33E_2\right) = \arg\max_{E_2\in[0,5.5]} (9.8E_2 + 39.6 - 10.33E_2) = \arg\max_{E_2\in[0,5.5]} (-0.53E_2 + 39.6) \\
E_2^{(3\text{ - PTC})*} = \arg\max_{E_2\in[0,5.5]} \left(V_2^* - 6.56E_2\right) = \arg\max_{E_2\in[0,5.5]} (9.8E_2 + 39.6 - 6.56E_2) = \arg\max_{E_2\in[0,5.5]} (3.24E_2 + 39.6)
\end{cases}
$$
$$
\Rightarrow E_2^{(2\text{ - PTC})*} = 0, E_2^{(3\text{ - PTC})*} = 5.5 \tag{X35}
$$

(2) scenario2: If $E_2 \in (5.5, 10]$, $V_2^* = 7.33E_2 + 53.2$

$$
\begin{cases}
E_2^{(2\text{ - PTC})*} = \arg\max_{E_2\in[5.5,10]} \left(V_2^* - 10.33E_2\right) = \arg\max_{E_2\in[5.5,10]} (7.33E_2 + 53.2 - 10.33E_2) = \arg\max_{E_2\in[5.5,10]} (-3E_2 + 53.2) \\
E_2^{(3\text{ - PTC})*} = \arg\max_{E_2\in[5.5,10]} \left(V_2^* - 6.56E_2\right) = \arg\max_{E_2\in[5.5,10]} (7.33E_2 + 53.2 - 6.56E_2) = \arg\max_{E_2\in[5.5,10]} (0.77E_2 + 53.2)
\end{cases}
$$
$$
\Rightarrow E_2^{(2\text{ - PTC})*} = 5.5, E_2^{(3\text{ - PTC})*} = 10 \tag{X36}
$$

Next, compare the max value and pick up the optimal references point between the above two scenarios.

(4) Compare $E_2^{(2\,-\,\text{PTC})*}$

$$
\left.\begin{cases}
1)\begin{cases} \text{If } E_2 \in [0,\ 5.5] \Rightarrow E_2^{(2\text{ - PTC})*} = 0 \\ E_2^{(2\text{ - PTC})*} = \arg\max_{E_2\in[0,5.5]} (-0.53E_2 + 39.6) \Rightarrow -0.53E_2+39.6|_{E_2^{(2\text{ - PTC})*} = 0} = 39.6 \end{cases} \\
2)\begin{cases} \text{If } E_2 \in [5.5,\ 10] \Rightarrow E_2^{(2\text{ - PTC})*} = 5.5 \\ E_2^{(2\text{ - PTC})*} = \arg\max_{E_2\in[5.5,10]} (-3E_2 + 53.2) \Rightarrow -3E_2+53.2|_{E_2^{(2\text{ - PTC})*} = 5.5} = 36.2 \end{cases}
\end{cases}\right\} \Rightarrow E_2^{(2\text{ - PTC})*} = 0 \tag{X37}
$$

(5) Compare $E_2^{(3\,-\,\text{PTC})*}$

$$
\left.\begin{cases}
1)\begin{cases} \text{If } E_2 \in [0,\ 5.5] \Rightarrow E_2^{(3\text{ - PTC})*} = 5.5 \\ E_2^{(3\text{ - PTC})*} = \arg\max_{E_2\in[0,5.5]} (3.24E_2 + 39.6) \Rightarrow 3.24E_2+39.6|_{E_2^{(3\text{ - PTC})*} = 5.5} = 57.42 \end{cases} \\
2)\begin{cases} \text{If } E_2 \in [5.5,\ 10] \Rightarrow E_2^{(3\text{ - PTC})*} = 10 \\ E_2^{(3\text{ - PTC})*} = \arg\max_{E_2\in[5.5,10]} (0.77E_2 + 53.2) \Rightarrow 0.77E_2+53.2|_{E_2^{(3\text{ - PTC})*} = 10} = 60.9 \end{cases}
\end{cases}\right\} \Rightarrow E_2^{(3\text{ - PTC})*} = 10 \tag{X38}
$$

Thus, we will get the optimal reference points at stage 2 that are shown as:

$$
E_2^{(2\,-\,\text{PTC})*} = 0, E_2^{(3\,-\,\text{PTC})*} = 10 \tag{X39}
$$

The optimal actions at stage 1 are shown as

$$
q_t^*(S_t) = \begin{cases}
\min\left\{E_{t+1}^{(2-PTC)*} - E_t, \overline{Q}^p\right\}, & E_t \in \left[0, E_{t+1}^{(2-PTC)*}\right] \left(\text{store renewable bring SOC up to } E_{t+1}^{(2-PTC)*}\right) \\
0, & E_t \in \left(E_{t+1}^{(2-PTC)*}, E_{t+1}^{(3-PTC)*}\right] \ (\text{keep SOC unchanged}) \\
\max\left\{E_{t+1}^{(3-PTC)*} - E_t, -\overline{Q}^g\right\}, & E_t \in \left(E_{t+1}^{(3-PTC)*}, \overline{E}\right] \left(disch\arg e \text{ make SOC down to } E_{t+1}^{(3-PTC)*}\right)
\end{cases}
$$
$$
\Rightarrow q_1^*(S_1) = 0, E_1 \in (0, 10] \tag{X40}
$$

The reward payoff functions at stage 1 are shown as follows:

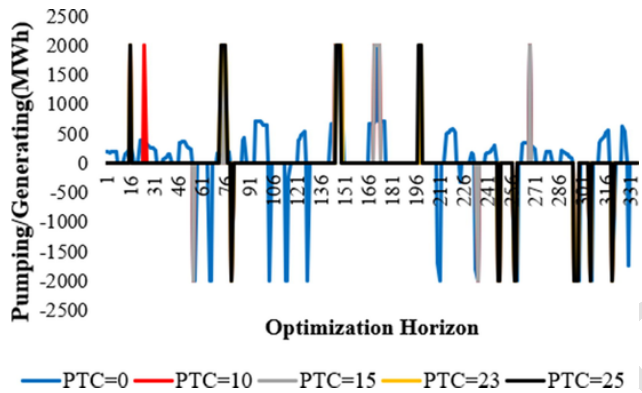

Fig. X1. The optimal actions under policy 1.

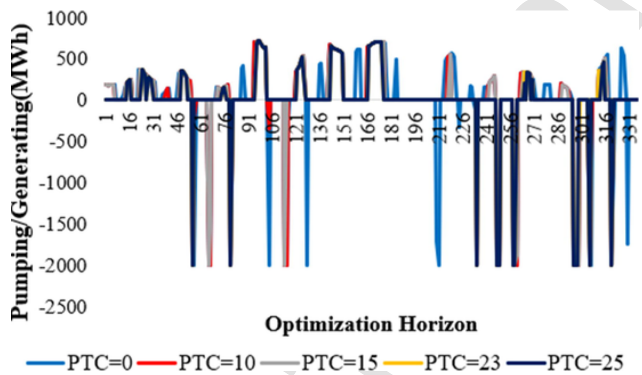

Fig. X2. The optimal actions under policy 2.

$$\begin{aligned} R_1^{(PTC)} &= -P_1 \cdot (q_1/\alpha - w_1) \cdot \rho \\ &\quad - s(q_1/\alpha - w_1) - c_w w_1 - c_p q_1 \\ &= -6 \cdot (0/0.9 - 3) \cdot 0.9 - 3(0/0.9 - 3) \\ &= 25.2,\ E_1 \\ &\in (0, \bar{E}] \end{aligned} \tag{X41}$$

Therefore, we will get the following optimal value functions at stage 1/initial stage.

$$\begin{aligned} V_1^* &= \max\{R_1^{(PTC)} + V_2^*\} \\ &= \begin{cases} 9.8E_2 + 39.6 + 25.2|_{E_2=E_1},\ E_1 \in (0, 5.5] \\ 7.33E_2 + 53.2 + 25.2|_{E_2=E_1},\ E_1 \in (5.5, 10] \end{cases} \\ &= \begin{cases} 9.8E_1 + 64.8,\ E_1 \in (0, 5.5] \\ 7.33E_1 + 78.4,\ E_1 \in (5.5, 10] \end{cases} \end{aligned} \tag{X42}$$

The corresponding optimal actions are shown:

In stage 1, $q_1^*(S_1) = 0,\ \text{if } E_1 \in (0, \bar{E}] (\textit{keep SOC unchanged})$

In stage 2, $q_2^*(S_2) = \{4.5,\ \text{if } \mathrm{E}_2 \in (0, 5.5];\ 10 - \mathrm{E}_2,\ \text{if } \mathrm{E}_2 \in (5.5, 10]$

In stage 3, $q_3^*(S_3) = -E_3,\ \text{if } E_3 \in (0, \overline{E}]$ (sell energy and make SOC down to 0 as close as possible)

To sum up, we get the following results:

If $E_1 = 1$ **(The initial SOC in the storage)**

Stage 1: If $E_1 = 1$, (sell wind generation 3 to the market, and keep SOC unchanged), then we will get $E_2 = 1$ (i.e., $q_1^* = 0,\ R_1 = 25.2$);

Stage 2: If $E_2 = 1$, (store all the wind generation 4.5), then, we will get $E_3 = 5.5$ (i.e., $q_2^* = 4.5, R_2 = -4.5$);

Stage 3: If $E_3 = 5.5$,(action: generating and selling), then, we have $E_4 = 0$ (i.e., $q_3^* = -5.5, R_3 = 53.9$).

Based on the forecasted price, total rewards are shown as $R = R_1 + R_2 + R_3 = 74.6 = V_1^*$.

If $E_1 = 5$ **(The initial SOC in the storage)**

Stage 1: If $E_1 = 5$,(do nothing), then, the relation of $E_2 = 5$ (i.e., $q_1^* = 0, R_1 = 25.2$) holds;

Stage 2: If $E_2 = 5$,(store all the wind generation 4.5), then there exists $E_3 = 9.5$ (i.e., $q_2^* = 4.5, R_2 = -4.5$);

Stage 3: If $E_3 = 9.5$,(generating and selling), then we have $E_4 = 0 = \underline{E}$ (i.e., $q_3^* = -9.5, R_3 = 93.1$).

Therefore, total rewards in three periods are shown as $R = R_1 + R_2 + R_3 = 113.8 = V_1^*$ if $E_1 = 5$.

Case 5 (Policy 2): When PTC $s = 1$

Recall the proof of Case 4, we will get the following results:

$$\begin{cases} E_4^{(2-PTC)*} = E_4^{(3-PTC)*} = 0; \\ E_3^{(2-PTC)*} = E_3^{(3-PTC)*} = 10; \\ E_2^{(2-PTC)*} = 0, E_2^{(3-PTC)*} = 10 \end{cases} \tag{X43}$$

The corresponding optimal value function at stage 1 are shown:

$$\begin{aligned} &V_1^* \\ &= \max\{R_1^{(PTC)} + V_2^*\} \\ &= \begin{cases} 8E_2 + 31.5 + 19.2|_{E_2=E_1}, \ E_1 \in (0, 5.5] \\ 5.11E_2 + 47.389 + 19.2|_{E_2=E_1}, E_1 \in (5.5, 10] \end{cases} \\ &= \begin{cases} 8E_1 + 50.7, \ E_1 \in (0, 5.5] \\ 5.11E_1 + 66.589, E_1 \in (5.5, 10] \end{cases} \end{aligned} \tag{X44}$$

In stage 1, $q_1^*(S_1) = 0$, if $E_1 \in (0, \bar{E}]$(*keep SOC unchanged*)

In stage 2, $q_2^*(S_2) = \{4.5$, if $E_2 \in [0, 5.5]$; $10 - E_2$, if $E_2 \in [5.5, 10]$

In stage 3, $q_3^*(S_3) = -E_3$, if $E_3 \in (0, \overline{E}]$ (sell energy and make SOC down to 0 as close as possible)

If $E_1 = 1$ **(The initial SOC in the storage)**

Stage 1: If $E_1 = 1$, $q_1^* = 0$, $R_1 = 19.2$; **Stage 2:** If $E_2 = 1$, $q_2^* = 4.5, R_2 = -4.5$;

Stage 3: If $E_3 = 5.5$, $q_3^* = -5.5, R_3 = 44$.

Based on the forecasted prices, total rewards are shown as $R = R_1 + R_2 + R_3 = 58.7 = V_1^*$.

If $E_1 = 5$ **(The initial SOC in the storage)**

Stage 1: If $E_1 = 5$, $(q_1^* = 0, R_1 = 19.2)$; **Stage 2:** If $E_2 = 5$, $(q_2^* = 4.5, R_2 = -4.5)$;

Stage 3: If $E_3 = 9.5$, $(q_3^* = -9.5, R_3 = 76)$.

Therefore, total rewards in three periods are shown as $R = R_1 + R_2 + R_3 = 90.7 = V_1^*$ if $E_1 = 5$.

Case 6(Policy 2): When PTC $s = 0$

Recall the proof of Case 4, we will get the following results:

$$\begin{cases} E_4^{(2-\mathrm{PTC})*} = E_4^{(3-\mathrm{PTC})*} = 0; \\ E_3^{(2-\mathrm{PTC})*} = E_3^{(3-\mathrm{PTC})*} = 10; \\ E_2^{(2-\mathrm{PTC})*} = 5.5, E_2^{(3-\mathrm{PTC})*} = 10 \end{cases} \tag{X45}$$

The corresponding optimal value function at stage 1 are shown:

$$V_1^* = \{7.1E_1 + 43.92, \text{if } \mathrm{E}_1 \in (0, 2.8]; \ 7E_1 + 44.2, \ \text{if } \mathrm{E}_1 \in (2.8, 5.5]; \ 4\mathrm{E}_1 + 60.7, \ \text{if } \mathrm{E}_1 \in (5.5, 10] \tag{X46}$$

In stage 1, $q_1^*(S_1) = \{2.7$, if $E_1 \in (0, 2.8]$; $5.5 - E_t$, *if* $E_1 \in (2.8, 5.5]$; 0, if $E_1 \in (5.5, 10]$;

In stage 2, $q_2^*(S_2) = \{4.5$, if $E_2 \in [0, 5.5]$; $10 - E_2$, if $E_2 \in [5.5, 10]$

In stage 3, $q_3^*(S_3) = -E_3$, if $E_3 \in (0, \bar{E}]$(*keep SOC unchanged*)

If $E_1 = 1$ **(The initial SOC in the storage)**

Stage 1: If $E_1 = 1$, $q_1^* = 2.7$, $R_1 = -2.7$; **Stage 2:** If $E_2 = 3.7$, $q_2^* = 4.5, R_2 = -4.5$;

Stage 3: If $E_3 = 8.2$, $q_3^* = -8.2, R_3 = 58.22$). Total rewards are $R = R_1 + R_2 + R_3 = 51.02 = V_1^*$.

If $E_1 = 5$ **(The initial SOC in the storage)**

Stage 1: If $E_1 = 5$, $q_1^* = 0.5, R_1 = 12.7$; **Stage 2:** If $E_2 = 5.5$,$q_2^* = 4.5, R_2 = -4.5$;

Stage 3: If $E_3 = 10$, $q_3^* = -10, R_3 = 71$.

Therefore, total rewards in three periods are shown as $R = R_1 + R_2 + R_3 = 79.2 = V_1^*$ if $E_1 = 5$.

Numerical simulation

The plots in Figs. X1 and X2 show the optimal scheduling under policy 1 and policy 2 with the initial energy in the storage of 2000 MW hours (i.e., 2GWh) when the transmission efficiency was $\rho = 0.9$.

The plots in Figs. X3 and X4 illustrate the optimal SOC curves under policy 1 and policy 2 that relate to the optimal operations in Figs. X1 and X2 with the initial energy in the storage of 2GWh.

When the merchant ignored the transmission loss (i.e., $\rho = 1$) , which is shown in Figs. X5 and X6 with different PTC policies when the initial energy (i.e., E1) in the storage is 2GWh.

The plots in Figs. X7 and X8 illustrate the optimal SOC curves with initial energy (i.e., E1) in the storage of 2000 MW hours under two PTC policies that corresponded to the optimal actions in Figs. X5 and X6 when the merchant ignored the transmission loss.

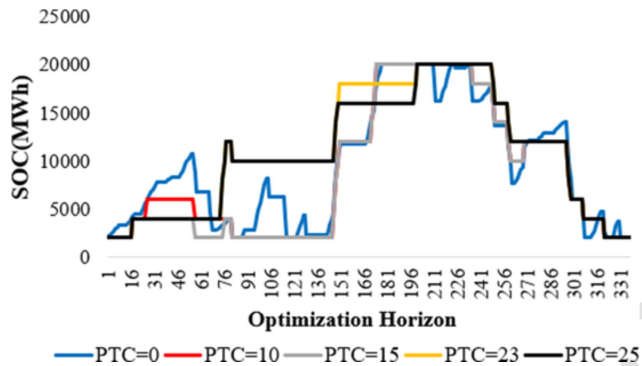

Fig. X3. The optimal SOC curves under policy 1.

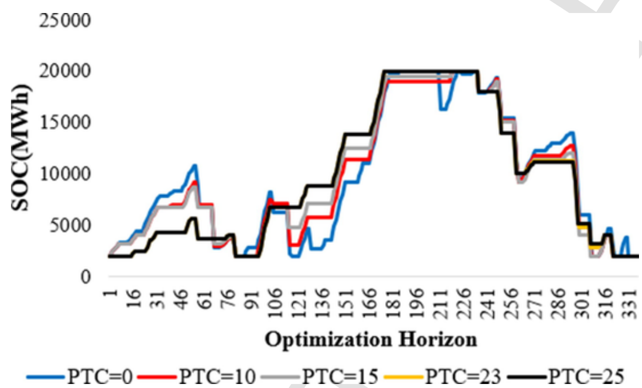

Fig. X4. The optimal SOC curves under policy 2.

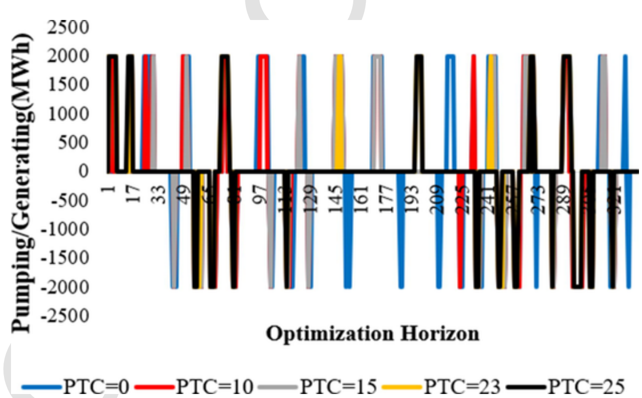

Fig. X5. The optimal actions under policy 1.

Table X1 reports simulation results for the number of idles actions—based on 336 decision periods in two weeks—of an electricity merchant with both storage and a wind farm under different PTC credit rates.

Table X2 demonstrates the Ludington PSH optimal profit results with initial energy in the storage of 2000 MW hours under two policies when the merchant considered and ignored the transmission loss, respectively.

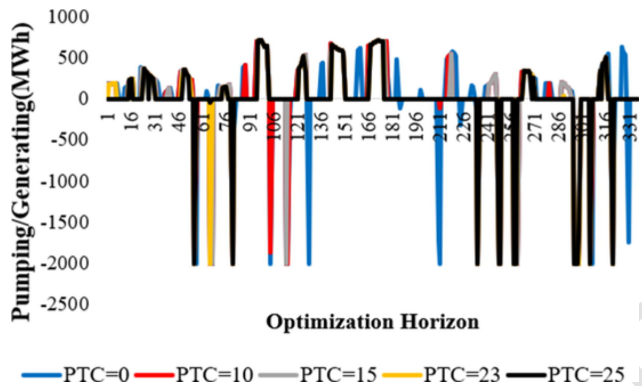

Fig. X6. The optimal actions under policy 2.

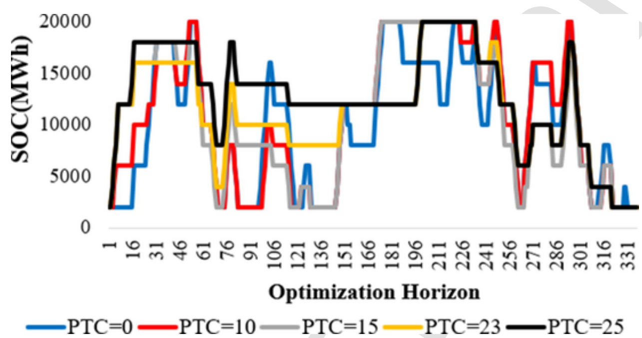

Fig. X7. The optimal SOC curves under policy 1.

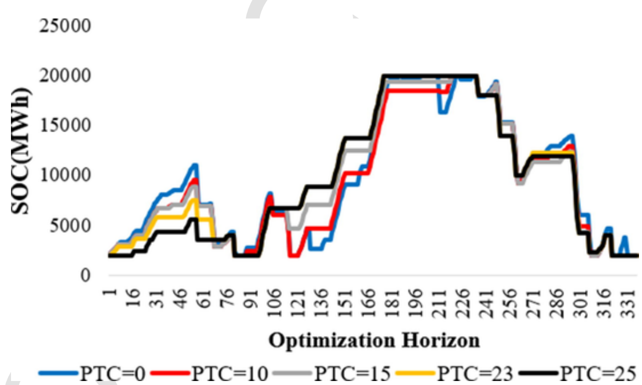

Fig. X8. The optimal SOC curves under policy 2.

Table X1
Number of Idle Actions with PTC Credit Rates Under Two Policies.

Transmission efficiency $\rho = 0.9$					
Policies/PTC	PTC = 0	PTC = 10$/MWh	PTC = 15$/MWh	PTC = 23$/MWh	PTC = 25$/MWh
Policy 1	183	312	312	316	316
Policy 2	173	220	236	272	274
Transmission efficiency $\rho = 1$					
Policies/PTC	PTC = 0	PTC = 10$/MWh	PTC = 15$/MWh	PTC = 23$/MWh	PTC = 25$/MWh
Policy 1	214	248	262	284	288
Policy 2	168	213	228	257	268

Table X2
Optimal Profit with PTC Credit Rates Under Two Policies.

Transmission efficiency $\rho = 0.9$					
Profit	PTC = 0	PTC = 10$/MWh	PTC = 15$/MWh	PTC = 23$/MWh	PTC = 25$/MWh
Profit on Policy 1($)	3,061,614	4,155,061	4,759,890	5,738,716	5,988,156
Profit on Policy 2($)	3,055,983	4,243,641	4,848,545	5,826,293	6,072,759
Transmission efficiency $\rho = 1$					
Policies	PTC = 0	PTC = 10$/MWh	PTC = 15$/MWh	PTC = 23$/MWh	PTC = 25$/MWh
Profit on Policy 1($)	3,632,275	4,688,021	5,252,604	6,210,561	6,456,628
Profit on Policy 2($)	3,408,305	4,591,765	5,194,291	6,167,203	6,412,381

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