On Information Transfer Based Characterization of Power System Stability

Subhrajit Sinha, Pranav Sharma, Umesh Vaidya and Venkataramana Ajjarapu

***Abstract*—In this paper, we present a novel approach to identify the generators and states responsible for the small-signal stability of power networks. To this end, the newly developed notion of information transfer between the states of a dynamical system is used. In particular, using the concept of information transfer, which characterizes influence between the various states of a dynamical system, we identify the generators and states which are responsible for causing instability of the power network. While characterizing influence from state to state, information transfer can also describe influence from state to modes thereby generalizing the well-known notion of participation factor while at the same time overcoming some of the limitations of the participation factor. The developed framework is applied to reproduce known results for the three bus system, identifying the various causes of instabilities, and is extended to IEEE 39 bus system.**

I. Introduction

The electric power grid is one of the largest network systems with thousands of components having complicated dynamics and topology. As such, control and maintaining reliable operation of a power network is challenging and difficult and one of the main challenges is the maintenance of stability of power networks. For a large system, it is usually challenging to identify the sources of instability and hence a *a priori* knowledge of the origins of instability would allow one to take precautionary measures before a power network becomes unstable.

Power system stability is the ability of the system to reach an operating equilibrium after disturbance from an initial operating condition where most system variables remain bounded in their practical limits [1]. Inherently, instability is a single phenomenon; however, for the ease of understanding, instabilities are classified based on time scale, magnitude and parameters causing them. In order to understand the behavior of a complex power network, we consider the linear behavior of the system in close affinity of an operating equilibrium. This study of power system is categorized as *small signal analysis* [1]. Small signal analysis captures system behavior and the interaction of dynamic components without analyzing complex non-linear system models. In [2], [3] the authors defined the concept of *participation factor* to analyze these linearized dynamic models. Participation factor is based on eigenvalue analysis and defines the influence of any single state on any system mode. This method is thus far prevalent in power system studies for model reduction, optimal control placement and identifying resonance interaction [4]. However, in [5], [6] authors identified anomalies in the participation factor definition and showed how the existing definition fails in even two dimensional linear systems. The authors further provided a new definition for participation factor which accounts for uncertainties in system states, which increases the computation involved in participation factor analysis. The other drawback associated with existing framework for modal analysis is that it only quantifies participation of an individual state into each mode. For a realistic power network with a considerable number of dynamical states, it becomes computationally challenging to evaluate system behavior at a macro level [7]. Thus a method is needed, where the influence of physical components and subsystems connected in a power system can be considered collectively.

In this paper, we propose a novel method to address the problem of identification of state(s) responsible for instability of a power network. In particular, we use the newly developed measure of causality and influence in a dynamical system [8], [9] to identify the state(s) causing instability. Causality and influence characterization has been an active area of research in various fields including neuroscience, biology, economics, etc. and as such there have been many different definitions and measures of causality with Granger causality [10], [11] being the most commonly used definition. Massey and Kramer [12], [13] generalized Shannon's information theory to incorporate a sense of direction and thus defined directed information. In the realm of dynamical systems, Liang and Kleeman formulated a new notion of causality [14] and showed that the measure captures some of the intuitions of causality. However, in [8], it was shown that in dynamical system setting, all these measures fail to capture the true notion of causality and in [8], [9], using the notion of information flow in a dynamical system, the authors provided a new definition of causality and showed how the new definition captures the correct causal structure in a dynamical system. Information transfer quantifies the influence of any state(s) on any other state(s), that is, how the evolution of a state(s) affect the evolution of any other state(s). In [15], [16] the authors used the notion of information flow in a control dynamical system for stability analysis of a power network. However, the results in [15], [16] are based on eigenvalue and eigenvector decomposition of the linearized model of the system and as such, as the size of the power network grows, it becomes computationally challenging.

The prime contribution of the paper is providing an alternative approach for influence characterization in a power network and use it for identification of state(s) responsible for instability. The significant advantages of this approach are

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two-fold. Firstly, this method does not require the computation of eigenvalue and eigenvectors, as required by the existing modal analysis. Secondly, unlike participation factor analysis, information transfer allows studying the influence of any subspace (states clubbed together) on any other subspace. This not only reduces the computation cost but also enables the study of the power network from a macro level (without focussing on individual states) such that each subspace has physical meaning. For example, one can study the information flow between the generator subspaces and identify which generator as a whole is responsible for instability. This has been illustrated in the simulations, where we study the IEEE 39 bus system.

The structure of this paper is as follows. In section II, we review the concept of information transfer and provide a physical interpretation for stability analysis and participation from information transfer standpoint. In section III we discuss the advantages of information transfer over the existing method of eigenvalue analysis and participation factor. In section IV we take a tutorial example of three bus system to explain the notion of information transfer for power system analysis for identifying states responsible for instability. Further, in section V we analyze the IEEE 39 bus system for identifying instabilities and responsible states (and generators). Finally, we conclude the paper in section VI, while highlighting the future implications of this work.

II. Information Transfer Based Participation and Stability Analysis

In this section, we briefly describe the concept of information transfer in a dynamical system [8], [9].

Consider the discrete time dynamical system

$$z(t+1) = S(z(t)) + \xi(t) \tag{1}$$

where $z = \begin{pmatrix} x^\top & y^\top \end{pmatrix}^\top \in \mathbb{R}^N$, $S : \mathbb{R}^N \to \mathbb{R}^N$ is assumed to be at least continuous and $\xi(t)$ is independent and identically distributed additive noise, which comes from the distribution g.

Information transfer from state (subspace) x to state (subspace) y gives a measure of how the evolution of x dynamics affect (influence) the evolution of y dynamics. In particular, we quantify this influence in terms of the entropy transferred from the x dynamics to the y dynamics, as the system (1) evolves in time. Note that, by entropy we mean the Shannon entropy and is measured in bits. Shannon entropy for a probability distribution is defined as follows :

Definition 1 (Shannon Entropy)**.** *Given a probability distribution $\rho(z)$, defined over the sample space Ω, the entropy of the distribution (H) is defined as*

$$H = -\int_\Omega \rho(z) \log \rho(z) dz \tag{2}$$

The entropy of a distribution is the measure of the information content of the distribution. Suppose there are two agents (states in the case of a dynamical system) which are interacting with each other. Each has its own entropy (information) and they *transfer* a part of their own information to the other agent via the interaction. We use this intuition to define the information transfer. In particular, information transfer from x to y is the amount of information (entropy) of x that is being transferred to y, the system (1) evolves in time. With this, we define the information transfer as follows.

Definition 2 (Information Transfer)**.** *The information transfer from a state x to state y $(T_{x\to y})$, as the dynamical system $z(t+1) = S(z(t)) + \xi(t)$ evolves from time step t to time step $t+1$ is defined as*

$$T_{x\to y} = H(y(t+1)|y(t)) - H_{\not{x}}(y(t+1)|y(t)) \tag{3}$$

where $z = \begin{pmatrix} x & y \end{pmatrix}^\top$, $H(y(t+1)|y(t))$ is the entropy of y at time $t+1$ conditioned on $y(t)$ and $H_{\not{x}}(y(t+1)|y(t))$ is the conditional entropy of $y(t+1)$, conditioned on $y(t)$, when x is held frozen.

The above definition of information transfer is for any time step t, transient or steady state. In this paper, we work mostly with steady state information transfer (except for the mass-spring system later in this section and the example on information transfer to a mode in section III), which is defined as follows.

Definition 3 (Steady State Information Transfer)**.** *The steady state information transfer from a state x to state y $(T^{ss}_{x\to y})$, for the dynamical system $z(t+1) = S(z(t)) + \xi(t)$ is defined as*

$$T^{ss}_{x\to y} = \lim_{t\to\infty} [H(y(t+1)|y(t)) - H_{\not{x}}(y(t+1)|y(t))] \tag{4}$$

provided the limit exists and is finite.

Note that unlike transient information transfer in Eq. (3), the steady state information transfer may or may not be well defined. However under the added assumption that the system dynamics are stable we can show that all the information transfers are well defined and finite (see Theorem 6 below). The intuition behind the definition of information transfer is the fact that the total entropy of y is the entropy of y when x is absent from the dynamics plus the entropy transferred from x to y. Hence, the information transfer from x to y gives the amount of entropy flowing from x to y and thus quantifies how much the x dynamics affects the y dynamics. With this, we define influence in a dynamical system as follows;

Definition 4 (Influence)**.** *We say a state (or subspace) x influences a state (or subspace) y if and only if the information transfer from x to y is non-zero.*

The larger the absolute value of $T_{x\to y}$, greater is the effect of x dynamics on y and hence larger is the influence of x on y. For general nonlinear systems it is often not possible to find close-form expressions for evolution of entropy, and hence the information transfer can be calculated numerically only. However, for linear systems of the form

$$z(t+1) = Az(t) + \sigma\xi(t) \tag{5}$$

where $z(t) \in \mathbb{R}^N$, $\sigma > 0$ is a constant and $\xi(t)$ is vector valued Gaussian random variable with zero mean and unit variance, the information transfer from any state z_i to z_j can be expressed as a closed form expression. In particular, we have the following theorem [8], [9].

Theorem 5. *Consider the linear dynamical system (5). We have the following expression for the transient information transfer between various subspaces*

$$[T_{x_1\to y}]_t^{t+1} = \frac{1}{2}\log\frac{|A_{yx}\Sigma_y^s(t)A_{yx}^\top + \sigma^2 I|}{|A_{yx_2}(\Sigma_y^s)_{yx_2}(t)A_{yx_2}^\top + \sigma^2 I|} \tag{6}$$

where

$$A = \begin{pmatrix} A_x & A_{xy} \\ A_{yx} & A_y \end{pmatrix} = \begin{pmatrix} A_{x_1} & A_{x_1x_2} & A_{x_1y} \\ A_{x_2x_1} & A_{x_2} & A_{x_2y} \\ A_{yx_1} & A_{yx_2} & A_y \end{pmatrix}, \tag{7}$$

$$\Sigma = \begin{pmatrix} \Sigma_x & \Sigma_{xy} \\ \Sigma_{xy}^\top & \Sigma_y \end{pmatrix} = \begin{pmatrix} \Sigma_{x_1} & \Sigma_{x_1x_2} & \Sigma_{x_1y} \\ \Sigma_{x_1x_2}^\top & \Sigma_{x_2} & \Sigma_{x_2y} \\ \Sigma_{x_1y}^\top & \Sigma_{x_2y}^\top & \Sigma_y \end{pmatrix}. \tag{8}$$

$\Sigma(t)$ is the covariance matrix of the states at time t ($\Sigma(t) = E[x(t)x(t)^\top]$) evolving according to the Lyapunov equation $\Sigma(t+1) = A\Sigma(t)A^\top + \xi(t)$, $\Sigma_y^s(t) = \Sigma_x(t) - \Sigma_{xy}(t)\Sigma_y(t)^{-1}\Sigma_{xy}(t)^\top$ is the Schur complement of $\Sigma_y(t)$ in the matrix $\Sigma(t)$, $|\cdot|$ is the determinant and $(\Sigma_y^s)_{yx_2}(t)$ is the Schur complement of $\Sigma_y(t)$ in the matrix

$$\begin{pmatrix} \Sigma_{x_2}(t) & \Sigma_{x_2y}(t) \\ \Sigma_{x_2y}^\top(t) & \Sigma_y(t) \end{pmatrix}.$$

Setting $z_i = x_1$, $z_j = y$ and $\{z_1,\cdots,z_N\}\setminus\{z_i,z_j\} = x_2$, one obtains the information transfer from any state z_i to any state z_j.

Moreover, when the steady state information transfer exists, then the steady state information transfer is given by

$$T_{x_1\to y}^{ss} = \frac{1}{2}\log\frac{|A_{yx}\Sigma_y^s A_{yx}^\top + \sigma^2 I|}{|A_{yx_2}(\Sigma_y^s)_{yx_2}A_{yx_2}^\top + \sigma^2 I|} \tag{9}$$

where Σ is the solution of the Lyapunov equation $\Sigma = A\Sigma A^\top + \sigma^2 I$, Σ_y^s is the Schur complement of Σ_y in the matrix Σ and $(\Sigma_y^s)_{yx_2}$ is the Schur complement of Σ_y in the matrix

$$\begin{pmatrix} \Sigma_{x_2} & \Sigma_{x_2y} \\ \Sigma_{x_2y}^\top & \Sigma_y \end{pmatrix}.$$

The information transfer thus defined can be extended to define information transfer between the various signals in a control dynamical system, namely information transfer from input to state, input to output and state to output. For details see [8].

To convey the meaning of information transfer, we consider a mass-spring-damper system depicted in Fig. 1.

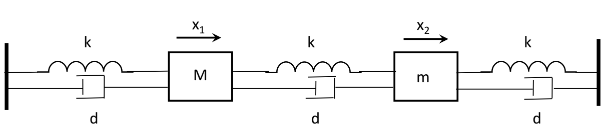


Fig. 1. Mass-spring-damper system

The equations of motion for the mass-spring system are

$$M\ddot{x}_1 + 2d\dot{x}_1 - d\dot{x} + +2kx_1 - kx_2 = 0 \tag{10}$$
$$m\ddot{x}_2 + 2d\dot{x}_2 - d\dot{x}_1 + 2kx_2 - kx_1 = 0 \tag{11}$$

where M, m are the masses, d is the damping coefficient and k is the spring constant. We assume that the damping coefficients of the dampers are equal and so are the spring constants of the springs. For simulation purposes, we choose $M = 10$, $m = 3$, $k = 20$ and $d = 1.5$ with appropriate units. Since $M > m$, a perturbation (perturbed so that it has some non-zero initial position) in the mass M will result in larger oscillations in the masses, compared to the case when the mass m is perturbed by the same amount. Hence, we can conclude that the mass M has a large influence on the smaller mass m, whereas, m has a much smaller influence on M.

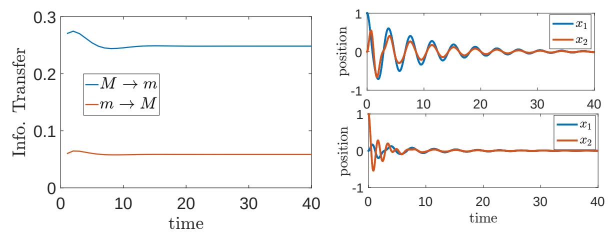


Fig. 2. (a) Information transfer between the two masses. (b) The top figure shows the position of the two masses when the bigger mass is at an initial position of 1 unit and the lower figure shows the position of the two masses when the smaller mass is at an initial position of 1 unit.

Information transfer between the two masses is shown in Fig. 2(a). The information transfer plotted are from $(x_1, \dot{x}_1)$ subspace to $(x_2, \dot{x}_2)$ subspace and vice versa. As can be seen from the figure, the larger mass has a greater influence on the smaller mass, as compared to the influence of smaller mass on the larger mass. This is also verified from the time-domain plots in Fig. 2(b). In both the plots, we plot the position of the two masses. In the top figure, the larger mass was initialized at 1 unit (that is, its initial position was set to 1 unit), and the system was allowed to evolve from this initial condition. In the bottom figure, the initial position of the smaller mass was initialized at 1 unit and the system was allowed to evolve from this initial condition. From Fig. 2(b), it can be observed that when the larger mass initialized at 1 unit, oscillations sustain for a longer time as compared to the case when the smaller mass is initialized at 1 unit. In fact, when M is initialized to the non-zero initial condition, the variance of the position of m is 0.0313, whereas when m is initialized to the same non-zero initial condition the variance of the position of M is 0.0030. Hence, time-domain simulations confirm the conclusions obtained using information transfer measure.

A. Information Transfer and Stability

As discussed earlier, information transfer can be used to measure the influence of one state variable on another state. However, the state to state information transfer can also be used as an indicator of the instability of a system. In particular we have the following theorem connecting information transfer and stability of the system matrix.

Theorem 6. *Consider the linear system*

$$x(t+1) = Ax(t) + \xi(t) \tag{12}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, $t \in \mathbb{Z}_{\geq 0}$ is the time parameter, taking values in non-negative integers, $A \in \mathbb{R}^{n\times n}$ is the system matrix and $\xi(t)$ is a zero mean i.i.d. Gaussian noise with covariance $Q = E[\xi(t)\xi(t)^\top]$. Then the system matrix A is Hurwitz if and only if all the steady state information transfers as defined in (6) are well defined and converge to a steady state value.

Proof. For the system (12), let $\Sigma(t) = E[x(t)x(t)^\top]$ be the covariance matrix of the states at time t. Then, since $x(t)$ and $\xi(t)$ are uncorrelated for all t, we have

$$\Sigma(t+1) = A^{t+1}\Sigma(0)(A^\top)^{t+1} + \sum_{k=0}^{t} A^k Q (A^\top)^k$$

where $\Sigma(0)$ is the covariance of the states at time 0. Now, both $\lim_{t\to\infty} A^{t+1}\Sigma(0)(A^\top)^{t+1}$ and $\lim_{t\to\infty}\sum_{k=0}^{t} A^k Q(A^\top)^k$ converges if and only if $\rho(A) < 1$, where $\rho(A)$ is the spectral radius of A. Hence the matrix $\lim_{t\to\infty}\Sigma(t+1)$ converges to a finite limit Σ if and only if $\rho(A) < 1$. The finite Σ, thus obtained, is the steady state covariance matrix. Hence the determinant is also finite. The information transfer from any state x_i to any state x_j, as the system evolves from time t to time $t+1$, for the system (12)[1] is

$$[T_{x_i\to x_j}]_t^{t+1} = \frac{1}{2}\log\frac{|A_{x_j x_{\not j}}\Sigma^s_{x_j}(t)A^\top_{x_j x_{\not j}} + \sigma^2 I|}{|A_{x_j x_{\not i \not j}}(\Sigma^s_{x_j})_{x_{\not i}}(t)A^\top_{x_j x_{\not i \not j}} + \sigma^2 I|} \quad (13)$$

where $A_{x_j x_{\not j}} \in \mathbb{R}^{n-1}$ is the j^{th} row of A with i^{th} element removed, $A_{x_j x_{\not i \not j}} \in \mathbb{R}^{n-2}$ is the j^{th} row of A with both i^{th} and j^{th} elements removed and $\Sigma^s_{x_j}(t)$ and $(\Sigma^s_{x_j})_{x_{\not i}}(t)$ are Schur complements of $\Sigma_{x_j}(t) = E[x_j(t)x_j(t)^\top]$ in $\Sigma(t)$ and $\Sigma_{x_{\not i}}(t)$ respectively. Here $\Sigma_{x_{\not i}}(t) \in \mathbb{R}^{(n-1)\times(n-1)}$ is obtained from $\Sigma(t)$ by deleting the i^{th} row and i^{th} column. The steady state information transfer is defined as the limit $\lim_{t\to\infty}[T_{x_i\to x_j}]_t^{t+1}$ and is obtained by replacing $\Sigma(t)$ by the steady state covariance matrix Σ in (13).

Now, the steady state covariance Σ exists and is finite if and only if A is stable. Hence, if A is stable, then the steady state covariance matrix exists and is finite and hence the Schur complements $\lim_{t\to\infty}\Sigma^s_{x_j}(t)$ and $\lim_{t\to\infty}(\Sigma^s_{x_j})_{x_{\not i}}(t)$ exists and is finite. Hence all the steady state information transfers $T_{x_i\to x_j}$ exists and is finite.

On the other hand, assume that all the steady state transfers $T_{x_i\to x_j}$ exist and are finite. Hence, all the Schur complements $\lim_{t\to\infty}\Sigma^s_{x_j}(t)$ and $\lim_{t\to\infty}(\Sigma^s_{x_j})_{x_{\not i}}(t)$ exists and are finite. Hence, $\lim_{t\to\infty}\Sigma_{x_j}(t)$ exists and is also finite. Hence $\lim_{t\to\infty}\Sigma(t) = \lim_{t\to\infty}\Sigma_{x_j}(t)\lim_{t\to\infty}\Sigma^s_{x_j}(t)$ exists and is finite. Now, $\lim_{t\to\infty}\Sigma(t)$ exists and is finite if and only if A is stable. Hence, if all the information transfers $T_{x_i\to x_j}$ exists and are finite, it implies that A is stable. Hence we have the steady state information transfer from any state x_i to x_j is finite if and only if A is stable. ☐

[1]We have a simplifying assumption that $Q = \sigma^2 I$. The information transfer formula remains valid for any i.i.d. Gaussian noise ξ_t with covariance matrix Q. The $\sigma^2 I$ part in the formula will be replaced by appropriate blocks of the matrix Q.

For example, consider a linear system

$$\begin{aligned} x(t+1) &= 0.4x(t) + 0.2y(t) + \xi_x(t) \\ y(t+1) &= \mu y(t) + \xi_y(t) \end{aligned} \quad (14)$$

where $\mu \in [-0.99, 0.99]$ and $\xi_x(t)$ and $\xi_y(t)$ are i.i.d. Gaussian noises of unit variance. The eigenvalues of the system are $(0.4, \mu)$ and hence as μ approaches 1, the system approaches instability. The instability occurs due to y dynamics and as μ increases, the entropy of y increases rapidly. Hence, the steady state information transfer from y to x also increases rapidly as $|\mu|$ approaches 1. This is shown in Fig. 3. Conversely, if the

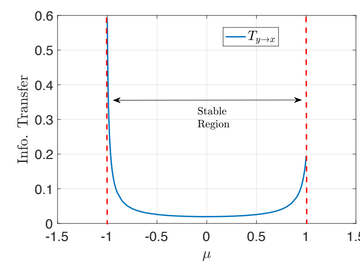


Fig. 3. Steady state information transfer increases rapidly as the system approaches instability.

information transfer from some state (subspace) to any other state (subspace) increases rapidly, it can be concluded that the system is approaching instability. The point to be noted is that the rapid increase in the steady state information transfer happens when the system is still operating in the stable zone. Hence, information transfer acts as an indicator and can be used to predict the onset of instability, allowing one to take preventive measures before the system becomes unstable.

III. Information Transfer and Participation Factor

Participation matrix P [17] for a linear system $\dot{x} = Ax$ is defined as $P = [p]_{ki} = \mathbf{u}^i_k \mathbf{w}^i_k$, where $\mathbf{u}^i_k$ ($\mathbf{w}^i_k$) is the k^{th} component of the i^{th} left (right) eigenvector corresponding to eigenvalue λ_i of the system A matrix . Here, A is assumed to have distinct eigenvalues and the eigenvectors are assumed to be normalized i.e.,

$$\mathbf{u}_i^\top \mathbf{w}_j = 1, \;\text{ if }\; i = j;\; 0 \;\text{ otherwise.}$$

Participation factor p_{ki} measures the relative participation of k^{th} state variable in the i^{th} mode. Hence, larger the participation factor of a state to a particular mode, larger the contribution of that state to that mode. However, as pointed in [4], [6], [18] participation factor fails to capture the contribution of states in some linear systems as illustrated in the following example highlighted in [6].

$$\dot{x} = \begin{pmatrix} -0.2231 & 3.4657 \\ 0 & -0.9163 \end{pmatrix} x \quad (15)$$

where $x \in \mathbb{R}^2$.

The eigenvalues of the system matrix are $\lambda_1 = -0.2231$ and $\lambda_2 = -0.9163$ and the participation matrix is $P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Hence participation factor predicts that x_1 does not contribute in the mode λ_2 and eigenvector corresponding to λ_2 and x_2 does not contribute in mode λ_1 and the corresponding eigenvector. However,

$$x_1(t) = [0.1961x_1(0) + 0.9806x_2(0)]e^{\lambda_1 t} - 0.9806e^{\lambda_2 t}x_2(0)$$

and so x_2 does *participate* in λ_1. Information transfer, on the other hand illustrates the fact that λ_1 is being influenced by x_2. For this, we discretize the continuous time system, with $\delta t = 1$ and look at the information transfer from x_2 to the eigenvector corresponding to λ_1. For details on computation of information transfer from state to output see [15], [8], [16].

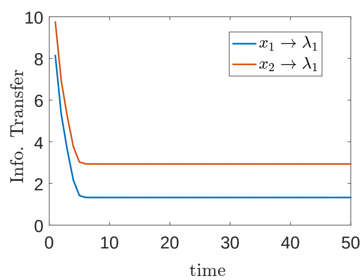


Fig. 4. Information Transfer from system states to the mode λ_1

Fig. 4 shows the information transfer from the states to the mode λ_1 and it can be seen that the information flow $T_{x_2 \to \lambda_1}$ is non-zero. Hence information transfer captures the influence of x_2 on mode λ_1.

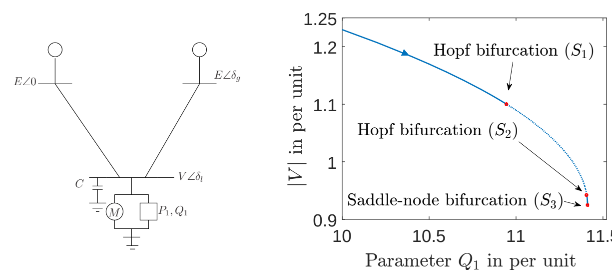
A. Subspace interaction in dynamic system

Information transfer $T_{x \to y}$, as defined in (3), is general enough in the sense that it not only quantifies the influence of one state on another but also from a combination of states (subspace) to any other combination of states (subspace). This definition has a great advantage in power system studies as it reduces the computational burden by utilizing the physical properties of the system and phenomenon. For example, in order to identify areas (generators) participating in an inter-area oscillatory mode using existing modal analysis method, one needs to compute all the eigenvalues and eigenvectors followed by participation factor of all the dynamic states of the system. We then select key states and thereby identify the generators and areas oscillating with respect to each other. However, as inter-area oscillations are properties of the bulk system, we can create subspaces by clustering all dynamic states in an area (or in a generator) into single clusters and then compute information transfer between these subspaces and inter-area modes. Information transfer directly identifies the key areas (generators) participating in a particular oscillatory mode. Thus, information transfer not only reduces the computation burden but also reveals the physical properties of the system in a more organic way.

B. Computational Cost

The known method of modal analysis applied to small signal stability analysis involves computation of eigenvalues and eigenvectors. For a linear system as given in equation (5), where $z(t) \in \mathbb{R}^n$, eigenvalue computation using known methods of Arnoldi iteration, QR algorithm, divide and conquer method and Jacobi eigenvalue algorithm is fairly complex. Eigenvalue computation involves an iterative method to obtain an equivalent diagonal or tridiagonal matrix for the given A matrix. Further, the computation of eigenpairs (eigenvalue and eigenvector) increases the computation complexity. As shown in Table I, the given computation cost of eigenpair computation is for each iteration and the number of iterations depends greatly on the size and structure of matrix [19], [20], [21], [22]. On the other hand information transfer computation is proportional to the order of $O(\bar{n}^{2.3})$, where $\bar{n}$ depends upon the physical nature of the dynamic model and the phenomena under observation. As illustrated in the next section, for 39 bus system, we can reduce the number of dynamic subspaces, to the number of generators, in order to identify critical generators and dynamic states in the system. Details are illustrated in Table I.

IV. Stability Characterization of 3-bus System



(a) (b)

Fig. 5. (a) 3-bus system. (b) Critical points of the system

Consider a power network with two generators and a load, as shown in Fig. 5(a). The load is modelled as an induction motor in parallel with a constant PQ load. The system is modelled as a four-dimensional dynamical system with the state being generator angle (δ_g), generator angular velocity (ω), the load angle (δ_l) and magnitude of load voltage (V). The dynamic equations for the system are

$$\dot{\delta}_g = \omega \tag{16}$$

$$\dot{\omega} = 16.66667\sin(\delta_l - \delta_g + 0.08727)V - 0.16667\omega + 1.88074 \tag{17}$$

$$\dot{\delta}_l = 496.87181V^2 - 166.66667\cos(\delta_l - \delta_g - 0.08727)V - 666.66667\cos(\delta_l - 0.20944)V \quad 93.33333V + 33.33333Q_1 + 43.33333 \tag{18}$$

$$\dot{V} = -78.76384V^2 + 26.21722\cos(\delta_l - \delta_g - 0.01241)V + 104.86887\cos(\delta_l - 0.13458)V + 14.52288V - 5.22876Q_1 - 7.03268 \tag{19}$$

For detailed analysis of the system equations we refer the interested reader to [23], [24].

The above power network has three *critical points*, namely S_1, S_2 and S_3, as shown in Fig. 5(b). At S_1 and S_2, a pair of imaginary eigenvalues cross the imaginary axis and at S_3,

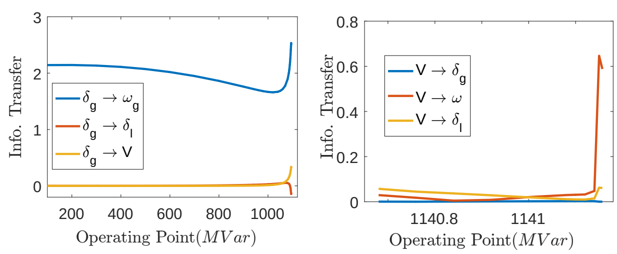
TABLE I
COMPUTATION COST FOR METHODS OF MODAL ANALYSIS

Eigenpair computation method\ Information transfer	Computation Cost (for each iteration)	Description
QR Algorithm	$\mathbb{O}(n^3)$	Factorize $A(=QR)$ into an orthogonal and triangular matrix and apply the next iteration to RQ.
Divide and Conquer Algorithm	$\mathbb{O}(n^3)$	Divide the matrix into subspaces of diagonalized matrices and then recombine.
Information Transfer	$\mathbb{O}(\bar{n}^{2.3})$	where $\bar{n} \in [log(n), n]$ depends upon the physical property of the system or phenomena under observation.

a real eigenvalue becomes zero. Hence, the system becomes unstable at S_1, remains unstable from S_1 to S_2, then regains stability after S_2 and again becomes unstable at S_3. It is known that the instability at S_1 is angle instability and the instability at S_3 is voltage instability. In this section, we demonstrate how information transfer not only identifies the instability but also identify the states responsible for instability.

A. State to State Information and Stability: 3-Bus System

It is known that in the three bus network, there are both angle and voltage instability. In particular, the instability that occurs at S_1 (Fig. 5) is angle instability and the instability that occurs at S_3 is voltage instability [24].



(a) (b)

Fig. 6. (a) Steady state information transfer over the operating points from δ_g before S_1. (b) Steady state information transfer over the operating points from V before S_3.

In Fig. 6(a) we show the information transfer from the angle variable of the generator to all the other states, as the system approaches the first Hopf bifurcation. It can be seen that the information transfer from the angle variable shows a sudden increase. This is consistent with the fact that at S_1, the system undergoes voltage instability and it is the angle variable that causes the instability. Similarly, as the system approaches voltage instability (S_3), the information transfer from the angle variable (V) to all the other states shows the sudden increase. This is shown in Fig. 6(b). Hence, one can identify the states which are responsible for the instability of the power network.

B. Information Transfer & Participation Factor: 3-Bus System

For calculating the participation factors of each state of the 3-bus system (16)-(19), we look at the linearized dynamics at the operating point $Q = 1141.1$ MVar. The rationale behind choosing this operating point is the fact that if Q is increased further, the system undergoes Saddle Node Bifurcation (voltage instability). It is well-known [24] that at SNB, it is the load voltage V that participates the most in the most unstable mode and this is shown in Table II.

TABLE II
PARTICIPATION AND INFORMATION TRANSFER TO MOST UNSTABLE MODE

State.\Index	Participation Factor	Information Transfer
δ_g	0.0137785	0.04
ω	0.00046	0.0094
δ_l	0.00072	0.0189
V	0.9850	1.39884

Algorithm 1 Information transfer computation in power system

1) Given a power system dynamic model as
$\dot{x} = f(x, y) \qquad g(x, y) = 0$
$x \in \mathbb{R}^n$ are the dynamic states and $y \in \mathbb{R}^l$ are the algebraic states of the system, obtain the linearized model at a given operating point (x_o, y_o) [25]. linearize the system by computing $\{\bar{A}, \bar{B}, \bar{C}, \bar{D}\}$ as:
$\bar{A} = \frac{\partial f(x,y)_{(x_o,y_o)}}{\partial x} \qquad \bar{B} = \frac{\partial f(x,y)_{(x_o,y_o)}}{\partial y}$
$\bar{C} = \frac{\partial g(x,y)_{(x_o,y_o)}}{\partial x} \qquad \bar{D} = \frac{\partial g(x,y)_{(x_o,y_o)}}{\partial y}$
2) Reducing system matrix
$$\dot{x} = A_{cont}x = \{\bar{A} - \bar{B}.(\bar{D}^{-1}\bar{C})\}x.$$
3) Obtain an equivalent discrete dynamic model, using time step τ as:
$$A = A_{disc} = e^{A_{cont}\tau}.$$

Computing information transfer

4) Compute steady state information transfer using formula (6) of theorem 5.

Information transfer from the states to the most unstable mode, at this operating point, is shown in Table II. For calculating the information transfer, we discretized the linearized continuous-time model with $\Delta t = 0.001$ and calculated the steady state information transfer from the states to the most unstable mode. It is observed that both information transfer and participation factor identify the load voltage as the state participating in the critical mode, that is leading the system to voltage collapse. Not only does information transfer recognize the load voltage as the state most responsible for instability, but it can be seen that the order of participation/influence is the same with both participation factor and information transfer. So far, we saw that participation factor and information transfer are providing the same insight. However, one of the critical advantages of the information transfer measure is the fact that one can look at information transfer between any combination of states (subspace) or from any combination of state to any output. Hence, in a system with a large number of states, one can treat each generator as a separate subspace and identify which generator is responsible for instability. Once the most critical generator is identified, one can zoom in to the states of the critical generator and determine the state(s) responsible for instability. This reduces the computation cost considerably. For more details on the relationship between participation factor and information transfer, we refer the interested reader to [16].

V. IEEE 39 Bus System

The model used in this section is based on the modeling described in [25]. The power network is described by a set of differential algebraic equations (DAE) and the power system dynamics is divided into three parts: differential equation model describing the generator and load dynamics, algebraic equations at the stator of the generator and algebraic equations describing the network power flow. We consider a power system model with n_g generator buses and n_l load buses. The generator dynamics at each generator bus can be represented as a 4^{th} order dynamical model:

$$\begin{aligned}
\frac{d\delta_i}{dt} &= \omega_i - \omega_s \\
\frac{d\omega_i}{dt} &= \frac{T_{m_i}}{M_i} - \frac{E'_{q_i} I_{q_i}}{M_i} - \frac{(X_{q_i} - X'_{d_i})}{M_i} I_{d_i} I_{q_i} - \frac{D_i(\omega_i - \omega_s)}{M_i} \\
\frac{dE'_{q_i}}{dt} &= -\frac{E'_{q_i}}{T'_{do_i}} - \frac{(X_{d_i} - X'_{d_i})}{T'_{do_i}} I_{d_i} + \frac{E_{fd_i}}{T'_{do_i}} \\
\frac{dE_{fd_i}}{dt} &= -\frac{E_{fd_i}}{T_{A_i}} + \frac{K_{A_i}}{T_{A_i}} (V_{ref_i} - V_i)
\end{aligned}$$

The algebraic equations at the stator of the generator are:

$$\begin{aligned}
&V_i \sin(\delta_i - \theta_i) + R_{s_i} I_{d_i} - X_{q_i} I_{q_i} = 0 \\
&E'_{q_i} - V_i \cos(\delta_i - \theta_i) - R_{s_i} I_{q_i} - X'_{d_i} I_{d_i} = 0 \\
&\text{for} \quad i = 1, \ldots, n_g.
\end{aligned} \tag{20}$$

The network equations corresponding to the real and reactive power at generator and load buses are shown below.

$$\begin{aligned}
&I_{d_i} V_i \sin(\delta_i - \theta_i) + I_{q_i} V_i \cos(\delta_i - \theta_i) + P_{L_i}(V_i) \\
&- \sum_{k=1}^{\bar{n}} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = 0 \\
&I_{d_i} V_i \cos(\delta_i - \theta_i) - I_{q_i} V_i \sin(\delta_i - \theta_i) + Q_{L_i}(V_i) \\
&- \sum_{k=1}^{\bar{n}} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = 0 \\
&\text{for} \;\; i = 1, \ldots, n_g. \\
&P_{L_i}(V_i) - \sum_{k=1}^{\bar{n}} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = 0 \\
&Q_{L_i}(V_i) - \sum_{k=1}^{\bar{n}} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = 0 \\
&\text{for} \;\; i = n_g + 1, \ldots, n_g + n_l.
\end{aligned} \tag{21}$$

here, δ_i, ω_i, E_{q_i}, and E_{fd_i} are the dynamic states of the generator and correspond to the generator rotor angle, the angular velocity of the rotor, the quadrature-axis induced emf and the emf of fast acting exciter connected to the generator respectively. The algebraic states I_{d_i} and I_{q_i} are the direct-axis and quadrature-axis currents induced in the generator respectively. Each bus voltage and its angle are denoted by V_i and θ_i. The parameters $T_{m_i}, V_{ref_i}, \omega_s, M_i$, and D_i are the mechanical inputs and machine parameters applied to the generator shaft, reference voltage, rated synchronous speed, generator inertia, and internal damping. The stator internal resistance is denoted by R_{s_i} and X_{q_i}, X_{d_i}, X'_{d_i} are the quadrature-axis salient reactance, direct-axis salient reactance and direct-axis transient reactance respectively. The exciter gain and time-constant are given by K_{A_i} and T_{A_i}.

A power system stabilizer (PSS), that acts as a local controller to the generator is designed based on the linearized DAEs. The input to the PSS controller is $\omega_i(t)$ and PSS output, $V_{ref_i}(t)$, is fed to the fast acting exciter of the generator. An IEEE Type-I PSS is considered here which consists of a wash-out filter and two phase-lead filters. The transfer function of PSS is given as follows.

$$\frac{\Delta V_{ref_i}(s)}{\Delta \omega_i(s)} = k_{pss} \frac{(1 + sT_{num})^2}{(1 + sT_{den})^2} \frac{sT_w}{1 + sT_w} \tag{22}$$

where k_{pss} is the PSS gain, T_w is the time constant of wash-out filter and T_{num}, T_{den} are time constants of phase-lead filter with $T_{num} > T_{den}$.

Elimination of the algebraic variables by Kron reduction, generates a reduced order dynamic model given by $\Delta \dot{x}_g = A_{gg} \Delta x_g + E_1 \Delta \tilde{u}$ where $\Delta x_g \in \mathbb{R}^{7n_g}$ and $\Delta \tilde{u} \in \mathbb{R}^{n_g}$.

In this section, we consider the IEEE 39 bus system, the line diagram of which is shown in Fig. 7. For the IEEE 39 bus system, we will observe small signal behavior of the system at various operating conditions with changes in load levels. As shown in figure 8, operating points are considered along the PV curve for the above described 39 bus system.

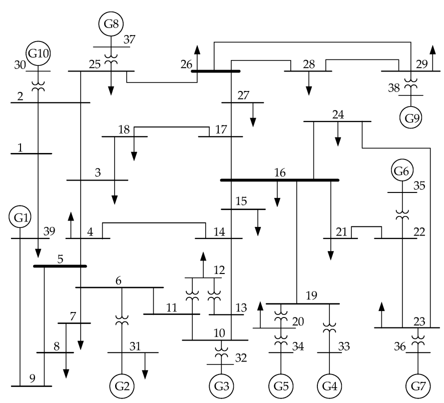


Fig. 7. IEEE 39 bus system.

The objective is to use the information transfer measure to identify the generator and the states of the generator which are responsible for instability. For determining the states responsible for instability, the concept of participation factor is generally used. But one drawback of participation factor is the fact that it can not compute the participation of a combination of states to any particular mode. However, our formulation of information transfer is free from such issues and one can compute the information transfer from any combination of states to any other combination of states. As such, one can

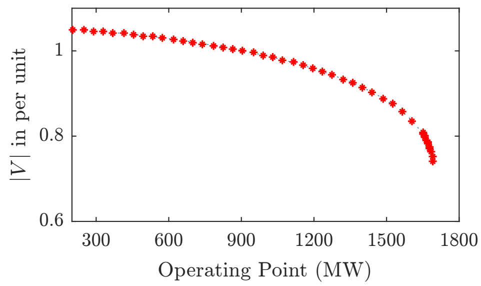
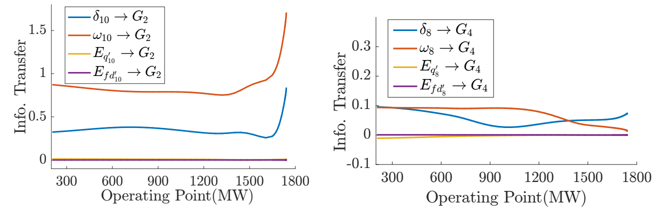


Fig. 8. Selected operating points for IEEE 39 bus system. Bus voltage and load level at Bus 28.

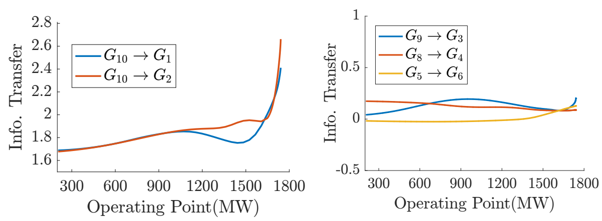


(a) (b)

Fig. 10. (a) Steady state information transfer over the operating points from the states of generator 10 to generators 2. (b) Steady state information transfer over the operating points from the states of generators 8 to generator 4.

combine the states of each generator together and look at the information transfer between the generators.

In the first set of simulations, we look at the information transfer between the generators and identify the generator from which the information transfer shows a sudden increase, as the network approaches instability. For simulation purposes, we linearized the system along the PV curve and used the linearized model to compute the information transfer between the generators. The discretization step used was 0.2 secs.



(a) (b)

Fig. 9. (a) Steady state information transfer over the operating points from generator 10 to generators 1 and 2. (b) Steady state information transfer over the operating points from generators 5, 8 and 9 to generators 3, 4 and 6 respectively.

Simulation results identified generator 10 as the generator which is responsible for instability. This is because, the information transfer from generator 10 showed a sudden rise, as the network approached instability. Information transfer from generator 10 to generators 1 and 2 is shown in Fig. 9(a) and from the figure it can be seen that the information transfer from generator 10 starts to increase rapidly. For comparison, we also plot the information transfer from generators 5, 8 and 9 to generators 3, 4 and 6 respectively in Fig. 9(b). It can be seen in Fig. 9(b) that the information transfers between these generators do not change that much, thereby implying that these generators are not responsible for instability.

Once we have identified the generator which is most responsible for instability, we zoom in to the generator and determine the states of the generator which are responsible for instability. We have considered only the four states of the generator and not the states of the PSS. In particular, we have studied the information transfer from the states of generator 10 to other generators and identified the states of generator 10, the information transfer from which to other generators show the sudden increase.

In Fig. 10(a) we plot the information transfer from the individual states of generator 10 to generator 2. From the figure, it can be seen that the information transfer from the angle and the angular speed variables show a sudden increase, whereas the other transfers remain almost same throughout the PV curve. Hence, we conclude that it is the angle and the angular speed variables which are most responsible for instability. This fact is also verified by participation factor analysis as shown in Table III, where it was found that the participation factor of the angle and angular speed of generator 10 to the most unstable mode is significantly high. For comparison, we also plot the information transfer from the states of generator 8 to generator 4 in Fig. 10(b). As expected, we find that the information transfer does not change much throughout the PV curve, and hence reaffirming the fact that generator 8 is not responsible for instability. The same conclusion holds for the states of the other generators as well.

TABLE III
PARTICIPATION FACTOR OF GENERATOR STATES IN CRITICAL MODE AT OPERATING POINT 1740.68 MW

Dynamic state	Participation factor of states in critical mode
$Gen_{10} : \delta$	0.470
$Gen_{10} : \omega$	0.464
$Gen_{10} : E'_{qi}$	0.021
$Gen_{10} : E_{fd}$	0.00014
$Gen_8 : \delta_g$	0.0016
$Gen_8 : \omega$	0.0016
$Gen_8 : E'_{qi}$	2.14×10^{-6}
$Gen_8 : E_{fd}$	1.427×10^{-6}

One of the key developments in the power industry in recent times is the synchrophasor technology. Phasor measurement units (PMUs) are capable of sampling system variables down to the level of hundredth of a second. Hence, there is a strong need for a framework to articulate system stability from the available measurements. In [26], [27], we have provided a data-driven approach to compute the information transfer measure from time series data. The data-driven approach for stability inference and characterization of influence in power networks is left for future investigations.

VI. CONCLUSION

In this paper, we introduce a new measure of information transfer in a dynamical system for stability characterization in a power network. In particular, we provide a conceptual basis on how causality characterization between dynamic states can reveal information about system instability and participating entities. Further, we illustrated the theory on three bus power network and recovered the known results on the cause of instability, voltage or angle, in the system. We compare the information transfer concept with the existing method of participation factor and show how information transfer measure captures non-zero influence when participation factor fails to do so. Moreover, we illustrated the theory on IEEE 39 bus system and identified the states and the generators responsible for the instability of the 39 bus system. Future research efforts will focus on proposing a data-driven approach for computing information transfer in power networks using results developed in [26], [27].

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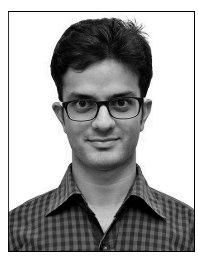
Subhrajit Sinha received his Ph.D. degree in electrical engineering from Iowa State University, Ames, IA, USA, in 2019. He is currently a Post Doctoral Research Associate in Pacific Northwest National Laboratory, Richland, WA, USA. His research interest lies in the intersection of information theory and control engineering and also in data-driven discovery and control of dynamical systems, with applications in power systems, building systems and biological systems.



Pranav Sharma ($S'18$) received the B.Tech. degree in electrical engineering from the Indian Institute of Technology- Roorkee, India. He is currently a Ph.D. student in the Department of Electrical and Computer Engineering at Iowa State University, Ames, IA, USA. His research interest includes data-driven methods for nonlinear system dynamic analysis, power system stability, PMU measurement-based data analytics and causal structure in power system dynamics.

Umesh Vaidya ($M07$) received the Ph.D. degree in mechanical engineering department from the University of California Santa Barbara, CA, USA, in 2004. He is an Associate Professor with the Department of Electrical and Computer Engineering, Iowa State University, Ames IA, USA. He was Research Engineer with United Technologies Research Center, East Hartford, CT, USA. His research interest is in the area of network controlled dynamical systems with applications to power systems, cyber physical systems, building systems, and aerospace systems.

Venkataramana Ajjarapu ($S86M86SM91F07$) received the Ph.D. degree in electrical engineering from the University of Waterloo, Waterloo, ON, Canada, in 1986. He is currently the David C. Nicholas Professor with the Department of ECpE, Iowa State University, Ames, IA, USA. His current research interests include voltage stability analysis and control, reactive power planning, data driven power system dynamic analysis and proactive distribution system integration.