

On volt-ampere characteristic of symmetric CCP

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ABSTRACT

The dynamic and static volt/ampere characteristics $I(V)$ are fundamental parameters of rf discharges. The discharge volt/ampere characteristics or impedance and their scaling laws are important for verification of discharge models and in the design of rf matching-tuning networks that couple plasma with an RF power source. It is pointed out that experiments with symmetrical capacitively coupled plasmas performed in a wide range of driving frequencies, gas types and pressures, and discharge voltages demonstrate a linear static volt-ampere characteristic. Such a behavior contradicts the widely accepted rf sheath models found in textbooks. The disagreement was shown to be due to the violation of the rf sheath charge conservation and, possibly, the neglect of ionization caused by oscillating electrons in the rf sheath models.

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Experiments and modeling of capacitively coupled plasmas (CCPs) started long before their practical application.^{1–5} Today, capacitive rf discharges are widely used in industry and still remain a subject of active research. It has been realized that near-electrode space charged sheaths (rf sheaths) play a fundamental role in CCP behavior, causing the rf sheath rectification of the applied rf voltage,^{2,3} plasma-sheath resonance,⁴ and collisionless (stochastic) rf power absorption.⁵

An analytical model of a symmetric planar CCP (referred in the literature as the basic CCP model) assuming uniform ion density distribution (ion matrix model) and an oscillating electron step-front shown in Fig. 1 was proposed in Ref. 6. This model was able to consistently connect the external CCP parameters (rf voltage V , or discharge current I , discharge gap L , type of gas and its pressure p , and rf frequency ω) with the internal CCP parameters (discharge current I , or discharge voltage V , plasma density n , voltage drop across the plasma V_p and across rf sheaths V_{sh} , and the time averaged capacitive sheath width S_0 corresponding to both sheaths' capacitance $C_{sh} = \epsilon_0 A / 2S_0$); here, ϵ_0 is the vacuum permittivity and A is the discharge cross section). Due to the continuity of the discharge current density, $J = \sigma_p E_p = \epsilon_0 \omega V_{sh} / 2S_0$, where σ_p and E_p are the plasma conductivity and the plasma rf electric field.

Under the action of rf voltage applied to CCP electrodes, the plasma-sheath interface $S(t)$ oscillates between its extremal positions S_1 and S_2 relative to unmovable ion background (see Fig. 1). The rf current in the CCP circuit $J(t) = J_0 \cos(\omega t)$ is conducted by the displacement current in the sheaths, $J(t) = \epsilon_0 \omega V_{sh} / 2S_0 = \text{en}(dS/dt)$, and by the conductive current in the plasma, $J = \text{env}_{rf}$ and thus,

$J_0 \cos(\omega t) = \text{en}(dS/dt) = \text{env}_{rf} \cos(\omega t)$. Here, v_{rf} is the electron oscillatory velocity at the plasma boundary.

According to this model, $S_0 = a + S_2$, where S_2 (shown in Fig. 1) is the minimal sheath width at the moment of the maximal rf electrode potential and $a = v_{rf}/\omega$ is the amplitude of electron oscillation at the plasma-sheath boundary. Starting from the floating sheath, when $S_2 = S_1$, the application of an applied rf voltage leads to an increase in S_1 and a decrease in S_2 . In the asymptotic limit of a very high rf voltage, $S_2 \rightarrow 0$, $S_0 \rightarrow a$, and $V_{sh} \rightarrow V$.

Due to the rf current continuity and $\nabla n_i = 0$, the sum of both rf sheath widths $S_a(t) + S_b(t) = 2S_0$ is time independent, which results in a strict linearity in the CCP dynamic volt-ampere characteristic. This is not the case for the static I/V characteristic of CCP, as shown in Fig. 2 for the generalized static $I(V)$ and $n(V)$ CCP characteristics at different ω/ν_{eff} .⁶ $J \propto n \propto (eV_{Rp}/T_e)[\omega_{pe}\Omega/(\omega\nu_{\text{eff}})]$.² Here, $V_{Rp} = \text{Re}(V_p) = V_p(1 + \omega^2/\nu_{\text{eff}}^2)^{-1/2}$ is the Ohmic part of the plasma voltage, T_e is the electron temperature in eV, ω_{pe} is the electron plasma frequency, $\Omega = v_{Te}/L$ is the electron bounce frequency with $v_{Te} = (8T_e/\pi m)^{1/2}$ being the mean electron thermal velocity, and ν_{eff} is the effective electron collision frequency accounting for both collisional and collisionless (stochastic) electron heating.

At $V^2 \gg V_p^2$, the discharge voltage V applied to rf electrodes is practically localized in rf sheaths ($V \approx V_{sh}$), and, according to the model,⁶ the static I/V characteristic is linear ($J \propto V$). This regime (corresponding to area A in Fig. 2) is typical for applications and has been the main subject of CCP research over the last few decades. The absorbed CCP rf power P and plasma density n are proportional to rf

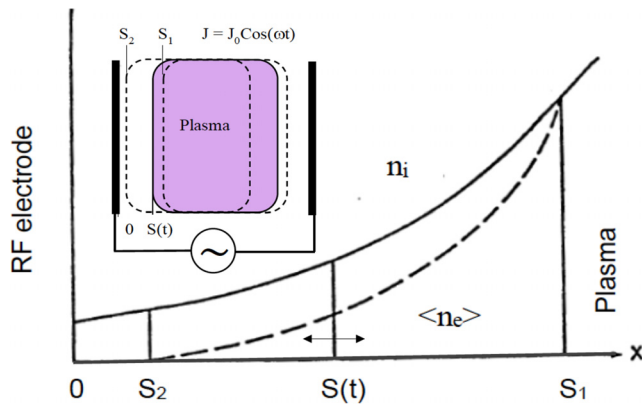


FIG. 1. CCP discharge gap and rf sheath structure with moving electron distribution step front density $S(t)$. The dashed line shows the time averaged electron density $\langle n_e \rangle$.

current. This is typical for weakly ionized gas discharge plasmas in the steady state when the plasma electric field is maintained at a nearly constant level by electron ionization and the electron energy balance.

The CCP existence (see Fig. 2) is limited by the condition $V/V_{Rp} \geq 1$ independently of the ω/ν_{eff} ratio. At minimal discharge voltage

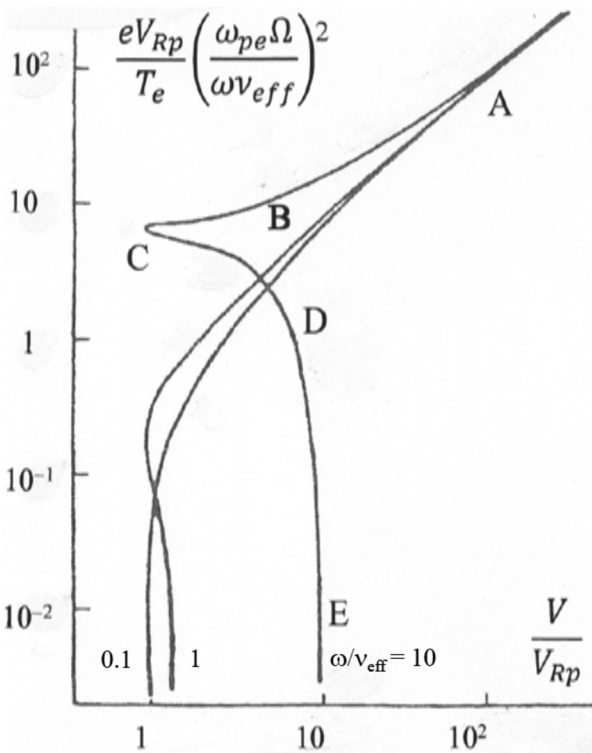


FIG. 2. The normalized current and plasma density dependence on the normalized discharge voltage. Reproduced with permission from V. Godyak, "Steady-state low pressure rf discharge," *Sov. Phys.-Plasma Phys.* 2, 78 (1976). Copyright 1976 AIP Publishing.

$V = V_{Rp}$, the plasma inductive reactance is compensated by the sheath capacitive reactance (plasma/sheath or geometric resonance) when $\omega^2 = \omega_r^2 \equiv \omega_{pe}^2(2S_0/L)$, and the CCP impedance Z is equal to the plasma Ohmic resistance ($Z = V_{Rp}/I$). The CCP resonant regime takes place at a relatively large ratio of ω/ν_{eff} and disappears when $(\omega/\nu_{eff})^2 \ll 1$, as seen in Fig. 2.

At $(\omega/\nu_{eff})^2 \gg 1$, the reduction of the discharge voltage leads to a slope reduction in the I/V characteristic (area B in Fig. 2) up to the point C, which corresponds to the minimal CCP sustaining voltage $V = V_{Rp}$ at the plasma-sheath resonance. At $\omega/\nu_{eff} \ll 1$, the resonance disappears, and the discharge current and plasma density are sharply decreasing with V (area E in Fig. 2) until the discharge extinction. In the area E, the plasma impedance $Z_p = Lm\nu_{eff}/Ane^2 \gg |Z_{sh}| = 2S_0/A\varepsilon_0\omega$; therefore, the discharge voltage is equal to the plasma voltage.

As seen in Fig. 2, at $\omega/\nu_{eff} \geq 1$, the model⁶ predicts double valued CCP current and plasma density, and a negative static I/V (near the resonance area B-C-D), when the CCP has an inductive impedance. Under near-resonant conditions, the discharge current and plasma density depend weakly on the discharge voltage and remain close to their resonance values I_r and n_r that are proportional to ω^3 . At $\omega/\nu_{eff} \gg 1$, in the area B-C-D, the CCP exhibits a self-organization feature, adjusting its parameters to remain close to their resonant values. Note that the double-valued, negative I/V , and functional dependence of $n_r \propto \omega^3$ and $J_r \propto \omega^3$ were confirmed in the experiments^{7,8} and reproduced in the numerical simulation.⁹ It is worth mentioning that at the very high frequency and at moderate rf power, industrial very high frequency CCPs (VHCCPs) should operate in the near-resonance regime, while CCP at the microwave frequency should operate in the D-E area, thus having an inductive impedance at $\omega < \omega_{pe}$.

The assumption of a uniform ion matrix neglecting ion acceleration in the strong dc electric field inside the sheath makes the basic CCP model⁶ unsatisfactory. A more consistent model for the symmetric planar CCP with the nonlinear rf sheath (where $\nabla n_i \neq 0$) accounting for the ion motion (dynamic model) under arbitrary rf voltage and arbitrary ion collisionality was formulated in Ref. 10. There, the ion density profile was considered at the kinetic level, which accurately accounts for (typical in experiment) rare ion collisions in the rf sheath. A considerable ion nonuniformity in the rf sheath has been found through a numerical solution of the model equations. It has been shown that the ion density near the rf electrode can be an order of magnitude smaller than that at the plasma-sheath interface for the collision-free ion motion, while under the strongly collisional ion motion, the ion density near the rf electrode dropped only twice. The last observation formed the basis for using the matrix sheath model for the analytical modeling of strongly collisional CCP.¹¹

The formulation of a consistent rf sheath model within a fluid approximation and an analytical solution for the collisionless rf sheath are given in Ref. 12. A more detailed analysis for the collisionless rf sheath and a numerical solution with arbitrary collisionality are given in Ref. 13. There, a comparison of the model^{12,13} with the widely accepted^{14,15} Lieberman's simplified asymptotic collisionless¹⁶ and highly collisional¹⁷ rf sheath models was presented.

The simplification in the models^{16,17} associated with neglecting the minimal sheath width by assuming $S_2 = 0$ seemed reasonable for relatively large rf discharge voltage (50–500 V).¹⁶ Note that in Refs. 6, 12, and 13, the value of S_2 was found from the electrical charge conservation in the sheath, i.e., no dc current to the rf electrodes

TABLE I. Analytical solutions for $\lambda_0(\rho)$, $\lambda_0(U_{sh})$, and I/V scaling for different rf sheath models.

1-Matrix asymptotic ⁶	2-Dynamic asymptotic ¹⁶	3-Dynamic arbitrary U_{sh} ¹²
$\lambda_0 = \rho$	$\lambda_0 = 0.34\rho^3 = 1.89H^{3/2}$	$\lambda_0 = \delta + 1.37\rho^2 + 0.34\rho^3$
$\lambda_0 \propto U_{sh}^{1/2}$ at $U_{sh} > 10^2$	$\lambda_0 \propto U_{sh}^{3/4}$ at $U_{sh} > 10^2$	$\lambda_0 \propto U_{sh}^{1/2}$ at $\delta \leq U_{sh} \leq 10^2$
$I \propto V_{sh}$	$I \propto V_{sh}^{1/2}$	$I \propto V_{sh}$

($I_i + \langle I_e(t) \rangle = 0$). It has been shown¹³ that for very high rf sheath voltage, the collisionless rf sheath model¹⁶ and the model^{12,13} converge. However, they differ significantly for the typical in practice discharge voltage below about 1 kV. Neglecting the small quantity S_2 in the models^{16,17} had resulted in several inconsistencies followed from these models: (a) there is no sheath without rf voltage and (b) a significant dc electron current flowing to each rf electrode.¹³ It has been shown^{13,18} that assuming of $S_2 = 0$ in the models^{16,17} can lead to a significant difference in CCP $I(V)$ scaling law compared to that following from the model.^{12,13} In spite of the differences in the value and the scaling of $I(V)$ followed from different models,^{12,16} the rf sheath dc characteristics V_0 and S_0 expressed in the form of the Child-Langmuir law differ very little.¹⁸ Here, V_0 is the dc voltage in the rf sheath.

With normalization $eV/T_e = U$, $S/\lambda_D = \lambda$, $a/\lambda_D = \rho$ and equalizing rf currents in the sheath to that in plasma, one can obtain a universal relation between the rf voltage across both sheaths $V_{sh} = U_{sh}T_e$ and their combined width $2S_0 = 2\lambda_0\lambda_D$; $U_{sh} = 2\lambda_0\rho$, where λ_D is the Debye length at the plasma-sheath boundary. Then, using the relation between λ_0 and U_{sh} from the considered above rf sheath models, one can find the scaling of the rf sheath volt-ampere characteristics that at $V^2 \gg V_{Rp}^2$ is the CCP characteristic.

The expressions for $\lambda_0 = U_{sh}/2\rho$, its scaling, and the scaling for rf sheath current are presented in Table I for different collisionless rf sheath models.^{6,12,16} In Table I, the upper row presents the analytical solutions for $\lambda_0(\rho)$. Here, $H = \rho^2/\pi$ is the normalization parameter in the model¹⁶ and $\delta \approx 3$ (for Argon ions) is the normalized floating sheath width without rf voltage. The second row presents the scaling for the asymptotic solutions,^{6,16} the approximation for the solution,¹² and the applicability of those solutions and approximations. The scaling of CCP volt-ampere characteristics is given in the third row.

The normalized sheath width λ_0 is shown in Fig. 3 as a function of the normalized sheath voltage U_{sh} from different rf sheath models. Here, the solutions^{6,12,16} (denoted correspondingly by numbers 1, 2, and 3, as in Table I) are presented together with the experiment¹⁹ and an approximation for the solution.¹²

As follows from Fig. 3, the experimental values of λ_0 found in CCP under conditions of collisionless rf sheath¹⁹ are very close to the analytical solution¹² (marked by 3). They are well approximated as $\lambda_0 \propto U_{sh}^{1/2}$, being about 2.5 times larger than those found from the asymptotic solution of the basic rf sheath model⁶ (with uniform ion distribution and $\lambda_2 = 0$ marked by 1). A different scaling gives the model¹⁶ (marked by 2), where $\lambda_0 \propto U_{sh}^{3/4}$. This model results in $I \propto V^{1/2}$, while models^{6,12} result in a linear volt-ampere characteristic, $I \propto V$ at moderate $U_{sh} \lesssim 10^2$. At a very large $U_{sh} \gtrsim 10^3$ when $\lambda_0 \rightarrow 0$, the values of λ_0 from the models^{6,16} slowly converge.¹³

Figure 3 shows good agreement between the experiment¹⁹ in mercury vapor at 40.8 MHz and the model.¹² Such agreement of both the scaling and the absolute values raises a question about the scaling

$I \propto V$ for other experiments and for the numerical modeling published for low-pressure CCPs where the collisionless rf sheath model¹² is applicable. Unfortunately, in the majority of the published CCP experiments and works on numerical modeling, the volt-ampere characteristics are ignored.

In Figs. 4–8, the available literature measurements of the volt-ampere characteristics for the symmetric CCPs are presented for a wide range of experimental conditions (geometry, gas pressure, frequency, and discharge voltage). The volt-ampere characteristics, $I(V)$, and the dependence of plasma density on discharge voltage, $n(V)$, are shown in Fig. 4 for a mercury CCP at vapor pressure between $(1.9 \times 10^{-4} - 2 \times 10^{-2})$ Torr and at driving frequency of 40.8 MHz.^{19,20} They demonstrate a very close to linear dependence of discharge current and plasma density on discharge voltage V at $V^2 \gg V_{Rp}^2$, where $V_{Rp} = \text{Re}(V_p)$ is the minimal discharge voltage equal to the real part of the plasma voltage. The linear I/V and n/V characteristics are observed not only for the collisionless ($\lambda_i > S_0$) rf sheath at 0.19 and 1.2 mTorr but also for the weakly collisional ($\lambda_i < S_0$) sheath at 5.4 and 20 mTorr.

The volt-ampere characteristics measured in a collision-dominated helium CCP are shown in Figs. 5 and 6 for $f = 40.8$ MHz¹¹

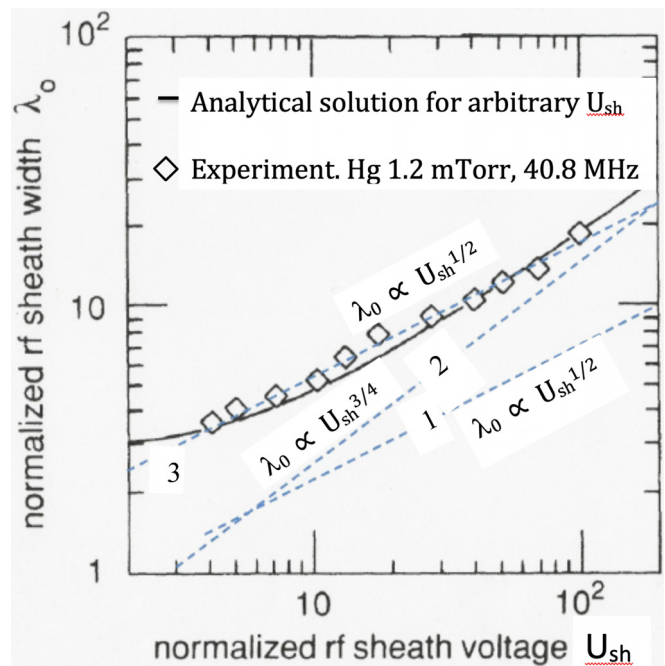


FIG. 3. Analytical solutions for $\lambda_0(U_{sh})$ from different rf sheath models presented in Table I.

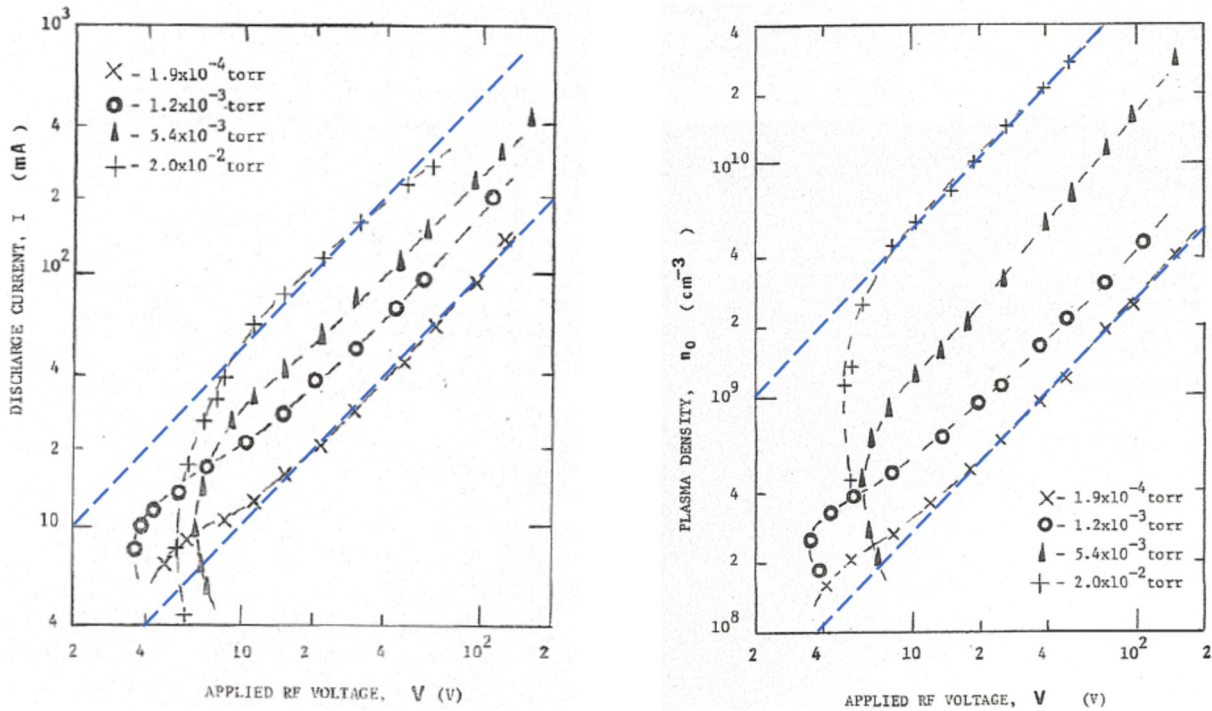


FIG. 4. *I*–*V* characteristics and plasma densities in CCP vs discharge voltage. The dashed lines represent a linear dependence. Hg, 40.8 MHz, *L* = 7.8 cm, *D* = 7 cm. Reproduced with permission from O. Popov and V. Godyak, “Power dissipated in low-pressure radio-frequency plasma,” *J. Appl. Phys.* **57**, 53 (1985). Copyright 1985 AIP Publishing.

and $f = 1.2$ MHz,²¹ respectively. In all cases, at $V^2 \gg V_{Rp}^2$, the volt-ampere characteristics are linear in a wide range of gas pressures and discharge voltages. Recall that at a relatively high gas pressure, ion collisions lead to the reduction of the ion density nonuniformity in rf sheaths, making it to be close to that in the asymptotic matrix model, which yields a strictly linear *I/V* characteristic.^{6,10,11} Surprisingly, the linear *I/V* characteristic is observed (Fig. 6) in the helium CCP at 0.5 Torr for the applied voltage in the range between 250 and 2200 V, when the CCP is expected to operate mainly in the γ -mode with an electron avalanche in rf sheaths.

The most comprehensive experimental study of electrical characteristics of a symmetrical CCP in a wide range of argon pressures (3 mTorr–3 Torr) and discharge voltages (10 V–1.5 kV) was published in Ref. 22. There, considerable attention was paid to the accuracy of experiment by compensating stray capacitance in the measurement circuit and the assurance of symmetrical drive of the CCP rf electrodes. Note that the geometric symmetry of the rf electrodes and the discharge gap do not guarantee the symmetric CCP operation. Due to rf plasma potential (mostly on the second harmonic $V_{2f} \approx 0.1$ V for a symmetrically driven CCP), the CCP asymmetric drive with one rf electrode being grounded leads to a significant rf plasma potential at the fundamental frequency *f* inducing radial rf displacement rf current. The radial rf current exists for the symmetrical CCP drive, but it is negligibly small and does not break the CCP symmetry.

The CCP discharge voltage *V*(*I*) and rf power *P*(*I*) shown in Fig. 7 demonstrate quite linear *I/V* characteristics at $V^2 \gg V_{Rp}^2$ for all discharge pressures and the discharge voltage of up to 1.5 kV, where both models^{12,16} predict $I \propto V^{1/2}$. They remain linear for the

collision-dominated rf sheath at higher argon pressure (similar to that in helium shown in Figs. 5 and 6) and in argon CCP in a wide range of rf frequencies²³ shown in Fig. 8.

The scaling $I \propto V^{3/4}$ was obtained for CCP¹⁴ with the sheath model¹⁶ at very low gas pressure when electron stochastic heating dominates, which also contradicts experiment at very low gas pressure and frequencies of 40.8 and 13.56 MHz.^{20,22} As to numerical modeling, amazingly, we could find only work,²⁴ and it demonstrates close to a linear CCP volt-ampere characteristic, although the linear dependence of plasma density on discharge voltage $n(V)$ was obtained in simulations^{25,26} that suggest a linear *V/A* characteristic, since $J = env_{rf}$ and the electron oscillatory velocity v_{rf} does not depend on the discharge current.

The linear *I/V* characteristic followed from the model^{12,13} for the collisionless rf sheath at $3 \leq U_{sh} \leq 10^2$ and $V^2 \gg V_{Rp}^2$, which corresponds to a discharge voltage of about 30–500 V, appears to be valid for CCP at much larger discharge voltages (up to 2.2 kV) and gas pressures (up to 1 Torr) when the rf sheath is highly collisional. Such a discrepancy between the rf sheath models^{6,12,16} with experiments that demonstrate linear *I/V*, we believe, is due to the neglect of ionization produced by electrons oscillating in the sheath (see Fig. 1). Neglecting ionization in the sheath is a common prerequisite for the sheath models. However, the simultaneous retaining of electrons in the Poisson equation and no ionization there make even traditional dc sheath model inconsistent.^{27,28}

In the dc sheath model^{14,15} at large voltages, electrons penetrate into the sheath at the distance of about λ_D . Thus, this inconsistency has a minor effect since $S \gg \lambda_D$, and the area occupied by the

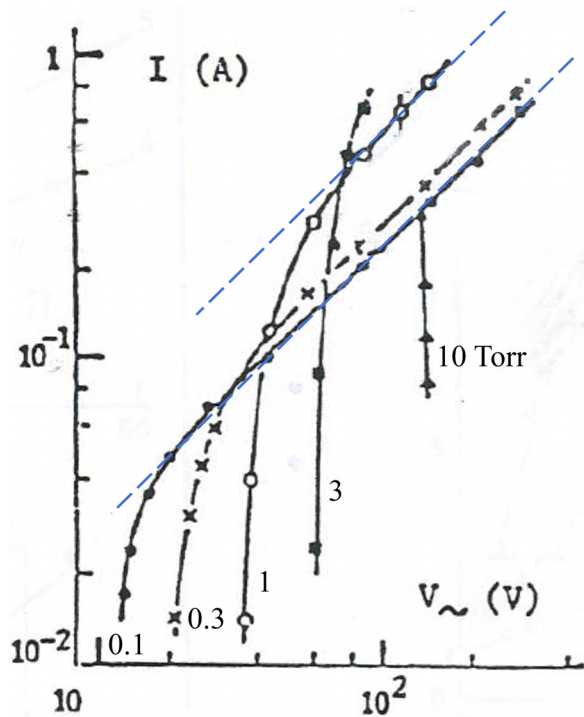


FIG. 5. IV characteristics in helium CCP at 40.8 MHz, $L = 7$ cm, $D = 10$ cm. Reproduced with permission from V. Godyak and A. Ganna, "Collisional rf discharge," *Sov. Phys.-Plasma Phys.* **6**, 372 (1980). Copyright 1980 AIP Publishing.

electrons is too small. However, in the rf sheath at a large rf voltage, the time-averaged electron population is spread over almost an entire rf sheath and generates slow ions, which compensate the ion density drop due to the ion acceleration. The ionization makes ion density in the sheath more uniform and similar to that at low and moderate rf voltages. The effect of ionization in the rf sheath is expected to be

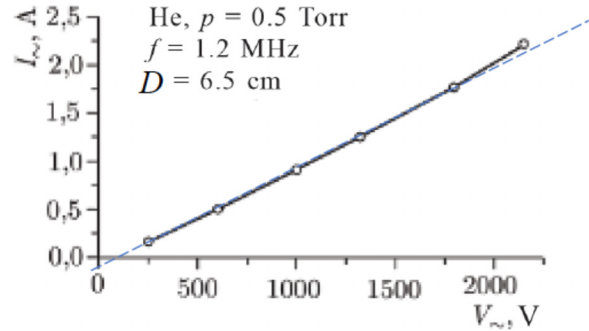


FIG. 6. IV characteristic in helium CCP at 1.2 MHz, 0.5 Torr, $D = 6.5$ cm. Reproduced with permission from V. Savinov, *Physics of Radiofrequency Capacitive Discharge* (CRC Press, Taylor & Francis, 2018). Copyright 2018 CRC Press, Taylor & Francis.

more pronounced at large rf voltages. This is quite opposite to the dc sheath where the effect of ionization is diminishing with increasing dc voltage.

At low gas pressures, for small and moderate sheath voltages, when $U_{sh} < 10^2$ (corresponding to $\rho < 2.8 < \delta \approx 3$), the rf sheath structure is close to the structure of the dc sheath at the floating voltage $V_f = (3-6)T_e/e$. Under these conditions, the ionization effect in the rf sheath should be minimal. The absence of considerable ionization seems to explain the agreement of the model^{12,13} with the experiment²⁰ at small and moderate rf sheath voltages. An additional ionization in a high-voltage rf sheath can be provided by the multiplication of the secondary-emission electrons (in γ -mode) at moderate and elevated gas pressures or by the fast ions (directly, or via the charge exchange with neutrals at low gas pressures).

The mentioned disagreement between the models in Refs. 12 and 16 at small and moderate rf sheath voltages is due to the violation of charge conservation in the sheath in model 16 by assuming $S_2 = 0$. Indeed, at small S_2 , the electron current to rf electrode I_{dc} is close to the electron saturation current with a strong exponential dependence on the sheath voltage. This conductivity current elevates the dc

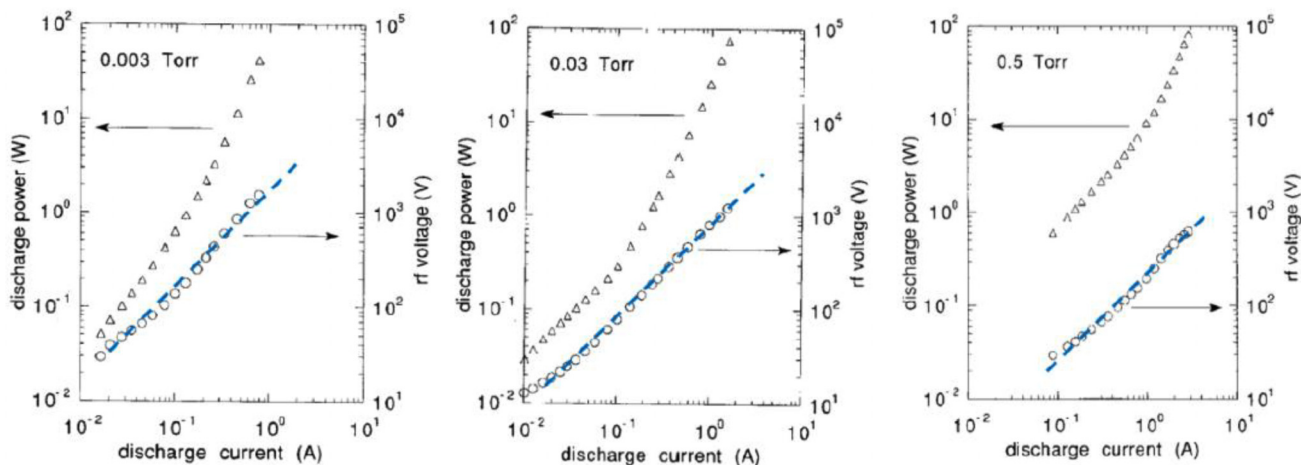


FIG. 7. IV characteristics in argon CCP at 13.56 MHz for 3, 30, and 500 mTorr. Reproduced with permission from Godyak et al., "Electrical characteristics of parallel-plate RF discharges in argon," *IEEE Trans. Plasma Sci.* **19**, 660 (1991). Copyright 1991 IEEE Publishing.

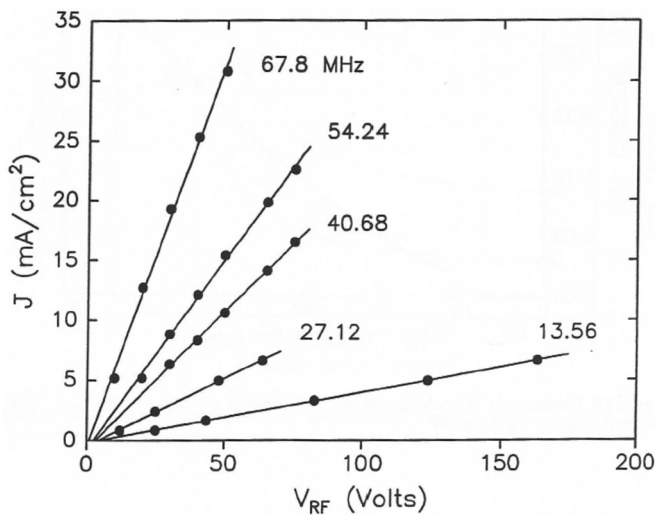


FIG. 8. I/V characteristics in argon CCP at several frequencies, $p = 0.25$ Torr, $L = 2$ cm. Reproduced with permission from M. Colgan and M. Meiyappan, in "Very high frequency capacitive plasma sources," in *High Density Plasma Sources*, edited by O. Popov (Noyes Publication, 1995). Copyright 1995 Noyes Publication.

negative voltage across the sheath, shifting the position of S_0 toward the plasma, thus leading to a larger value and weaker growing of S_0 with V_{sh} in the model¹² than that in the model.¹⁶ Note that the assumption $S_2 = 0$ is frequently adopted in many other rf sheath models making them inconsistent.

In summary:

1. The available experiments in CCP obtained in a wide range of driving frequencies, gas pressures, gas types, and rf discharge voltages demonstrate a very close to a linear static volt-ampere characteristic ($I \propto V$).
2. At low gas pressures and moderate discharge voltages ($V_{RF}^2 \ll V^2$ and $eV/T_e \lesssim 10^2$) corresponding to the discharge voltage of up to few hundred volts, the scaling law obtained in the experiments is in good agreement with the collisionless rf sheath model^{12,13} and disagrees with the commonly accepted model,¹⁶ which gives $I \propto V^{1/2}$.
3. The difference in the models^{12,16} is due to the assumption¹⁶ of the sheath collapsing ($S_2 = 0$) at the moment of the maximal rf electrode potential. This assumption leads to the violation of the electrical charge conservation in the rf sheath and to the sheath absence without rf voltage.
4. The disagreements between the experiments and all rf models considered here at high gas pressures and high discharge voltages are attributed to the neglect of ionization in all the existing rf sheath models.

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