Mixed-Mode and Mode-II Fatigue Crack Growth in Woven Composites

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ABSTRACT

A woven carbon/epoxy composite was subjected to fatigue crack growth under mixed Mode-I/Mode-II loading to obtain crack growth behavior at different cyclic strain energies. Owing to the woven structure of the material, pure Mode-II fracture is usually a difficult proposition because of friction, interference, and interlock of woven tows in adjacent plies at an interlaminar crack. These limitations were overcome by the use of a novel form of mixed Mode-I/Mode-II specimen, which imposes sufficient crack surface opening (Mode-I) to alleviate ply-ply interactions, but not so much as to obscure the sliding (Mode-II) response. Comparison with pure Mode-I fatigue crack growth data, in conjunction with a fracture interaction criterion, provided a means to extract the Mode-II behavior.

Introduction

Composites comprised of woven fibers embody a significant portion of commercial composite material production, owing to the high performance of finished parts and the relative ease of handling the raw materials (preforms, prepreg). However, in spite of their unquestionable practicality, the additional geometric complexity introduced by the woven structure of the fibers (crimps) in such composites has many confounding consequences on the characterization and testing of these materials.

In the case of fracture, the complexity of the woven structure has different effects depending on the mode of crack propagation. For instance, in the case of Mode-I crack growth (opening mode), the propagation of an interlaminar crack around the crimps of the woven structure has an effect of introducing significant tortuosity to the crack path, and this often will manifest itself as a 'stepwise' or 'zig-zag' load-displacement character in comparison to a more uniform unidirectional material (Fig. 1) that lacks the more complicated geometry of the fiber arrangement. Such effects can often be accommodated reasonably well in data analysis by averaging methods which look at overall material response rather than local behavior at any given instant of crack propagation around a feature of the woven structure [1-3].

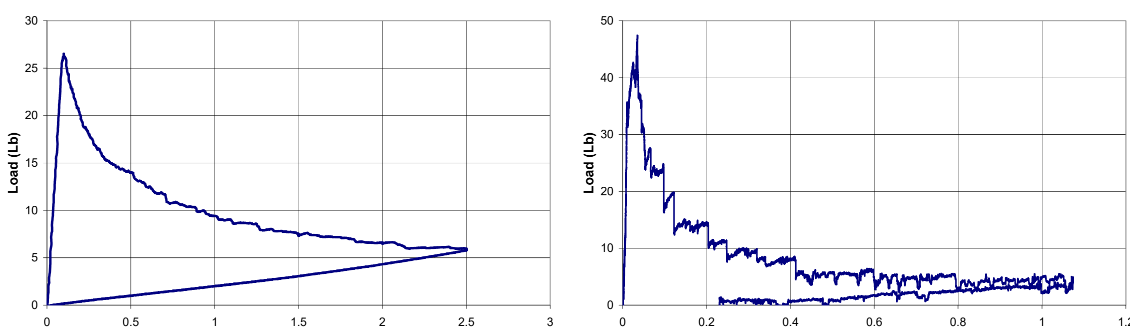

Fig. 1 – Representative load-displacement data from similar Mode-I fixed-width double cantilever beam (DCB) fracture specimens. Left: unidirectional carbon/epoxy composite, Right: woven carbon/epoxy composite, showing 'stepwise' behavior

In the case of a Mode-II process (sliding mode), the situations is, however, quite worse. One of the more significant challenges in Mode-II fracture testing of woven composites is the 'interlock' phenomenon associated with the woven structure. In essence, the crimps of the fabric weave, in adjacent plies, have a strong tendency to physically interfere with one another, opposing sliding of one ply against another. Structurally, this is beneficial, as it presents a mechanism of resistance to the sliding motion required for extension of a crack in Mode-II (sliding mode) that is more robust than in unidirectional composites (Fig. 2). From a materials testing standpoint, however, it is quite confounding, as it prevents the successful execution of many forms of convenient Mode-II fracture test because the nascent crack fails to propagate properly beyond initial release, and initial release is sometimes suspect.

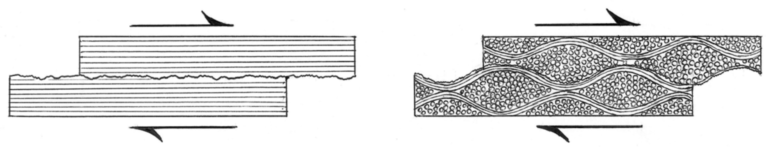

Fig. 2 – Effect of material structure on a Mode-II (sliding) interlaminar fracture process.
Left: unidirectional material showing relatively uninhibited planar growth
Right: woven material showing effect of mechanical interference of crimps impeding movement of adjacent crack faces

Because Mode-I crack propagation (opening mode) is not so fundamentally opposed or thwarted by the more complicated path a crack must take in a woven composite, this effect of ply-on-ply interlock can generally be ignored or treated as irrelevant in data analysis. It can then readily be seen how imposing some amount of Mode-I opening onto an otherwise "pure" Mode-II sliding test might give a means of alleviating the interlock effect while still preserving enough of the Mode-II behavior of a test to measure, which is the principal aim of this research.

Material Preparation

Two types of carbon/epoxy composite were employed in this study to achieve two different aims. A unidirectional composite material was prepared from prepreg of Aksaca A42 fibers with Dow P6300 IMR resin as the matrix material. Curing was performed in a heated hydraulic press to produce finished sheets of composite at a final fiber volume fraction of approximately 51%. A woven composite material was prepared from dry 5-harness satin AS4 fiber preform (as Hexcel AGP370-5H) and an anhydride-cured epoxy (Huntsman GY6010-HY917-DY070 in weight proportions 100:90:1) in a wet layup process with subsequent vacuum debulking and curing in a heated hydraulic press. In both cases, the curing temperature was 150°C. Composite sheets as cast were nominally 300mm x 300mm x 2.5mm. Samples were abrasively machined from these sheets using diamond abrasive cutting wheels and SiC abrasive papers to achieve desired geometries.

Mode-I Testing

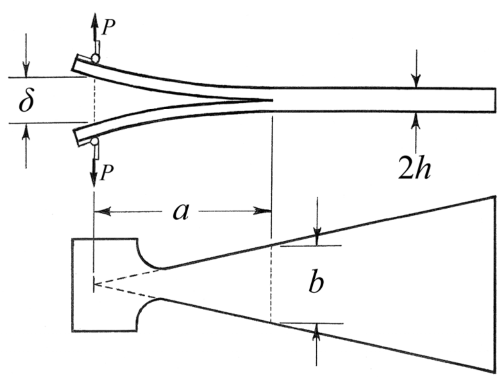

Fig. 3 – Illustration of typical width-tapered DCB Mode-I fracture specimen

To calibrate the response of the mixed-mode specimen at the heart of the study, pure single-mode characterization tests were first carried out on the unidirectional material, as its simpler microstructure typically gives experimental data that are easier to interpret than woven material. The Mode-I critical strain energy release rate (G_{Ic}) was obtained by testing of width-tapered DCB specimens [4] (Fig. 3) under quasi-static conditions, obtaining load-displacement curves as in Fig. 4 (left).

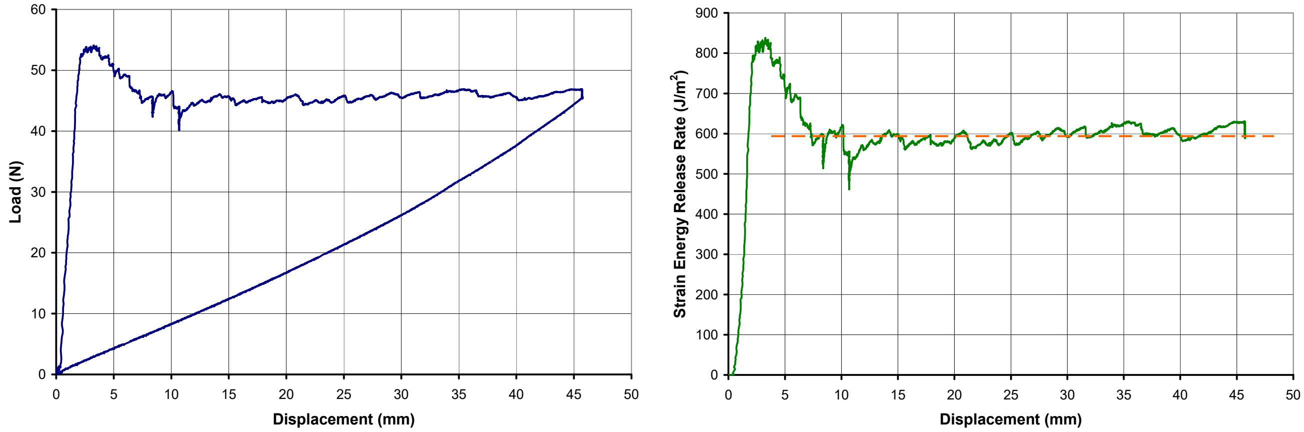

Fig. 4 - Representative curves obtained from Mode-I static fracture testing of unidirectional composite. Left: Load-displacement curve. Right: Plot of instantaneous strain energy release rate G_I during test

For the width-tapered DCB specimen, the specimen compliance is found from Timoshenko beam theory to be

$$C = \frac{\delta}{P} = \frac{12}{E_1 h^3}\left[(a^2 - s^2)k + \frac{2s}{3B}\right] + \frac{12}{5hG_{13}}\left[\ln\left(\frac{a}{s}\right) + \frac{s}{B}\right] \tag{1}$$

where δ is the end deflection of the specimen
P is the applied load
E_1 is the longitudinal elastic modulus of the composite material
G_{13} is the shear modulus of the composite material in plane of bending
a is the crack length
s is the tab length
B is the tab width
$k=a/b$ is the width taper ratio
$2h$ is the specimen thickness

Because testing is conducted in a uniaxial manner, the compliance method may be applied to obtain the Mode-I strain energy release rate G_I at any given instant as

$$G_I = \frac{1}{2}\frac{P^2}{b}\frac{dC}{da} = \frac{12P^2 k}{E_1 h a^2}\left[k\left(\frac{a}{h}\right)^2 + \frac{1}{10}\left(\frac{E_1}{G_{13}}\right)\right] \tag{2}$$

It is readily apparent from this expression that, for conditions where the sample aspect ratio is high ($a >> h$) or where shear effects may otherwise be neglected (e.g. as $G_{13} \to \infty$), this reduces to the expression

$$G_I \cong \frac{12P^2k^2}{E_1 h^3} \tag{3}$$

It is worth noting that this expression does not contain the crack length a and that the width-tapered DCB specimen is thus generally insensitive to the crack length, requiring no effort to measure it in order to obtain an accurate value for the critical strain energy release rate G_{Ic}. Applying Eq. 3 to the load displacement data gives a plot of the strain energy release rate as a function of sample displacement, showing a relatively constant character.

Similar tests were subsequently performed on woven material, giving load-displacement curves and strain energy release rate response as in Fig. 5.

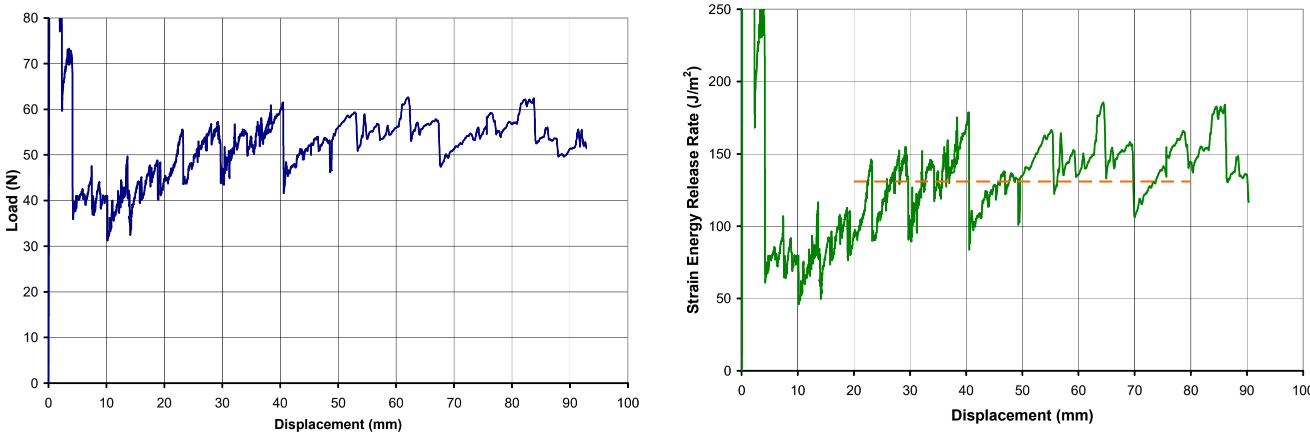

Fig. 5 - Representative curves obtained from Mode-I static fracture testing of woven composite.
Left: Load-displacement curve. Right: Plot of instantaneous strain energy release rate G_I during test

Mode-II Testing

Pure Mode-II static fracture tests were conducted using the ubiquitous edge notched flexure (ENF) specimen [5] to provide the remaining necessary information to calibrate the mixed-mode response. The Mode-II critical strain energy release rate (G_{IIc}) was obtained by testing of these specimens (Fig. 6) under quasi-static conditions, obtaining load-displacement curves as in Fig. 7.

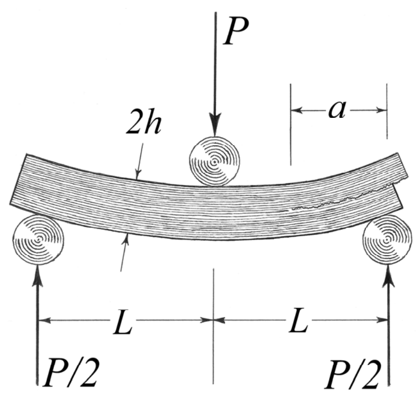

Fig. 6– Illustration of typical fixed-width edge notched flexure (ENF) Mode-II fracture specimen

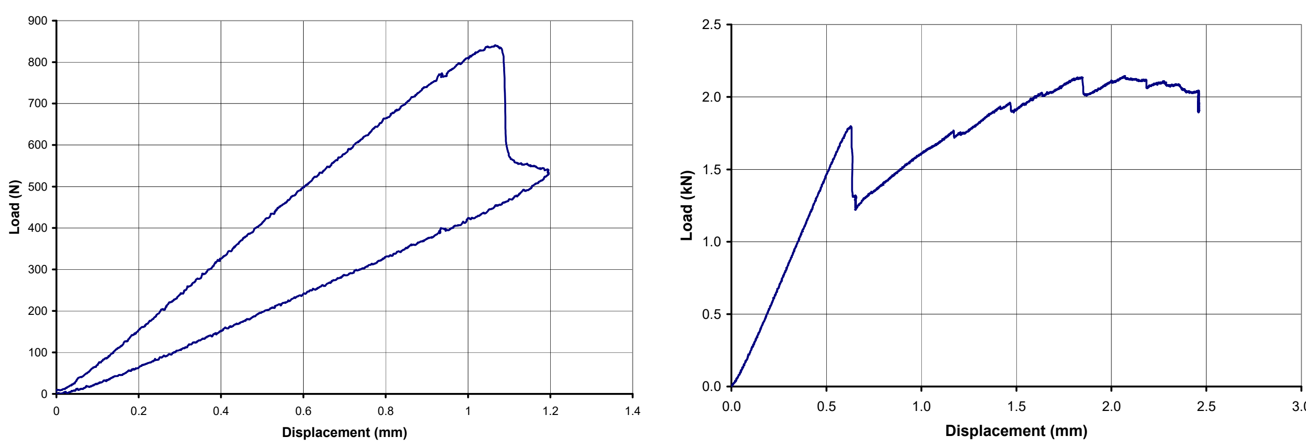

Fig. 7 – Representative load-displacement plots from Mode-II ENF static test .
Left: unidirectional composite; Right: woven composite

For the ENF specimen, the specimen compliance is found from Timoshenko beam theory to be

$$C = \frac{\delta}{P} = \frac{1}{8bh}\left[\frac{2L^3 + 3a^2}{E_1 h^2} + \frac{12L + 9a}{5G_{13}}\right] \tag{4}$$

where L is the half-span of the ENF specimen
b is the specimen width

The compliance method is again readily invoked to obtain the Mode-II strain energy release rate of

$$G_{II} = \frac{1}{2}\frac{P^2}{b}\frac{dC}{da} = \frac{9P^2}{16E_1 bh}\left[\left(\frac{a}{h}\right)^2 + \frac{1}{5}\left(\frac{E_1}{G_{13}}\right)\right] \tag{5}$$

Unlike the Mode-I DCB, the Mode-II ENF generally lacks a stable crack growth character for initial crack lengths shorter than $a \cong 0.7L$ [5]. Hence, only the initial release is of significance, and one may only rely on the peak load prior to release to obtain the Mode-II critical strain energy release rate. The expected load-displacement character (Fig. 8) is one in which there exists a steep drop in load at first release, followed by a monotonically declining load as the crack is extended further. It is noteworthy that the unidirectional material largely exhibits this physical behavior (Fig. 7 left – a small portion of the declining load after peak can be seen) in the load-displacement curves as one would expect, but the woven material does not. In fact, the load drop at release in the woven material is comparatively small, and if a woven sample is deformed beyond this point, it exhibits an increasing load characteristic (Fig. 7 right) that does not agree with the expected declining load and stable crack growth response obtained from theory. In essence, this is a consequence of the ply-on-ply interlock which is the confounding effect at the heart of many problems in Mode-II fracture testing of woven composites.

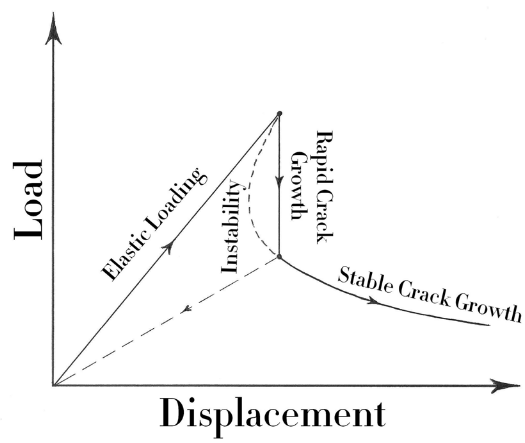

Fig. 8 – Illustration of idealized ENF specimen load-displacement response and associated physical phenomena

Mixed-Mode Testing

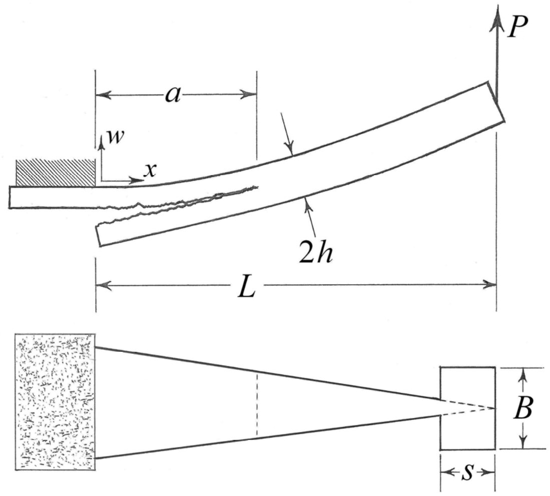

Fig. 9 – Illustration of mixed-mode fracture specimen

For the mixed-mode static tests (and subsequent fatigue crack growth tests), a somewhat novel specimen geometry was employed (Fig. 9). Conceptually, the specimen resembles its pure Mode-I width-tapered DCB cousin (as in Fig. 3), but with a few modifications to introduce a mixture of Mode-I and Mode-II. The method of imposing multiple modes is most readily understood by decomposing the end loading conditions into two more obvious cases of pure Mode-I and pure Mode-II behavior (Fig. 10).

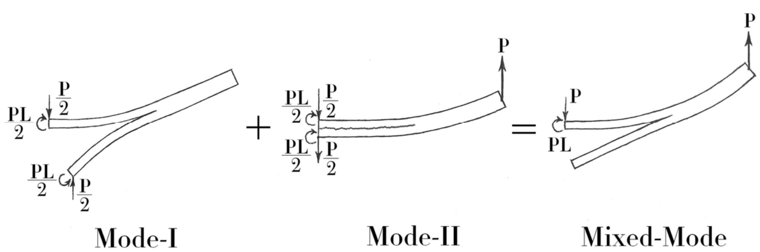

Fig. 10 – Illustration of linear superposition of pure Mode-I and pure Mode-II specimen end loadings which give rise to the loading state found in the mixed-mode specimen

The compliance of the physical specimen (mixed mode) is found from Timoshenko beam theory to be

$$C = \frac{\delta}{P} = \frac{k}{2\kappa h G_{13}}\left[1 + \ln\left(\frac{L^2}{s(L-a)}\right)\right] + \frac{k}{4E_1 h^3}\left(3L_2 + 42La - 21a^2 - 3s^2\right) + \frac{2s^3}{4E_1 Bh^3} + \frac{s}{2\kappa BhG_{13}} \qquad (6)$$

where δ is the end deflection of the specimen
P is the applied load at the end of the specimen
L is the span between the fixed and simple loading points
a is the length of the crack
B is the width of the loading tab
s is the length of the loading tab
2*h* is the total specimen thickness
κ is the Timoshenko shear coefficient (*κ* =5/6 for rectangular cross section)
G_{13} is the shear modulus
E_1 is the longitudinal material elastic modulus

Applying the compliance method gives a total strain energy release rate of

$$G_T = \frac{P^2 k^2}{4h}\left[\frac{6}{5G}\left(\frac{1}{L-a}\right)^2 + \frac{21}{Eh^2}\right] \qquad (7)$$

which, under conditions of negligible shear effects may be approximated as

$$G_T \cong \frac{21P^2k^2}{4E_1h^3} \tag{8}$$

A similar analysis is readily performed on the Mode-I and Mode-II component cases (as in Fig. 9), giving Mode-I and Mode-II component strain energy release rates of

$$G_I \cong \frac{3P^2k^2}{E_1h^3} \tag{9}$$

and

$$G_{II} \cong \frac{9P^2k^2}{4E_1h^3} \tag{10}$$

From this analysis, it is possible to then compute the ratio of the Mode-I strain energy release rate component to the total & the Mode-II strain energy release rate component to the total, giving

$$\frac{G_I}{G_T} = \frac{4}{7} \quad \text{and} \quad \frac{G_{II}}{G_T} = \frac{3}{7} \tag{11,12}$$

showing this specimen configuration to possess a constant proportion of Mode-I and Mode-II components regardless of crack length.

There are several sample geometries which may accomplish a similar kind of effect of imposing multiple modes [6-8], but the one presented here was selected because of attractive features which make it unique. In essence, the sample mimics the geometry of its pure Mode-I width-tapered DCB cousin as a means of obviating the need to directly measure the crack length. It also contains an inherent and constant ratio of fracture mode energies as the crack progresses, which is useful when conducting a test to interrogate the interaction of modes, as the test is uniform in its mode-mixing behavior from beginning to end.

The largest deficiency of this attack is the need to ultimately apply some form of mode mixing criterion to explain the interaction of the Mode-I and Mode-II energy release processes during cracking. There exist several models which can accomplish this [9-11], but unfortunately they are all semi-empirical in nature. The theory of Benzeggagh and Kenane was chosen for this work because of prior experimental efforts that had shown its applicability [12].

In its original form, the Benzeggagh-Kenane semi-empirical relation describing fracture mode interaction is

$$G_T = G_I + (G_{II} - G_I)\left(\frac{G_{II}}{G_T}\right)^m \tag{13}$$

where G_T is the total specimen (mixed mode) strain energy release rate
G_I is the Mode-I strain energy release rate component
G_{II} is the Mode-II strain energy release rate component
m is an empirical interaction parameter

Because the ratio between the Mode-II strain energy release rate component G_I and the total strain energy release rate G_T are essentially fixed (Eqs. 11-12), this relation collapses to a simple linear rule-of-mixtures expression as

$$G_T = G_I\left[1 - \left(\frac{3}{7}\right)^m\right] + G_{II}\left(\frac{3}{7}\right)^m \tag{14}$$

Hence, given measurements for G_{Ic}, G_{IIc}, and G_{Tc} by experiment, the mode mixing parameter is obtained as

$$m = -\frac{\log\left(\dfrac{G_{Tc} - G_{Ic}}{G_{IIc} - G_{Ic}}\right)}{\log\left(\dfrac{3}{7}\right)} \tag{15}$$

Samples of the mixed mode type were then tested under displacement-controlled conditions, giving load-displacement curves as in Fig. 10. Ideally, these specimens should have given a constant plateau-load behavior upon onset of crack growth, as in the Mode-I width-tapered DCB tests (Fig. 5). However, there was a general trend toward monotonically increasing load in all samples tested, and the woven samples exhibited much more severe ‘zig-zag’ or ‘stepwise’ jumps than in Mode-I tests.

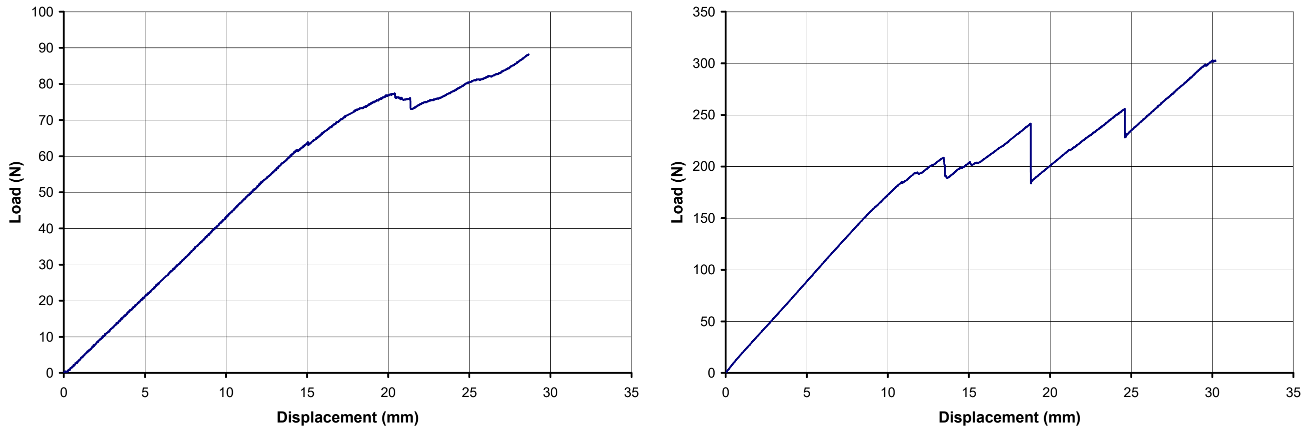

Fig. 10 – Representative mixed-mode specimen load-displacement curves. Left: unidirectional composite; Right: woven composite

Collecting the data from the pure Mode-I width-tapered DCB tests, the pure Mode-II ENF tests, and the mixed-mode tests, the critical strain energy release rates for the two materials were found to be as follows

Property	Value	CoV
Mode-I (G_{Ic})	537 J/m^2	12.8%
Mode-II (G_{Ic})	913 J/m^2	17.6%
Mixed-Mode (G_T)	1100 J/m^2	29.1%

Table 1 – Measured strain energy release rates for unidirectional carbon/epoxy composite

Property	Value	CoV
Mode-I (G_{Ic})	163 J/m^2	19.0%
Mode-II (G_{Ic})	590 J/m^2	30.2%
Mixed-Mode (G_T)	2020 J/m^2	33.4%

Table 2 – Measured strain energy release rates for woven carbon/epoxy composite

Application of the data in Tables 1-2 to Eqn. 15 then gave Benzeggagh-Kenane interaction parameters of
m=0.476 for the unidirectional composite
m=1.735 for the woven composite

This result is not altogether surprising, because the matrix behavior of the two materials is rather different, with the unidirectional material possessing a much less brittle matrix than the woven material. Furthermore, the result for the woven material is generally in agreement with Benzeggagh & Kenane’s assertion that m=2 is generally a good value for most brittle materials [10].

Mixed-Mode Fatigue Crack Growth

An interesting feature of the mixed mode specimen (and of some other width-tapered specimens [13]) is the behavior under conditions of an applied cyclic load ΔP permits continuous testing at a constant crack growth rate. In describing fatigue crack growth in a brittle material, it is often convenient to invoke the semi-empirical Paris law to describe the relationship between applied cyclic strain energy and rate of crack extension per cycle. The Paris law may be stated as

$$\frac{da}{dN} = C(\Delta K)^n \qquad (16)$$

where a is the crack length
N is the number of cycles
ΔK is the cyclic applied stress intensity

C and *n* are empirically determined constants

Rearranging the Benzeggagh-Kenane relation using the expressions for the Mode-I strain energy release rate component G_I (Eq. 9) and the total strain energy release rate G_T (Eq. 8) of the mixed mode specimen gives an expression for the Mode-II component alone as

$$\frac{P^2k^2}{E_1h^3}\left[\frac{9}{4}\left(\frac{7}{3}\right)^m+3\right]=G_{II} \tag{17}$$

Because beam bending specimens generally embody a case of an Eulerian beam, they may be treated as possessing a plane-stress case, and the relation between the stress intensity factors K_i and strain energy release rates G_i ($i=I,II,T$) is merely

$$\frac{K_i^2}{E_1}=G_i \tag{18}$$

If this is used to rewrite Eqn. 17 in terms of the stress intensity factor K_{II}, the expression becomes

$$K_{II}=\frac{Pk}{h^{3/2}}\left[\frac{9}{4}\left(\frac{7}{3}\right)^m+3\right]^{1/2}$$

If, however, a cyclic load is applied to the specimen ($\Delta P=P_{\max}-P_{\min}$), this expression simply becomes

$$\Delta K_{II}=(P_{\max}-P_{\min})\frac{k}{h^{3/2}}\left[\frac{9}{4}\left(\frac{7}{3}\right)^m+3\right]^{1/2} \tag{19}$$

This expression can be substituted into the Paris Law relation (Eq. 16), and thus a test conducted on a mixed-mode specimen under conditions of constant cyclic load amplitude ΔP should give data for a single crack growth rate da/dN related to the pure Mode-II response, giving a means of obtaining Mode-II cyclic crack growth behavior. This is significant because the mixed-mode specimen provides a means of alleviating the composite ply 'interlock' phenomenon that often obscures the true Mode-II fatigue crack growth behavior.

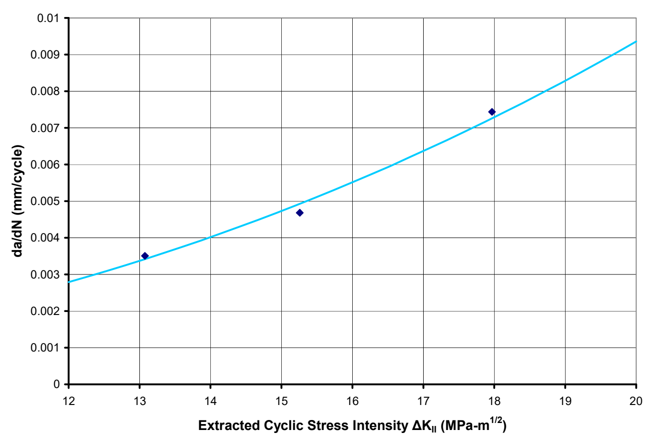

Fig. 11 – Extracted Mode-II Fatigue Crack Growth Behavior from Mixed-Mode Testing and Paris Law Fit

Preliminary tests were then conducted using mixed-mode samples at different cyclic load amplitudes ΔP under tension-tension conditions with a load ratio $R=P_{\min}/P_{\max}$ of 0.1. The resultant measured specimen cyclic crack growth rates da/dN were then used to calculate the Paris Law parameters for the extracted Mode-II response using Eqn. 19. The result of these calculations, as well as pure Mode-I Paris Law parameters obtained from prior work using only width-tapered DCB specimens (for comparison purposes) on the same material [14], are given in Table 3.

Mode	C	n
I	2.049×10^{-7}	10.55
II	7.720×10^{-6}	2.37

Table 3 – Woven composite parameters for Paris Law. Mode-II values extracted from mixed-mode sample behavior. Mode-I values from simple width-tapered DCB tests [14] on same material.
N.B. Parameters *C* and *n* are for ΔK in MPa-m$^{1/2}$ and da/dN in mm/cycle

Conclusions

While the woven structure of some composites presents a unique challenge in the area of fracture testing, especially in cases of sliding-mode crack propagation, this research shows that it may be possible to alleviate some of these complications through the technique of mode mixing in a fracture test. The technique requires a greater amount of testing effort to obtain data for the pure Mode-I and Mode-II response of the material, which is not necessarily trivial to obtain, and also requires the use of a mixed-mode interaction criterion which is not as fundamental as other phenomena involved in fracture problems,. However, the benefit of utilizing a mixed-mode test is readily seen in situations where the woven structure of the material renders more conventional methods of fatigue crack growth testing ineffective.

The non-ideal response of the mixed-mode specimen, with monotonic increase in load over the full breadth of its range, implies some deviation from the fracture mechanics model, which is still cause for concern. Most likely, there is a complication arising from the fact that the available energy in the sample to create new crack surface (the strain energy associated with bending of the uncracked section) is constantly in decline as the crack progresses. One would expect that this would not matter, as the mixed-mode sample is width tapered and presents a smaller crack width as the crack progresses as well – the two phenomena should be in balance with one another. However, this basic process runs in opposition to samples like the Mode-I width-tapered DCB where the available energy is constantly increasing, while the crack width is increasing, and where the viability of the specimen geometry has been observed. The final implication is that in well-behaved width-tapered samples where the energy and crack width simultaneously increase, the incremental elastic energy increase most likely overtakes the increase in crack width by some small amount due to other effects (e.g. nonlinear beam behavior at large deflections). If the direction of crack propagation were reversed, it might alleviate this difficulty, and render the mixed-mode sample more predictable in its behavior.

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