A Flexible Operation of Distributed Generation in Distribution Networks with Dynamic Boundaries

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***Abstract*—Distributed generators performing black start to form isolated microgrids offer a flexible and resilient solution to service restoration in distribution systems. However, the flexibility of distributed generators has not been thoroughly investigated and utilized in existing works. To address this issue, this letter presents a new model to the reconfiguration formulation in active distribution networks considering different operation modes of distributed generators. Compared with the existing models, this study provides a new formulation supporting different DGs' operation modes and fully making use of their flexibility. Illustrative results on IEEE 34-test systems verify the effectiveness of the proposed model.**

***Index Terms*—Boundaries of microgrids, distributed generation, distribution systems, mixed-integer linear programming (MILP), load restoration**

I. INTRODUCTION

Flexibility and reliability are the critical factors for the operation of an active distribution system in future smart grids [1]. Increasing penetration of distributed generators (DGs) with advanced management systems brings a new measure to increase system resilience through partitioning distribution systems into isolated subsystems in the context of microgrids (MGs), while reconfiguring networks dynamically by controllable switches. The network reconfiguration problem is generally modeled by a complex combinational problem and solved by heuristic, mathematical approaches and MILP from different perspectives [2]–[4], in which MILP is commonly used. However, none of them supports modelling different operation modes of DGs and integrates them into the problem formulation. Although in [4], the authors considered different operation modes in service restoration, DGs are operated and fixed as the black-start mode and dispatchable mode (PQ mode) only, which is not accurate since different types of DGs would have various operation modes in practice [5]. As in [6], depending on various DG control, the DG can be in either constant power factor (PQ mode) or constant voltage (PV mode). Therefore, this letter proposes a new comprehensive formulation considering different operation modes of DGs, i.e., the black-start mode and the dispatchable mode (including PV and PQ mode), to fully make use of the flexibility of different DG modes in service restoration. With the introduced flexibility, the proposed formulation can help the operator with the identified DG operation and the determination for the selection and number of DGs in PV mode or PQ mode from the candidate eligible DGs.

II. PROBLEM FORMULATION

A distribution system can be partitioned into multiple isolated MGs by using controllable switches. An isolated MG performs as a local supplier to inner user demands, where only one DG in the MG can be operated as the black-start mode to ensure a stable system. The switches enable the boundaries between isolated MGs to change dynamically, and therefore DGs have to change their operation mode accordingly, resulting in a changing network topology to adapt to the varying system condition. This section introduces a new model for the flexible change of DGs' operation modes, i.e., the black-start mode and the dispatchable mode, including the PV mode and PQ mode.

A. Distribution System Model

A distribution system is represented by a graph $\mathcal{G}(\mathcal{N},\ \mathcal{E})$, where $\mathcal{N} := [1,\dots,N_n]$ is the set of all buses and the edge set $\mathcal{E} := \{(i,\ j) : i \in \mathcal{N}, j \in \mathcal{N}, i \neq j\}$ denotes the set of branch lines in the system. Let $\mathcal{N}^{\mathrm{DG}} \subset \mathcal{N}$ and $\mathcal{N}^{\mathrm{L}} \subset \mathcal{N}$ denote the set of DGs and loads, respectively with $|\cdot|$ being the cardinality of a set. $\mathcal{N}^s(j)$ and $\mathcal{N}^p(j)$ represent respectively the set of all children and parents of jth node. A time step $t \in \mathcal{T} := [1,\dots,T]$ denotes the operation horizon.

B. Network Constraints

1) Linear DistFlow Constraints: The linearised DistFlow model is used in this study to formulate the operation constraints for radial distribution systems, which has been widely adopted by dropping negligible non-linear terms in the DistFlow equations. Using binary variables, the following model is introduced,

$$\begin{cases} P^{\mathrm{DG}}_{j,t} - P^{\mathrm{L}}_{j,t} = \displaystyle\sum_{h\in\mathcal{N}^s(j)} P_{hj,t} - \sum_{i\in\mathcal{N}^p(j)} P_{ij,t},\ \forall i \in \mathcal{N},\ \forall t \in \mathcal{T} \\ -\underline{P}_{ij,t} b_{ij,t} \le P_{ij,t} \le \bar{P}_{ij,t} b_{ij,t},\ \forall (i,j) \in \mathcal{E},\ \forall t \in \mathcal{T} \end{cases} \tag{1}$$

$$\begin{cases} Q^{\mathrm{DG}}_{j,t} - Q^{\mathrm{L}}_{j,t} = \displaystyle\sum_{h\in\mathcal{N}^s(j)} Q_{hj,t} - \sum_{i\in\mathcal{N}^p(j)} Q_{ij,t},\ \forall i \in \mathcal{N},\ \forall t \in \mathcal{T} \\ -\underline{Q}_{ij} b_{ij,t} \le Q_{ij,t} \le \bar{Q}_{ij} b_{ij,t},\ \forall (i,j) \in \mathcal{E},\ \forall t \in \mathcal{T} \end{cases} \tag{2}$$

$$\begin{cases} 2(r_{ij}P_{ij,t} + x_{ij}Q_{ij,t}) - M(1 - b_{ij,t}) \le U_{i,t} - U_{j,t} \\ \quad \le 2(r_{ij}P_{ij,t} + x_{ij}Q_{ij,t}) + M(1 - b_{ij,t})\ \forall (i,j) \in \mathcal{E},\ \forall t \in \mathcal{T} \end{cases} \tag{3}$$

$$x_{i,t}\underline{U} \le U_{i,t} \le x_{i,t}\bar{U},\ \forall i \in \mathcal{N},\ \forall t \in \mathcal{T} \tag{4}$$

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where $P^{\text{DG}}_{j,t}$ and $Q^{\text{DG}}_{j,t}$ are the active and reactive power output from DG of jth bus at tth step; $P^{L}_{j,t}$ and $Q^{L}_{j,t}$ are the restored load of jth bus at tth step; $P_{ij,t}$ and $Q_{ij,t}$ are the active and reactive power flow from ith bus to jth bus at tth step with $\underline{P}_{ij}/\underline{Q}_{ij}$ and $\bar{P}_{ij}/\ \bar{Q}_{ij}$ being the lower and upper bounds on the active/reactive power flow of ijth branch; r_{ij} and x_{ij} are the resistance and reactance of ijth branch; $b_{ij,t}$ is the status of ijth branch at tth step i.e., if $b_{ij,t} = 0$ the branch is opened and vice versa when the branch is closed; M is a large parameter for the Big-M formulation; $U_{i,t}$ is the squared voltage of ith bus at tth step with $\underline{U}$ and $\bar{U}$ being the lower and upper bounds on the voltage following [4] ; (1) and (2) represent the active and reactive power balance with the corresponding power flow limits, where $b_{ij,t}$ is used to indicate that only closed branches can carry power flow. (3) denotes the voltage difference between two end nodes of a closed branch; (4) limits the voltage of energised nodes to a safety range limits. The active power output from jth DG is limited by its maximum value, i.e., $P^{\text{DG}}_{j,t} \le \overline{P}^{\text{DG}}_{j,t}$.

Remark 2.1: Note that following a similar approach as in [7], the apparent power flow of the branches can be transferred to the upper and lower bounds for active/reactive power flow. The readers please refer to [7] for the details

2) Connectivity and Topology Constraints: The connectivity and topology constraints are adopted to ensure the network connectivity and realise the partition of the distribution network into multiple isolated MGs while considering the essential radial topology. These constraints are widely used in the resilient operation of active distribution networks which are also included in this study. However, the focus of this study is on providing a flexible operation framework for DGs and therefore these constraints are not listed here for the sake of simplification. The interested readers can refer to [4] for further details on these constraints.

C. DG Operation Constraints

According to [4], during distribution network reconfiguration, DGs can be operated in either the black-start mode or the dispatchable mode. However, the operation mode of a DG is usually fixed during the service window and this approach is not applicable to active distribution networks with dynamic boundaries. With the changing status of smart switches, different MG groups will be formed based on the current system condition, in which only one DG can be operated in the black-start mode and the rest of DGs have to be switched to dispatchable accordingly. Furthermore, operating DGs with power electronics interfaces (e.g., a photovoltaic system with a voltage controlled inverter) only in the constant power factor mode is not accurate and hence an additional operation mode (e.g., PV mode) should be allowed. Once the black-start DG is decided, the operation mode of dispatchable DGs will vary from constant voltage to constant power factor. Additional binary variables that are decided based on the voltage and reactive power generation from DGs are introduced to the conventional distribution network reconfiguration.

To ensure the flexible operation of DGs, the following equations are introduced and added into the formulation using binary variables, $\forall i \in \mathcal{N}^{\text{DG}}, \forall t \in \mathcal{T}$,

$$\begin{cases} U_{i,t} \le g_{i,t}U_{\text{base}} + U_{i,0} - z_{i,t} - (z^l_{i,t} + z^u_{i,t}) \\ \qquad + (x^l_{i,t} + x^u_{i,t})\bar{U}, \\ U_{i,t} \ge g_{i,t}U_{\text{base}} + U_{i,0} - z_{i,t} - (z^l_{i,t} + z^u_{i,t}) \\ \qquad + (x^l_{i,t} + x^u_{i,t})\underline{U}, \end{cases} \tag{5}$$

$$\begin{cases} U_{i,t} \ge z^l_{i,t} + (1 - x^l_{i,t})\underline{U} \\ U_{i,t} \le z^u_{i,t} + (1 - x^u_{i,t})\bar{U}, \end{cases} \tag{6}$$

$$\underline{Q}^{\text{DG}}_i + (\bar{Q}^{\text{DG}}_i - \underline{Q}^{\text{DG}}_i)x^u_{i,t} \le Q^{\text{DG}}_{i,t} \le \bar{Q}^{\text{DG}}_i + (\underline{Q}^{\text{DG}}_i - \bar{Q}^{\text{DG}}_i)x^l_{i,t}, \tag{7}$$

$$x^l_{i,t} + x^u_{i,t} \le 1 - g_{i,t}, \tag{8}$$

$$\begin{cases} g_{i,t}\underline{U} \le z_{i,t} \le g_{i,t}\bar{U} \\ U_{i,0} - (1 - g_{i,t})\bar{U} \le z_{i,t} \le U_{i,0} - (1 - g_{i,t})\underline{U} \\ x^l_{i,t}\underline{U} \le z^l_{i,t} \le x^l_{i,t}\bar{U}, \\ U_{i,0} - (1 - x^l_{i,t})\bar{U} \le z^l_{i,t} \le U_{i,0} - (1 - x^l_{i,t})\underline{U} \\ x^l_{i,t}\underline{U} \le z^u_{i,t} \le x^l_{i,t}\bar{U}, \\ U_{i,0} - (1 - x^u_{i,t})\bar{U} \le z^u_{i,t} \le U_{i,0} - (1 - x^u_{i,t})\underline{U} \end{cases} \tag{9}$$

where U_{base} is the base voltage of a DG in the black-start mode. A variable $U_{i,0}$ is introduced to indicate the desired voltage of a DG in the PV mode at ith bus. When a DG is switched to the PV mode, the system therefore can select the desired voltage based on the current system condition rather than fixing this value during the entire process, resulting in a better voltage performance. Several binary variables are introduced to DGs indicating their operation mode. Specifically, if $g_{i,t} = 1$, the DG will be operated in the black-start mode and otherwise it will be a dispatchable DG that can be operated in either the PV mode or the PQ mode based on its reactive output. $x^l_{i,t}$ and $x^u_{i,t}$ are the binary variables based on the DG reactive power, where $x^u_{i,t} = 1$ if the reactive power of a DG reaches its upper bound otherwise $x^u_{i,t} = 0$ that is the opposite for $x^l_{i,t}$. (5) is the voltage constraint of DG buses in different operation modes. In particular, if $g_{i,t} = 1$, the voltage of a black-start DG is set to the base voltage. Otherwise $g_{i,t} = 0$, if both $x^l_{i,t} = 0$ and $x^u_{i,t} = 0$, a dispatchable DG will be operated in the PV mode and its voltage will be fixed to $U_{i,0}$. If the reactive power from a dispatchable DG exceeds its either lower or upper bound, $x^l_{i,t}$ or $x^u_{i,t}$ will be one and hence, the voltage will be released from a fixed point to the acceptable range (i.e., $[\underline{U}_i,\ \bar{U}_i]$). For a dispatchable DG ($g_{i,t} = 0$), (6) represents the voltage should less than the specified value when the reactive power reaches its upper bound and it should be higher when reaching its lower bound. (7) denotes the operation range of the reactive power with $\underline{Q}^{\text{DG}}_i$ and $\bar{Q}^{\text{DG}}_i$ being the lower and upper bounds, respectively. It shows that as long as the reactive power generation does not exceed the bound, a DG will be in the PQ mode and its reactive power can vary within the normal range from $[\underline{Q}^{\text{DG}}_i\ \bar{Q}^{\text{DG}}_i]$. In case of any violation, the reactive power will be fixed at the lower bound if $x^l_{i,t} = 1$ or the upper bound if $x^u_{i,t} = 1$ accordingly. Hence, a dispatchable DG is switched to the PQ mode. Note that only one case of violation can happen at each step and a DG can only switch between the PQ and PV modes if it is not a black-start DG, which is

TABLE I
DG MODE SWITCH AND RESTORED LOAD AT EACH TIME

h (hrs)	DG Mode			Restored Load (kW)	Current system load (kW)	Line-Off	Line-On
	Black-start	Dispatchable					
		PV	PQ				
1	DG1, DG2, DG3, DG5		DG4	1279.94	1279.95		
2	DG1, DG3, DG5	DG2, DG4		1300.01	1300.01	SW_{836}^{860}	SW_{858}^{832}, SW_{860}^{834}
3	DG3, DG5	DG1, DG4	DG2	1254.75	1254.75		SW_{824}^{816}
4	DG5	DG3, DG4	DG1, DG2	1266.14	1266.14		SW_{836}^{860}
5	DG2, DG5	DG3, DG4	DG1	1187.49	1187.49	SW_{858}^{832}	
6	DG2, DG3, DG5	DG4	DG1	1182.90	1182.90	SW_{860}^{834}	
7	DG1, DG5	DG2	DG3, DG4	1193.91	1193.92	SW_{854}^{830}	SW_{858}^{832}, SW_{860}^{834}
8	DG2, DG3, DG5		DG1, DG4	1182.16	1182.16	SW_{844}^{842}, SW_{858}^{832}	SW_{854}^{830}
9	DG5	DG1, DG3	DG2, DG4	1192.50	1192.50		SW_{844}^{842}, SW_{858}^{832}
10	DG3, DG5	DG1,DG4	DG2	1263.41	1263.41	SW_{836}^{860}	
11	DG2, DG3, DG5	DG1, DG4		1356.79	1356.80	SW_{890}^{888}	
12	DG1, DG5	DG2, DG3	DG4	1417.31	1417.32	SW_{854}^{830}	SW_{836}^{860}, SW_{890}^{888}

ensured by (8). Let $z_{i,t} = g_{i,t}U_{i,0}$, $z_{i,t}^{l} = x_{i,t}^{l}U_{i,0}$, $z_{i,t}^{u} = x_{i,t}^{u}U_{i,0}$, and the constraints in (9) are introduced to linearize the products of $g_{i,t}U_{i,0}$, $x_{i,t}^{l}U_{i,0}$, $x_{i,t}^{u}U_{i,0}$, respectively.

D. Objective Function

To maximize the total restored energy over the entire time window, the following objective function is defined including ω_j as the load priority:

$$\max \sum_{j\in\mathcal{N}^L}\sum_{t\in\mathcal{T}} \omega_j P_{j,t}^{L}\Delta t$$

$$\begin{aligned} \text{s.t.,}\quad & \text{Network Constraints : (1)} - \text{(4)} \\ & \text{Connectivity and Topology Constraints in [4]} \\ & \text{DG Operation Constraints : (5)} - \text{(9).} \end{aligned}$$

where Δt is the length of time interval. The introduction of ω_j is to identify the critical loads and non-critical loads based on their importance.

III. NUMERICAL RESULTS

The effectiveness of the proposed framework is demonstrated by an illustrative study on a modified IEEE 34-bus test system including 11 switchable lines. The MILP problem is solved by CPLEX 12.9 on a personal laptop with a 2.6 GHz Intel Core i7 and 16 GB of RAM. The system has five DGs assuming DG4 (350kW capacity) only can be operated in the dispatchable mode (PV/PQ mode) and the rest of DGs (DG1 450kW; DG2 400kW; DG3 400kW; DG5 500kW) can switch between the black-start mode and the dispatchable mode (PV/PQ mode). The specified voltage of DGs in the PV mode is assumed to be 1 p.u. The power factor of DGs in the PQ mode is set to 0.9. The reactive power limit of DGs in the PV mode is calculated by setting the limit of their power factors between [0.8, 1.0] [5]. The system is initialized by the sequential restoration method introduced by [4], as in Fig. 1, which is partitioned into four isolated MGs by smart switches (red squares).

To verify the proposed model in a dynamic configuration, it is implemented for a 12hrs (8:00am - 8:00pm) continuous-operation of a distribution network. The hourly PV generation and load demand are adopted from [8] that is scaled from W to kW given by the current system load in Table I. The result is illustrated in Table I, where $SW_{\square}^{\triangle}$ refers to the smart switch with a superscript and a subscript representing one end bus $\triangle$ and the other end bus $\{\square\}$, respectively. As shown in Table I, the mode of DG4 is only switched between the PV and PQ modes, and in the meantime, the rest of DGs can switch between any modes in each time step. The distribution network is partitioned into different isolated MG configurations based on the system loading condition. The results verify that the proposed framework can achieve the flexible switching of the DG mode based on its working condition while ensuring the system loads are fully restored (comparing restored load and current system load).

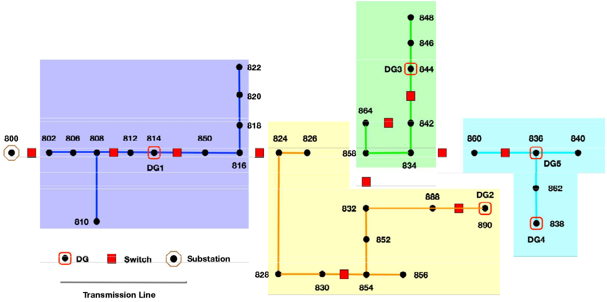
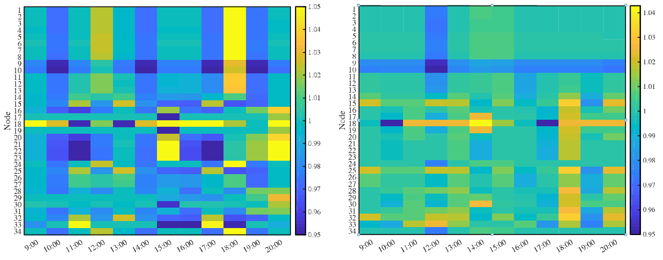


Fig. 1. The initial system configuration at 1st time step



(a) case 1

(b) case 2

Fig. 2. Voltage magnitudes of (a) two operation modes and (b) the proposed flexible framework

Fig. 2 illustrates a comparison study including: a) DGs are only switched between the black-start mode and PQ mode; b) the proposed flexible framework. It is shown that enabling the flexibility for the DG operation results in a better voltage performance. In particular, when the system is heavily loaded, using the PQ mode only may not be enough to enforce the voltage limit, but having the PV mode can improve the overall voltage profile.

IV. CONCLUDING REMARK

This letter proposes a new formulation considering different operation modes of DGs. The capability to include the proposed model into the distribution system reconfiguration is investigated through a numerical study. In future, the application of the proposed model needs to be investigated with more considerations, e.g., the MG dynamics and load shedding.

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