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# Time Behavior of a Critical Reactor Pulsed with a Large Square Wave Source

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Naval Application Memo No. 8.

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FROM: C. Barnett

SUBJECT: Time Behavior of a Critical Reactor Pulsed with a Large Square Wave Source.

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Reference: NAM No. 3 by J. Hadley, "Reactor Vulnerability to Neutron Flux".

In the referenced memo a discussion is given of what happens to the power level of a reactor when an instantaneous very large neutron source is introduced. The motivation for the problem arises from the question of what happens when a nuclear bomb is detonated near a reactor powered missile.

The present note looks at the same problem as that which is discussed in NAM No. 3 but in slightly more detail. The results obtained are essentially the same as those of NAM No. 3; somewhat more attention is given to the nature of the assumptions made.

The kinetics model used is the space-time separable, one delayed group model. Space-time separability is almost certainly a poor assumption in this case. When a very large neutron wavefront impinges on a reactor, the subsequent short time behavior can hardly be that predicted by this simple model. One would expect very large local power densities in one part of the core before another part would see anything unusual. The other assumption,

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one delayed group, is not necessary but makes the mathematics simpler, and in view of the first assumption is probably quite acceptable. In spite of these simplifications the model gives results which should have qualitative physical relevance.

The problem to be solved is:

$$\frac{dn}{dt} = [k(1-\beta)-1]\frac{n}{\ell^*} + \lambda C + A \tag{1}$$

$$\frac{dC}{dt} = k\beta\frac{n}{\ell^*} - \lambda C \tag{2}$$

where $n$ ≡ total reactor neutron population at time t

$C$ ≡ total reactor precursor population at time t

$\ell^*$ ≡ mean prompt neutron lifetime

$\beta$ ≡ delayed fraction

$\lambda$ ≡ the one delayed group decay constant

$k$ ≡ neutron multiplication factor

$$A = \begin{cases} 0 & t<0 \\ A_0 & 0 \leq t \leq t_1 \\ 0 & t > t_1 \end{cases}$$

The forcing function is assumed to be a square wave beginning at time t = 0 and ending at time $t = t_1$.

To solve Eq. 1 and 2 employ the Laplace transform technique. The transformed equations are:

$$s\bar{n} - n_0 = [K(1-\beta)-1]\frac{\bar{n}}{\ell^*} + \lambda\bar{C} + \int_0^\infty A(t)\,e^{-st}\,dt \tag{3}$$

$$s\bar{C} - C_0 = K\frac{\beta}{\ell^*}\bar{n} - \lambda\bar{C} \tag{4}$$

Where $s \equiv$ independent variable in transform space

$\bar{n} \equiv \mathcal{L}[n(t)]$

$\bar{C} \equiv \mathcal{L}[C(t)]$

Carrying out the integration indicated in (3) yields

$$s\bar{n} - n_0 = [K(1-\beta)-1]\frac{\bar{n}}{\ell^*} + \lambda\bar{C} + A_0\left(\frac{1-e^{-st_1}}{s}\right) \tag{3'}$$

Equations (3') and (4) are simultaneous linear equations in $\bar{n}$ and $\bar{C}$. Solving for $\bar{n}$

$$\bar{n} = \frac{(\lambda+s)\left[A_0\left(\frac{1-e^{-st_1}}{s}\right)+n_0\right] + \lambda C_0}{(\lambda+s)\left\{s - \frac{[K(1-\beta)-1]}{\ell^*}\right\} - K\frac{\beta\lambda}{\ell^*}} \tag{5}$$

which with manipulation and the definition

$$a \equiv \frac{K(1-\beta)-1}{\ell^*}$$

becomes

$$\bar{n} = \frac{(s+\lambda)A_0\left(\frac{1-e^{-st_1}}{s}\right)}{(s+\lambda)(s-a) - \frac{K\beta\lambda}{\ell^*}} + \frac{(s+\lambda)n_0}{(s+\lambda)(s-a) - \frac{K\beta\lambda}{\ell^*}} + \frac{\lambda C_0}{(s+\lambda)(s-a) - \frac{K\beta\lambda}{\ell^*}} \qquad (6)$$

Before taking the inverse transform of (6) it is convenient to assume K = 1. This isn't necessary as far as finding the inverse transform is concerned, but it does make the result simpler, and this assumption will have to be made sooner or later anyway in order to eliminate Co from the solution. That is, it will be necessary to assume that the reactor has been exactly critical for a long enough time prior to the pulse to insure that precursor population is saturated. In that case Co is simply related to $n_0$ and can be eliminated from the result. With the assumption K = 1 the denominator common to each of the three terms in (6) becomes

$$\left[(s+\lambda)(s-a) - \frac{K\beta\lambda}{\ell^*}\right] \longrightarrow s\left[s - \left(-\frac{\beta}{\ell^*} - \lambda\right)\right],$$

and with the definition $b \equiv -\left(\frac{\beta}{\ell^*} + \lambda\right)$ plus more manipulation, (6) takes a form which is convenient for finding the inverse transform

$$\bar{n} = \frac{A_0 + n_0\lambda + C_0\lambda}{s(s-b)} + \frac{n_0}{s-b} + \frac{A_0\lambda}{s^2(s-b)} - \frac{A_0 e^{-st_1}}{s(s-b)} - \frac{A_0\lambda e^{-st_1}}{s^2(s-b)} \qquad (7)$$

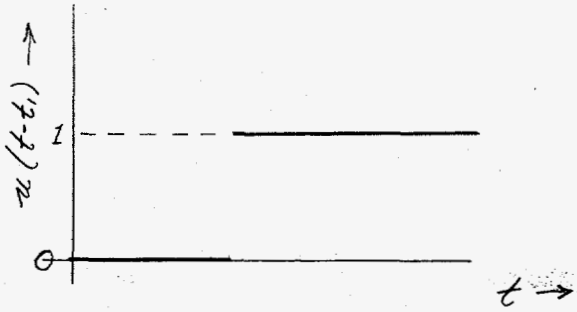
From tables of Laplace transforms, one finds the inverse transform of (7) to be

$$n(t) = (A_0 + n_0\lambda + C_0\lambda)\left(\frac{1}{b}e^{bt} - \frac{1}{b}\right) + n_0 e^{bt} + A_0\lambda\left(\frac{1}{b^2}e^{bt} - \frac{1}{b^2} - \frac{1}{b}t\right)$$

$$-A_0\left(\frac{1}{b}e^{b(t-t_1)} - \frac{1}{b}\right)u(t-t_1) - A_0\lambda\left(\frac{1}{b^2}e^{b(t-t_1)} - \frac{1}{b^2} - \frac{1}{b}(t-t_1)\right)u(t-t_1) \quad (8)$$

where $u(t-t_1)$ = unit step function shifted $t_1$ units to the right of the origin, i.e.

$$u(t-t_1) = \begin{cases} 0 & t < t_1 \\ 1 & t > t_1 \end{cases}$$




Now invoke the assumption that k was exactly one for some time prior to the arrival of the source pulse. In that case at time t = 0.

$$\left.\frac{dc}{dt}\right]_{t=0} = 0 = \frac{\beta n_0}{\ell^*} - \lambda C_0 \longrightarrow C_0 = n_0\frac{\beta}{\lambda\ell^*} \quad (9)$$

Substitution of (9) into (8) and further manipulation yields the following two equations for the neutron population as a function of time. The first is valid during the source pulse time interval the second is valid after the pulse is removed.

$$n(t) = n_0 + A_0 \lambda \tau \left[ \frac{(1-\lambda\tau)\left(1-e^{-\frac{t}{\tau}}\right)}{\lambda} + t \right], \quad t < t_1 \qquad 10.1$$

$$n(t) = n_0 + A_0 \lambda \tau \left[ \frac{(1-\lambda\tau)\left(1-e^{-\frac{t_1}{\tau}}\right)}{\lambda} e^{-\left(\frac{t-t_1}{\tau}\right)} + t_1 \right], \quad t > t_1 \qquad 10.2$$

where $\tau \equiv -\frac{1}{b} = \frac{1}{\frac{\beta}{\ell^*} + \lambda}$ is a sort of "characteristic time" for the reactor time behavior.

That Eq. 10 is a solution to the system (1) and (2) with the added assumption of steady state prior to t = 0 can be verified by substitution into (1) and (2). This method of checking requires first finding the solution for C(t), however. This won't be done here. It is sufficient to check the behavior of 10.1 and 10.2 in some limiting cases.

1. At t = 0 $\quad n(t)\Big]_{10.1} = n_0$

2. At $t = t_1$, i.e. at the end of the pulse, both 10.1 and 10.2 give the same result for n(t).

3. $\lim_{t \to \infty} n(t)\Big]_{10.2} = n_0 + A_0 t_1 \lambda \tau$

After the disturbance has passed, the steady state neutron population has an added increment of

$$A_0 t_1 \lambda \tau = \frac{A_0 t_1}{1 + \frac{\beta}{\ell^* \lambda}} .$$

[redacted]

Notice that a hypothetical reactor with no delayed fraction $(\beta \to 0)$ would have an increase in neutron population equal to the total number of neutrons contained in the pulse. Notice also that the added increment in population increases with an increase in $\ell^*$. Evidently a longer prompt lifetime reactor "remembers" its past history better than a short lifetime reactor does.

4. If $\frac{t_1}{\tau} << 1$, the highest neutron population attained is $n(t_1)\Big]_{10.1} = n_0 + A_0 t_1$. That is, if the pulse width is small compared to the reactor characteristic time, the peak power is independent of the prompt lifetime and delayed group characteristics.

As far as damage to the reactor is concerned an important quantity is the total amount of energy generated during the transient resulting from the source pulse. The assumption that power is proportional to neutron population allows calculation of this quantity. It is convenient, first, to put equations 10.1 and 10.2 into a dimensionless form. Define the following:

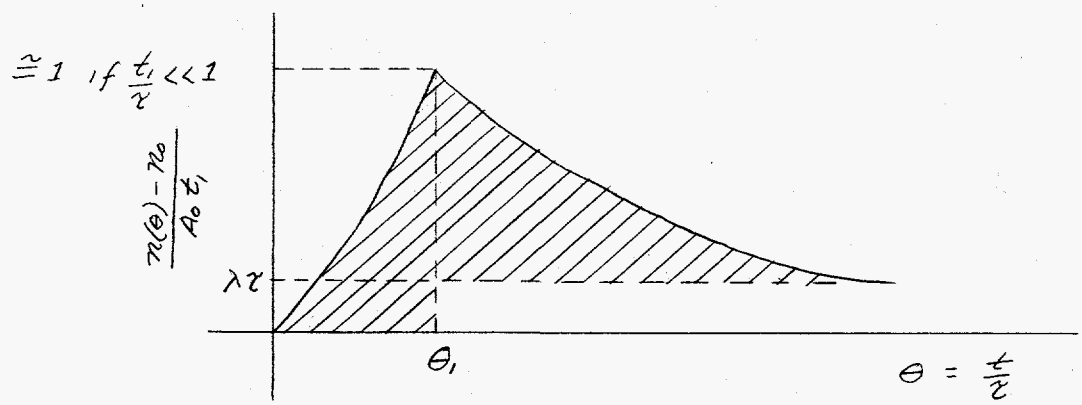
$$\theta \equiv \frac{t}{\tau} \qquad\qquad \theta_1 \equiv \frac{t_1}{\tau_1}$$

With these definitions and a little more manipulation, 10.1 and 10.2 become

$$\frac{n(\theta) - n_0}{A_0 t_1} = \frac{\lambda\tau}{\theta_1}\left[\left(\frac{1-\lambda\tau}{\lambda\tau}\right)\left(1 - e^{-\theta}\right) + \theta\right], \quad \theta < \theta_1 \qquad 11.1$$

$$\frac{n(\theta) - n_0}{A_0 t_1} = \frac{\lambda\tau}{\theta_1}\left[\left(\frac{1-\lambda\tau}{\lambda\tau}\right)\left(1 - e^{-\theta_1}\right) e^{-(\theta-\theta_1)} + \theta_1\right], \quad \theta > \theta_1 \qquad 11.2$$

The left hand sides of these equations are dimensionless and are proportional to the incremental increase in power level. Preliminary to determining the energy generated by the pulse, consider the rough sketch below of Eq. 11.




The energy generated during the transient is proportional to the area under the curve shown in the figure. In computing the contribution due to the decay after the source is removed it is convenient to subtract out the small residual which persists as $\theta \to \infty$. This allows the upper limit of integration to extend to $\infty$ and yields a closed form result. Hence, the area computed in the following calculation is the one shaded in the sketch - the long time contribution is subtracted out.

The integrals to be determined are

$$\int_{\theta=0}^{\theta=\theta_1} \left[\frac{n(\theta)-n_0}{A_0 t_1}\right]_{11.1} d\theta + \int_{\theta=\theta_1}^{\theta=\infty} \left\{\left[\frac{n(\theta)-n_0}{A_0 t_1}\right]_{11.2} - \lambda\tau\right\} d\theta$$

$$= \int_0^{\theta_1} \frac{\lambda\tau}{\theta_1}\left[\left(\frac{1-\lambda\tau}{\lambda\tau}\right)\left(1-e^{-\theta}\right)+\theta\right] d\theta + \int_{\theta_1}^{\infty} \frac{\lambda\tau}{\theta_1}\left(\frac{1-\lambda\tau}{\lambda\tau}\right)\left(e^{\theta_1}-1\right)e^{-\theta}\, d\theta$$

$$= 1 + \lambda\tau\left(\frac{\theta_1}{2} - 1\right) \qquad (12)$$

The contribution that was subtracted out to make the integration convenient can be added back in at this point. In order to do this it is necessary to define a time $\Theta$ after which the disturbance is assumed to be non-existent. If $\Theta = \theta_1 + 5$, the tail of the disturbance is essentially gone ( $e^{-5} = 0.007$ ). So if the disturbance is over at $\Theta = \theta_1 + 5$, the contribution to be added to Eq. 12 is $5\lambda\tau$. Then the total extra energy generated by the disturbance is proportional to

$$I \equiv \int_{\theta=0}^{\theta=\theta_1+5} \left[\frac{n(\theta)-n_0}{A_0 t_1}\right] d\theta \cong 1 + \lambda\tau\left(\frac{\theta_1}{2}-1\right) + 5\lambda\tau = 1 + \lambda\tau\left(\frac{\theta_1}{2}+4\right) \tag{13}$$

Changing the form of (13) back to familiar reactor parameters yields

$$I = 1 + \frac{\lambda t_1}{2} + \frac{4}{1+\frac{\beta/\lambda}{\ell^*}} \tag{13.1}$$

$$\text{Now} \quad \tau \int_0^{\theta_1+5} \left[\frac{n(\theta)-n_0}{A_0 t_1}\right] d\theta = \int_{t=0}^{t=t_1+5\tau} \left[\frac{n(t)-n_0}{A_0 t_1}\right] dt = \frac{1}{\frac{\beta}{\ell^*}+\lambda} I$$

$$\therefore \int_{t=0}^{t=t_1+5\tau} \left[n(t)-n_0\right] dt = \frac{A_0 t_1}{\frac{\beta}{\ell^*}+\lambda}\left(1 + \frac{\lambda t_1}{2} + \frac{4}{1+\frac{\beta/\lambda}{\ell^*}}\right) \tag{14}$$

The term within the parentheses in Eq. 14 is a function of pulse width, reactor prompt lifetime and reactor delayed group characteristics. In fact, however, for any reasonable values which these parameters may assume the term in parenthesis isn't a function of any of them. The only term of significance is the first. For example, assume the same values for reactor parameters as those used in NAM No. 3 ($\ell^* = 10^{-4}$ sec, $\beta = 0.007$, $\lambda = 0.1/\text{sec}$). Then the parenthesis term becomes

$$1 + \frac{t_1}{20\,\text{sec}} + \frac{4}{701}$$

Notice that the duration of the pulse would have to be a few seconds before a significant contribution would arise. This is much longer than any pulse to be expected from a nuclear explosion.

Finally, the total extra energy generated due to the large pulse of neutrons is

$$\Delta E = C\,(A_0 t_1)\,\frac{1}{\frac{\beta}{\ell^*} + \lambda} \tag{15}$$

Where C is the proportionality factor between total reactor neutron population and reactor power. This result, except for the appearance of $\lambda$ which is small compared to $\frac{\beta}{\ell^*}$ is identical to the result obtained in NAM No. 3.

## Discussion

The results arrived at above were based upon the assumption of a square wave forcing function. However, they suggest that for any sensible pulse widths to be expected the total energy generated is probably quite insensitive to the shape of the pulse. The peak power attained is somewhat more sensitive to pulse width which suggests that peak power in the general case would be sensitive to pulse shape.

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