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INITIAL HEATING CONSIDERATIONS FOR POROUS MATERIALS DESCRIBABLE WITH EXPONENTIAL P-α RELATION (U)

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ABSTRACT (CFRD)

The work by Herrmann to develop a means for predicting the dynamic responses of initially heated porous materials is extended to include porous materials that can be described with an exponential relation between the distention ratio, α, and pressure, P. Examples are presented that illustrate how Hugoniots can vary with initial heating. In addition, instantaneous heating is considered, and examples are shown in which the generated pressure is related to the degree of heating.

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INITIAL HEATING CONSIDERATIONS FOR POROUS MATERIALS DESCRIBABLE WITH EXPONENTIAL P-α RELATION

Introduction

Porous, or distended, materials have long been of interest because of their excellent stress pulse attenuation properties and because of the low stress levels that are generated as a result of rapid increases in internal energy. In keeping with this interest, considerable effort has been expended in recent years to characterize the dynamic responses of these materials to a variety of different stimuli. The need for predicting the dynamic behavior of porous materials has been a strong stimulus to the development of analytical models that can be used in numerical calculative schemes.

One such model, which is commonly referred to as the P-α model, was developed by Herrmann[1-3] and has been widely used to obtain numerical solutions to wave propagation problems. The formulation of the P-α model is essentially based on the premise that the equation of state of the porous material can be determined from the equation of state of the corresponding solid material provided the distention ratio, α, can be specified as a function of pressure, P, and energy, E.

The procedure that is generally followed is to determine the pressure dependence of α from laboratory shock loading experiments, i.e., from the Hugoniot behavior. The energy dependence of α is considerably more elusive, primarily because very few experimental data are presently available to provide guidance in establishing a quantitatively consistent formulation. Early versions of the P-α model[1,2] did not have provisions for specifying an explicit dependence of α on energy and thus had their greatest utility in dealing with problems where temperatures were near room temperature, i.e., near the temperature where the Hugoniot measurements were made. In a number of applications, however, the explicit dependence of α on E can be an important consideration. For example, the response of a porous material to rapid increases in internal energy is strongly dependent on the explicit relation between α and E, particularly if the energy range of interest is

near the region of melt. In Reference 3, Herrmann presents one method for incorporating explicit energy dependence into the P-α model. This was accomplished by considering the dependence of the Hugoniot on initial heating. The formalism developed in Reference 3, however, is specifically applicable to porous materials with Hugoniots that can be described with a quadratic relation between α and P. Not all porous materials fall into this category. For example, the Hugoniot of porous copper cannot be adequately described with a quadratic relation but instead is more accurately described using an exponential relation.[4] A number of other porous materials[5] can also be described with an exponential relation.

The purpose of this report, then, is to present a method for introducing explicit energy dependence into the P-α model for cases where the Hugoniots of the porous materials can be described with an exponential relation between α and P. This work is an extension of Herrmann's work. The major difference lies primarily in the manner of treating the porous material response near the energy region of melt.

A number of examples are shown, all of which are for a porous copper. While Hugoniot properties of the porous copper have been determined at room temperature,[4] the behavior of this material at elevated temperatures is completely unknown. Thus, no claims are made for the quantitative accuracy of the examples. Rather, the work is intended to illustrate plausible responses of porous materials, with emphasis placed on the latitude that is possible by varying the functional energy dependence of the various parameters that effect the dynamic response and to illustrate the need for experimental data.

Dependence of Porous Material Hugoniot on Initial Heating

In the P-α model the distention ration, α, is defined as

$$\alpha = V/V_s \quad , \tag{1}$$

where V is the specific volume of the porous material at a given pressure*, P, and specific internal energy, E, state and V_s is the specific volume of the corresponding solid material at the same pressure and energy state. The underlying assumption in the P-α model is that the equation of state of the porous material can

*In this report pressure and stress will be considered to be synonomous.

be expressed with the same functional relation as is used for the solid material provided one accounts for the variation of α with P and E. That is, if

$$P = f(V_s, E) \tag{2}$$

represents the equation of state of the solid material, then

$$P = f(V/\alpha, E) \tag{3}$$

represents the equation of state of the porous material. In this work, the equation of state of the solid material is assumed to be the Mie-Grüneisen equation, expressed as

$$P - P_H = \frac{\gamma}{V_s}(E - E_H) \quad , \tag{4}$$

where the Hugoniot of the solid material is used as a reference curve. P_H and E_H, respectively, are the pressure and specific internal energy along the Hugoniot of the solid (evaluated at V_s) and γ is the Grüneisen parameter. The quantity, γ/V_s, has been taken to be constant and equal to its zero stress value, γ_o/V_{so}. Assuming a linear relation between shock wave velocity, U_s, and particle velocity, u_p,

$$U_s = C_o + Su_p \quad , \tag{5}$$

where C_o and S are constants characteristic of a material, the Rankine-Hugoniot equations for conservation of mass and momentum can be combined with Equation 5 to yield

$$P_H = \frac{C_o^2(V_{so} - V_s)}{\left[V_{so} - S(V_{so} - V_s)\right]^2} \tag{6}$$

for the pressure along the Hugoniot. The specific internal energy along the Hugoniot can be expressed as

$$E_H = P_H(V_{so} - V_s)/2 \quad . \tag{7}$$

When combined, Equations 4, 6, and 7 define the function $f(V_s, E)$ in Equation 2. To complete the description of the equation of state of the porous material, it is necessary to specify the functional dependence of α on P and E.

In the simplest formulation, α can be assumed to be a function of P only, which is what was done in the earliest versions[1, 2] of the P-α model. The mathematical formalism developed for this simple case provides a convenient means for introducing explicit energy dependence. Before proceeding with explicit energy considerations, a brief summary of commonly used methods for expressing the variation of α with P will be given.

Generally, when treating porous materials, precursor waves must be considered so it is convenient to divide the analysis into two stress regions: one region for stress levels below the precursor amplitude and one for higher stress levels. Below the precursor amplitude the variation of α with P is usually assumed to be linear, i.e.,

$$\alpha = \alpha_o - (\alpha_o - \alpha_1)P/P_1 , \tag{8}$$

where α_o and α_1 are values for α at zero stress and at the precursor amplitude, respectively, and P_1 is the precursor amplitude. For stress levels above the precursor amplitude, several relations have been used. Among the more common are the quadratic relation,

$$\alpha = 1 + (\alpha_1 - 1)\left(\frac{P_s - P}{P_s - P_1}\right)^2 , \tag{9}$$

and the exponential relation,

$$\alpha = 1 + (\alpha_1 - 1) \exp\left[-\hat{a}(P - P_1)\right] . \tag{10}$$

In Equation 9, P_s is the stress level where compaction becomes complete (i.e., where $\alpha = 1$ or where $V = V_s$). In Equation 10, $\hat{a}$ is a parameter that controls the rate with which a fully compacted state is reached with increasing stress. Equation 9 was considered by Herrmann to introduce explicit energy dependence into the P-α model. Equation 10 will be considered in this work. One of the

main differences between the two equations is that a fully compacted state is never actually reached with the exponential equation. It is this difference that necessitates the different approach taken in this work as compared with the approach taken by Herrmann.

In Herrmann's work, three quantities were assumed to depend on the initial temperature (or energy) of the porous material, namely V_{s0}, P_1, and P_s. If the material is heated to a temperature, T_i, above room temperature, T_{rm}, the new specific volume of the solid at zero stress is,

$$V_{so}(T_i) = V_{so}(T_{rm})\left[1 + \beta\left\{T_i - T_{rm}\right\}\right] , \tag{11}$$

where β is the volumetric thermal expansion coefficient (assumed to be constant). With no change in the initial distention ratio, the new initial specific volume of the porous material is $V_o(T_i) = \alpha_o V_{so}(T_i)$. Other parameters (like C_o, S, and γ_o) related to the response of the solid material were assumed by Hermann to be independent of temperature. Both parameters related to the compression of the porous material (P_1 and P_s) were assumed to decrease with increasing temperature and vanish at some energy where the strength of the material becomes negligible.

The same basic procedures followed by Herrmann will be used here except that the energy dependence of the compression of the porous material above the precursor amplitude will be introduced through the parameter $\hat{a}$, rather than through P_s. Concepts of the formulation will be discussed through use of specific mathematical expressions defining the dependence of the quantities of interest on energy. No loss of generality results from this approach if the mathematical expressions are understood to be illustrative only. These expressions can be replaced by more appropriate expressions if future experimental data so dictates.

The initial specific internal energy of a heated specimen will be expressed as E_i, with $E_i = E_i(T_{rm}) = 0$ representing the room temperature energy. The energy level where the strength becomes negligible is taken to be the incipient melt energy, E_{IM}, and the ratio, E_i/E_{IM}, will be designated as Q.

A relation hypothesized to represent the decrease in precursor amplitude with increasing energy for $Q < 1$ is

$$P_1 = P_{1o}(1 - Q^2) \quad , \tag{12}$$

where P_{1o} is the precursor amplitude at room temperature. For $Q \geq 1$, P_1 is zero. The basic equation assumed here to represent the variation $\hat{a}$ with increasing energy is

$$\hat{a} = \hat{a}_o/(1 - Q^2) \quad , \tag{13}$$

where $\hat{a}_o$ is the value of $\hat{a}$ at room temperature. When $Q = 1$, P_1 vanishes and $\hat{a}$ goes to infinity. This implies that a state of complete compaction is reached even for infinitesimal increases in stress. Possible rate dependent and small residual strength effects may impose modifications to this description. Mathematically it is possible to retain some residual strength effects by defining a new relation for $\hat{a}$ as the value of Q approaches unity, i.e., for all values of Q greater than some value, Q^*. This relation would have the property that $\hat{a}$ always remain finite. A possible form for such a relation is

$$\hat{a} = \hat{a}_o(k_1 + k_2 Q) \quad . \tag{14}$$

The constants, k_1 and k_2, can be determined by requiring that the functions in Equations 13 and 14 and their derivatives with respect to Q be continuous at $Q = Q^*$. With these conditions, the constants are

$$k_1 = \left(1 - 3Q^{*2}\right)/\left(1 - Q^{*2}\right)^2 \tag{15}$$

$$K_2 = 2Q^{*2}/\left(1 - Q^{*2}\right)^2 \quad . \tag{16}$$

Thus, for examples discussed here, $\hat{a}$ is defined by Equation 13 for $Q < Q^*$ and by Equation 14 for $Q \geq Q^*$. The influence of the quantity Q^* is not great when considering the dependence of the Hugoniot on initial heating but does have important consequences for cases where instantaneous heating occurs. This will be discussed later in the report.

The reason for choosing the particular form for $\hat{a}$ given in Equation 13 can be seen by considering the stress level where the porous material has been compacted to some arbitrary state near complete compaction, say where $(\alpha - 1)/(\alpha_1 - 1) = 0.01$. This stress level (call it $P_{0.01}$) can be determined from Equation 10 to be

$$P_{0.01} - P_1 = \ln(0.01)/\hat{a} \tag{17}$$

or

$$P_{0.01} - P_1 = \frac{\ln(0.01)}{\hat{a}_o}(1 - Q^2) \quad . \tag{18}$$

This type of behavior is similar to the behavior that would result if $(P_s - P_1)$ is assumed to depend quadratically on Q and (nearly) corresponds to one of the cases considered by Herrmann. The quadratic function of Q in Equation 18 (and also in Equation 12), while intended primarily for illustrative purposes, is however felt to be quite reasonable. These relations show that the strength of the material decreases rather slowly for small increases in internal energy. When the initial energy approaches the incipient melt energy, however, the loss of strength is more pronounced.

To illustrate how the above defined initial heating considerations affect a porous material Hugoniot, a number of calculations were made for a porous copper. These calculations were performed using the computer program (called TEMPPL) given in Appendix A of this report. Values for the various constants required in the calculation are given in Table I. The results of the calculations are shown on Figure 1. Values of Q equal to 0, 0.5, 0.75, and 0.95 were considered. The assumed value of Q^* for these calculations was 0.95 so Equation 14 had no effect on the results. Figure 1 is a graph showing stress-specific volume Hugoniots for the porous copper. As is evident, the initial specific volume of the porous material increases as the initial energy (or the value of Q) is increased. The Hugoniot of solid copper is also displaced toward larger specific volumes as the initial energy is increased, however, this is not shown on the figure. The important features of Figure 1 are that the precursor amplitude decreases with increasing initial energy and that essentially fully compacted states are reached at lower stress levels as the initial energy is increased. It should be again

emphasized that the behavior depicted on Figure 1 is based on the energy dependence assumed in Equations 12 and 13.

TABLE I

Values of Constants Used for Porous Copper in the Calculations

Solid Copper Parameters	Porous Copper Parameters
$\rho_{so} = 1/V_{so} = 8.93\ g/cm^3$	$\rho_o = 1/V_o = 6.43\ g/cm^3$
$C_o = 4.022\ mm/\mu sec$	$P_{10} = 1.35\ kbar$
$S = 1.48$	$^{**}U_{s1} = 1.93\ mm/\mu sec$
$\gamma_o = 1.96$	$\hat{a} = 0.254\ kbar^{-1}$
$\beta = 6.0 \times 10^{-5}\ {}^oC^{-1}$	
$^*C_p = 4.39 \times 10^6\ ergs/g/{}^oC$	
$E_{IM} = 4.7 \times 10^9\ erg/g$	

*Specific heat.
**Precursor wave velocity (assumed to be independent of temperature).

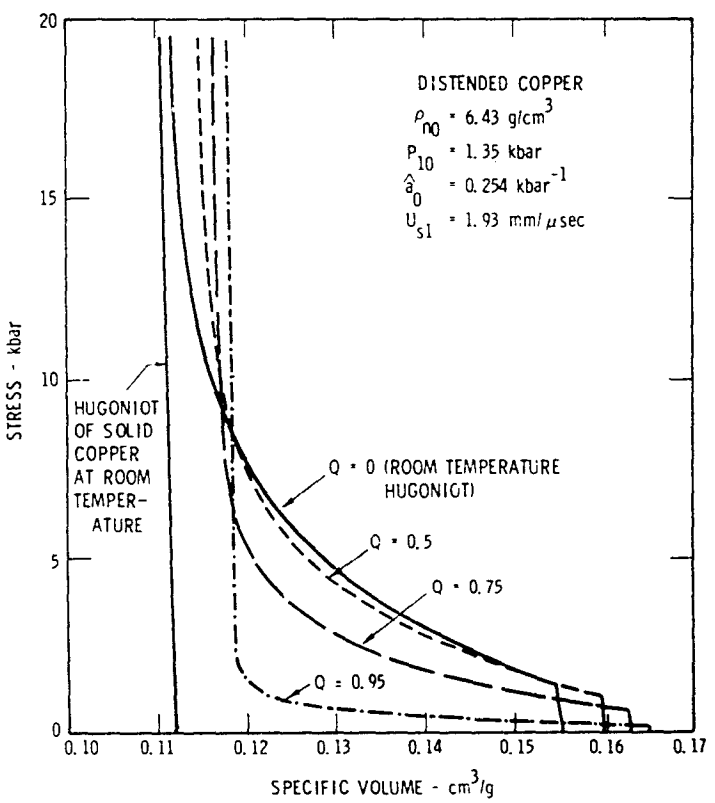

Figure 1.

Stress-Specific Volume Graph Showing Effect of Initial Heating on Hugoniot of Distended Copper

Instantaneous Heating Considerations

To consider the effect of instantaneous heating on the response of a porous material, it is convenient to define a new quantity as an independent variable in Equations 2 and 3. This quantity is

$$\eta = 1 - V_s/V_{so} \quad , \tag{19}$$

which becomes

$$\eta = 1 - \frac{V\alpha_o}{\alpha V_o} \tag{20}$$

when expressed in terms of the specific volumes of the porous material. Since the heating process is instantaneous, no time is available for a volume change. Thus, $V = V_o$ and

$$\eta = 1 - \frac{\alpha_o}{\alpha} \quad . \tag{21}$$

When the pressure and energy along the Hugoniot of the solid are expressed in terms of the quantity η, Equations 6 and 7 become

$$P_H = \frac{C_o^2}{V_{so}} \frac{\eta}{(1 - S\eta)^2} \tag{22}$$

and

$$E_H = P_H V_{so} \eta/2 \quad . \tag{23}$$

Substituting Equations 22 and 23 into Equation 4, the relation between pressure generated in the porous material and the increase in energy becomes

$$P = \frac{\gamma_o}{V_{so}} E + \frac{C_o^2}{V_{so}} \frac{\eta}{(1 - S\eta)^2} (1 - \gamma_o \eta/2) \quad . \tag{24}$$

Since η, through Equation 21, depends on α and hence on both P and E, Equation 24 must be solved using an iterative procedure. A computer program (called TDGEFF) to accomplish this is given in Appendix B of this report. This program has been used to obtain plots of generated pressure versus the increase in specific internal energy for porous copper. The results of the calculations are shown on Figure 2.

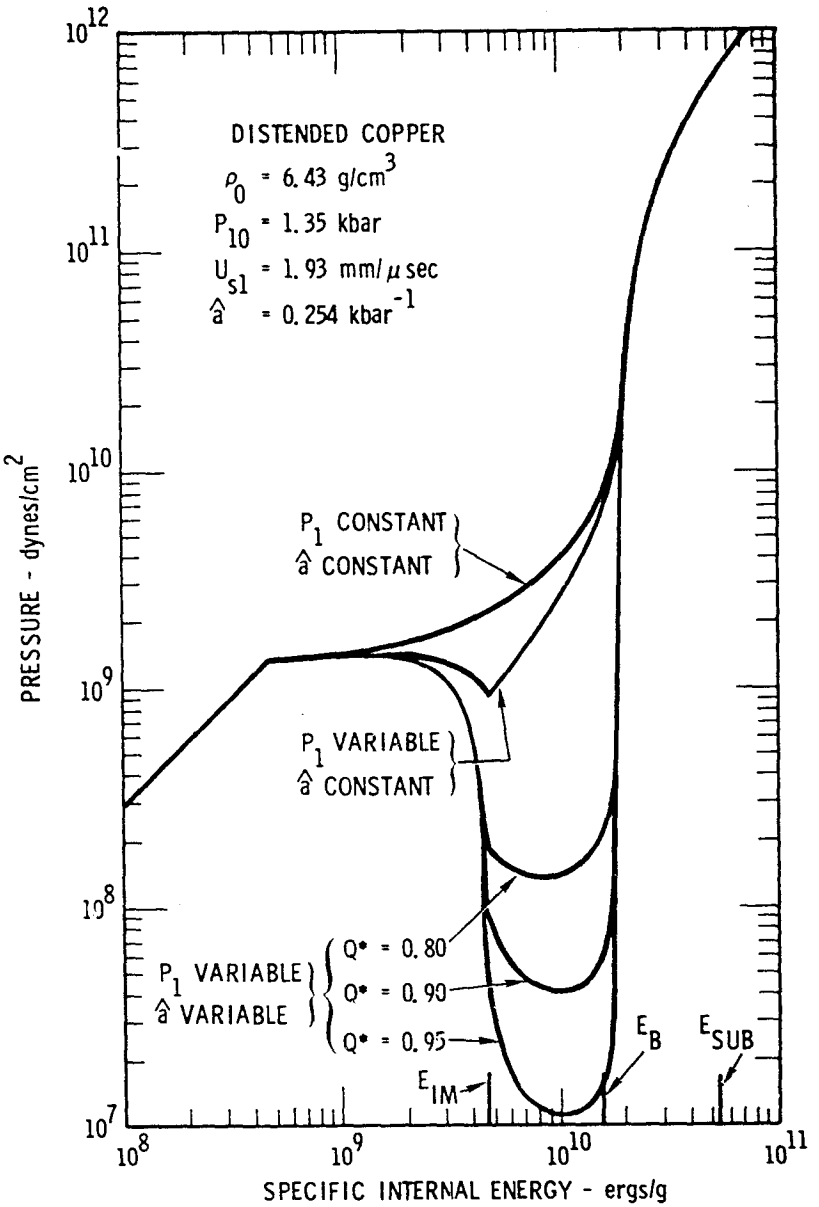

Figure 2. Plot of Pressure Generated Versus Increase in Specific Internal Energy for Case of Instantaneous Heating

The pressure dependence of α for these curves is that given by Equation 8 and 10. The energy dependence is introduced through Equations 12 through 14, except for two cases where explicit energy dependence is suppressed or partially suppressed (the two uppermost curves). All of the curves on Figure 2 depict essentially the same behavior in the lowest energy region and in the highest energy region. At low energies, a linear relation between pressure and energy is observed that exists until the precursor amplitude is exceeded. This region can be interpreted as an elastic region. Further increases in energy have the effect of causing the fully dense portions of the porous material to flow plastically into the voids. The effect of the energy dependence of α becomes evident for energy levels above the linear region and into the melt region.

The uppermost curve on Figure 2 illustrates the behavior that results when both P_1 and $\hat{a}$ are considered to be independent of energy. The pressure for this case increases monatomically with increasing energy, which is probably not physically realistic.

The effect of allowing only the precursor amplitude to decrease with energy is depicted by the second curve from the top of Figure 2. Here moderate decreases in pressure occur as the energy is increased up to the level E_{IM}, where P_1 assumes a constant value of zero. Further increases in energy produce increases in pressure.

The effect of allowing the parameter $\hat{a}$ to increase with energy is illustrated with the three lowest curves on Figure 2. The effect of the parameter Q^* is also illustrated here. If Q^* were equal to unity, $\hat{a}$ would approach an infinite value as E approaches E_{IM} and the pressure generated would approach zero. By assigning various values to Q^* which are less than unity, the minimum pressure can be adjusted. Figure 3 shows values of $\hat{a}$ as a function of Q for the three values of Q^* considered on Figure 2.

In the highest energy region shown on Figure 2, all of the curves become coincident. This occurs when the value for α approaches unity, or when the voids are filled. Thus, the energy dependence assumed for α is no longer of consequence insofar as Equation 24 is concerned and η assumes a constant value

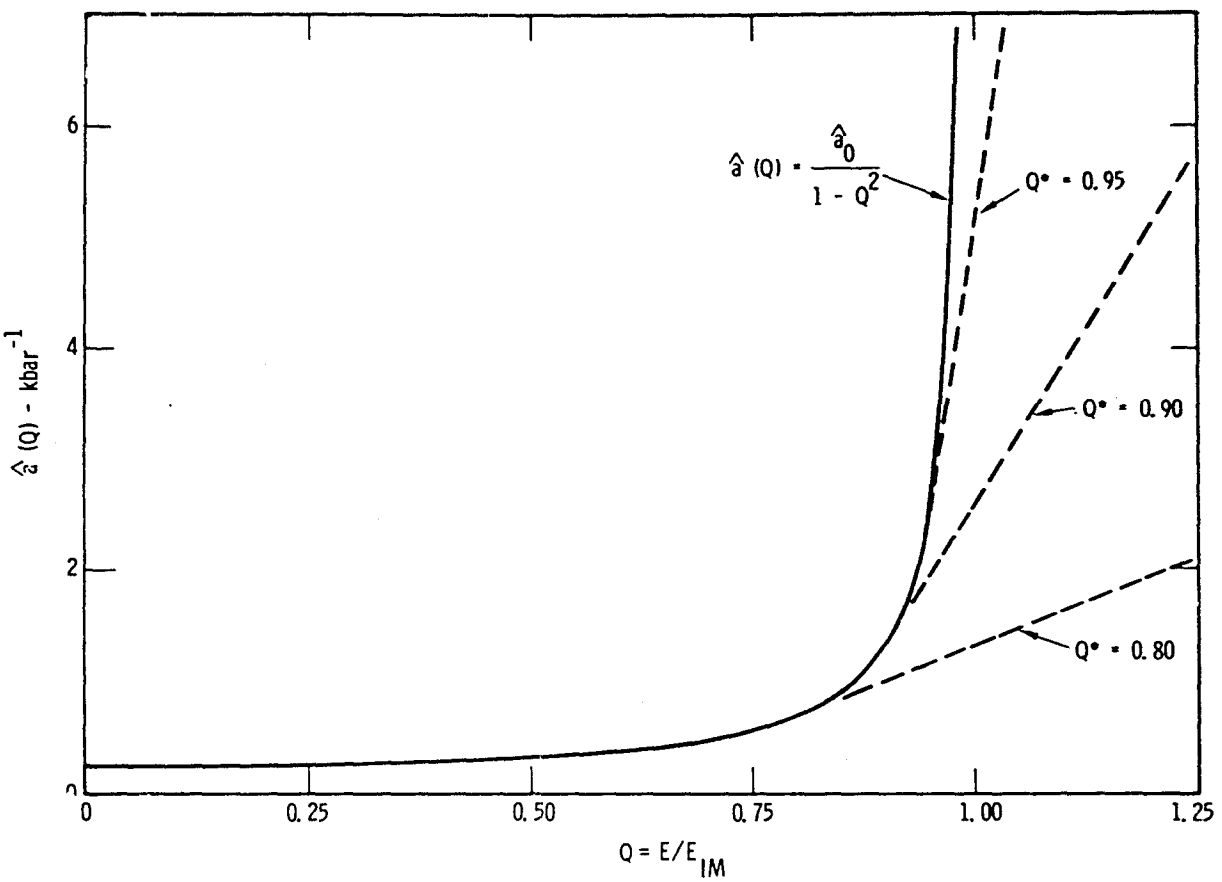

Figure 3. Values Used For â For Cases Where Q* is Equal to 0.80, 0.90, and 0.95

of $1 - \alpha_o$. The rapid increase in pressure with energy occurs at an energy level just greater than the boiling point of copper E_B. This is reasonable since the vapor can then fill the voids; however, that the model should predict this is considered to be fortuitous. No provisions are presently incorporated into the model which relate E_B to the energy level where α becomes equal to unity. The energy level, E_{SUB}, shown on the right of Figure 2, is the sublimation energy of copper. The equation of state to be used in the higher energy region should properly include vapor considerations to account for the mixed phase region between E_B and E_{SUB} and the all vapor region above E_{SUB}. Because the equation of state considered here does not account for vapor, the high energy region shown on Figure 2 does not represent a truly realistic situation.

It is of interest also to consider an effective Grüneisen parameter for a porous material. This quantity can be defined as

$$\gamma_{eff} = PV_o/E \quad . \tag{25}$$

Values for γ_{eff} as a function of energy are shown on Figure 4 for the cases considered on Figure 2. At low energy levels (in the elastic region), the value for γ_{eff} is essentially constant. Beyond the elastic region, values for γ_{eff} decrease and the behavior for all cases considered is similar until the energy level approaches E_{IM}. At that time rather dramatic decreases in the value of γ_{eff} are seen for the cases where both P_1 and $\hat{a}$ are considered to be functions of energy. For the cases where P_1 and/or $\hat{a}$ are held constant, the rapid decrease in the value of γ_{eff} does not occur. At the highest energy levels shown on Figure 4, values for γ_{eff} increase rapidly and all cases become equivalent.

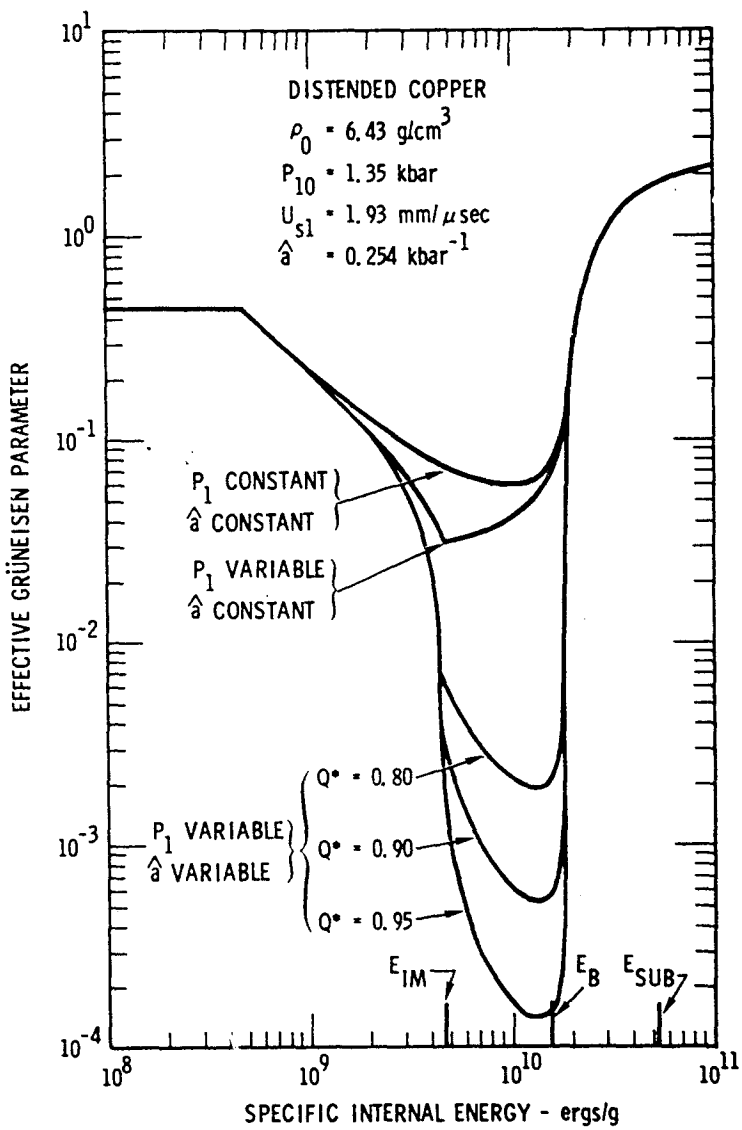

Figure 4. Values of γ_{eff} Versus Increase in Specific Internal Energy for Case of Instantaneous Heating

Discussion

The work presented in this report illustrates the general features of the dynamic behavior of porous materials when subjected to conditions of initial heating. Explicit energy dependence has been introduced by considering the dependence of the Hugoniot of the porous material on initial temperature. In so doing, only the parameters (i.e., P_1 and $\hat{a}$) that produce the most significant effects have been assumed to depend on energy. Other quantities (e.g., the velocity of the precursor wave in the porous material and the quantities related to the Hugoniot of the solid material) have been assumed to be independent of initial temperature, which is justified since reasonable variations of these quantities with energy do not significantly alter any of the results presented in this report.

In general, the predicted responses of heat porous materials presented here are considered to be realistic but not necessarily exact. An exact description of porous material behavior under conditions of initial heating must await the results of future experimental studies. One of the objectives of this work is to point out the need for such experiments. Two types of experiments are required. One is the conventional Hugoniot experiment (probably of the gas gun impact variety) where the porous specimens are initially heated to various temperatures between room temperatures and the melting temperature. Such experiments would yield information about the actual variation of P_1 and $\hat{a}$ with energy. The other type of experiment required is that where the porous specimen is instantaneously (or very rapidly) heated, as for example with an electron beam. These latter experiments would verify that the approach taken in this work to predict pressure levels is in fact valid or would define areas where improvement is necessary.

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Appendix A

This Appendix describes the program TEMPPL which computes the Hugoniot of a porous material that has been raised to an initial energy, E_i. An exponential relation between α and P is assumed. The program is written in BASIC language for Sandia's PDP10 time sharing computer. A listing of the program follows:

```
TEMPPL          09:51          04-DEC-70

10 REM TEMPPL RRBOADE 1224
20 READ J1
30 READ R0,C0,S,G0
40 READ Q9,C9,B9,Q8
50 READ R6,S6,P6,A6
60 LET K1=(1-3*Q8↑2)/(1-Q8↑2)↑2
70 LET K2=2*Q8/(1-Q8↑2)↑2
80 LET V0=1/R0
90 LET C=C0/10
100 LET V6=1/R6
110 LET S7=S6/10
120 LET P7=P6/1000
130 LET L0=V6/V0
140 PRINT"              SOLID MATERIAL PARAMETERS"
150 PRINT
160 PRINT "  RHO(0)","  V(0)","  C0","  S","  GAMMA(0)"
170 PRINT R0,V0,C0,S,G0
180 PRINT
190 PRINT "  CP","  BETA","  E(IM)"
200 PRINT C9,B9,Q9
210 PRINT
220 PRINT
230 PRINT "              POROUS MATERIAL PARAMETERS"
240 PRINT
250 PRINT "  RHO(0)","  V(0)","  C1","  P1(0)","  ALPHA(0)"
260 PRINT R6,V6,S6,P6,L0
270 PRINT
280 PRINT "  Q(STAR)","  K1","  K2"
290 PRINT Q8,K1,K2
300 PRINT
310 PRINT
320 PRINT
330 READ Q
340 LET P8=P7*(1-Q↑2)
350 IF Q>Q8 THEN 380
360 LET A=A6/(1-Q↑2)
370 GO TO 390
380 LET A=A6*(K1+K2*Q)
390 LET W1=Q*Q9
400 LET T=W1/C9
410 LET V1=V0*(1+B9*T)
420 LET V7=V1*L0
430 LET R7=1/V7
440 LET U7=P8/(R7*S7)
450 LET V8=V7*(1-U7/S7)
460 LET G=G0/(2*V1)
470 LET K0=C*C/V1
480 LET D3=P8*(1-G*(V7-V8))
490 LET D1=S*S*D3+G*K0*V1
500 LET D2=2*S*D3+K0
510 LET D4=(D2/(2*D1))↑2-D3/D1
520 LET N1=D2/(2*D1)-SQR(D4)
530 LET V2=V1*(1-N1)
540 LET L1=V8/V2
550 PRINT "  PARAMETERS AT T=";T*2;"DEGREES C"
560 PRINT "E=";Q;"E(IM) =";W1;"ERGS/G"
570 PRINT "V(SO) AT E =";V1;"CC/G"
580 PRINT
590 PRINT "              POROUS MATERIAL PARAMETERS AT T"
600 PRINT "  RHO(0)","  V(0)","  P1","  A"
610 PRINT 1/V7,V7,1000*P8,A
620 PRINT
630 PRINT "  V(1)","  R(1)","  UP(1)","  ALPHA(1)"
640 PRINT V8,1/V8,10*U7,L1
650 PRINT
660 PRINT
670 PRINT "  P","  V","  US","  UP","  ALPHA"
680 LET F=P8
690 LET A=A*1000
700 LET F1=.1/A
710 FOR J=0 TO J1
720 LET F=F+J*F1
730 LET F4=F
740 IF F=0 THEN 900
750 FOR X=1 TO 30
760 LET L2=1+(L1-1)*EXP(-A*(F-P8))
770 LET H1=2*S+K0/F4
780 LET N2=(H1-SQR(H1↑2-4*S*S))/(2*S*S)
790 LET V2=V1*(1-N2)
800 LET V9=L2*V2
810 LET F5=(F*(V8-V9-1/G)+P8*(V7-V9))/(V1-V2-1/G)
820 IF ABS((F5-F4)/F5)<=.000001 THEN 870
830 LET F8=F4
840 LET F4=F5
850 NEXT X
860 PRINT "ITERATION DID NOT CONVERGE"
870 LET S8=U7+V8*SQR((F-P8)/(V8-V9))
880 LET U8=U7+SQR((F-P8)*(V8-V9))
890 PRINT 1000*F,V9,10*S8,10*U8,L2
900 NEXT J
910 PRINT
920 PRINT
930 GO TO 330
940 DATA
950 DATA
960 DATA
970 DATA
980 DATA
2000 END
```

Input instructions can be made in statement numbers 940 through 980 as follows:

Statement 940: Enter J1, the number of points to be calculated. The first calculation is made at the precursor amplitude. Subsequent calculations are made at increments of stress that increase in such a way that 20 calculated points usually cover the stress range of incomplete compaction quite adequately.

Statement 950: Enter solid material parameters for room temperature; ρ_{so} (initial density in g/cm^3), C_o (bulk sound speed in mm/μsec), S (slope of $U_s - u_p$ line), and γ_o (zero stress Grüneisen parameter).

Statement 960: Enter the parameters; E_{IM} (incipient melt energy in ergs/g), C_p (specific heat of solid in ergs/g/^oC), β (volumetric thermal expansion coefficient in $^oC^{-1}$), and Q* (see Equations 13 and 14 of text).

Statement 970: Enter porous material parameters for room temperature; ρ_o (initial density in g/cm^3), U_{s1} (velocity of precursor wave in mm/μsec), P_{1o} (precursor amplitude in kbar), and $\hat{a}_o$ (parameter in Equations 13 and 14 in kbar^{-1}).

Statement 980: Enter values for $Q = E_i/E_{IM}$ to be calculated.

The printed output of the program includes values for most of the important parameters affecting the solution and values for specific volume (V), shock wave velocity (US), particle velocity (UP), and α (ALPHA) for various values of pressure (P) above the precursor amplitude. The first values printed for US and UP are not correct since the pressure is at the precursor amplitude and no additional shock wave exists as yet at that level. A sample calculation is shown in the following:

[redacted]

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```
940 DATA 20
950 DATA 8.93,4.022,1.48,1.96
960 DATA 4.7E9,.439E7,60.E-6,.95
970 DATA 6.43,1.93,1.35,.254
980 DATA .5
RUN

TEMPPL          13:04          27-NOV-70


          SOLID MATERIAL PARAMETERS

 RHO(0)          V(0)            C0              S               GAMMA(0)
8.93             0.111982        4.022           1.48            1.96

 CP              BETA            E(IM)
4390000          6.00000 E-5     4.70000 E+9


          POROUS MATERIAL PARAMETERS

 RHO(0)          V(0)            C1              P1(0)           ALPHA(0)
6.43             0.155521        1.93            1.35            1.3888

 Q(STAR)         K1              K2
0.95            -179.619         199.869


  PARAMETERS AT T= 555.308DEGREES C
E= 0.5E(IM) = 2.35000 E+9ERGS/G
V(S0) AT E = 0.115579CC/G

             POROUS MATERIAL PARAMETERS AT T
 RHO(0)          V(0)            P1              A
6.22991          0.160516        1.0125          0.338667

 V(1)            E(1)            UP(1)           ALPHA(1)
0.159816         6.25721         8.42086 E-3     1.38374


 P               V               US              UF              ALPHA
1.0125           0.159816        8.42086 E-3     8.42086 E-3     1.38374
1.30778          0.155574        0.430098        1.96118 E-2     1.34722
1.89833          0.148265        0.451004        4.04079 E-2     1.28428
2.78415          0.139707        0.482791        6.81080 E-2     1.2106
3.96526          0.131635        0.525737        9.96410 E-2     1.14117
5.44163          0.125159        0.57975         0.132315        1.08562
7.21329          0.120626        0.644124        0.164309        1.04699
9.28022          0.117806        0.717403        0.194788        1.02334
11.6424          0.116216        0.797537        0.223703        1.01049
14.2999          0.115373        0.882275        0.251429        1.00426
17.2527          0.114921        0.969626        0.278439        1.00157
20.5007          0.114644        1.05814         0.305122        1.00052
24.044           0.114432        1.14691         0.331726        1.00016
27.8826          0.114236        1.23548         0.358384        1.00004
32.0164          0.114037        1.32364         0.385159        1.00001
36.4456          0.11383         1.41127         0.412081        1.
41.17            0.113613        1.49835         0.439165        1.
46.1897          0.113385        1.58486         0.466418        1.
51.5046          0.113147        1.67076         0.493846        1.
57.1149          0.112901        1.75607         0.521456        1
63.0204          0.112645        1.84076         0.549251        1
```

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[redacted]

Appendix B

This appendix describes the computer program TDGEFF which computes the pressure generated in a porous material that has been instantaneously heated to an energy level, E. An exponential relation between α and P is assumed. The program also computes the value of the effective Grüneisen parameter, γ_{eff}, as a function of E. Like the program described in Appendix A, TDGEFF is written in BASIC language for Sandia's PDP10 time sharing system. A listing of the program follows:

```
TDGEFF        13:36        27-NOV-70

10 REM TDGEFF BOADF 1224
20 PRINT "UNLESS SPECIFIED, UNITS ARE G/CC, MM/USEC, OR KBAR(OR 1/KBAR)"
30 PRINT
40 PRINT
50 READ R0,C0,S,G0
60 READ Q8,E1,E2,N1,N2
70 READ R6,P6,S6,A0
80 LET V0=1/R0
90 LET C=C0*1.E5
100 LET V6=1/R6
110 LET P7=P6*1.E9
120 LET S7=S6*1.E5
130 LET A1=A0*1.E-9
140 LET L0=V6/V0
150 LET G=G0/(2*V0)
160 LET K0=C*C/V0
170 LET K1=(1-3*Q8↑2)/(1-Q8↑2)↑2
180 LET K2=2*Q8/(1-Q8↑2)↑2
190 PRINT "          SOLID MATERIAL PARAMETERS"
200 PRINT "  RHO(0)","  C0","  S","  GAMMA(0)","  E(IM)-ERGS/G"
210 PRINT R0,C0,S,G0,E2
220 PRINT
230 PRINT
240 PRINT "          POROUS MATERIAL PARAMETERS"
250 PRINT "  RHO(0)","  P1(0)","  US(1)","  ALPHA(0)","  A(0)"
260 PRINT R6,P6,S6,L0,A0
270 PRINT
280 PRINT "  Q(STAR)","  K1","  K2"
290 PRINT Q8,K1,K2
300 PRINT
310 PRINT
320 LET N3=10↑(1/100)
330 LET E=E1/N3
340 LET G6=G0
350 PRINT "  CGS UNITS"
360 PRINT "  E","  P","  ALPHA","  GAMMA(EFF)"
370 LET N9=1
380 FOR N4=1 TO 100*N1
400 LET E=E*N3
410 LET P=G6*E/V6
420 LET Q=E/E2
430 IF Q<1 THEN 460
440 LET P8=0
450 GO TO 470
460 LET P8=P7*(1-Q↑2)
470 IF Q>Q8 THEN 500
480 LET A=A1/(1-Q↑2)
490 GO TO 510
500 LET A=A1*(K1+K2*Q)
510 LET V7=V6*(1-P8/(R6*S7↑2))
520 LET D3=P8*(1-G*(V6-V7))
530 LET D1=S*S*D3+G*K0*V0
540 LET D2=2*S*D3+K0
550 LET D4=(D2/(2*D1))↑2-D3/D1
560 LET N=D2/(2*D1)-SQR(D4)
570 LET V1=V0*(1-N)
580 LET L1=V7/V1
590 IF P<P8 THEN 650
600 IF ABS(A*(P-P8))>25 THEN 630
610 LET L=1+(L1-1)*EXP(-A*(P-P8))
620 GO TO 670
630 LET L=1
640 GO TO 670
650 IF P8=0 THEN 610
660 LET L=L0-(L0-L1)*P/P8
670 GO SUB 960
680 LET F5=P-F1-F2*E
690 LET P4=P
700 LET P=P4*.999
710 FOR M=1 TO 50
720 IF P<P8 THEN 780
730 IF ABS(A*(P-P8))>25 THEN 760
740 LET L=1+(L1-1)*EXP(-A*(P-P8))
750 GO TO 800
760 LET L=1
770 GO TO 800
780 IF P8=0 THEN 730
790 LET L=L0-(L0-L1)*P/P8
800 GO SUB 960
810 LET F6=P-F1-F2*E
820 LET P5=P
830 LET P=P5-F6*(P5-P4)/(F6-F5)
840 IF ABS((P-P5)/P)<=.00005 THEN 890
850 LET P4=P5
860 LET F5=F6
870 NEXT M
880 PRINT "ITERATION DID NOT CONVERGE"
890 LET G6=V6*P/E
900 IF N4>=N9 THEN 920
910 GO TO 940
920 PRINT E,P,L,G6
930 LET N9=N9+100/N2
940 NEXT N4
950 GO TO 2000
960 LET N=1-L0/L
970 LET P1=K0*N/(1-S*N)↑2
980 LET W1=P1*V0*N/2
990 LET F2=2*G
1000 LET F1=P1-F2*W1
1010 RETURN
1020 DATA
1030 DATA
1040 DATA
2000 END
```

Input instructions can be made through statements 1020 through 1040 as follows:

Statement 1020: Enter solid material parameters for room temperature; ρ_{so} (initial density in g/cm^3), C_o (bulk sound speed in mm/μsec), S (slope of U_s - U_p line), and γ_o (zero stress Gruneisen parameter).

Statement 1030: Enter the parameters; Q* (see Equations 13 and 14 of text), E1 (minimum energy to be considered in ergs/g), E_{IM} (incipient melt energy in ergs/g), N1 (number of orders of magnitude over which energy variation is to be considered), and N2 (number of calculations in each order of magnitude).

Statement 1040: Enter porous material parameters at room temperature; ρ_o (initial density in g/cm^3), P_{10} (precursor amplitude in kbar), U_{s1} (precursor wave velocity in mm/μsec), and $\hat{a}_o$ (parameter in Equations 13 and 14 of text in kbar^{-1}).

The printed output of the program includes most of the input parameters and calculated values for generated pressure (P), α (ALPHA), and the effective Grüneisen parameter (GAMMA(EFF)) for the specified values of energy (E). A sample calculation follows:

[redacted]

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```
1020 DATA 8.93,4.022,1.48,1.96
1030 DATA .9,1.E8,4.7E9,3,8
1040 DATA 6.43,1.35,1.93,.254
RUN

TDGEFF          13:32          27-NOV-70


UNLESS SPECIFIED, UNITS ARE G/CC, MM/USEC, OR MBAR(OR 1/MBAR)


          SOLID MATERIAL PARAMETERS
 RHO(0)          C0              S               GAMMA(0)        E(IM)-ERGS/G
 8.93            4.022           1.48            1.96            4.70000 E+9


          POROUS MATERIAL PARAMETERS
 RHO(0)          P1(0)           US(1)           ALPHA(0)        A(0)
 6.43            1.35            1.93            1.3888          0.254

 O(STAR)         K1              K2
 0.9             -39.6122        49.8615


 CGS UNITS
 P               P               ALPHA           GAMMA(EFF)
 1.00000 E+8     2.69761 E+8     1.3874          0.450639
 1.34896 E+8     3.90990 E+8     1.38691         0.450769
 1.77828 E+8     5.15607 E+8     1.3863          0.450929
 2.39883 E+8     6.95900 E+8     1.38540         0.451166
 3.16228 E+8     9.17954 E+8     1.38435         0.45145
 4.26579 E+8     1.23943 E+9     1.38279         0.451869
 5.62341 E+8     1.34857 E+9     1.38059         0.37296
 7.58577 E+8     1.36699 E+9     1.37726         0.280256
 1.00000 E+9     1.38259 E+9     1.37313         0.215022
 1.34896 E+9     1.38993 E+9     1.36711         0.160244
 1.77828 E+9     1.37107 E+9     1.3596          0.119908
 2.39883 E+9     1.28007 E+9     1.34855         8.29897 E-2
 3.16228 E+9     1.04260 E+9     1.33457         5.12753 E-2
 4.26579 E+9     3.85936 E+8     1.31355         1.40703 E-2
 5.62341 E+9     59396085        1.28733         1.64266 E-3
 7.58577 E+9     43239183        1.24821         8.86476 E-4
 9.99999 E+9     4.02664 E+7     1.19699         6.26228 E-4
 1.34896 E+10    46621180        1.11596         5.30575 E-4
 1.77828 E+10    1.35538 E+8     1.0023          1.18536 E-3
 2.39883 E+10    1.07348 E+11    1               0.695961
 3.16227 E+10    2.40972 E+11    1               1.18511
 4.26579 E+10    4.34119 E+11    1               1.5827
 5.62341 E+10    6.71740 E+11    1               1.85776
 7.58577 E+10    1.01521 E+12    1               2.08135


TIME:  2.03 SECS.

READY
```

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