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Technical Memorandum

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PIEZOELECTRIC CRYSTAL - EQUIVALENT CIRCUIT

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Name: US Layne	
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Name: R.B. Craner	OK for Approved

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ABSTRACT

An equivalent circuit for a piezoelectric crystal is presented. Some results of an experimental study of the effects of capacitive and resistance loading on the output of a crystal are given.

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The electrical circuit which represents a piezoelectric crystal is complicated by the interaction between the electrical and mechanical components of the system. However, as often is the case when electrical and mechanical systems interact, the mechanical components may be reflected as electrical components into the electrical circuits. An analysis of the system then reduces to a study of the resulting electrical circuit.

Morse[1] derives an equivalent circuit for a piezoelectric crystal for the case of longitudinal vibrations. This same circuit applies to a crystal for thickness vibrations after a slight modification of the constants involved. Thickness vibration means that the primary mechanical motion is in the same direction as the applied electrical field. The circuit is shown below. In this circuit $\tau = \frac{sh}{Ad}$ and $\sigma = \tau^2$

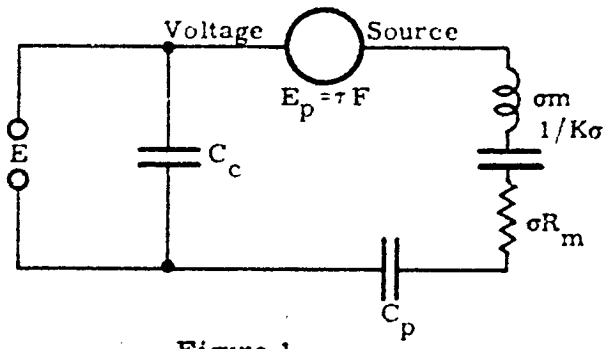

Figure 1

where

s = elastic modulus = reciprocal of Young's modulus

h = thickness of crystal

A = cross-sectional area of crystal

d = piezoelectric constant

The two capacities C_c and C_p are given by the relations $C_p = \frac{Ad^2}{sh}$ and $C_c = \frac{A}{4\pi h}(1 + 4\pi K_1) - C_p$ where K_1 is the dielectric susceptibility of the crystal, as measured in the audio frequency range. The components σm, $1/K\sigma$ and σR_m are the effective impedances of the mechanical system as they are reflected into the electrical circuit. The constants m, K and R_m are the mass, the spring constant and the resistance or damping constant of the mechanical load on the crystal. These are the constants which appear in the mechanical equation of motion

$$m \frac{d^2x}{dt^2} + R_m \frac{dx}{dt} + Kx = F(\omega t).$$

The mass of the mechanical load is reflected into the electrical circuit as an inductance.

[1] Morse, P.M., Vibrations and Sound, McGraw-Hill, New York.

The reciprocal of the spring constant reflects into the circuit as a capacitance and the damping constant reflects as a resistance. The equivalent capacity C_p is analogous to the capacity $1/K\sigma$ and is inversely proportional to the spring constant of the crystal itself.

The voltage source is in series with the equivalent impedance of the mechanical load. This voltage source supplies a voltage E_p which is proportional to the force F on the crystal. The open circuit voltage of the crystal then is that portion of E_p which appears across C_c. Thus, for a given mechanical load and a given force, one can calculate the open circuit voltage of the crystal. Conversely, if a given voltage E is applied to the crystal one may calculate the force which the crystal will exert on the mechanical load. The current in the mechanical arm of the circuit is proportional to the velocity of the face of the crystal.

We notice several important features of this circuit. In the case of large mechanical loads, the impedance in the mechanical arm of the circuit becomes very large so that the equivalent electrical circuit reduces to the single capacitor C_c, this being the clamped capacity of the crystal. If the crystal is unloaded and the external force is zero, the equivalent circuit consists of the two capacitors C_c and C_p in parallel. A calculation of the capacity of a crystal having a diameter of 1/2 inch and a thickness of 1/4 inch using Brush constants gives the values $C_p = 68\mu\mu f$, $C_x = C_c + C_p = 300\ \mu\mu f$. The measured capacity of several Brush crystals having these dimensions varied from 290 to 310 $\mu\mu f$. A calculation of the capacity of a crystal having a diameter of 3/8 inch and a thickness of 1/2 inch using Centralab constants gives the values $C_p = 19\ \mu\mu f$ and $C_x = C_c + C_p = 69\mu\mu f$. The measured capacity of a Centralab crystal with these dimensions was 71 $\mu\mu f$.

It should be pointed out that the circuit holds only for frequencies smaller than the resonance frequency of the crystal. At higher frequencies, the stress, strain, polarization, etc., which appear in the equations used to derive this equivalent circuit, are not uniform throughout the crystal. However, it is significant that the circuit predicts a mechanical resonance when the equivalent reactance in the mechanical arm is zero and that at a higher frequency, parallel resonance will occur with the shunt capacity C_c.

It should also be remembered that the dielectric susceptibility, elastic modulus, etc., are not really constants but vary with temperature and stress. However, over the ranges of temperature and stresses which are useful in contact fuzing, they are essentially constant for barium titanate and may be considered as such.

In a previous analysis of contact fuzing circuits, Claassen and Goodman[2] used an equivalent circuit which consisted of a voltage generator in series with the crystal capacity. The present circuit can be shown to reduce to that equivalent circuit for small mechanical loads in which the impedances representing the mechanical load are negligible. For negligible mechanical loads the present circuit reduces to the circuit shown below.

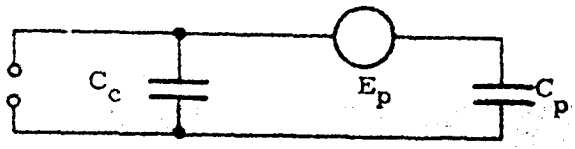

Figure 2

[2] Memo: R. S. Claassen and A. Goodman to distribution, Ref. Sym: 5143-(50), dated October 20, 1952

Applying Thevenin's Theorem, as far as a load impedance connected across C_c is concerned, the network is equivalent to a generator of voltage $V_{in} = E_p \frac{C_p}{C_x}$ and having an internal impedance C_x where $C_x = C_p + C_c$.

By substituting the values of E_p, C_p and C_c, the open circuit voltage of the crystal becomes

$$V_{in} = \frac{4\pi}{1+4\pi K_1} \frac{dF}{} \quad \frac{h}{A}$$

Thus, in order to obtain maximum voltage for a given force, one should minimize the area of the crystal and maximize its length, as is to be expected.

The main interest in contact fuzing is the first output voltage pulse of the crystal which is largely due to transients. However, the equations given by Claassen and Goodman were derived on the basis of steady-state conditions. If the forcing function were known, it should be possible to solve for the complete solution of the differential equation of the circuit. This solution would contain both the transient and steady-state solutions. The forcing function, of course, depends on a great many parameters of the impacting system. The influence of these parameters on the output of the crystal is not now fully understood. However, further work is being done in an attempt to determine the influencing parameters and the effect of these parameters on the problem.

Although the Claassen and Goodman equations were derived on the basis of steady-state conditions, the voltage equation in the case of capacitive loading should still be valid in the transient condition since we have essentially a capacity divider and a capacity divider is frequency independent. However, in the case of resistive loading, we may expect something quite different.

Taking first the case of capacitive loading, one may rearrange the equation $V_i = V_{in} \frac{C_x}{c_x + C_L}$ to get $C_L = \frac{C_x V_{in}}{V_1} - C_x$. One sees that a plot of C_L vs. $1/V_1$ should result in a straight line which intersects the $1/V_1$ axis at $1/V_{in}$ and intersects the C_1 axis at $-C_x$. To verify this equation several types of pickups were mounted on rods and plates which were struck with a pendulum in the set up described by Goodman.[3] The pickups were loaded with a variable capacitor and the output was fed to an oscilloscope through a 10-1 attenuator. The attenuator was used to raise the load resistance to 10 meg ohms so that resistive loading could be neglected. Figure 4 shows the variation of $1/V_1$ with C_L for an XMC-300 mounted on the end of an aluminum rod 24 inches long. Figure 5 shows the results of the same pickups mounted on a 90° bracket and this mounted on an aluminum plate. The intercept of the line on the C_L axis in Figure 4 indicates a capacity of 90 $\mu\mu f$. Figure 5 indicates a capacity of 93 $\mu\mu f$ for the XMC-300. The measured capacity was 89 $\mu\mu f$ which includes 71 $\mu\mu f$ crystal capacity and 18 $\mu\mu f$ stray capacity in the pickups. Figure 6 gives similar results for a skin-type compression pickup. It has an indicated capacity of 300 $\mu\mu f$ and a measured capacity of 310 $\mu\mu f$. The variation of voltage with capacity loading for the XMC-300 mounted on the plate is shown photographically in Figure 8.

[3] Technical Memorandum 34-52-51, Albert Goodman, December 15, 1952.

The response of the system to resistive loading is quite different from capacitive loading, being dependent on the nature of the force function. If we again assume small mechanical loads, the circuit reduces to that shown below.

If

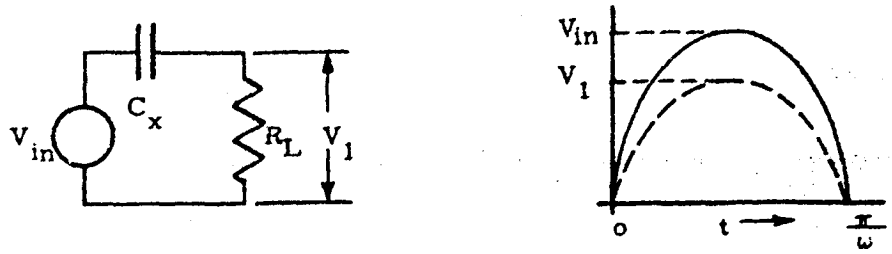

Figure 3

the force function is a sine wave then V_{in} will also be a sine wave. Initially, the entire input voltage will be across the resistance load. However, as charge builds up on C_x the voltage across the load will fall behind the input voltage. Mathematically, this can be expressed as $V_{in} - V_1 = \frac{Q_x}{C_x}$ where

$$Q_x = \int I dt = \int \frac{V_1}{R_L} dt.$$

Although the voltage across the load is not a sine wave, we assume that it is as a first approximation. Thus, at the peak of the first pulse

$$Q_x = \frac{1}{R} \int_0^{\pi/2\omega} V_i \sin \omega t \; dt$$

or $Q_x = \frac{V_1}{\omega R}$. Now we have that

$$V_{in} = (1 + \frac{1}{\omega R_L C_x}) V_1$$

or written in another form

$$\frac{1}{V_1} = \frac{1}{\omega C_x V_{in}} \frac{1}{R_L} + \frac{1}{V_{in}} .$$

Thus, one sees that a plot of $1/V_1$ vs. $1/R_L$ should result in a straight line if this equation holds. The intercept of the line with the $1/V_1$ axis would give $1/V_{in}$ and one may use this and the slope of the line to calculate the frequency of the force function, since the slope is equal to $\frac{1}{2\pi f C_x V_{in}}$.

A check of this equation is shown in Figure 7. The curve shows the variation of $1/V_1$ vs. $1/R_L$ for an XMC-300, mounted on the end of an aluminum rod. The recording equipment was the same as that used in the case of capacitive loading. The slope of the line indicates a frequency of 4.5 kc. The measured frequency found from the pulse width was 4.6 kc. The agreement is surprisingly good but must not be taken

as an indication that the problem of resistance loads is fully understood. For if one were to assume that the force function were a sine wave, then the exact solution can be obtained from the differential equation of the circuit. This solution is

$$V_1 = \frac{\omega V_{in}}{1+\omega^2} \left[-e^{-t/R_L C_x} + \cos \omega t + \frac{1}{\omega} \sin \omega t \right] .$$

The experimental data agree better with the previous simple approximation than they do with this more complicated general solution.

For this study of resistive loading the type pickup and method of impact were chosen to give the simplest output waveform for analysis purposes. In actual fuzing problems the waveform will be much more complicated. Further study is being made on effects of resistive loading to provide a useful method for analyzing the effects of resistive loading under actual impact conditions.

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FIGURE 4

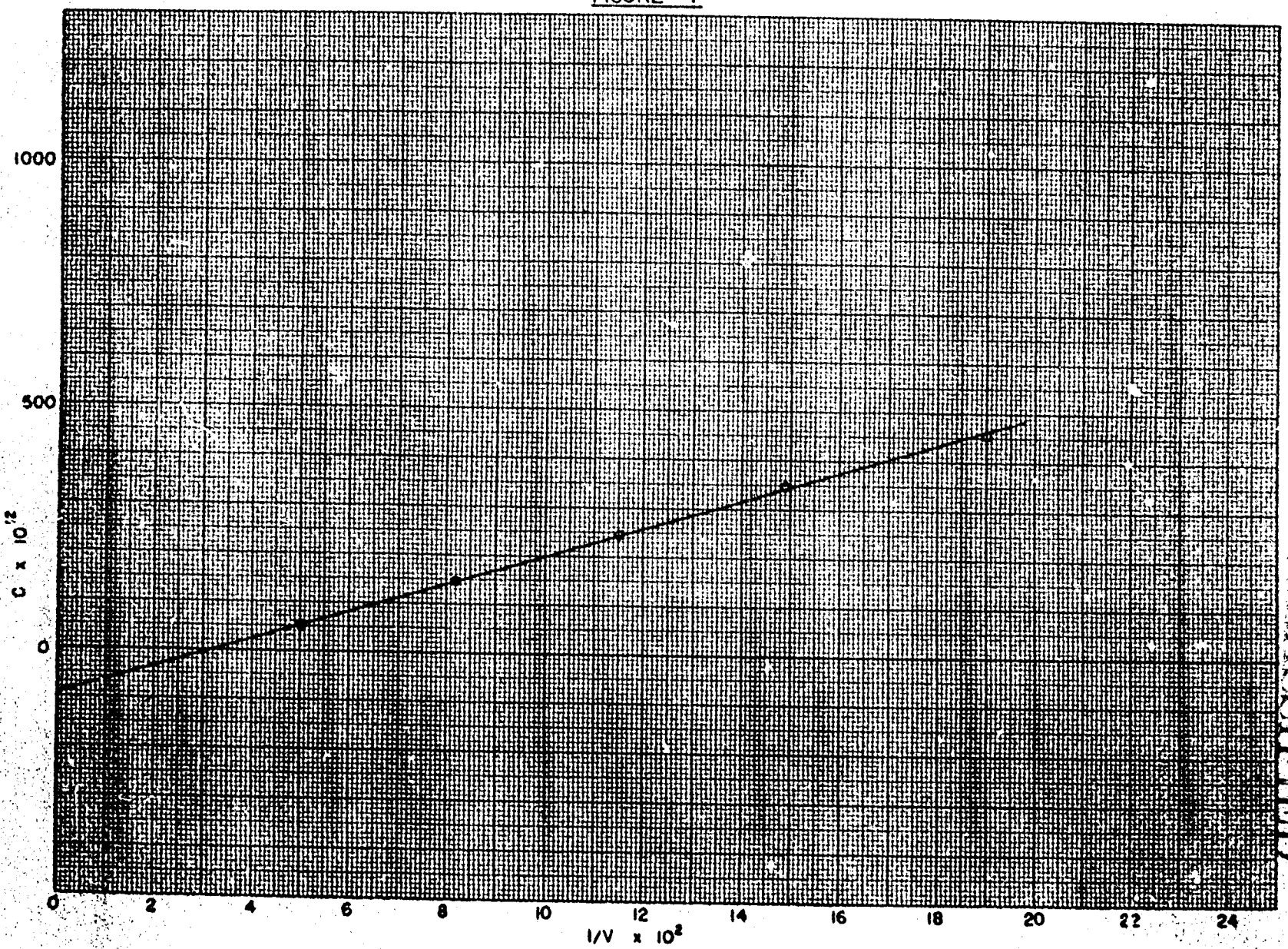

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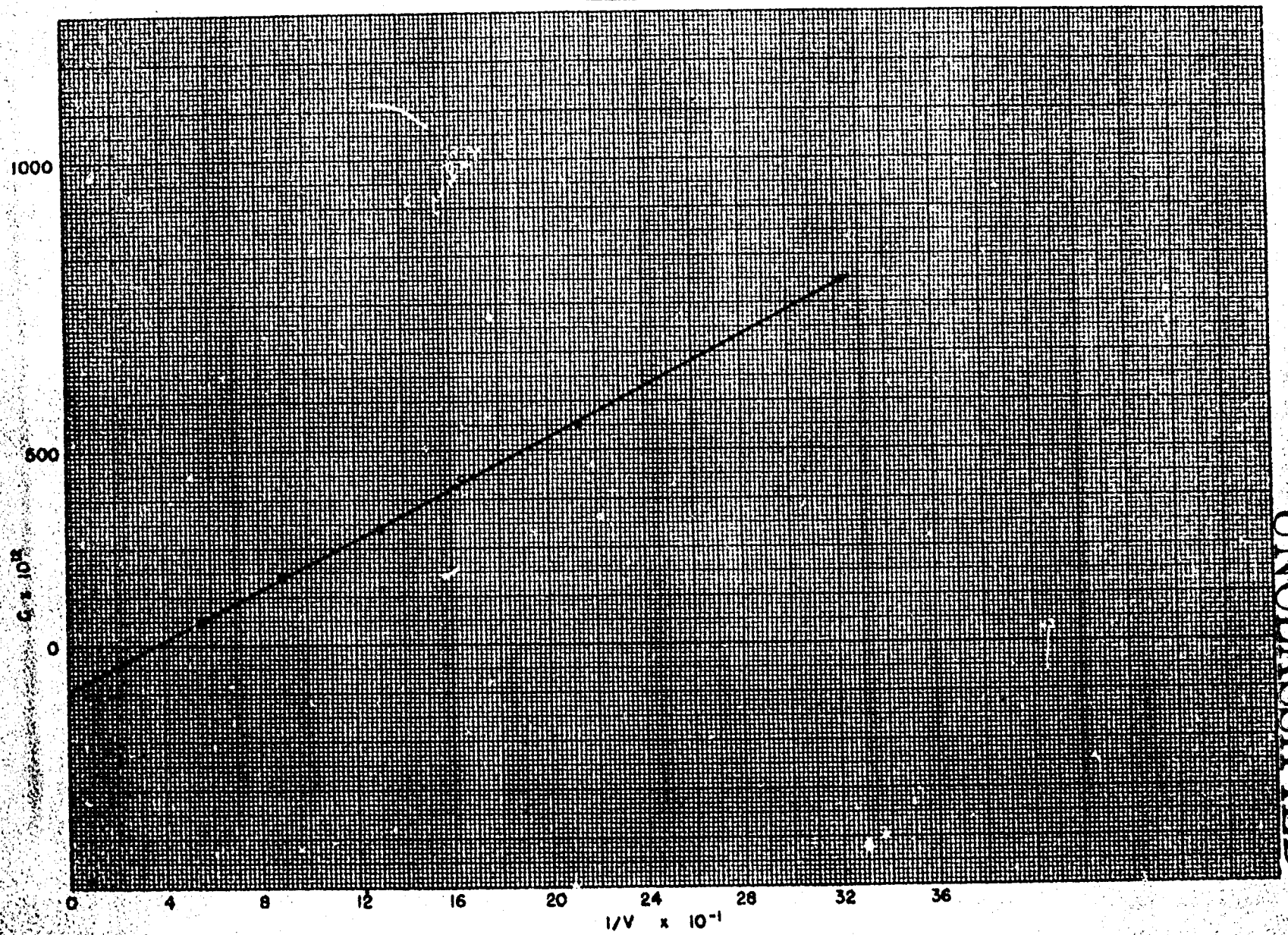
FIGURE 5



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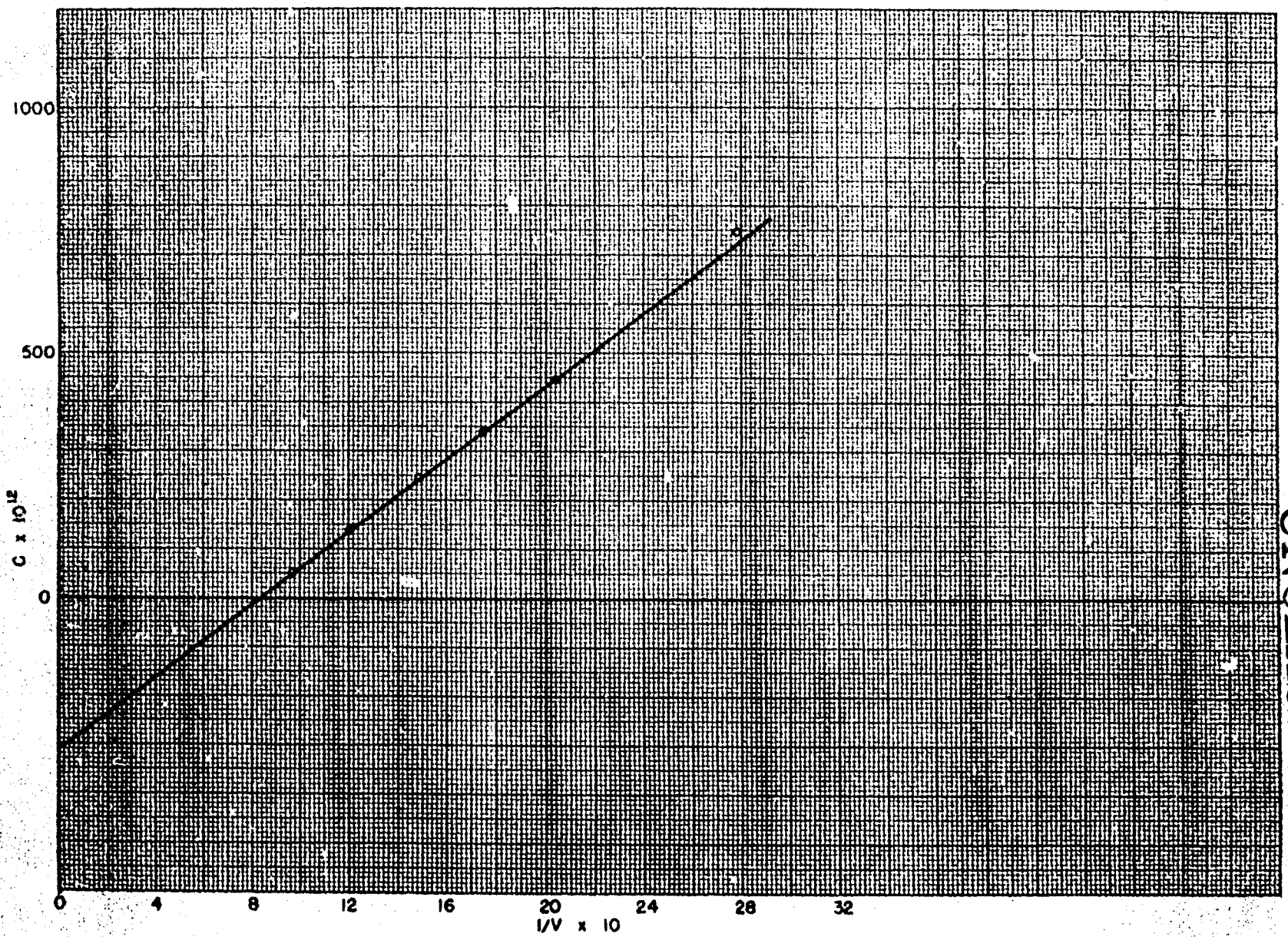

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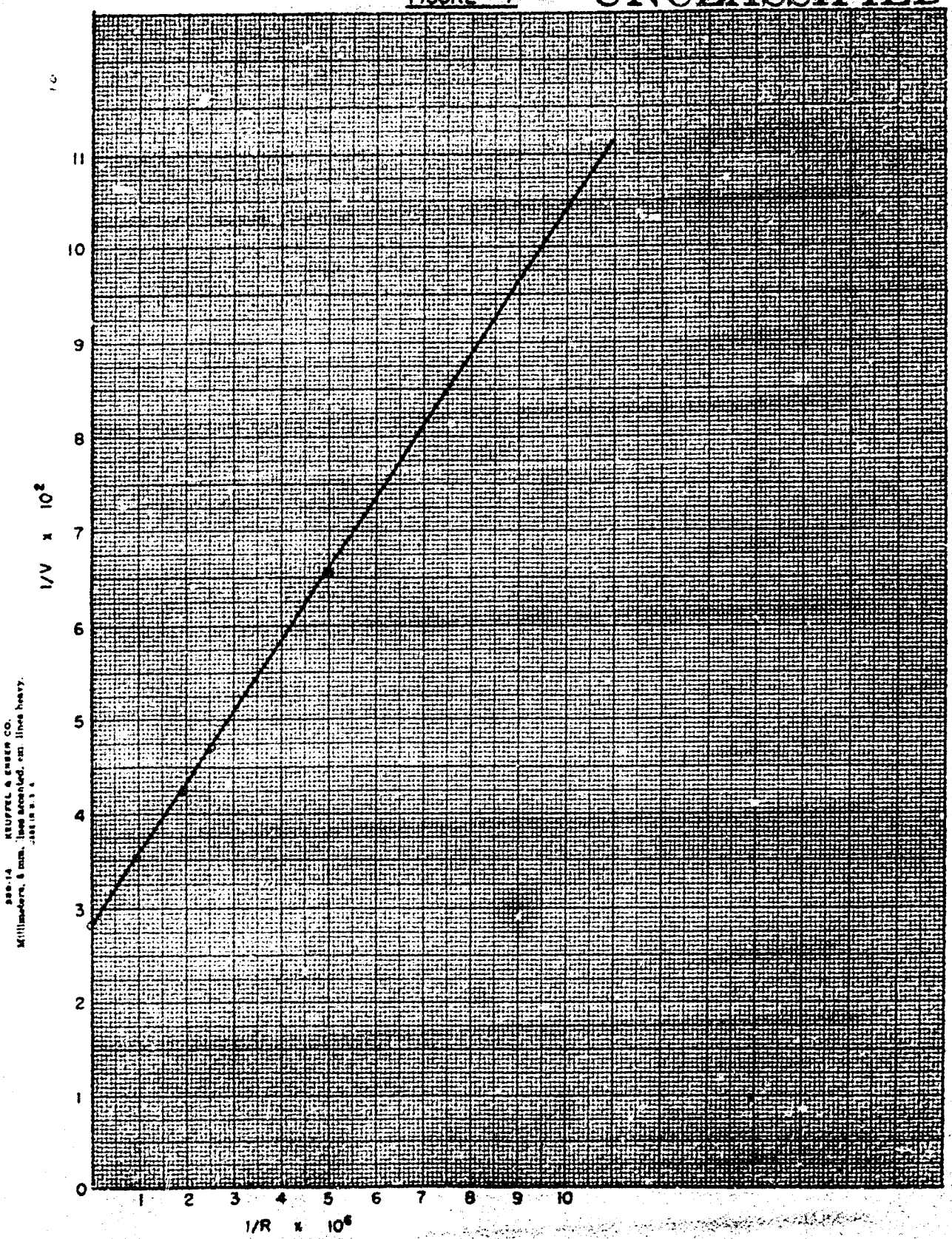
FIGURE 7



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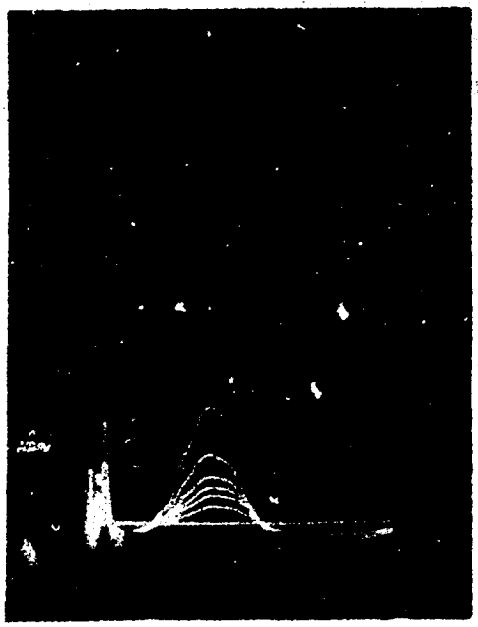
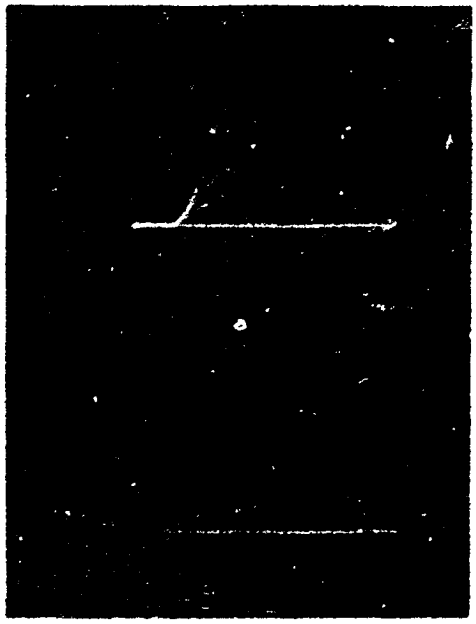


Variation of Voltage with Capacity Loading



Variation of Voltage with Resistance Loading

FIGURE 8