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I. Siddique, Z. Xu

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Alternative methods to measure global polarization of Λ hyperons

Irfan Siddique,[1] Zuo-tang Liang,[2] Michael Annan Lisa,[3] Qun Wang,[1] and Zhang-bu Xu[4, 2]

[1] *Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[2] *School of Physics & Key Laboratory of Particle Physics and Particle Irradiation (MOE)*
[3] *Physics Department, The Ohio State University, Columbus, Ohio 43210, USA*
[4] *Physics Department, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*

We propose alternative methods to measure the global polarization of Λ hyperons. These methods involve event averages of proton's and Λ's momenta in the lab frame. We carry out simulations using these methods and show that all of them work equivalently well in obtaining the global polarization of Λ hyperons.

I. INTRODUCTION

It is well-known that rotation and polarization are inherently correlated: the rotation of an uncharged object can lead to spontaneous magnetization and polarization, and vice versa [1, 2]. We expect that the same phenomena exist in heavy ion collisions. It is straightforward to estimate that huge global angular momenta are generated in non-central heavy ion collisions at high energies [3–8]. How such huge global angular momenta are converted to the particle polarization in the hot and dense matter and how to measure the global polarization are two core questions to be answered. To address the first question, there are some theoretical models in the market, e.g., the microscopic spin-orbital coupling model [3, 4, 8, 9], the statistical-hydro model [10–13] and the kinetic model with Wigner functions [14–17], see Ref. [18] for a recent review. For the second question, one can use the weak decay property of Λ hyperons to measure the global polarization [3, 4]: the parity-breaking weak decay of Λ into a proton and a pion is self-analysing since the daughter proton is emitted preferentially along Λ's spin in Λ's rest frame [5, 19]. The global polarization of a vector meson can be measured through the angular distribution of its decay products which is related to some elements of its spin density matrix [4].

Recently the global polarization of Λ and $\bar{\Lambda}$ has been measured at collisional energies below 62.4 GeV [20, 21]. The average values of the global polarization for Λ and $\bar{\Lambda}$ are $\mathscr{P}_\Lambda = (1.08 \pm 0.15)\%$ and $\mathscr{P}_{\bar{\Lambda}} = (1.38 \pm 0.30)\%$. The polarization of $\bar{\Lambda}$ is a little larger than that of Λ which is thought to be caused by a negative (positive) magnetic moment of $\Lambda(\bar{\Lambda})$ in magnetic fields. But such a difference is negligible within the error bars and magnetic fields extracted from the data are consistent to zero. The global polarization of Λ and $\bar{\Lambda}$ decreases with collisional energies. This is due to that the Bjorken scaling works better at higher energies than lower energies. From the data one can estimate the local vorticity: $\omega = (9 \pm 1) \times 10^{21}\ \mathrm{s}^{-1}$, implying that the matter created in ultra-relativistic heavy ion collisions is the most vortical fluid that ever exists in nature. The vorticity field of the quark gluon plasma has been studied by many authors in a variety of methods including hydrodynamical models [22–24] and transport models [25, 26]. The global polarization of Λ and $\bar{\Lambda}$ has also been calculated by hydrodynamical models [27, 28], the transport model [29] the chiral kinetic model [30].

The method used in the STAR measurement is through the event average of $\sin\left(\phi_{\mathrm{p}}^* - \psi_{\mathrm{RP}}\right)$, where ϕ_{p}^* and ψ_{RP} are the azimuthal angle of the proton momemtum in Λ's rest frame and that of the reaction plane respectively [20, 21]. The orientation of the reaction plane cannot be directly measured but through that of the event plane determined from the direct flow. Therefore a reaction plane resolution factor was introduced to account for the finite resolution of the reaction plane by the detector [20, 21].

In this paper, we propose alternative methods to measure the global polarization of Λ and $\bar{\Lambda}$ hyperons based on Lorentz transformation. The advantages of these methods are that all event averages are taken over momenta in the lab frame instead of Λ's rest frame. We compare these methods by simulations and show that all of them work equivalently well in obtaining the global polarization of Λ hyperons.

II. HYPERON'S WEAK DECAY AND POLARIZATION

The polarization of the Λ (and $\bar{\Lambda}$) hyperons can be measured by its parity-breaking weak decay $\Lambda \to \mathrm{p} + \pi^-$. The daughter protons are emitted preferentially along the Λ's polarization in Λ's rest frame. The angular distribution of the daughter proton reads

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha_H \mathscr{P}_\Lambda \frac{\mathbf{n}^* \cdot \mathbf{p}^*}{|\mathbf{p}^*|}\right), \tag{1}$$

where α_H is the hyperon decay parameter, $\mathscr{P}_\Lambda$ is the Lambda global polarization, $\mathbf{n}^*$, $\mathbf{p}^*$ and Ω^* are the Λ's polarization, the proton's momentum and its solid angles respectively in the rest frame of the hyperon which are labeled by the superscript $'*'$. We note that Eq. (1) is Lorentz invariant by observing

$$\begin{aligned}\mathbf{n}^* \cdot \mathbf{p}^* &= -n_\mu p^\mu = -n \cdot p, \\ E_\mathrm{p}^* &= \frac{1}{2m_\Lambda}(m_\Lambda^2 + m_\mathrm{p}^2 - m_\pi^2), \\ |\mathbf{p}^*| &= \frac{1}{2m_\Lambda}\sqrt{[m_\Lambda^2 - (m_\mathrm{p} - m_\pi)^2][m_\Lambda^2 - (m_\mathrm{p} + m_\pi)^2]},\end{aligned} \tag{2}$$

where p^μ and p_Λ^μ are the four-momentum of the proton and the hyperon in any frame respectively, n^μ is the space-like four-vector of the hyperon's polarization in a general frame. We now focus on the lab frame and the hyperon's rest frame. We now use p^μ, p_Λ^μ and n^μ to label quantities in the lab frame and all quantities with the superscript $'*'$ are those in the hyperon's rest frame. We have Lorentz transformation for the Λ's polarization,

$$n^\mu = \Lambda^\mu_{\ \nu}(-\mathbf{v}_\Lambda) n^{*\nu}, \tag{3}$$

where $\Lambda^\mu_{\ \nu}(-\mathbf{v}_\Lambda)$ is Lorentz transformation with $\mathbf{v}_\Lambda = \mathbf{p}_\Lambda / E_\Lambda$. The Λ's polarization in the rest frame $n^{*\nu}$ has the form $n^{*\mu} = (0, \mathbf{n}^*)$ where $\mathbf{n}^*$ is the three-vector of the polarization with $|\mathbf{n}^*|^2 < 1$. From Eq. (3) we have

$$n^\mu = (n_0, \mathbf{n}) = \left(\frac{\mathbf{n}^* \cdot \mathbf{p}_\Lambda}{m_\Lambda}, \mathbf{n}^* + \frac{(\mathbf{n}^* \cdot \mathbf{p}_\Lambda)\mathbf{p}_\Lambda}{m_\Lambda(m_\Lambda + E_\Lambda)} \right). \tag{4}$$

We can also express $n^{*\mu}$ in terms of n^μ,

$$n^{*\mu} = \Lambda^\mu_{\ \nu}(\mathbf{v}_\Lambda) n^\nu, \tag{5}$$

or explicitly,

$$n^{*\mu} = (0, \mathbf{n}^*) = \left(0, \mathbf{n} - \frac{\mathbf{p}_\Lambda(\mathbf{n} \cdot \mathbf{p}_\Lambda)}{E_\Lambda(E_\Lambda + m_\Lambda)} \right). \tag{6}$$

The polarization four-vector of one particle is always orthogonal to its four-momentum, $n \cdot p_\Lambda = n^0 E_\Lambda - \mathbf{n} \cdot \mathbf{p}_\Lambda = 0$, so we can express n^0 in term of $\mathbf{n}$, $n^0 = \mathbf{n} \cdot \mathbf{v}_\Lambda$. One can verify that n^μ in Eq. (4) does satisfy $n^0 = \mathbf{n} \cdot \mathbf{v}_\Lambda$. From $(n^0)^2 - |\mathbf{n}|^2 = -|\mathbf{n}^*|^2$ and $n^0 = \mathbf{n} \cdot \mathbf{v}_\Lambda$, we can solve $|\mathbf{n}|^2$ as

$$|\mathbf{n}|^2 = \frac{|\mathbf{n}^*|^2}{1 - |\mathbf{v}_\Lambda|^2(\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}_\Lambda)^2}. \tag{7}$$

We see that when $|\mathbf{v}_\Lambda|^2(\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}_\Lambda)^2 \to 1$, $|\mathbf{n}|^2 \to \infty$, so $|\mathbf{n}|^2$ is not bound. In case of transverse polarization, i.e. $\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}_\Lambda = 0$, we have $|\mathbf{n}|^2 = |\mathbf{n}^*|^2 < 1$.

In the lab frame, a 3-dimensional vector (e.g. impact parameter, global angular momentum, beam) can be written as $\mathbf{a} = \mathbf{a}_x \mathbf{e}_x + \mathbf{a}_y \mathbf{e}_y + \mathbf{a}_z \mathbf{e}_z$ with $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ being three basis directions.

III. PREVIOUS METHOD TO MEASURE HYPERON'S POLARIZATION

In this section we introduce the method used in STAR's measurement of the Λ's polarization [21]. From Eq. (13) we can determine the Λ's polarization in the rest frame by taking the event average over the direction of the proton momentum $\hat{\mathbf{p}}^*$. Then we make projection onto the direction of the global angular momentum $\mathbf{e}_L$,

$$\begin{aligned}\mathscr{P}_\Lambda &= \frac{3}{\alpha_H} \langle \hat{\mathbf{p}}^* \cdot \mathbf{e}_L \rangle_{\mathrm{ev}} \\ &= \frac{3}{\alpha_H} \langle \cos\theta^* \rangle_{\mathrm{ev}}\end{aligned} \tag{8}$$

where θ^* is the angle in Λ's rest frame between the proton momentum and the global angular momentum corresponding to the reaction plane. We have the following relation

$$\cos\theta^* = \sin\theta_\mathrm{p}^* \sin\left(\phi_\mathrm{p}^* - \psi_{\mathrm{RP}}\right), \tag{9}$$

where θ_{p}^* and ϕ_{p}^* are the polar and azimuthal angle of $\hat{\mathbf{p}}^*$ respectively, and ψ_{RP} is the azimuthal angle of the reaction plane. Then we can integrate over θ_{p}^* of Eq. (1) to obtain

$$\frac{dN}{d\phi_p^*} = \int_0^\pi d\theta_{\mathrm{p}}^* \sin\theta_{\mathrm{p}}^* \frac{dN}{d\Omega^*} = \frac{1}{2\pi} + \frac{1}{8}\alpha_H \mathscr{P}_\Lambda \sin\left(\phi_{\mathrm{p}}^* - \psi_{\mathrm{RP}}\right), \tag{10}$$

which gives the polarization in terms of the azimuthal angle of the daughter proton,

$$\mathscr{P}_\Lambda = \frac{8}{\pi\alpha_H}\left\langle \sin\left(\phi_{\mathrm{p}}^* - \psi_{\mathrm{RP}}\right)\right\rangle_{\mathrm{ev}}, \tag{11}$$

with

$$\left\langle \sin\left(\phi_{\mathrm{p}}^* - \psi_{\mathrm{RP}}\right)\right\rangle_{\mathrm{ev}} = \int_0^{2\pi} d\phi_p^* \frac{dN}{d\phi_p^*} \sin\left(\phi_{\mathrm{p}}^* - \psi_{\mathrm{RP}}\right). \tag{12}$$

In the STAR experiment, the azimuthal angle of the reaction plane cannot be directly measured but through the measurement of the event plane by direct flows. This introduces a reaction plane resolution factor in the denominator in Eq. (11), $R_{\mathrm{EP}}^{(1)} = \left\langle \cos\left(\psi_{\mathrm{RP}} - \psi_{\mathrm{EP}}^{(1)}\right)\right\rangle_{\mathrm{ev}}$, where $\psi_{\mathrm{EP}}^{(1)}$ is the azimuthal angle of the event plane determined by the direct flows.

IV. ALTERNATIVE METHODS

In this section, we introduce alternative methods to measure the Λ's polarization. The advantage of these methods is that the polarization can be measured through the proton's momentum in the lab frame.

We start with the formula for Λ's polarization vector in its rest frame,

$$\overrightarrow{\mathscr{P}}_\Lambda = \frac{3}{\alpha_H}\left\langle \hat{\mathbf{p}}^*\right\rangle_{\mathrm{ev}}. \tag{13}$$

We can project the above onto the direction of the global polarization which we assume to be along the y-axis, see Fig. (1).

Now we try to evaluate $\left\langle \hat{\mathbf{p}}^*\right\rangle_{\mathrm{ev}}$. To this end, we use following Lorentz transformation for the proton's momentum,

$$\mathbf{p} = \mathbf{p}^* + \frac{\mathbf{p}_\Lambda(\mathbf{p}^*\cdot\mathbf{p}_\Lambda)}{m_\Lambda\left(E_\Lambda + m_\Lambda\right)} + \frac{E_{\mathrm{p}}^*}{m_\Lambda}\mathbf{p}_\Lambda, \tag{14}$$

where E_{p}^* is determined by the masses of the proton, pion and Λ as in Eq. (2). Now we take the event average of $\left\langle \mathbf{p}\right\rangle_{\mathrm{ev}}$,

$$\left\langle \mathbf{p}\right\rangle_{\mathrm{ev}} = \left\langle \mathbf{p}^*\right\rangle_{\mathrm{ev}} + \left\langle \frac{\mathbf{p}_\Lambda(\mathbf{p}^*\cdot\mathbf{p}_\Lambda)}{m_\Lambda\left(E_\Lambda + m_\Lambda\right)}\right\rangle_{\mathrm{ev}}, \tag{15}$$

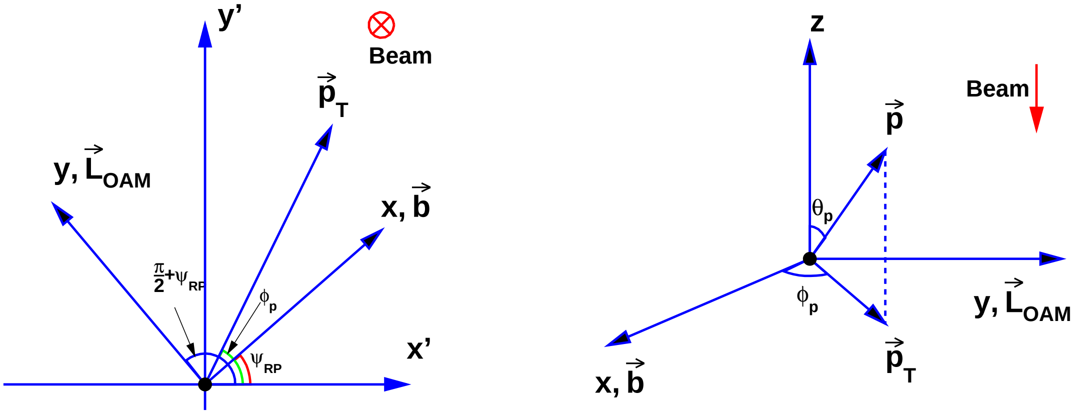
where we have used $\left\langle \mathbf{p}_\Lambda\right\rangle_{\mathrm{ev}} = 0$.

In order to evaluate the second event average in the right-hand-side of Eq. (15), we make two assumptions: (1) $\mathbf{p}_\Lambda$ and $\mathbf{p}^*$ are statistically independent, so we have $\left\langle \mathbf{p}_\Lambda(\mathbf{p}_\Lambda\cdot\mathbf{p}^*)\right\rangle_{\mathrm{ev}} \approx \mathbf{e}_i\left\langle \mathbf{p}_\Lambda^i\mathbf{p}_\Lambda^j\right\rangle_{\mathrm{ev}}\left\langle \mathbf{p}_j^*\right\rangle_{\mathrm{ev}}$, where $\mathbf{p}_\Lambda = \mathbf{e}_i\mathbf{p}_\Lambda^i$ with $i = x, y, z$; (2) $\left\langle \mathbf{p}_\Lambda^i\mathbf{p}_\Lambda^j\right\rangle_{\mathrm{ev}} = \left\langle |\mathbf{p}_\Lambda^i|^2\right\rangle_{\mathrm{ev}}\delta_{ij}$. Then Eq. (15) becomes

$$\begin{aligned}
\left\langle \hat{\mathbf{p}}_x^*\right\rangle_{\mathrm{ev}} &\approx \frac{1}{|\mathbf{p}^*|}\left(1 + \left\langle \frac{|\mathbf{p}_\Lambda^x|^2}{\left(E_\Lambda + m_\Lambda\right)m_\Lambda}\right\rangle_{\mathrm{ev}}\right)^{-1}\left\langle \mathbf{p}_x\right\rangle_{\mathrm{ev}}, \\
\left\langle \hat{\mathbf{p}}_y^*\right\rangle_{\mathrm{ev}} &\approx \frac{1}{|\mathbf{p}^*|}\left(1 + \left\langle \frac{|\mathbf{p}_\Lambda^y|^2}{\left(E_\Lambda + m_\Lambda\right)m_\Lambda}\right\rangle_{\mathrm{ev}}\right)^{-1}\left\langle \mathbf{p}_y\right\rangle_{\mathrm{ev}}, \\
\left\langle \hat{\mathbf{p}}_z^*\right\rangle_{\mathrm{ev}} &\approx \frac{1}{|\mathbf{p}^*|}\left(1 + \left\langle \frac{|\mathbf{p}_\Lambda^z|^2}{\left(E_\Lambda + m_\Lambda\right)m_\Lambda}\right\rangle_{\mathrm{ev}}\right)^{-1}\left\langle \mathbf{p}_z\right\rangle_{\mathrm{ev}}.
\end{aligned} \tag{16}$$

Now we choose the new coordinate system as in Fig. (1): the impact parameter vector is along the x-axis, the global orbital momentum is along the y-axis and the beam direction is along the negative z-axis. The old coordinate system

Figure 1: In the coordinate system (x, y, z), the beam direction is along the negative z-direction, the impact parameter vector is in the x-direction, and the orbital angular momentum is in the y-direction. The direction of the proton momentum can be described by the polar angle θ_{p} and the azimuthal angle ϕ_{p}. The coordinate system (x', y', z') is used in experiment. The z'-axis is just the z-axis. The azimuthal angle of the impact parameter vector in the (x', y', z') system is ψ_{RP}.



is the one that is used in the experiment: the beam direction is along the negative z-axis, the impact parameter vector (reaction plane) has azimuthal angle ψ_{RP} relative to the x-axis. In the new coordinate system, we have $\mathbf{p}^x_{\Lambda,\mathrm{p}} = |\mathbf{p}^T_{\Lambda,\mathrm{p}}| \cos(\phi_{\Lambda,\mathrm{p}} - \psi_{\mathrm{RP}})$ and $\mathbf{p}^y_{\Lambda,\mathrm{p}} = |\mathbf{p}^T_{\Lambda,\mathrm{p}}| \sin(\phi_{\Lambda,\mathrm{p}} - \psi_{\mathrm{RP}})$ with $\phi_{\Lambda,\mathrm{p}}$ being the azimuthal angle of the Λ hyperon and proton respectively.

We can further simplifiy Eq. (16) by using the elliptic flow coefficients. The distribution of $\mathbf{p}_\Lambda$ is not isotropic but satisfies

$$\begin{aligned} \left\langle|\mathbf{p}^x_\Lambda|^2\right\rangle_{\mathrm{ev}} &\approx \left\langle|\mathbf{p}^T_\Lambda|^2\right\rangle_{\mathrm{ev}} \left\langle\cos^2(\phi_\Lambda - \psi_{\mathrm{RP}})\right\rangle_{\mathrm{ev}} \approx \left\langle|\mathbf{p}^T_\Lambda|^2\right\rangle_{\mathrm{ev}} \frac{1}{2}\left(1+v_2^\Lambda\right), \\ \left\langle|\mathbf{p}^y_\Lambda|^2\right\rangle_{\mathrm{ev}} &\approx \left\langle|\mathbf{p}^T_\Lambda|^2\right\rangle_{\mathrm{ev}} \left\langle\sin^2(\phi_\Lambda - \psi_{\mathrm{RP}})\right\rangle_{\mathrm{ev}} \approx \left\langle|\mathbf{p}^T_\Lambda|^2\right\rangle_{\mathrm{ev}} \frac{1}{2}\left(1-v_2^\Lambda\right), \end{aligned} \tag{17}$$

where v_2^Λ is the elliptic flow of the Λ hyperon. Since the global angular momentum is along the y-axis, we have $\langle\mathbf{p}_x\rangle_{\mathrm{ev}} = \langle\mathbf{p}_z\rangle_{\mathrm{ev}} = 0$, so only non-vanishing component is

$$\begin{aligned} \left\langle\hat{\mathbf{p}}^*_y\right\rangle_{\mathrm{ev}} &\approx \frac{1}{|\mathbf{p}^*|}\left(1+\left\langle\frac{|\mathbf{p}^T_\Lambda|^2\sin^2(\phi_\Lambda-\psi_{\mathrm{RP}})}{(E_\Lambda+m_\Lambda)\,m_\Lambda}\right\rangle_{\mathrm{ev}}\right)^{-1} \left\langle|\mathbf{p}_T|\sin(\phi_{\mathrm{p}}-\psi_{\mathrm{RP}})\right\rangle_{\mathrm{ev}} \\ &\approx \frac{1}{|\mathbf{p}^*|}\left[1+\frac{1}{2}\left(1-v_2^\Lambda\right)\left\langle\frac{|\mathbf{p}^T_\Lambda|^2}{(E_\Lambda+m_\Lambda)\,m_\Lambda}\right\rangle_{\mathrm{ev}}\right]^{-1} \left\langle|\mathbf{p}_T|\sin(\phi_{\mathrm{p}}-\psi_{\mathrm{RP}})\right\rangle_{\mathrm{ev}} \end{aligned} \tag{18}$$

In the central rapidity region we have $|\mathbf{p}^z_\Lambda| \ll |\mathbf{p}^T_\Lambda|$ and then $|\mathbf{p}^T_\Lambda| \approx |\mathbf{p}_\Lambda|$, Eq. (18) becomes

$$\left\langle\hat{\mathbf{p}}^*_y\right\rangle_{\mathrm{ev}} \approx \frac{1}{|\mathbf{p}^*|}\left[1+\frac{1}{2}(1-v_2^\Lambda)\left(\langle\gamma_\Lambda\rangle_{\mathrm{ev}}-1\right)\right]^{-1} \left\langle|\mathbf{p}_T|\sin(\phi_{\mathrm{p}}-\psi_{\mathrm{RP}})\right\rangle_{\mathrm{ev}} \tag{19}$$

In non-relativistic limit when $\gamma_\Lambda \approx 1$ or $|\mathbf{v}_\Lambda| \approx 0$, we obtain

$$\left\langle\hat{\mathbf{p}}^*_y\right\rangle_{\mathrm{ev}} \approx \frac{1}{|\mathbf{p}^*|}\left\langle|\mathbf{p}_T|\sin(\phi_{\mathrm{p}}-\psi_{\mathrm{RP}})\right\rangle_{\mathrm{ev}} \tag{20}$$

The difference from the previous method is that we now take event average over the proton's momenta in the lab frame.

Another method is to use the Lorentz transformation for the energy associated with Eq. (14)

$$E_{\mathrm{p}} = \gamma_\Lambda E^*_{\mathrm{p}} + \frac{\mathbf{p}^*\cdot\mathbf{p}_\Lambda}{m_\Lambda} \tag{21}$$

to replace $(\mathbf{p}^*\cdot\mathbf{p}_\Lambda)/m_\Lambda$ with $E_{\mathrm{p}} - \gamma_\Lambda E^*_{\mathrm{p}}$ in Eq. (14). Then Eq. (14) becomes

$$\begin{aligned} \mathbf{p} &= \mathbf{p}^* + (E_{\mathrm{p}} - \gamma_\Lambda E^*_{\mathrm{p}})\frac{\mathbf{p}_\Lambda}{E_\Lambda+m_\Lambda} + \frac{E^*_{\mathrm{p}}}{m_\Lambda}\mathbf{p}_\Lambda \\ &= \mathbf{p}^* + \frac{E_{\mathrm{p}}}{E_\Lambda+m_\Lambda}\mathbf{p}_\Lambda + \frac{E^*_{\mathrm{p}}}{E_\Lambda+m_\Lambda}\mathbf{p}_\Lambda. \end{aligned} \tag{22}$$

Table I: The simulation results for the global polarizaton of the Λ hyperon. We set $\mathscr{P}_\Lambda = 1/3$, i.e. the Λ hyperons are complete polarized. By analyzing the momentum distribution of daughter protons in the lab frame, we can determine the Λ's polarization. The results of four methods are presented: the method by Eqs. (8,11) used in the STAR experiment [21] and three by Eqs. (18,23,19) proposed in this paper. The numbers of events collected are 4×10^4 at 200 GeV and 2.5×10^4 at other energies. The results of method 1-4 are from events in the full rapidity range, while those of method 5 are in the rapidity range $[-0.5, 0.5]$.

Energy GeV	method 1 Eq. (8)	method 2 Eq. (11)	method 3 Eq. (18)	method 4 Eq.(23)	method 5 Eq. (19)	Number of Λs (full rapidity)
200	0.33581	0.335851	0.3324	0.33014	0.308495	1304795
180	0.330877	0.33141	0.326565	0.329057	0.306966	927717
140	0.338745	0.337673	0.338942	0.335862	0.351934	892533
120	0.333962	0.333688	0.329696	0.334152	0.318965	995522
100	0.336686	0.334685	0.34669	0.34522	0.360992	971596
62.4	0.331964	0.33118	0.324133	0.333466	0.353216	918787
40	0.330536	0.330302	0.332092	0.331782	0.323459	795837
39	0.337252	0.337516	0.332983	0.331683	0.312195	847367
19.6	0.328531	0.328434	0.339587	0.328939	0.31276	707868
7.7	0.341257	0.3417	0.364069	0.34862	0.302301	434697

When taking the event average, using $\langle \mathbf{p}_\Lambda/(E_\Lambda + m_\Lambda)\rangle \approx 0$, we obtain

$$\begin{aligned} \langle \mathbf{p}^*\rangle_{\mathrm{ev}} &= \langle \mathbf{p}\rangle_{\mathrm{ev}} - \left\langle \frac{E_\Lambda}{E_\Lambda + m_\Lambda} E_{\mathrm{p}} \mathbf{v}_\Lambda \right\rangle_{\mathrm{ev}} \\ &= m_{\mathrm{p}} \left\langle \gamma_{\mathrm{p}} \left(\mathbf{v}_{\mathrm{p}} - \frac{\gamma_\Lambda}{\gamma_\Lambda + 1} \mathbf{v}_\Lambda \right) \right\rangle_{\mathrm{ev}}, \end{aligned} \tag{23}$$

where γ_{p} and γ_Λ are Lorentz contraction factors for the proton and Λ repectively. We see the right-hand-side of the above equation involves only momenta in the lab frame. We can project Eq. (23) onto the y-direction (the direction of the orbital angular momentum) to obtain $\langle \mathbf{p}_y^*\rangle_{\mathrm{ev}}$.

With $\langle \hat{\mathbf{p}}_y^*\rangle_{\mathrm{ev}}$ given in one of Eqs. (18,19,23), we can obtain the global polarization of Λ from through Eq. (13). In the next section we will compare these methods by simulations.

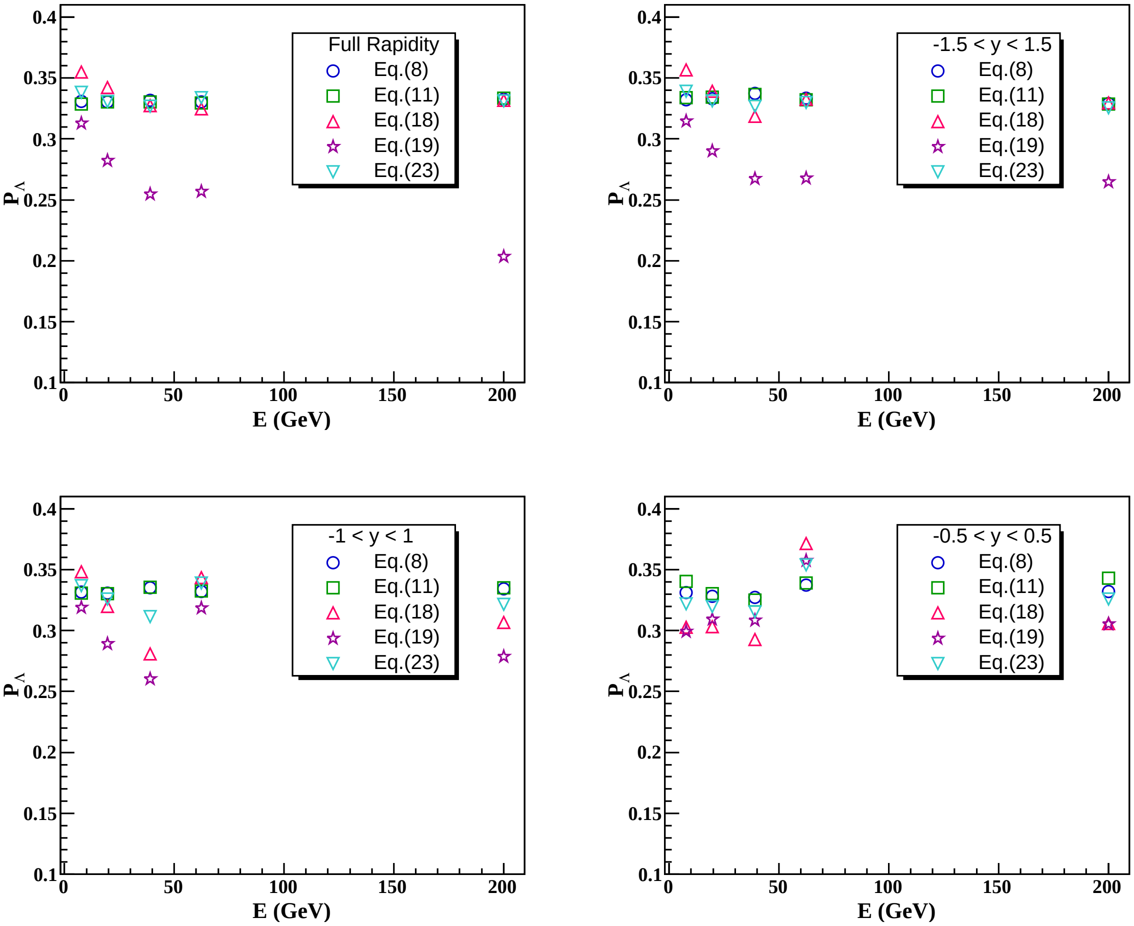
V. SIMULATION RESULTS WITH URQMD

The UrQMD model [31, 32] has been used for producing an ensemble of Λ's four-momemta $(E_\Lambda, \mathbf{p}_\Lambda)$ for Au+Au collisions with the impact parameter 6 fm for collisional energies listed in Table I. In each event there are a few Λ hyperons produced. All these hyperons are collected. Each hyperon is allowed to decay into a proton and a pion whose angular distribution in Λ's rest frame is given by

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha_H \mathscr{P}_\Lambda \frac{\mathbf{n}^* \cdot \mathbf{p}^*}{|\mathbf{p}^*|}\right), \tag{24}$$

where $\mathscr{P}_\Lambda$ denotes the Λ's polarization. By taking a specific value of $\mathscr{P}_\Lambda$, we can then sample proton's momenta in Λ's rest frames. For each $\mathit{\Lambda}$ hyperon, the proton's momentum in its rest frame is then boosted back to the lab frame. In this way we create an ensemble of proton momenta in the lab frame. With the ensemble of momenta for protons and Λs, we can obtain $\langle \mathbf{p}_y^*\rangle_{\mathrm{ev}}$. Here we choose the direction of the global angular momentum along the y-direction. Finally we obtain $\mathscr{P}_\Lambda$ by Eq. (13). The simulation results for the global polarizaton of Λ hyperons using methods corresponding to Eqs. (8,11,18,23,19) are shown in Table I. We see in the table that all these methods work equivalently well as the STAR method. Figure 2 shows the dependence of simulation results on rapidity ranges. We can see that all these methods work well for the rapidity ranges chosen except that using Eq. (19) in the full rapidity range, $[-1.5, 1.5]$ and $[-1, 1]$. This is understandable since Eq. (19) is only valid in central rapidity. We can see that the method using Eq. (19) does work well in the central rapidity range $[-0.5.0.5]$.

Figure 2: The dependence of simulation results on rapidity ranges for the global polarizaton of the Λ hyperon. The same parameters and number of events are used as in Table I.



VI. SUMMARY

The previous method used in the STAR experiment to measure the global Λ polarization is through the event average of $\sin\left(\phi_{\mathrm{p}}^{*}-\psi_{\mathrm{RP}}\right)$, where ϕ_{p}^{*} and ψ_{RP} are the azimuthal angle of the proton momemtum in Λ's rest frame and that of the reaction plane respectively. We propose several alternative methods to measure the global Λ polarization in the lab frame. Based on Lorentz transformation for momenta, we can express the gobal polarization in terms of momenta of protons and Λ hyperons in the lab frame. So the event average can be taken over quantities in the lab frame. To test how well these methods are for measuring the global polarization compared to the STAR's method, we use the UrQMD model to produce an ensemble of Λ's momenta and then sample the angular distribution of protons and pions following the weak decay formula for Λ hyperons. By taking event average over quantities as functions of momenta of protons and Λs in the lab frame we can determine the global polarization by these methods as well as by the STAR's. The simulation results show that all these methods work equivalently well as the STAR method.

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