UNCLASSIFIED

21885 14-29 C, 4A

[redacted]

[redacted]

[redacted]

Technical Memorandum

[redacted]

SANDIA SYSTEMATIC DECLASSIFICATION REVIEW	
1st Review Date: 3-24-95	Determination (Circle Numbers):
Authority: ☐ADC ☒ADD W. Layne	1 Classification Retained
Name	2 Classification Changed to
	3 Contains No DOE Classified Information
2nd Review Date	4 Coordinate With:
Authority: ADD	5 Contains UCAI?
Name	6 Comments:

SC TM-30-53-51

DEGRADATION OF LINEARLY PROPAGATED ATOMIC-BLAST EFFECTS BY STRUCTURES OR HILLY TERRAIN

James H. Smith

INVENTORIED
AUG 1 1953
3427-1

SANDIA SYSTEMATIC DECLASSIFICATION REVIEW DOWNGRADING OR DECLASSIFICATION STAMP	
Emelda Selph 4/4/97	AUTHORITY: R.B. Craner
PERSON CHANGING MARKING 5/5/97	RECORD ID: 97SN1096
Pamela M. Gallegos	
PERSON VERIFYING MARKING & DATE	DATED: 3/27/97

ABSTRACT

The degradation factor was computed for linearly propagated blast waves (created by an atomic burst) striking a circular plot of land having a V-shaped or sawtooth type of contour. A corresponding but briefer study was made for sinusoidal terrain. Curves for both studies are given showing the degradation factor as a function of burst height and major hill dimensions.

INVENTORIED
2510 JAN 12 1954

Inventoried
September 1960

Case No. 418.0

April 11, 1953

[redacted]

905-133

[redacted]

UNCLASSIFIED

TABLE OF CONTENTS

	Page
INTRODUCTION	4
RESULTS OF THE INVESTIGATION	
Part 1: V-Shaped Terrain	6
Part 2: Sinusoidal Terrain	10
APPENDIX A	
Derivation of the Expressions for the Degradation Factor for V-Shaped Hills whose Ridges and Valleys are Concentric about Ground Zero	12
APPENDIX B	
Sample Computations for the Graphs of Equations 29 and 31 in Figure 2	22
APPENDIX C	
Determination of the Degradation Factor for Sinusoidal Terrain	31

LIST OF ILLUSTRATIONS

Figure		Page
1	Circular Plot of Terrain Having V-Shaped (Sawtooth) Type of Contour	7
2	Degradation Factor (A_e/A) versus Ratio of Burst Height to Hill Height (h/H) for Concentric V-Shaped Hills	9
3	Degradation Factor (L_e/L) versus Ratio of Burst Height to Hill Height (h/H) for Sinusoidal-Shaped Hills	11
4	Atomic Burst over Circular Terrain Having Uniform V-Shaped Contour	13
5	Diagram for the Calculation of S_e	15

INTRODUCTION

This report assumes that the blast waves created by an atomic burst travel only along straight lines emanating from the center point of the burst (that is, blast waves propagate like sound waves and do not bend around corners). This assumption is not realistic insofar as actual blast waves are concerned, but is reasonably valid for radiation effects. All reflections of the blast waves from surfaces were assumed negligible.

In general, the degradation factor is defined as the ratio A_e/A, where A_e is the total area that is directly exposed to the blast waves, and A is the total area of the surface within the range of the blast and of which A_e is a part.

Originally this study was to be extended to other than sawtooth and sinusoidal types of terrain and to structures, the latter being represented by a square or rectangular wave type of profile. However, this complicated analytical study was not made for such a complex analysis did not seem worthwhile in view of the unrealistic assumptions required.

The results of the studies made can be of value in some future study concerning radiation, for example, where the assumptions would be reasonably valid. Recently, there has been considerable interest in the degradation of thermal and other radiation effects produced by atomic weapons. Consequently, even though an answer to the immediate problem of actual blast effects was not produced, this investigation was considered to have been quite worth-while.

[redacted]

[redacted]

The work was concluded with a study of the Nagasaki atomic bomb damage map to determine whether any correlation exists between the damage and the contours indicated thereon. No correlation could be established, principally because much of the hilly area of interest was initially devoid of any structures and, hence, the damage that might have occurred in these regions was undeterminable.

[redacted]

[redacted]

[redacted]

RESULTS OF THE INVESTIGATION

PART I

V-SHAPED TERRAIN

Appendix A shows that for a circular plot of land having a V-shaped or sawtooth type of contour (Fig. 1) the degradation factor is

$$\frac{A_e}{A} = \left(\frac{B}{R}\right)^2 \left\{ \left(N - \frac{1}{2}\right)^2 + \sum_m \left[\frac{m - 1 - \frac{h}{2H}}{m - 2 + \frac{h}{2H}} + 2m \right] \left[\frac{\frac{h}{H} - 1}{m - 2 + \frac{h}{2H}} \right] \right\} \quad h > H \qquad (1)$$

where B = one-half wavelength of the V-shaped contour (feet)

R = radius defining extent of blast (feet)

(See Fig. 1. For simplicity, it is assumed that $\frac{R - \frac{B}{2}}{2B}$ is an integer.)

N = a factor determined as discussed below

m = $N+1,\ N+3,\ N+5,\ N+7,\ \ldots\ \frac{R - \frac{1}{2}B}{B}$

h = burst height of weapon (feet)

H = maximum height of a hill (feet), that is, amplitude of the V-shaped contour.

If $\frac{h}{2H}$ is an odd integer, then $N = \frac{h}{2H} + 2$; if $\frac{h}{2H}$ is an even integer, excluding zero, then $N = \frac{h}{2H} + 1$; if $\frac{h}{2H}$ is not an integer but is greater than one, N is the higher of the two successive odd integers between which $\frac{h}{2H}$ lies; if $\frac{1}{2} < \frac{h}{2H} < 1$, then $N = 1$. If $\frac{h}{2H} \leqq \frac{1}{2}$, that is, $h \leqq H$, then the first hill to the right of GZ in Fig. 1 shields all portions of the other hills farther to the right, and the degradation factor becomes

$$\frac{A_e}{A} = \frac{B^2}{4R^2} \qquad (2)$$

[redacted]

[redacted]

[redacted]

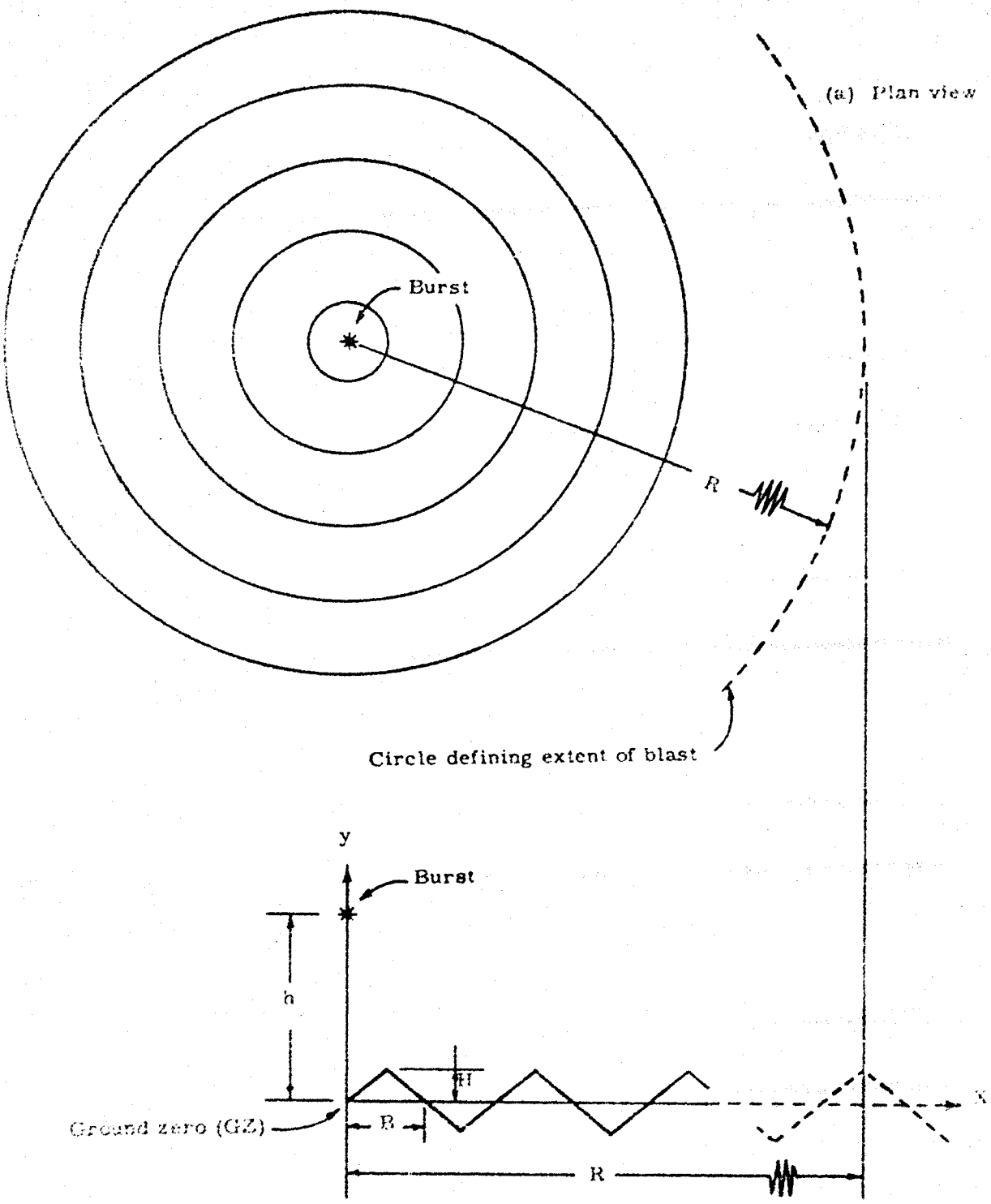

Fig. 1. -- A circular plot of terrain having a V-shaped (sawtooth) type of contour.

[redacted]

If h >> H, Eq 1 simplifies a little, resulting in

$$\frac{A_e}{A} = \left(\frac{B}{R}\right)^2 \left\{ \left(N - \frac{1}{2}\right)^2 + \sum_m \left[\frac{m - 1 - \frac{h}{2H}}{m - 2 + \frac{h}{2H}} + 2m \right] \left[\frac{1}{\left(\frac{H}{h}\right)m + \frac{1}{2}} \right] \right\} \qquad (3)$$

Figure 2 shows an approximate relationship between the degradation factor (A_e/A) and the burst-height-to-hill-height ratio (h/H) for the concentric V-shaped terrain shown in Fig. 1. Calculations of the points which determine the curves of Fig. 2 are shown in Appendix B of this report. The three solid-line curves were derived from data based on Eq 1 and on a pdh curve for 12 psi and 1-kt yield in a memorandum by G. T. Pelsor.[1] Burst and hill heights are indicated for each of these curves. The broken-line curve is for a 13,000-ft blast range and a hill for which H is 1000 feet and B is 2000 feet. No pdh curve was used to calculate the coordinates of the x-points which determine the broken-line curve. (See part 2 of Appendix B.) The broken-line curve has been drawn more smoothly than the points determining it would require.

The primary difficulty in calculating data to plot the solid-line curves is that the pdh curve does not provide many points which fulfill the requirements that $A_e/A < 1$, and $\frac{R - \frac{B}{2}}{2B}$ must be an integer. Because of the sharp peaks and valleys of the hills, A_e/A is not expected to be a completely smooth function of h/H in any instance, but since data for so few points were computed it seemed best to draw average curves through the points determined.

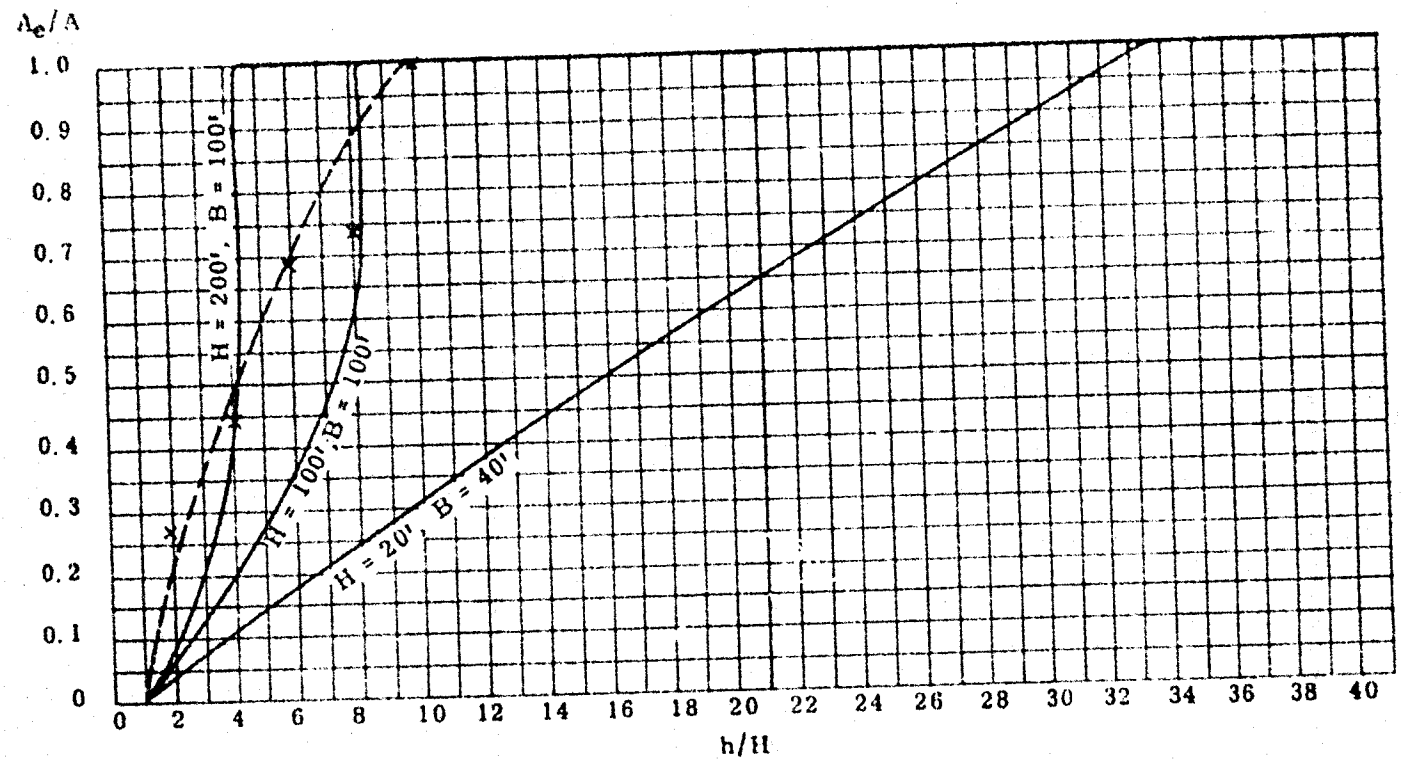

Fig. 2 -- Degradation factor A_e/A vs ratio of burst height to hill height, h/H, for concentric V-shaped hills

[redacted]

PART 2

SINUSOIDAL TERRAIN

Terrain having a sinusoidal contour was studied very briefly. The profile of this terrain is similar to that shown in part b of Fig. 1, except that the V-shaped wave is replaced by a sine wave whose wave length is 2B. For simplicity, the degradation factor is expressed as L_e/L, rather than A_e/A, where L_e is the length of the profile directly exposed to the blast waves, and L is the total profile length within the range of the blast and of which L_e is a part.

No simple expression for L_e/L could be derived. A point-by-point numerical solution was required, the details of which are given in Appendix C. Curves expressing L_e/L as a function of h/H for a blast range fixed at 10,000 feet are shown in Fig. 3, where B is one half the wavelength and H is the amplitude of the sine wave representing the terrain, and the terms 'peak' and 'valley' imply that the location of the burst relative to the sinusoidal terrain is over a peak or over the center of a valley. When the burst occurs over a peak, the condition h/H = 1 implies a contact burst upon the peak itself. Hence, the curves of L_e/L for bursts over a peak do not exist for values of h/H less than one. When the blast occurs over the center of a valley, L_e/L is negligible for values of h/H less than about 0.9.

[redacted]

APPENDIX A

DERIVATION OF THE EXPRESSIONS FOR THE DEGRADATION FACTOR FOR V-SHAPED HILLS WHOSE RIDGES AND VALLEYS ARE CONCENTRIC ABOUT GROUND ZERO

This appendix contains a study of some of the effects resulting from a weapon bursting over a circular plot of terrain which has a uniform V-shaped contour as shown in Fig. 4. The entire circular plot of land is that which would be generated by rotating the contour of Fig. 4 horizontally about the point GZ (ground zero for the burst.)

The effects of the burst that are of interest here are those which propagate linearly from the center of the burst. It is desired to derive a factor for determining to what extent the presence of the V-shaped hills degrades the energy reaching the terrain.

The symbols in Fig. 4 are defined as follows:

B = half the wavelength of the V-shaped contour

GZ = ground zero and the origin of the coordinates

n = number of a peak or valley as illustrated

S_e = slant height of any area directly exposed to blast waves

N = minimum value of n that produces a shielded length $S_s > 0$

r_e = x intercept of far side of hill which is the limiting case such that the profile of the far side, if extended toward the burst point, passes through the burst point.

r = x intercept of any hill for which $S_s > 0$

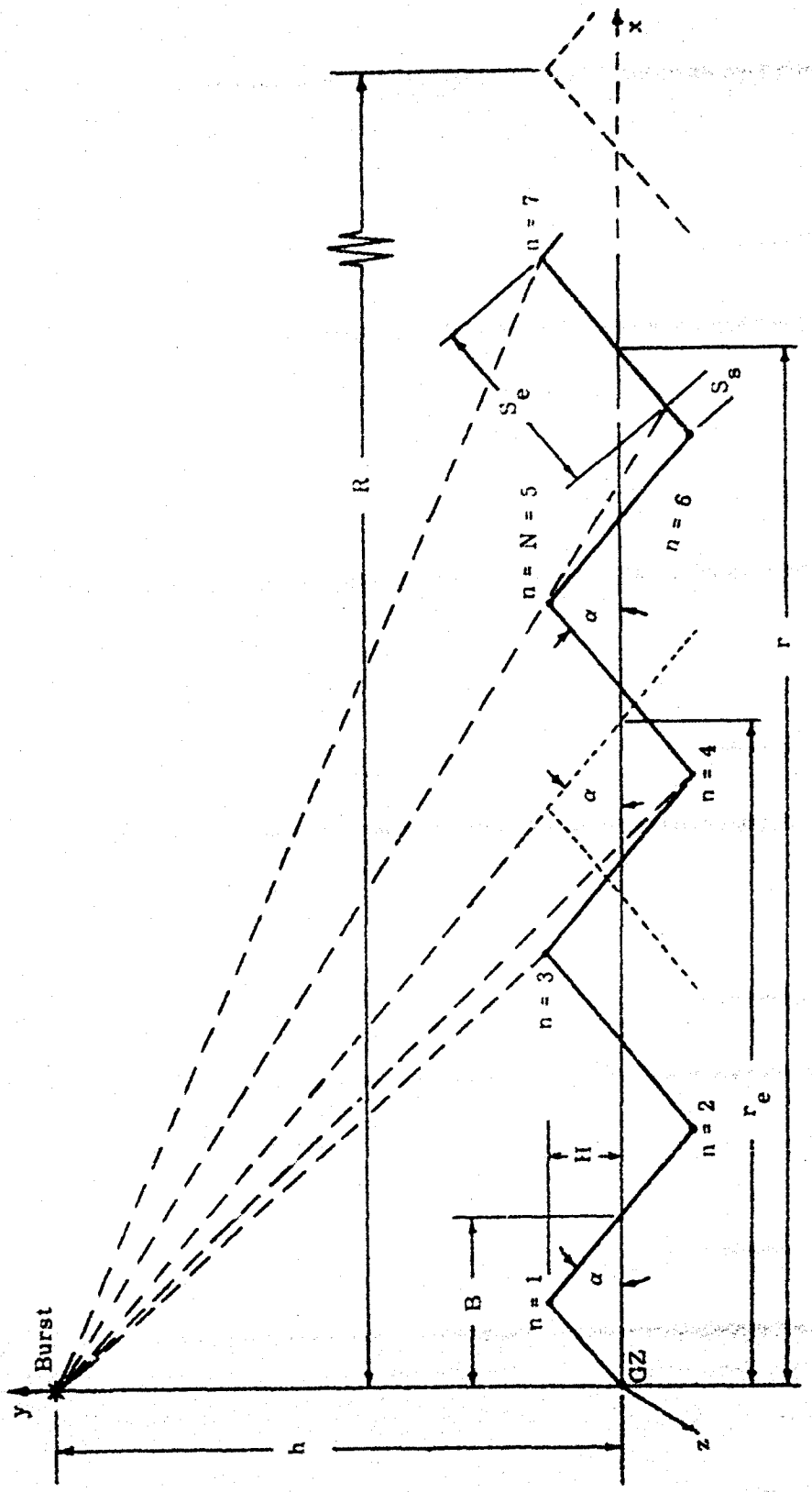

Fig. 4. -- Atomic burst over a circular plot of terrain having a uniform V-shaped contour

R = x coordinate of peak of any hill so located that effect of blast has dropped below any selected amount. For simplicity it is assumed that $\frac{R - \frac{B}{2}}{2B}$ is an integer.

From the geometry of Fig. 4, it follows that

$$\tan \alpha = \frac{h}{r_e} = \frac{2H}{B} \tag{4}$$

Hence,

$$r_e = \frac{Bh}{2H} \tag{5}$$

For the derivations to follow, part of Fig. 4 is redrawn and called Fig. 5. The equation of the line through the points having coordinates (0, h) and $(r - \frac{3B}{2}, H)$ is

$$y - h = \frac{2(h-H)x}{3B-2r}$$

or

$$(y-h)(3B-2r) = 2(h-H)x \tag{6}$$

The equation of the line containing S_e is

$$y = \frac{2H}{B}(x - r)$$

or

$$x = \frac{B}{2H}\, y + r \tag{7}$$

To determine the coordinates of the point of intersection P of the two above lines, Eq 6 and 7 must be solved simultaneously. The resulting value of y may be designated as y_P (the y coordinate of P), thus

$$y_P = \frac{(3Bh - 2Hr)H}{4BH - 2Hr - Bh} \tag{8}$$

The vertical projection of S_e is $H - y_P$, the negative sign indicating that the magnitude of y_P is negative when P is below the x-axis or positive when P is above the

[redacted]

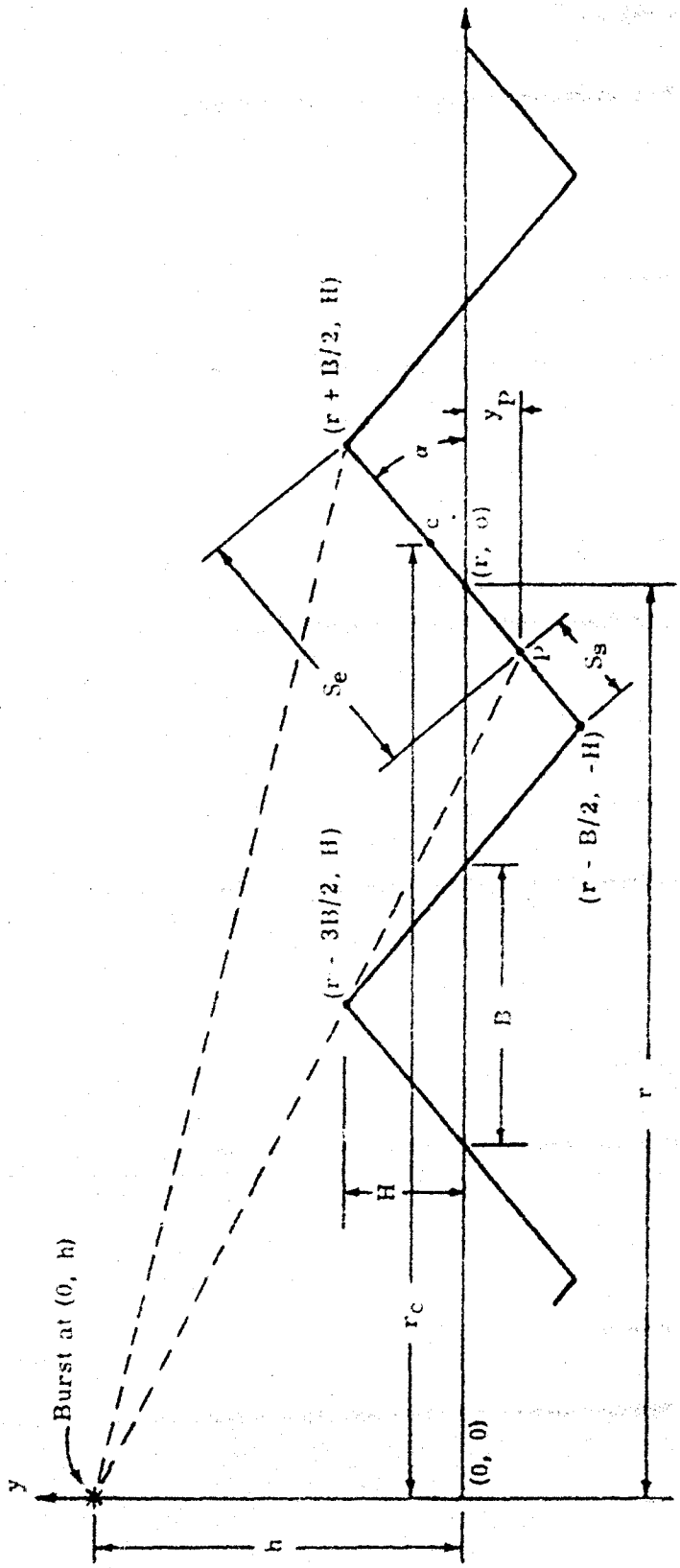

Fig. 5.-- Diagram for the calculation of length L_r

[redacted]

x-axis. The length of S_e itself can be found from the following expression:

$$S_e = (H - y_p) \csc \alpha \tag{9}$$

where

$$\csc \alpha = \frac{\sqrt{B^2 + 4H^2}}{2H} \tag{10}$$

Hence,

$$S_e = \frac{\sqrt{B^2 + 4H^2}}{2} - \frac{3Bh - 2Hr}{4BH - 2Hr - Bh} \cdot \frac{\sqrt{B^2 + 4H^2}}{2}$$

$$= \frac{\sqrt{B^2 + 4H^2}}{2}\left[1 - \frac{3Bh - 2Hr}{4BH - 2Hr - Bh}\right] \tag{11}$$

This expression can be further reduced to

$$S_e = \frac{2B(h-H)}{B(h-4H) + 2Hr}\sqrt{B^2 + 4H^2} \tag{12}$$

or

$$S_e = \frac{2\sqrt{B^2 + 4H^2}}{1 - \frac{3}{\frac{h}{H} - 1} + \frac{2r}{(\frac{h}{H} - 1)B}} \tag{13}$$

Each of the hills shown in Fig. 4 to have part of its near side shielded from the blast waves has an integral value of r which when placed in Eq 12 or 13 gives the value of S_e for the hill. The value of r to be used in any instance can be expressed in terms of B and the number n of an adjacent peak or valley.

The computation of r_e/B, or its equivalent, $\frac{h}{2H}$, permits N, the minimum value of n, to be determined.

If $\frac{h}{2H}$ is an odd integer, $N = \frac{h}{2H} + 2$; if $\frac{h}{2H}$ is an even integer, excluding zero, $N = \frac{h}{2H} + 1$; if $\frac{h}{2H}$ is not an integer but is greater than one, N is the higher of the two successive odd integers between which $\frac{h}{2H}$ lies; if $\frac{1}{2} < \frac{h}{2H} < 1$, $N = 1$. If

$\frac{h}{2H} \geqq \frac{1}{2}$, that is $h \geqq H$, the first hill from GZ shields all portions of the other hills farther out. This situation is discussed by itself on p. 20.

If, through error, too small a value of r were to be placed in Eq 12 or 13, an absurd value of S_e would be obtained. The values of r to be used are (N + 1)B, (N + 3)B, (N + 5)B, (N + 7)B, ...(R - B/2). Let r = mB, where m takes on the successive values N + 1. N + 3, N + 5, N + 7, . . . $\frac{R - \frac{B}{2}}{B}$.

Then Eq 12 becomes

$$\Sigma S_e = \left[2(h-H) \sqrt{B^2 + 4H^2} \right] \sum_m \frac{1}{h - 4H + 2mH} \qquad (14)$$

and Eq 13 becomes

$$\Sigma S_e = \sum_m \frac{2\sqrt{B^2 + 4H^2}}{1 - \frac{3}{\frac{h}{H} - 1} + \frac{2m}{\frac{h}{H} - 1}} \qquad (15)$$

When h >> H, > Eq 15 can be expressed as:

$$\Sigma S_e = \sum_m \frac{2h\sqrt{B^2 + 4H^2}}{h + 2mH - 3H} \qquad (16)$$

With h >> H, h + 2mH >> 3H, and hence Eq 16 can be simplified as:

$$\Sigma S_e = \sum_m \frac{2h\sqrt{B^2 + 4H^2}}{h + 2mH} \qquad h >> H \qquad (17)$$

Simple expressions do not exist for the summations of the harmonic series appearing in Eq 14 through 17.

The terrain of concern here is half of a wafer whose contour along any radius lying in the x, z plane is shown in Fig. 4. It is necessary to compute the total

surface area that is directly exposed to the blast waves.

In Fig. 5 let r_c be the x coordinate of c, the midpoint of S_e. Then the area of any one surface of slant height S_e on the half wafer is

$$a_e = \pi r_c S_e \tag{18}$$

It is desirable to derive a general expression for r_c.

From p. 14, the vertical projection of S_e is $H - y_P$. It follows that the y coordinate of c is

$$y_c = H - \frac{H - y_P}{2} = \frac{H + y_P}{2} \tag{19}$$

The x coordinate of c can be found by substituting y_c for y in Eq 7; thus

$$r_c = \frac{B}{4H}(H + y_P) + r \tag{20}$$

Substituting y_P from Eq 8 into Eq 20 results in

$$r_c = \frac{B}{4} + \frac{(3Bh - 2Hr)B}{4(4BH - 2Hr - Bh} + r$$

$$= \frac{B}{2}\left[\frac{2BH - 2Hr + Bh}{4BH - 2Hr - Bh}\right] + r \tag{21}$$

If mB is substituted for r,

$$\Sigma r_c = \frac{B}{2} \sum_m \left[\frac{2Hm - 2H - h}{2Hm - 4H + h} + 2m\right] \tag{22}$$

Let A_{NR} be the total of all a_e from N out to range R. Then, from Eq 14 and 18,

$$A_{NR} = \pi B(h - H)\sqrt{B^2 + 4H^2} \sum_m \left[\frac{2H(m-1)-h}{2H(m-2)+h} + 2m\right]\left[\frac{1}{2H(m-2)+h}\right] \tag{23}$$

To determine the total area between GZ and R that is directly exposed to the blast

waves, it is necessary to add to A_{NR} the area A_{ON} of the portion of the half wafer between GZ and N.

From Fig. 4,

$$A_{ON} = \frac{\pi B}{4} \cdot \frac{\sqrt{B^2 + 4H^2}}{2} + \pi B \sqrt{B^2 + 4H^2} + 2\pi B \sqrt{B^2 + 4H^2}$$

$$+ 3 \pi B \sqrt{B^2 + 4H^2} + \ldots\ldots + \pi (N - 1) B \sqrt{B^2 + 4H^2}$$

$$= \pi B \sqrt{B^2 + 4H^2} \left[\frac{1}{8} + 1 + 2 + 3 + \ldots\ldots + (N - 1)\right] \tag{24}$$

Except for the first term, the bracketed expression in Eq 24 is an arithmetic series in which the last term and the number of terms are both equal to (N - 1). Summing the arithmetic series above results in

$$A_{ON} = \pi B \sqrt{B^2 + 4H^2} \left[\frac{(2N - 1)^2}{8}\right] \tag{25}$$

The total area on one half of the wafer directly exposed to the blast waves is

$$A_e = A_{ON} + A_{NR} \tag{26}$$

A_e is to be expressed as a fraction of A, where A is the total surface area of the half wafer between GZ and R. The expression for A is the same as that for A_{ON} in Eq 24, except that the series goes through $\frac{R - \frac{B}{2}}{B}$ instead of just through (n - 1). Hence,

$$A = \pi B \sqrt{B^2 + 4H^2} \left[\frac{1}{8} + 1 + 2 + 3 + 4 + 5 + \ldots \frac{R - \frac{B}{2}}{B}\right] \tag{27}$$

Summing the arithmetic series in Eq 27 results in

$$A = \pi B \sqrt{B^2 + 4H^2} \left[\frac{1}{2} \left(\frac{R}{B}\right)^2\right] \tag{28}$$

[redacted]

It follows that

$$\frac{A_e}{A} = \left(\frac{B}{R}\right)^2 \left\{ \left(N - \frac{1}{2}\right)^2 + \sum_m \left[\frac{m - 1 - \frac{h}{2H}}{m - 2 + \frac{h}{2H}} + 2m \right] \left[\frac{\frac{h}{H} - 1}{m - 2 + \frac{h}{2H}} \right] \right\} \tag{29}$$

If Eq 17 is used instead of Eq 14 to derive Eq 23, then, for $h >> H$, Eq 29 becomes

$$\frac{A_e}{A} = \left(\frac{B}{R}\right)^2 \left\{ \left(N - \frac{1}{2}\right)^2 + \sum_m \left[\frac{m - 1 - \frac{h}{2H}}{m - 2 + \frac{h}{2H}} + 2m \right] \left[\frac{1}{\left(\frac{H}{h}\right) m + \frac{1}{2}} \right] \right\} \tag{30}$$

When h is less than H, S_e (in Fig. 5) computed from Eq 12 has no practical significance, and Eq 29 and 30 are not valid. When $h \leqq H$ (the case of $\frac{h}{2H} = \frac{1}{2}$ on p. 17), the only portion of the terrain subject to blast waves is the near side of the first hill from GZ (in Fig. 4). A_e/A is now computed for this situation. A is still as it was in the previous study and can be computed from Eq 28. The requirement that $\frac{R - \frac{B}{2}}{2B}$ be an integer is of no significance when $h \leqq H$. Here

$$A_e = \frac{\pi B}{8} \sqrt{B^2 + 4H^2}$$

and hence

$$\frac{A_e}{A} = \frac{B^2}{4R^2} \qquad h \leqq H \tag{31}$$

Note that Eq 31 can be obtained from either Eq 29 or 30 by considering the summations to be zero and by letting N = 1. When h = H, and N = 1, the summation terms in Eq 29 and 30 are equal to zero, and these equations reduce to Eq 31 as the limiting relation.

If A_e/A is to be computed for some given constant overpressure on the earth's surface, Eq 29, 30, and 31 should be used in conjunction with the corresponding pdh curve. At present, all pdh curves are for flat terrain, and do not apply exactly to the terrain for which Eq 29, 30, and 31 were derived. Nevertheless, a pdh curve provides a systematic means for estimating a fairly realistic value of R to use with each value of h. Care must be used in selecting B and R so that the relation between them satisfies the requirement that $\frac{R - \frac{B}{2}}{2B}$ is an integer.

From Fig. 4 it is seen that for $A_e/A = 1$ it is required that

$$h \geqq (R - \frac{3}{2} B) \tan \alpha \tag{32}$$

or

$$h \geqq \frac{(2R - 3B)H}{B} \tag{33}$$

or, by rearrangement,

$$R - \left(\frac{B}{2H}\right) h \leqq \frac{3B}{2} \tag{34}$$

It is worth-while, therefore, to apply Eq 34 at the beginning of the process of computing A_e/A.

APPENDIX B

SAMPLE COMPUTATIONS FOR THE GRAPHS OF EQUATIONS 29 AND 31* IN FIG. 2

1. For the Three Curves Based on the 12-PSI PDH Curve for a 1-kt Yield

   a. Arbitrarily choose the pdh curve for a 12-psi overpressure obtained from a 1-kt yield. If B = 40 feet, and if values of d are selected as listed below, the relation that $\frac{R - \frac{B}{2}}{2B}$ must be an integer is satisfied at each value of d. These values of d are used as R in Eq 29 and 31. The value of h read from the pdh curve for each value of d is used with its value of d = R. Arbitrarily let H = 20 feet.

d = R(ft)	$R - \frac{B}{2}$ (ft)	$\frac{R - 20}{2B}$	h (ft)
100	80	1	935
180	160	2	920
260	240	3	890
340	320	4	885
420	400	5	870
500	480	6	860
580	560	7	840
660	640	8	820
740	720	9	700
820	800	10	250
900	880	11	---
860	---	--	20

*Equations 29 and 31 are the same as Eq 1 and 2.

For $h \leqq 20$ feet, applying Eq 31 results in

$$\frac{A_e}{A} = \frac{B^2}{4R^2} = \frac{40^2}{4R^2} = \frac{400}{R^2}$$

At h = 20 feet, R = d = 860 feet on the pdh curve.

Hence,

$$\frac{A_e}{A} = \frac{400}{(860)^2} = 0.000541$$

When Eq 34 is applied,

$$R - \left(\frac{40}{2 \times 20}\right) h \leqq \frac{3 \times 40}{2}$$

Hence,

$$R - h \leqq 60$$

Therefore, compute $\frac{A_e}{A}$ for $R - h > 60$.

The only place on p. 22 where $R - h > 60$ is where R = 820 feet and h = 250 feet.

$$\frac{h}{2H} = \frac{250}{2 \times 20} = 6.25$$

Hence,

N = 7, and m = 8, 10, 12, 14, 16, 18, 20. The greatest value of

$$m = \frac{R - \frac{B}{2}}{B} = \frac{820 - 20}{40} = 20.$$

It is necessary now to evaluate Eq 29 for this case.

$$\left[\frac{m - 1 - \frac{h}{2H}}{m - 2 + \frac{h}{2H}} + 2m\right]\left[\frac{\frac{h}{H} - 1}{m - 2 + \frac{h}{2H}}\right] = \underbrace{\left[\frac{m - 1 - 6.25}{m - 2 + 6.25} + 2m\right]}_{M}\underbrace{\left[\frac{11.50}{m - 2 + 6.25}\right]}_{Q}$$

[redacted]

m	M	Q	MQ
8	16.06	0.9388	15.08
10	20.19	0.8070	16.30
12	24.29	0.7077	17.19
14	28.37	0.6301	17.88
16	32.43	0.5679	18.42
18	36.48	0.5169	18.86
20	40.53	0.4742	19.22
			Σ MQ = $\sum_m$ = 122.95

$$(N - \frac{1}{2})^2 = (7 - \frac{1}{2})^2 = 42.25$$

$$(N - \frac{1}{2})^2 + \sum_m = 42.25 + 122.95 = 165.20$$

$$(\frac{B}{R})^2 = (\frac{40}{820})^2 = 0.002380$$

Therefore,

$$\frac{A_e}{A} = 0.002380 \times 165.20 = 0.393$$

[redacted]

TABULATION OF RESULTS

h (ft)	h/H	A_e/A	$\frac{H}{B} = \frac{20}{40} = \frac{1}{2}$
935	46.8	1.00	
920	46.0	1.00	
890	44.5	1.00	
885	44.2	1.00	
870	43.5	1.00	
860	43.0	1.00	
840	42.0	1.00	
820	41.0	1.00	
700	35.0	1.00	
740	34.0	1.00	
250	12.5	0.393	
20 or less	1.00 or less	0.000541	

b. Again use the pdh curve for 12 psi for 1-kt yield. Let B = 100 feet and H = 100 feet.

$$\frac{R - \frac{B}{2}}{2B} = \frac{R - 50}{200}$$

R(ft)	$\frac{R-50}{200}$	h(ft)
250	1	910
450	2	870
650	3	830
850	4	65
840	-	100

For $h \leqq 100$ feet, applying Eq 31 results in

$$\frac{A_e}{A} = \frac{(100)^2}{4 \times (840)^2} = 0.00354$$

Apply Eq 34

$$R - \left(\frac{100}{2 \times 100}\right)h \leqq \frac{3 \times 100}{2}$$

Hence,

$$R - \frac{h}{2} \leqq 150$$

Therefore, compute

$$\frac{A_e}{A} \text{ for } R - \frac{h}{2} > 150.$$

For

$$R - \frac{h}{2} \leqq 150, \frac{A_e}{A} = 1; \text{ or at } R = 570, \frac{h}{2} = \frac{840}{2} = 420,$$

and

$$R - \frac{h}{2} = 150.$$

For only two of the sets of R and H values shown on p. 25 is $R - \frac{h}{2}$ greater than 150: one is for R = 650 feet, h = 830 feet; the other is for R = 850 feet, h = 65 feet.

When

$$R = 650 \text{ feet and } h = 830 \text{ feet.}$$

$$\frac{h}{2H} = \frac{830}{2 \times 100} = 4.15$$

Hence,

$$N = 5, \text{ and } m = 6.$$

[redacted]

The greatest value of $m = \dfrac{R - \frac{B}{2}}{2} = \dfrac{650 - 50}{100} = 6.$

$$M = \left[\frac{m - 1 - \frac{h}{2H}}{m - 2 + \frac{h}{2H}} + 2m\right] = \frac{6 - 1 - 4.15}{6 - 2 + 4.15} + 12 = 12.10$$

$$Q = \frac{\frac{h}{H} - 1}{m - 2 + \frac{h}{2H}} = \frac{7.30}{8.15} = 0.896$$

$$\underset{m}{\Sigma} = MQ = 12.10 \times 0.896 = 10.84$$

$$(N - \tfrac{1}{2})^2 = (5 - \tfrac{1}{2})^2 = 20.25$$

$$(N - \tfrac{1}{2})^2 + \underset{m}{\Sigma} = 20.25 + 10.84 = 31.09$$

$$(\tfrac{B}{R})^2 = (\tfrac{100}{650})^2 = 0.02367$$

$$\therefore \frac{A_e}{A} = 0.02367 \times 31.09 = 0.736$$

When R = 850 feet, and h = 65 feet,

$$\frac{h}{2H} = \frac{65}{2 \times 100} = 0.325$$

Hence,

$$\frac{h}{2H} < \frac{1}{2}\text{, and Eq 31 applies.}$$

$$\frac{h}{2H} < \frac{1}{2} \text{ is equivalent to } h < H \text{ or } h < 100.$$

From p. 26 it is seen that $\frac{A_e}{A} = 0.00354$.

TABULATION OF RESULTS

h (ft)	$\frac{h}{H} = \frac{h}{100}$	$\frac{A_e}{A}$	$\frac{H}{B} = \frac{100}{100} = 1$
910	9.10	1.00	
870	8.70	1.00	
840	8.40	1.00	
830	8.30	0.736	
65 or less	1.00 or less	0.00354	

c. Again use the pdh curve for 12 psi for a 1-kt yield. Let B = 100 feet and H = 200 feet. The results are

h (ft)	$\frac{h}{H} = \frac{h}{200}$	$\frac{A_e}{A}$	$\frac{H}{B} = \frac{200}{100} = 2$
876	4.38	1.00	
870	4.35	0.633	
830	4.15	0.452	
200 or less	1.00	0.00354	

2. For the single curve having R fixed at 13,000 feet

Let B = 2000 feet and H = 1000 feet. This time R is fixed at 13,000 feet, while h is allowed to vary from zero to 12,000 feet.

Then

$$\frac{R - \frac{B}{2}}{2B} = \frac{13,000 - 1000}{4000} = 3$$

For $h \leqq 1000$ feet $(\frac{h}{H} \leqq 1)$. From Eq 31,

$$\frac{A_e}{A} = \frac{(2000)^2}{4(13,000)^2} = 0.00592$$

If Eq 34 is applied:

$$13,000 - \frac{2000}{2 \times 1000} h \leqq \frac{3 \times 2000}{2}$$

$$13,000 - h \leqq 3000$$

$$h \geqq 10,000$$

This means that $\frac{A_e}{A} = 1$ for $h \geqq 10,000$ feet $(\frac{h}{H} = 10)$.

Compute $\frac{A_e}{A}$ for h = 2000, 4000, 6000, and 8000 feet.

When

$$h = 2000 \text{ feet } (\frac{h}{H} = 2),$$

$$\frac{h}{2H} = \frac{2000}{2 \times 1000} = 1$$

Hence,

$$N = 3, \text{ and } m = 4, 6.$$

The greatest value of $m = \frac{R - \frac{B}{2}}{B} = \frac{13,000 - 1000}{2000} = 6$

[redacted]

For m = 4:

$$M = \frac{3 - 1}{2 + 1} + 8 = \frac{26}{3}$$

$$Q = \frac{2 - 1}{3} = \frac{1}{3}$$

$$MQ = \frac{26}{9}$$

For m = 6:

$$M = \frac{5 - 1}{4 + 1} + 12 = \frac{64}{5}$$

$$Q = \frac{2 - 1}{5} = \frac{1}{5}$$

$$MQ = \frac{64}{5} \times \frac{1}{5} = \frac{64}{25}$$

$$\sum_m = \frac{26}{9} + \frac{64}{25} = 5.449$$

$$(N - \frac{1}{2})^2 = (3 - \frac{1}{2})^2 = 6.250$$

$$(N - \frac{1}{2})^2 + \sum_m = 6.250 + 5.449 = 11.699$$

$$(\frac{B}{R})^2 = (\frac{2000}{13,000})^2 = 0.02367$$

$$\therefore \frac{A_e}{A} = 0.02367 \times 11.699 = 0.277$$

TABULATED RESULTS

h (ft)	h/H	$\frac{A_e}{A}$
10,000	10	1.00
8,000	8	0.73
6,000	6	0.687
4,000	4	0.442
2,000	2	0.277
1,000 or less	1 or less	0.00592

APPENDIX C

DETERMINATION OF THE DEGRADATION FACTOR FOR SINUSOIDAL TERRAIN

A semigraphical method was used to determine the degradation factor for sinusoidal terrain. First, an illustration similar to Fig. 4 was drawn to scale for each sine-wave type of contour. As in Fig. 4, straight lines emanating from the burst point were drawn to determine which regions on each sine wave would be directly exposed to the blast. Then the lengths of the exposed regions on each wave were computed and totaled, the total being called L_e, where L_e is defined more explicitly on page 10. The total profile length L for each wave was computed for the given range of the blast. The degradation factor for each contour is L_e/L.

The length of a sine curve between any two points whose x coordinates are x_1 and x_2 $(x_2 > x_1)$ is

$$s = \sqrt{\left(\frac{B}{\pi}\right)^2 + H^2} \left[E\left(\sqrt{\frac{\pi^2 H^2}{B^2 + \pi^2 H^2}}, \frac{\pi x_2}{B}\right) - E\left(\sqrt{\frac{\pi^2 H^2}{B^2 + \pi^2 H^2}}, \frac{\pi x_1}{B}\right) \right] \quad (35)$$

where 2B is the wavelength and H is the amplitude of the wave, and E is an elliptic integral of the second kind. The derivation of Eq 35 is given below. Tables in Ref. 2 were used to evaluate the elliptic integrals.

Using x, y coordinates as in Fig. 4,

$$s = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad (36)$$

The equation of the sine wave representing the terrain is

$$y = H \sin \frac{\pi x}{B} \quad (37)$$

Hence,

$$s = \int_{x_1}^{x_2} \sqrt{1 + \frac{\pi^2 H^2}{B^2} \cos^2 \frac{\pi x}{B}} \; dx \tag{38}$$

but

$$\cos^2 \frac{\pi x}{B} = 1 - \sin^2 \frac{\pi x}{B} \text{ , and hence}$$

$$s = \int_{x_1}^{x_2} \sqrt{1 + \frac{\pi^2 H^2}{B^2} - \frac{\pi^2 H^2}{B^2} \sin^2 \frac{\pi x}{B}} \; dx \tag{39}$$

By algebraic manipulation Eq 39 reduces to

$$s = \sqrt{\left(\frac{B}{\pi}\right)^2 + H^2} \int_{x_1}^{x_2} \sqrt{1 - \frac{\pi^2 H^2}{B^2 + \pi^2 H^2} \sin^2 \frac{\pi x}{B}} \left(\frac{\pi}{B} \; dx\right) \tag{40}$$

This integral is an elliptic integral of the second kind. For it to be valid it is required that

$$0 < \sqrt{\frac{\pi^2 H^2}{B^2 + \pi^2 H^2}} < 1,$$

or

$$0 < \sqrt{\frac{1}{\frac{B^2}{\pi^2 H^2} + 1}} < 1. \tag{41}$$

[redacted]

For Eq 41 to be true, $\frac{B^2}{\pi^2 H^2}$ and its equivalent, $\frac{1}{\pi^2 (\frac{H}{B})^2}$, must be finite and greater than zero. It follows that $\frac{H}{B}$ must be finite and greater than zero, which requirement will be satisfied by any existing hill.

JAMES H. SMITH - 5132

DISTRIBUTION:
1/11A - S. C. Hight, 5100
2/11A - E. F. Cox, 5110
3/11A - R. W. Shephard, 5120
4/11A - K. W. Erickson, 5130
5/11A - G. E. Hansche, 5140
6/11A - B. F. Murphey, 5111
7/11A - M. L. Merritt, 5112
8/11A - J. H. Smith, 5330
9/11A - W. E. Boyes, 5131
10,11/11A - G. Byrne, 1921-3

Case No. 418.0
April 11, 1953

BIBLIOGRAPHY

1. Felsor, G. T., to Petersen, R. P., Measured Pressures at the Recent Buster and Jangle Full-Scale Tests, 5121-(57), Z-76661, February 18, 1952.

2. Jahnke, E., and Emde, F., Tables of Functions, Dover Publications, 1943.

3. Rumbaugh, L. H., Green, J. B., Turkel, S. H., and Brackney, H. W., Tactical Employment of Atomic Weapons, Appendix A, ORO-R-2(FEC), March 1, 1951, Operations Research Office, The Johns Hopkins University.