

Theory of the strange metal $\text{Sr}_3\text{Ru}_2\text{O}_7$

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The bilayer perovskite $\text{Sr}_3\text{Ru}_2\text{O}_7$ has been widely studied as a canonical strange metal. It exhibits T -linear resistivity and a $T \log(1/T)$ electronic specific heat in a field-tuned quantum critical fan. Criticality is known to occur in “hot” Fermi pockets with a high density of states close to the Fermi energy. We show that while these hot pockets occupy a small fraction of the Brillouin zone, they are responsible for the anomalous transport and thermodynamics of the material. Specifically, a scattering process in which two electrons from the large, “cold” Fermi surfaces scatter into one hot and one cold electron renders the ostensibly noncritical cold fermions a marginal Fermi liquid. From this fact the transport and thermodynamic phase diagram is reproduced in detail. Finally, we show that the same scattering mechanism into hot electrons that are instead localized near a 2D van Hove singularity explains the anomalous transport observed in strained Sr_2RuO_4 .

strange metals | quantum criticality | linear resistivity | transport

The conventional theory of metallic transport predicts that the approach to the residual resistivity at low temperatures should follow one of several simple power laws. If electron–electron scattering dominates, then $\rho \sim T^2$ is expected, with each factor of T coming from the suppression of final electron states due to Pauli exclusion. Dominant electron–phonon scattering instead leads to $\rho \sim T^5$ in three dimensions. “Strange metals” can be defined as metals exhibiting an anomalous temperature dependence of the low-temperature resistivity. The T -linear behavior $\rho \sim T$ has been widely observed, but other scalings including $\rho \sim T^{3/2}$ and $\rho \sim T^2 \log T$ have also been reported. These are all stronger than the conventional T^2 scaling due to electronic scattering. A common scenario is that the anomalous scaling is observed above a “transport temperature” T_{tr} and that $T_{\text{tr}} \rightarrow 0$ at some point in the phase diagram, often associated with quantum criticality (1).

A compelling theory of a strange metal must connect the anomalous transport behavior to other measurements, including thermodynamic and spectroscopic probes. Given the similarity of the strange metal phenomenon across materials, a thorough grounding of anomalous transport in specific microscopic electronic properties of a single material has the potential to uncover mechanisms that may be at work more broadly. Here we show how such a comprehensive theory can be achieved for the bilayer perovskite $\text{Sr}_3\text{Ru}_2\text{O}_7$. This material has been widely studied as a prototypical strange metal (2), with T -linear resistivity in a quantum critical fan and diverging effective electron masses as the critical field is approached at low temperature (Fig. 1).

The key fact will be that $\text{Sr}_3\text{Ru}_2\text{O}_7$ is known from photoemission measurements to exhibit very shallow, small Fermi pockets. These lead to a sharp peak in the density of states close to the chemical potential (3). The distance of the peak from the chemical potential is comparable to the scale T_{tr} above which strange metal transport is observed at zero field. Furthermore, it is the “hot” electrons in these pockets that go critical as the field is tuned. These facts suggest that the hot (h) electron pockets should be responsible for the anomalous transport, but a suitable mechanism has been lacking. The hot electrons are irrelevant as carriers of charge, because they are short circuited by the faster “cold” (c) electrons (4, 5). Instead, a distinctive $cc \rightarrow ch$

scattering process, in which one cold electron is scattered by another into a hot pocket, leads to strange metal transport by the cold electrons. As previously noted in the context of cuprates (6, 7), electrons in a peak close to the chemical potential are classical above a low-energy scale, so there is no phase space suppression for scattering into the hot electron pocket and hence $\rho \sim T$.

In addition to the T -linear resistivity (2, 8, 9), $cc \rightarrow ch$ scattering above the temperature T_{tr} produces the observed anomalous specific heat (10) and optical conductivity (11) of $\text{Sr}_3\text{Ru}_2\text{O}_7$. At temperatures below T_{tr} , the dominance of $cc \rightarrow ch$ scattering as the critical field is approached leads to a violation of the Kadowaki–Woods relation $A \sim \gamma^2$ between the resistivity $\rho \sim A T^2$ and specific heat coefficient $\gamma \equiv c/T$. Fig. 2 shows that instead $A \sim \Delta\gamma$ for $\text{Sr}_3\text{Ru}_2\text{O}_7$ (2, 10, 12), as our model predicts. Here $\Delta\gamma$ is the enhancement of the specific heat as the critical field is approached.

The scattering mechanism we have identified may be relevant in other contexts. We will show that the same $cc \rightarrow ch$ scattering explains the anomalous $\rho \sim T^2 \log T$ of a different ruthenate, Sr_2RuO_4 , as it crosses a Lifshitz transition (13). Beyond ruthenates, in many families of strange metals strong quantum critical scattering is directly experienced only by a relatively small fraction of the electrons, localized in “hot regions” or in “hot bands.” The conceptual understanding of transport $\text{Sr}_3\text{Ru}_2\text{O}_7$ developed here gives a solid starting point for unraveling the mystery of strange metals more broadly.

Quantum Criticality in $\text{Sr}_3\text{Ru}_2\text{O}_7$

Criticality in $\text{Sr}_3\text{Ru}_2\text{O}_7$ is field tuned, with a critical c -axis field of $H_c \approx 7.9$ T (2), and displays several canonical features of metallic quantum criticality. The transport and thermodynamic phase diagram is sketched in Fig. 1. In more detail:

Significance

The behavior of “strange metals” has eluded theoretical understanding for some time. It has remained unclear whether strange metals are fundamentally novel forms of matter or whether some unidentified variant of well-understood physics is at work. We show how the behavior of strontium ruthenate, a widely studied strange metal, follows in detail from well-established electronic properties. Specifically, strontium ruthenate contains “hot” electrons that are less quantum mechanical than the other “cold” electrons. A scattering process in which a cold electron becomes hot after colliding with a second cold electron is unusually strong, because the hot electrons are not subject to the Pauli exclusion principle. This fact is seen to underpin the strange metallicity of strontium ruthenate.

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The authors declare no competing interest.

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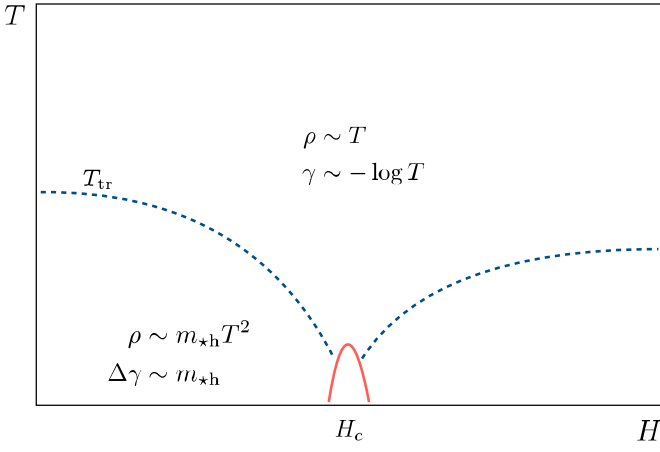


Fig. 1. Schematic phase diagram of $\text{Sr}_3\text{Ru}_2\text{O}_7$ as a function of field and temperature. There is a marginal Fermi liquid of cold electrons above T_{tr} . In the conventional Fermi liquid below T_{tr} , transport and specific heat are dominated by the diverging hot electron mass m_{*h} as $H \rightarrow H_c$.

- 1) At $H = 0$ the resistivity is T linear above $T_{\text{tr}} \sim 20$ K and goes as T^2 at the lowest temperatures. As $H \rightarrow H_c$, the scale T_{tr} apparently collapses toward zero (2, 8, 9).
- 2) As $H \rightarrow H_c$ at low temperatures, an effective mass m_* is strongly enhanced. The growth of m_* is seen in the coefficient A of the low-temperature resistivity $\rho \sim AT^2$ (2), NMR spin relaxation (14), and the low-temperature specific heat coefficient $\gamma \equiv c/T$ (10, 12). However, we see in Fig. 2 that the specific heat enhancement $\Delta\gamma$ follows the scaling $A \sim \Delta\gamma$, distinct from the conventional Kadowaki–Woods relation $A \sim \gamma^2$.
- 3) At $H = H_c$ a low-temperature divergence $\gamma \sim \log T$ is observed (10, 15), until cut off at a spin-ordering transition at a temperature of about 1 K (16).
- 4) An extended Drude fit to $\sigma(\omega)$ in the T -linear regime reveals a scattering rate $\Gamma(\omega) \sim \omega$ for $\omega \gtrsim T$ (11).

Hot Pockets and $cc \rightarrow ch$ Scattering

Quantum criticality in $\text{Sr}_3\text{Ru}_2\text{O}_7$ involves a dichotomy between cold and hot electrons. The hot electrons have a large density of states already in zero field, and only the hot electrons go critical as a function of field. Specifically:

- 1) Angle-resolved photoemission (ARPES) reveals a complicated fermiology with multiple bands (3, 17). Quantum oscillations show that all except one set of bands—the γ_2 pockets, which are not seen in Shubnikov–de Haas oscillations—do not exhibit a strongly divergent mass at the critical field (18). The γ_2 pockets are remarkably shallow and give rise to a peak in the density of states at a few millielectronvolts from the chemical potential at zero field (3, 17).
- 2) The coefficient γ as a function of temperature shows a maximum at $T_{\text{pk}} \approx 7$ K at zero field (10). A similar peak is seen at $T'_{\text{pk}} \approx 17$ K in the magnetic susceptibility (19, 20). As $H \rightarrow H_c$, T_{pk} collapses to zero.

At zero field the hot electron peak in the density of states seen in photoemission quantitatively explains the peaks seen in the specific heat coefficient γ and magnetic susceptibility χ . Using the measured density of states of the γ_2 pockets (3) in standard free-electron formulas for γ and χ leads to peaks at temperatures of 9 K and 19 K, respectively. These are in good agreement with the zero-field experimental values mentioned above. This is the first imprint of the hot electrons on physical observables. In the remainder we show that $cc \rightarrow ch$ scattering into the hot electron pockets fundamentally determines the transport behavior both above and below T_{tr} .

Fig. 3 illustrates the interplay of hot and cold electrons. The Fermi surface mostly consists of regions with a high Fermi velocity, as well as a set of “hot pockets” with a large density of states peaked near or at the chemical potential. The essential feature for the present discussion is that the high density of states is localized in a small part of the Brillouin zone, whereas the light portions of the Fermi surface are much more extended.

The density of states of the “hot electrons” is high, whereas the “cold electrons” away from the hot spots dominate the current due to their higher Fermi velocity. It is therefore natural for the transport lifetime to be determined by scattering processes where both cold and hot electrons are involved. Several such processes are possible: A cold electron can scatter off a hot one, denoted $ch \rightarrow ch$, and a pair of cold electrons can scatter into a cold and a hot electron ($cc \rightarrow ch$), etc.

The $ch \rightarrow cc$ or $cc \rightarrow ch$ processes involve large momentum transfer and contribute significantly to transport. Moreover, for a large enough cold Fermi surface, all cold electrons can participate in $cc \rightarrow ch$ processes and no “short circuiting” occurs. Other types of scattering are either small angle, and do not efficiently degrade current, or not active across the entire Fermi surface. We discuss this further in *SI Appendix*. In the following we show that $cc \rightarrow ch$ scattering reproduces the phase diagram in Fig. 1.

A Marginal Fermi Liquid from Classical Fermions

The peak in the density of states due to the hot γ_2 pockets (at $H = 0$, from ARPES) occurs at a distance from the chemical potential that roughly coincides with the temperature above which T -linear resistivity is observed. Let the distance of the peak from the chemical potential be ϵ_h and its width be W_h (Fig. 3). At zero field, $W_h \lesssim |\epsilon_h|$. At temperatures above $|\epsilon_h| + W_h$ the γ_2 fermions are classical and nondegenerate. A $cc \rightarrow ch$ scattering process can therefore be expected to be suppressed by T rather than T^2 . We now verify that this is the case by showing that this scattering leads to marginal Fermi liquid (MFL) (21) behavior of the cold fermions, as has been previously noted (6, 7).

To connect with the MFL cleanly, we write the scattering process as the interaction of a cold fermion with a bosonic mode

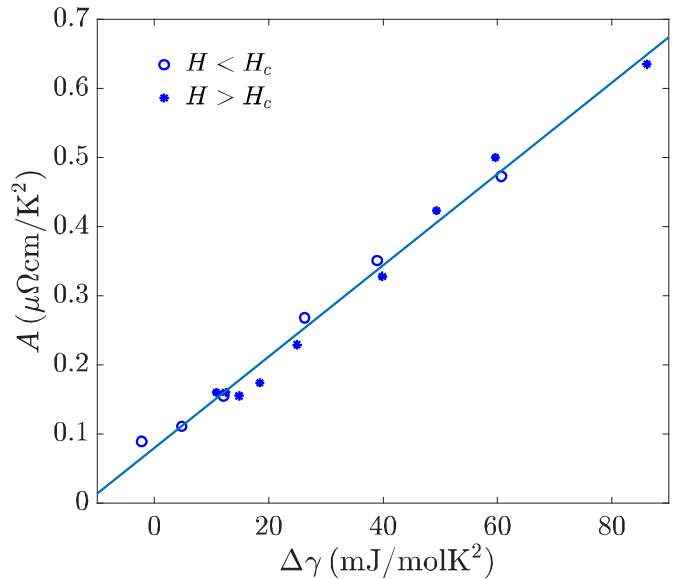


Fig. 2. As $H \rightarrow H_c$ at low temperatures, the coefficient A of the T^2 resistivity increases by a factor of ~ 7 (2), and is linearly proportional to the specific heat enhancement $\Delta\gamma$ (10, 12).

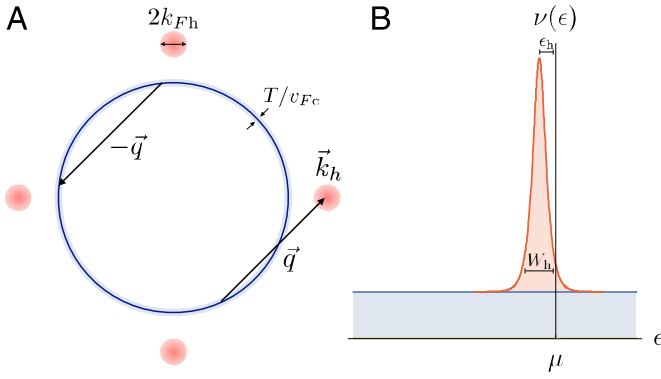


Fig. 3. (A) Illustration of $cc \rightarrow ch$ scattering, in which one of the fermions is scattered from the cold Fermi surface (blue) into the hot region (red). In general, the hot regions could also be on the same Fermi sheet as the cold fermions. (B) A sharp peak in the density of states $\nu(\epsilon)$ close to the chemical potential, due to the hot fermions.

generated by the cold-hot fermion polarizability. We treat the interaction perturbatively, starting from free cold fermions with spectral weight $\text{Im } G_c^R(\omega, \vec{k}) = \pi \delta(\omega - \epsilon_{c,\vec{k}})$. Due to the interaction, the cold fermions acquire the self-energy (we work in d spatial dimensions for generality)

$$\Sigma_c''(\omega, \vec{k}) = \lambda^2 \int \frac{d^d q}{(2\pi)^d} \int \frac{d\Omega}{\pi} \frac{f(\omega - \Omega)b(\Omega)}{f(\omega)} \times \text{Im } G_c^R(\omega - \Omega, \vec{k} - \vec{q}) \Pi_{ch}''(\Omega, \vec{q}). \quad [1]$$

In this form, Eq. 1 is transparently Fermi's golden rule: λ is the four-fermion coupling constant and the Fermi-Dirac and Bose-Einstein distributions are $f(\Omega) = 1/(e^{\Omega/T} + 1)$ and $b(\Omega) = 1/(e^{\Omega/T} - 1)$. In textbooks (22), [1] is commonly expressed differently, using the identity $f(\omega - \Omega)b(\Omega)/f(\omega) = f(\Omega - \omega) + b(\Omega)$. The polarizability is

$$\Pi_{ch}''(\Omega, \vec{q}) = \int \frac{d^d k'}{(2\pi)^d} \int \frac{d\Omega'}{\pi} \frac{f(\Omega' + \Omega)f(-\Omega')}{b(\Omega)} \times \text{Im } G_c^R(\Omega', \vec{k}') \text{Im } G_h^R(\Omega' + \Omega, \vec{k}' + \vec{q}). \quad [2]$$

Here $\text{Im } G_h^R(\Omega' + \Omega, \vec{k}' + \vec{q})$ is the spectral weight of the hot fermions, which we can keep general. Similarly to above, we have written Eq. 2 in a manifestly physical way, related to common formulations through the identity $f(\Omega' + \Omega)f(-\Omega')/b(\Omega) = f(\Omega') - f(\Omega + \Omega')$.

At temperatures greater than $|\epsilon_h| + W_h$, we have $T \sim |\omega| \sim |\Omega| \sim |\Omega'| \gg |\Omega' + \Omega|$ wherever $\text{Im } G_h^R(\Omega' + \Omega, \vec{k}' + \vec{q})$ is nonzero. This leads to simplifications in [2]. The cold fermions are at energies $T \ll v_{Fc} k_{Fc} = E_{Fc}$ (where k_{Fc} , v_{Fc} , and E_{Fc} denote the cold fermions' Fermi momentum, Fermi velocity, and Fermi energy, respectively), which allows the cold fermion dispersion to be linearized about the Fermi energy. In this way we obtain the explicitly MFL form

$$\Pi_{ch}''(\Omega, \vec{q}) \approx F(\vec{q}) \tanh \frac{\Omega}{2T}. \quad [3]$$

Taking the cold Fermi surface to be round for simplicity,

$$F(\vec{q}) \approx \frac{\pi}{2} \int \frac{d^d k'}{(2\pi)^d} \delta(|\vec{k}'| - k_{Fc}) \times \int \frac{d\Omega'}{\pi} \frac{1}{v_{Fc}} \text{Im } G_h^R(\Omega' + \Omega, \vec{k}' + \vec{q}). \quad [4]$$

Eq. 3 was noted in ref. 6. It is then well known that inserting the MFL polarization **3** into the expression **1** for the self-energy leads to $\Sigma_c'' \sim \max(T, \omega)$ (21). Thus, there is a T -linear scattering rate that becomes ω linear at higher frequencies. This explains the experimental resistivity and optical conductivity noted above. A careful evaluation of the $cc \rightarrow ch$ scattering rate in *SI Appendix*, using the experimentally determined density of states in the γ_2 pockets (3) to evaluate [2], shows that T -linear scattering is found above precisely the observed $T_{tr} \approx 20$ K. Because $cc \rightarrow ch$ scattering is large angle, this result for the single-particle scattering rate also determines the transport lifetime.

The result **3** is obtained for any form of the hot fermion Green's function. The only assumption is that the temperature is above the characteristic energy scales of the hot fermions (such as W_h and ϵ_h). We can estimate the magnitude of the scattering rate from [4] by taking the hot band to be flat, so that $\text{Im } G_h^R(\omega, \vec{k}) = \pi \delta(\omega)$ for some region of area/volume k_{Fh}^d in the Brillouin zone. The hot regions are smaller than the cold Fermi sea, so that $k_{Fh} \ll k_{Fc}$. In this case, the phase space that the cold fermion can scatter into is constrained by the size of the hot regions; evaluating the integrals one finds the scattering rate $\Gamma_c \equiv \Sigma_c''(\omega = 0) \sim [\lambda^2 k_{Fc}^{d-2} k_{Fh}^d / v_{Fc}^2] T$. We give some details of the kinematics leading to this result in *SI Appendix*. Specializing to $d = 2$ and restoring units:

$$\Gamma_c \sim \lambda^2 \frac{k_{Fh}^2 k_B T}{v_{Fc}^2 \hbar}. \quad [5]$$

For small k_{Fh} , the prefactor of $k_B T / \hbar$ in [5] is small and the perturbative computation is controlled. For $\text{Sr}_3\text{Ru}_2\text{O}_7$ we can estimate the prefactor using numbers from ref. 9. There are eight hot pockets with $\sum_i k_{Fh,i}^2 \approx 0.03 \text{ \AA}^{-2}$. The inverse velocity of the cold Fermi surfaces in [5] controls the available phase space for scattering. Averaging over bands we estimate $\langle v_{Fc}^{-1} \rangle \approx 0.5 \times 10^{-4} \text{ s/m}$. The microscopic coupling λ has units of \hbar per mass. Again averaging over bands we estimate $\lambda \sim (v_{Fc} / k_{Fc}) \approx 0.1 \hbar / m_e$. These numbers lead to $\Gamma \sim k_B T / \hbar$, consistent with the observed ‘‘Planckian’’ scattering rate (9, 23).

A similar computation can be done for the $ch \rightarrow ch$ process, in which a cold electron scatters off an electron in the hot region. Both the initial and final states of the hot electron have no Fermi-Dirac suppression. One finds $\Sigma_c''(\omega, \vec{k}) \sim \lambda^2 k_{Fh}^2 \int \frac{d^2 q}{(2\pi)^2} \delta(\omega - \epsilon_{c,\vec{k}-\vec{q}})$. This scattering therefore behaves as elastic disorder, leading to a scattering rate proportional to $\lambda^2 k_{Fh}^3 / v_{Fc}$ (the q integral is restricted to a region of the same size as the hot region). It contributes a temperature-independent term to resistivity. However, because $k_{Fh} \ll k_{Fc}$, this is necessarily small-angle scattering for the cold electrons. Therefore, the effective transport scattering rate is suppressed by an additional factor of $(k_{Fh} / k_{Fc})^2$, so that $\Gamma_{tr} \sim k_{Fh}^4 / k_{Fc}^4 \times k_{Fh} v_{Fc} / \hbar$ (assuming for simplicity that $\lambda \sim v_{Fc} / k_{Fc}$ is set by a single cold Fermi surface).

There is also a conventional T^2 contribution to the resistivity from $cc \rightarrow cc$ scattering. For the MFL T -linear scattering to dominate, the phase space for scattering into cold fermions must be smaller than that for scattering into hot fermions: $k_{Fc} k_B T / v_{Fc} \ll k_{Fh}^2$. Geometrically, $cc \rightarrow ch$ scattering dominates when the area of the hot region is greater than the area of the thermally broadened cold annulus shown in Fig. 3. This requirement, that $k_B T \ll k_{Fh}^2 / k_{Fc}^2 \times E_{Fc}$, can be satisfied despite k_{Fh} being small because E_{Fc} is a high-energy scale. In *SI Appendix* we verify that the phase space for $cc \rightarrow ch$ scattering indeed dominates in the T -linear regime of $\text{Sr}_3\text{Ru}_2\text{O}_7$.

Low-Temperature Approach to the Critical Field

The maximum in the density of states in Fig. 3 is at a distance $|\epsilon_h|$ from the chemical potential. At temperatures well below this

scale all of the fermions are degenerate, and electron–electron scattering leads to the usual T^2 resistivity. If $|\epsilon_h| < W_h$, however, there remains a degenerate Fermi pocket of hot fermions that makes a large contribution to the density of states and $cc \rightarrow ch$ scattering is still important. In this regime $\omega \sim \Omega \sim \Omega' \sim \Omega + \Omega' \sim T$, and the formulas above give the scattering rate (reinstating factors of k_B and \hbar in the final term)

$$\Gamma_c \sim \lambda^2 \frac{k_{Fh}}{v_{Fh} v_{Fc}^2} T^2 \sim \frac{m_{*h}}{m_{*c}} \frac{(k_B T)^2}{\hbar E_{Fc}}, \quad [6]$$

where we define the mass of the heavy electrons through their density of states at the Fermi level, $m_{*h} = \nu_h(0)/2\pi$, and similarly for m_{*c} . In the final term in [6] we again assumed for simplicity that $\lambda \sim \hbar/m_{*c}$ is set by a single cold Fermi surface. See *SI Appendix* for details of the kinematics leading to [6]. The important factor is the ratio m_{*h}/m_{*c} that, we will posit, becomes large as the critical field is approached. The scattering rate Γ_c determines the resistivity through the Drude formula $\rho = m_{*c} \Gamma_c / (n_c e^2)$. The resulting scaling of the resistivity with the large hot mass is therefore $\rho \propto m_{*h} T^2$. In contrast, if all fermions have a strongly enhanced mass (become hot) or if $ch \rightarrow ch$ scattering dominates (for example, if the hot band is large in the Brillouin zone), then $\rho \propto m_{*h}^2 T^2$.

At the same low temperatures, the electronic specific heat is also given by the conventional Fermi liquid formula. If $m_{*h} \gg m_{*c}$, then hot electrons dominate the specific heat, even while cold electrons dominate transport. The enhancement of the specific heat coefficient γ due to the increasing mass of the hot fermions is $\Delta\gamma \sim k_B^2 m_{*h}$. Together with the results in the previous paragraph, this leads to $A \sim \Delta\gamma$, where A is the coefficient in the resistivity $\rho \sim AT^2$. The notation $\Delta\gamma$ refers to the fact that the specific heat of the cold fermions has been subtracted out.* We have recovered precisely the observed linear scaling shown in Fig. 2. The key input is the hypothesis that m_{*h} becomes large as $H \rightarrow H_c$, which leads to the dominance of $cc \rightarrow ch$ scattering. This scaling is distinct from the Kadowaki–Woods relation $A \sim \gamma^2$ (24), which instead follows from the resistivity $\rho \propto m_{*h}^2 T^2$.

Microscopically, the mass enhancement has been assumed to be tied up with metamagnetic quantum criticality of the hot electrons (2, 25–27). The results above are not sensitive to the details of these physics beyond requiring a divergent m_{*h} in a small region of the Brillouin zone. However, in using [4] we are also assuming that the integrated single-particle spectral weight of the hot electrons remains finite as their mass diverges. This leads to a picture in which the growth of the mass is primarily a band structure effect associated with a strongly enhanced density of states at the Fermi energy as $H \rightarrow H_c$ (28). This spectral weight must survive the presence of any quantum critical scattering.

The Collapse of T_{tr} at the Critical Field

The hot electrons dominate the specific heat at low temperatures, especially as the critical field is approached. At temperatures above the peak in the density of states their contribution becomes small: $\gamma_{hot} \sim k_{Fh}^2 \max(W_h^2, \epsilon_h^2) / (k_B T^3)$. In computing γ_{hot} here the chemical potential is kept fixed and constant, controlled by the large total number of cold electrons. The cold fermions dominate the specific heat in this regime. The MFL scattering rate $\Sigma_c' \sim \max(T, \omega)$ implies a logarithmic effective mass enhancement so that $\Delta\gamma_{cold} \sim k_B^2 \Delta m_{*c} \sim k_B^2 m_{*c}$.

$k_{Fh}^2/k_{Fc}^2 \cdot \log(\Lambda/T)$. Here m_{*c} is the unrenormalized cold electron mass. The energy cutoff scale $\Lambda \sim v_{Fc} k_{Fh}$ because MFL scattering $\Gamma_c \sim \omega$ requires $\omega/v_{Fc} \lesssim k_{Fh}$, for the scattering phase space to be determined by the hot fermion pocket (*SI Appendix*). Precisely such a logarithmic temperature dependence is seen clearly at low temperatures at the critical field, as we recalled above. The onset of logarithmic behavior is also visible in the data away from the critical field, at temperatures above T_{tr} . At temperatures below T_{tr} , within the hot electron peak in the density of states, the hot fermions become degenerate and the cold fermion mass enhancement is cut off as $\gamma_{cold} \sim k_B^2 m_{*c} \cdot k_{Fh}^2/k_{Fc}^2 \cdot \log(\Lambda/(|\epsilon_h| + W_h))$. The relative contribution of hot and cold electrons to the low-temperature γ at the $H \approx H_c$ is discussed further in *SI Appendix*.

The observed behaviors of the specific heat and transport data shown in Fig. 1 are entirely reproduced by the model of dominant $cc \rightarrow ch$ scattering if $T_{tr} \sim |\epsilon_h| + W_h$ collapses toward zero at the critical field. This collapse is cut off near the critical field by the onset of spin ordering at around 1 K, as shown in Fig. 1. Therefore the width of the peak in the density of states need not strictly go to zero.

The data suggest that it is predominantly the scale $|\epsilon_h|$ that drives the collapse of T_{tr} toward the critical field. We recalled above that a peak in the specific heat is seen at a temperature T_{pk} . This Schottky anomaly-like peak in $\gamma(T)$ is quantitatively explained at zero field by the peak in the density of states seen in ARPES (3). As the temperature is raised more states in the hot region become accessible and hence the specific heat coefficient increases until the temperature crosses $\sim \max(|\epsilon_h|, W_h)$, beyond which the specific heat starts to decrease (and become dominated by the cold electrons). The simultaneous collapse of T_{pk} and T_{tr} as the critical field is approached is therefore naturally explained by $|\epsilon_h| \rightarrow 0$. Such behavior is consistent with a recent band structure analysis (28). As noted above, the essential requirement on the width W_h of the peak is that it should be of order 1 K at the critical field, so that the MFL behavior can persist all of the way down to the onset of the low-temperature ordered phase.

As $\epsilon_h \rightarrow 0$, the density of states at the chemical potential increases. This leads to an increase in the specific heat coefficient at low temperature, tying the collapse of T_{tr} to the low-temperature mass enhancement we described above.

In the scenario we have outlined, nondegenerate hot fermions are important even at low temperatures at the critical field. In this case, as noted above, elastic $ch \rightarrow ch$ scattering gives an additional contribution to the residual, temperature-independent resistivity. A strong peak in the residual resistivity at the critical field is indeed seen in the data (2). Furthermore, the Lorenz ratio at low temperatures shows increased elastic scattering at the critical field (29).

Our discussion has been purely in terms of the fermionic band structure. Direct contributions to transport and thermodynamics from quantum critical collective modes are not necessary to explain the data, as considered in refs. 30 and 31. Nonetheless, we mention two ways in which such bosonic physics could additionally be present. First, quantum critical fluctuations of an overdamped bosonic order parameter with $z_B = 2$ in two dimensions would contribute to a logarithmic specific heat at low temperatures (32). Second, order parameter fluctuations provide an additional scattering mechanism. A different source of T -linear scattering will be called for at the lowest temperatures if W_h does not in fact collapse down to around 1 K at the critical field (as we assumed above).

Scattering into a Divergent Density of States

Taking a step back from $\text{Sr}_3\text{Ru}_2\text{O}_7$, nondegenerate fermions will be present in a localized region of the Brillouin zone whenever a divergence in the density of states occurs at the chemical

*As m_{*h} diverges, m_{*c} also becomes enhanced due to $cc \rightarrow ch$ scattering. We will see shortly that the enhancement is only logarithmic: $\Delta m_{*c} \propto k_{Fh}^2 \log(m_{*h} v_{Fc}/k_{Fh})$. Close to the critical field this growth is parametrically weaker than that of m_{*h} , plausibly explaining why it is not seen in quantum oscillations (18). That said, our interpretation of the specific heat data in *SI Appendix* suggests that $\Delta m_{*c} \sim m_{*c}$ is not negligible.

potential, such as at a van Hove point. In general, a dispersion with the scaling $\epsilon_{h,\vec{k}} \sim k^z$ (this need not be a minimum in k , e.g., at a van Hove point $\epsilon_{h,\vec{k}} \sim k_x^2 - k_y^2$) leads to a density of states

$$\nu_h(\epsilon) \sim \epsilon^{d/z-1}, \quad [7]$$

which is divergent for $z > d$ (for $z = d$ there can be a logarithmic divergence). While this section shares with our earlier discussion the importance of nondegenerate fermions, it describes a different scenario. The MFL discussed above arises at temperatures above a sharp peak in the density of states. The physics discussed here instead arise at temperature within a broader, power law or logarithmic, peak that is located at the Fermi energy. The density of states of the hot electrons diverges while W_h remains finite.

The lifetime of cold fermions due to $cc \rightarrow ch$ scattering can again be computed from [1] and [2] above. The energies are now $\omega \sim \Omega \sim \Omega' \sim T \ll E_{Fc}$. The dispersion of the cold electrons can be linearized about their Fermi surfaces, as previously. The hot electrons obey

$$\text{Im } G_h^R(\omega, \vec{k}) = \frac{1}{\omega} \left(\frac{W_h}{\omega} \right)^\eta G \left(\frac{\omega}{k^z} \frac{k_{Fh}^z}{W_h} \right), \quad [8]$$

for some scaling function G and, for generality, allowing for an anomalous dimension η for the hot electrons. Here k_{Fh} is a characteristic momentum scale of the hot fermions. This anomalous dimension further shifts the exponent in the density of states 7, so that $\nu_h(\epsilon) \sim \int d^d k \text{Im } G_h^R(\epsilon, \vec{k}) \sim \epsilon^{d/z-1-\eta}$. The simplest case of free electrons is $\text{Im } G_h^R(\omega, \vec{k}) = \pi \delta(\omega - k^z W_h/k_{Fh}^z)$. The scattering kinematics are similar to the cases considered previously and are described in detail in *SI Appendix*. The resulting scattering rate follows from scaling: Of the two momentum integrals in [1] and [2], one is over the hot region in the Brillouin zone, with $|\vec{k}' + \vec{q}| \sim T^{1/z}$, while the other is over the cold Fermi surface. The constraint that $\vec{k}, \vec{k} - \vec{q}$ and \vec{k}' must be near the Fermi surface means this latter integral is over a region of size k_{Fc}^{d-2} . All told, we obtain the decay rate

$$\Gamma_c \sim \lambda^2 \frac{k_{Fc}^{d-2}}{v_{Fc}^2} T \int_{-T}^T d\epsilon \nu_h(\epsilon) \sim \lambda^2 \frac{k_{Fh}^d k_{Fc}^{d-2}}{v_{Fc}^2} \frac{k_B T}{\hbar} \left(\frac{k_B T}{W_h} \right)^{d/z-\eta}. \quad [9]$$

The scattering rate 9 is the number of states in the peak that are thermally accessible. In the final term we reinserted factors of \hbar and k_B .

There can be logarithmic corrections to [9]. For example, a van Hove point in two dimensions has $z = 2$ with the dispersion $\omega \sim k_x^2 - k_y^2 = k^2 \cos(2\theta)$. This leads to a logarithmic divergence in the integrals over q' in [2] at $\theta \approx \pm\pi/4$. Assuming the anomalous dimension η is small, we obtain $\Gamma_c \sim T^2 \log(W_h/T)$. Another interesting general case is a multicritical point with $z = 4$ in two dimensions (28, 33). As long as the anomalous dimension η is small, this gives $\Gamma_c \sim T^{3/2}$. Both of these two scalings of Γ_c with temperature have been obtained previously in ref. 34 from a variational Boltzmann equation computation. Band structure effects on transport are also considered in refs. 35 and 36. In our discussion above, the scattering mechanism responsible for the resistivity is physically transparent.

Scattering into the van Hove Point in Sr_2RuO_4

Under uniaxial pressure, Sr_2RuO_4 undergoes a Lifshitz transition, traversing a van Hove singularity at a critical compressive strain ϵ_{vH} (37). Resistivity measurements on very pure samples show a characteristic strange metal “fan” emanating from a critical strain at zero temperature (13). Similar behavior has been found upon traversing the van Hove point by chemical doping

(38, 39) or by epitaxial strain (40). Outside the fan, the resistivity $\rho \sim T^2$. In the uniaxial strain experiment the resistivity near the critical strain fits $\rho \sim T^2 \log(T_{vH}/T)$, with $T_{vH} = 230$ K, to excellent accuracy over a range of 40 times the residual resistivity (13). This is precisely the behavior obtained above for $cc \rightarrow ch$ scattering near a van Hove singularity. The single-particle scattering rate scales as T due to $ch \rightarrow ch$ processes (41).

At high temperatures, the resistivity of unstrained Sr_2RuO_4 gradually crosses over from quadratic to linear temperature dependence. The linear regime onsets at about 600 K (42). It is interesting to note that this temperature is roughly comparable to the scale T_{vH} mentioned above. This conceivably suggests that the high-temperature T -linear behavior might also be due to $cc \rightarrow ch$ scattering into the band containing the van Hove point, although the hot band is not small in this case.

Outlook

We have shown that the strange metal phenomenology of two strontium ruthenates can be understood from the $cc \rightarrow ch$ scattering of cold electrons into small regions of hot, nondegenerate electrons. This simple process is a fermionic analogue of the well-established T -linear scattering rate that arises from scattering off classical bosons. It is distinct, however, from scattering off a classical fermion, which would correspond to $ch \rightarrow ch$.[†]

In $\text{Sr}_3\text{Ru}_2\text{O}_7$ in particular, the proposed dominance of $cc \rightarrow ch$ scattering successfully explains quantitative properties of dc and optical charge transport and specific heat. It will be interesting to see whether more subtle observables such as the Hall coefficient, which shows a dramatic change in behavior across T_{tr} (46, 47), can also be understood from this perspective. The thermopower is also a potentially sensitive probe of the presence of nondegenerate electrons.

Our description of $\text{Sr}_3\text{Ru}_2\text{O}_7$ is structured around the presence of a peak in the density of states at zero field that collapses toward the Fermi energy as $H \rightarrow H_c$. These statements are well grounded experimentally. The most speculative aspect of our discussion has been the assumption that the width of this peak becomes small (of order 1 K) at the critical field. This is necessary within our framework to explain how the T -linear resistivity and $T \log 1/T$ specific heat are able to extend down to the lowest temperatures. Direct experimental confirmation of the narrowing of the peak as it moves toward the Fermi energy is needed to fully justify (or falsify) this aspect of our description. Existing scanning tunneling microscopy (STM) data as a function of field do not clearly show the peak moving toward the Fermi energy (48) and are therefore difficult to square with several aspects of the phenomenology.

Turning to other families of strange metals, heavy fermion systems contain narrow peaks in the density of states. A dichotomy between hot and cold fermions may be at work in those cases also, and hence they are candidates for the approach we have taken here. For more general strange metals, however, the following caveat should be kept in mind. While controlled quantum Monte Carlo modeling of, e.g., density wave or Ising nematic criticality in metals clearly shows distinct hot and cold regions of the Fermi surface (49), the hot regions do not necessarily lead to peaks in the density of states because the spectral weight of the hot fermions can vanish in tandem with a divergent mass (50). It remains to be understood how variants of the $cc \rightarrow ch$ scattering mechanism identified in this work could underpin the strange metallic physics arising in those cases.

[†]In SDW transitions, scattering off composite operators at a hot spot has been argued to lead to “lukewarm” fermions over the entire Fermi surface (43–45). This is analogous to $ch \rightarrow ch$ scattering in our framework. In particular, the fact that the scattering is small angle means that it does not strongly impact transport. It may be interesting to look at $cc \rightarrow ch$ scattering in those models.

Data Availability

The data and codes used in this work are available upon request.

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1. S. Sachdev, B. Keimer, Quantum criticality. *Phys. Today*, **64**, 29–35 (2011).
2. S. A. Grigera *et al.*, Magnetic field-tuned quantum criticality in the metallic ruthenate $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Science* **294**, 329–332 (2001).
3. A. Tamai *et al.*, Fermi surface and van Hove singularities in the itinerant metamagnet $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. Lett.* **101**, 026407 (2008).
4. R. Hlubina, T. M. Rice, Resistivity as a function of temperature for models with hot spots on the Fermi surface. *Phys. Rev. B* **51**, 9253–9260 (1995).
5. A. Rosch, Interplay of disorder and spin fluctuations in the resistivity near a quantum critical point. *Phys. Rev. Lett.* **82**, 4280–4283 (1999).
6. C. M. Varma, “Theoretical framework for the normal state of copper oxide metals” in *Strongly Correlated Electronic Systems, the Los Alamos Symposium 1991*, A. F. Hebard *et al.*, Eds. (Addison-Wesley, Reading MA, 1994), pp. 573–603.
7. A. E. Ruckenstein, C. M. Varma, A theory of marginal Fermi-liquids. *Phys. C Supercond.* **185–189**, 134–140 (1991).
8. L. Capogna *et al.*, Sensitivity to disorder of the metallic state in the ruthenates. *Phys. Rev. Lett.* **88**, 076602 (2002).
9. J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie, Similarity of scattering rates in metals showing T-linear resistivity. *Science* **339**, 804–807 (2013).
10. A. W. Rost *et al.*, Thermodynamics of phase formation in the quantum critical metal $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Proc. Natl. Acad. Sci. U.S.A.* **108**, 16549–16553 (2011).
11. C. Mirri *et al.*, Anisotropic optical conductivity of $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. B* **78**, 155132 (2008).
12. A. W. Rost, R. S. Perry, J.-F. Mercure, A. P. Mackenzie, S. A. Grigera, Entropy landscape of phase formation associated with quantum criticality in $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Science* **325**, 1360–1363 (2009).
13. M. E. Barber, A. S. Gibbs, Y. Maeno, A. P. Mackenzie, C. W. Hicks, Resistivity in the vicinity of a van Hove singularity: Sr_2RuO_4 under uniaxial pressure. *Phys. Rev. Lett.* **120**, 076602 (2018).
14. K. Kitagawa *et al.*, Metamagnetic quantum criticality revealed by ^{17}O -NMR in the itinerant metamagnet $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. Lett.* **95**, 127001 (2005).
15. D. Sun, A. W. Rost, R. S. Perry, A. P. Mackenzie, M. Brando, Low temperature thermodynamic investigation of the phase diagram of $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. B* **97**, 115101 (2018).
16. C. Lester *et al.*, Field-tunable spin-density-wave phases in $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Nat. Mater.* **14**, 373–378 (2015).
17. M. P. Allan *et al.*, Formation of heavy d-electron quasiparticles in $\text{Sr}_3\text{Ru}_2\text{O}_7$. *New J. Phys.* **15**, 063029 (2013).
18. J.-F. Mercure *et al.*, Quantum oscillations near the metamagnetic transition in $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. B* **81**, 235103 (2010).
19. R. J. Cava *et al.*, $\text{Sr}_2\text{RuO}_4 \cdot 0.25 \text{CO}_2$ and the synthesis and elementary properties of $\text{Sr}_3\text{Ru}_2\text{O}_7$. *J. Solid State Chem.* **116**, 141–145 (1995).
20. S.-I. Ikeda, Y. Maeno, S. Nakatsuji, M. Kosaka, Y. Uwatoko, Ground state in $\text{Sr}_3\text{Ru}_2\text{O}_7$: Fermi liquid close to a ferromagnetic instability. *Phys. Rev. B* **62**, R6089–R6092 (2000).
21. C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, A. E. Ruckenstein, Phenomenology of the normal state of Cu-O high-temperature superconductors. *Phys. Rev. Lett.* **63**, 1996–1999 (1989).
22. G. D. Mahan, *Many-Body Physics* (Springer, 2000).
23. J. Zaanen, Why the temperature is high. *Nature* **430**, 512–513 (2004).
24. K. Kadowaki, S. B. Woods, Universal relationship of the resistivity and specific heat in heavy-fermion compounds. *Solid State Commun.* **58**, 507–509 (1986).
25. R. S. Perry *et al.*, Metamagnetism and critical fluctuations in high quality single crystals of the bilayer ruthenate $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. Lett.* **86**, 2661–2664 (2001).
26. A. J. Millis, A. J. Schofield, G. G. Lonzarich, S. A. Grigera, Metamagnetic quantum criticality in metals. *Phys. Rev. Lett.* **88**, 217204 (2002).
27. Y. Tokiwa, M. Mchlatw, R. S. Perry, P. Gegenwart, Multiple metamagnetic quantum criticality in $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. Lett.* **116**, 226402 (2016).
28. D. V. Efremov *et al.*, Multicritical Fermi surface topological transitions. *Phys. Rev. Lett.* **123**, 207202 (2018).
29. F. Ronning *et al.*, Thermal conductivity in the vicinity of the quantum critical end point in $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. Lett.* **97**, 067005 (2006).
30. A. W. Rost *et al.*, Power law specific heat divergence in $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Status Solidi B* **247**, 513–515 (2010).
31. A. P. Mackenzie *et al.*, Quantum criticality and the formation of a putative electronic liquid crystal in $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. C Supercond.* **481**, 207–214 (2012).
32. A. J. Millis, Effect of a nonzero temperature on quantum critical points in itinerant fermion systems. *Phys. Rev. B* **48**, 7183–7196 (1993).
33. N. F. Q. Yuan, H. Isobe, L. Fu, Magic of high order van Hove singularity. arXiv: 1901.05432 (16 January 2019).
34. R. Hlubina, Effect of impurities on the transport properties in the van Hove scenario. *Phys. Rev. B* **53**, 11344–11347 (1996).
35. J. M. Buhmann, Unconventional scaling of resistivity in two-dimensional Fermi liquids. *Phys. Rev. B* **88**, 245128 (2013).
36. F. Š. Herman, J. Buhmann, M. H. Fischer, M. Sigrist, Deviation from Fermi-liquid transport behavior in the vicinity of a van Hove singularity. *Phys. Rev. B* **99**, 184107 (2019).
37. A. Steppe *et al.*, Strong peak in T_C of Sr_2RuO_4 under uniaxial pressure. *Science* **355**, eaaf9398 (2017).
38. N. Kikugawa, C. Bergemann, A. P. Mackenzie, Y. Maeno, Band-selective modification of the magnetic fluctuations in Sr_2RuO_4 : A study of substitution effects. *Phys. Rev. B* **70**, 134520 (2004).
39. K. M. Shen *et al.*, Evolution of the Fermi surface and quasiparticle renormalization through a van Hove singularity in $\text{Sr}_{2-y}\text{La}_y\text{RuO}_4$. *Phys. Rev. Lett.* **99**, 187001 (2007).
40. B. Burganov *et al.*, Strain control of fermiology and many-body interactions in two-dimensional ruthenates. *Phys. Rev. Lett.* **116**, 197003 (2016).
41. P. C. Pattnaik, C. L. Kane, D. M. Newns, C. C. Tsuei, Evidence for the van Hove scenario in high-temperature superconductivity from quasiparticle-lifetime broadening. *Phys. Rev. B* **45**, 5714–5717 (1992).
42. A. W. Tyler, A. P. Mackenzie, S. NishiZaki, Y. Maeno, High-temperature resistivity of Sr_2RuO_4 : Bad metallic transport in a good metal. *Phys. Rev. B* **58**, R10107–R10110 (1998).
43. S. A. Hartnoll, D. M. Hofman, M. A. Metlitski, S. Sachdev, Quantum critical response at the onset of spin-density-wave order in two-dimensional metals. *Phys. Rev. B* **84**, 125115 (2011).
44. A. V. Chubukov, D. L. Maslov, V. I. Yudson, Optical conductivity of a two-dimensional metal at the onset of spin-density-wave order. *Phys. Rev. B* **89**, 155126 (2014).
45. E. Abrahams, J. Schmalian, P. Wölfle, Strong-coupling theory of heavy-fermion criticality. *Phys. Rev. B* **90**, 045105 (2014).
46. R. S. Perry *et al.*, Hall effect of $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. B Condens. Matter* **284–288**, 1469–1470 (2000).
47. Y. Liu *et al.*, Electrical transport properties of single-crystal $\text{Sr}_3\text{Ru}_2\text{O}_7$: The possible existence of an antiferromagnetic instability at low temperatures. *Phys. Rev. B* **63**, 174435 (2001).
48. K. Iwaya *et al.*, Local tunneling spectroscopy across a metamagnetic critical point in the bilayer ruthenate $\text{Sr}_3\text{Ru}_2\text{O}_7$. *Phys. Rev. Lett.* **99**, 057208 (2007).
49. E. Berg, S. Lederer, Y. Schattner, S. Trebst, Monte Carlo studies of quantum critical metals. *Annu. Rev. Condens. Matter Phys.* **10**, 63–84 (2019).
50. Y. Schattner, M. H. Gerlach, S. Trebst, E. Berg, Competing orders in a nearly antiferromagnetic metal. *Phys. Rev. Lett.* **117**, 097002 (2016).