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RELATIVE FUZING LOSSES FOR DIFFERENT TYPES OF TARGETS

L. V. Skinner

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ABSTRACT

The loss in blast damage due to fuzing error is shown to be greater for an infinite area target than for any other type of target of a practical nature. As this target is the easiest type to calculate, the problem of determining the most stringent requirements on the fuzing system accuracy is simplified.

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RELATIVE FUZING LOSSES FOR DIFFERENT TYPES OF TARGETS

I INTRODUCTION

It has been noted from various computations of expected blast damage for air burst weapons, that the loss in damage due to fuzing error is greater for large area targets than for other types of targets. It is the purpose of this memorandum to show theoretically that infinite area targets, as defined herein, do suffer a greater loss in damage due to fuzing error, than other types of targets which are likely to be selected for attack. As infinite area targets are of considerable practical importance and as they are easier to calculate than other types of targets, this simplifies the problem of determining the most stringent requirements on the fuzing system accuracy.

II DEFINITIONS AND SYMBOLS

The following definitions will be used:
An "infinite area target" is an area uniformly covered with structures of uniform toughness which is sufficiently large that there is a negligible probability of the area of effect of a weapon extending outside the target area. If a circle with radius of 3 times the circular probable error (CEP) plus the lethal radius of the weapon, or larger, may be inscribed within the target area, it will be considered to be an infinite area target.

A "point target" is a target the major dimension of which is small in comparison with both the CEP and the lethal radius of the weapon. This is equivalent to saying that the probability of the lethal area covering only a fraction of the target area is negligibly small.

A "complex target" is an array of target elements each of which fulfills the conditions for a point target.

A "finite area target" is an area target which is larger than a point target but smaller than an infinite area target.

The "expected damage" to a point target is the probability that the target will be covered by the lethal area of a weapon. For an area target it is the expected target area covered by the lethal area of a weapon.

A "fuzing distribution" is the probability density of the fuze operation as a function of height above ground. The distribution may not be normal.

The "lethal radius" of a weapon for a particular type structure is the horizontal distance from ground zero (GZ) of the weapon at which the overpressure is expected to be just sufficient to destroy the structure.

The "loss in damage due to a fuzing distribution" is

$$\left[1 - \frac{\text{Expected damage with fuzing distribution}}{\text{Expected damage with optimum fuzing}}\right] \qquad (1)$$

The following symbols and abbreviations will be used:

CEP - Circular probable error of bombing

D - Expected damage to a target

GZ - Ground zero, point on ground directly under burst point of a weapon

h - Burst height above ground

IGZ - Intended ground zero

$I_n(x)$ - Modified Bessel function of the first kind of integral order "n"

$p = p(r)$ - Probability that the lethal area of a weapon will cover a target element as a function of lethal radius

P(h) - Probability density of fuze operation as a function of height above ground

$r = r(h)$ - Lethal radius of a weapon as a function of burst height

r_m - Maximum lethal radius of a weapon

R - Radius of a circular target

ρ - Distance from intended ground zero to a target element

$S = S(r)$ - Expected overlap of the lethal area of a weapon and a circular target as a function of lethal radius

$\sigma = \text{CEP}/1.177$ - Standard deviation of a circular Gaussian bombing distribution

Other symbols will be defined when first used.

III COMPARISON OF INFINITE AREA AND POINT TARGET

The expected damage to an infinite area target with any arbitrary fuzing distribution P(h) is

$$D_\infty = \pi \int_{-\infty}^{\infty} \left[r(h)\right]^2 P(h)\, dh. \qquad (2)$$

Since the CEP of the bombing does not appear in the equation this is the easiest target to calculate. With optimum fuzing "r" becomes "r_m" and the expected damage is

$$\left(D_\infty\right)_m = \pi r_m^2 \qquad (3)$$

The probability that a point target located at the IGZ will be covered by the lethal area of a weapon is[1]

$$1 - e^{-\frac{1}{2}\left[\frac{r(h)}{\sigma}\right]^2}$$

The expected damage to the target with a fuzing distribution is

$$D_o = \int_{-\infty}^{\infty} \left[1 - e^{-\frac{1}{2}\frac{r^2}{\sigma^2}}\right] P(h)\, dh. \qquad (4)$$

With optimum fuzing the expected damage becomes

$$\left(D_o\right)_m = 1 - e^{-\frac{1}{2}\frac{r_m^2}{\sigma 2}} \qquad (5)$$

It will now be shown that for a given fuzing distribution P(h) and overpressure curve r(h) that the loss in damage due to fuzing error is always greater for an infinite area target than for a point target regardless of the CEP.

Stated mathematically in terms of (1), (2), (3), (4) and (5) we must show that

$$1 - \frac{D_o}{\left(D_o\right)_m} < 1 - \frac{D\infty}{\left(D_\infty\right)_m},$$

(1) The Circular Coverage Function, H. H. Germond, Project Rand RM-330, 26 January 1950.

which is equivalent to showing that

$$\frac{D_o}{(D_o)_m} > \frac{D_\infty}{(D_\infty)_m} . \tag{6}$$

Since the integrand is always positive for both (2) and (4) a sufficient condition for (6) to be true is that

$$\frac{1 - e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}}{(D_o)_m} > \frac{r^2}{r_m^2} ,$$

or that

$$F(r) \equiv \frac{r_m^2}{(D_o)_m} \cdot \frac{1 - e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}}{r^2} > 1 \tag{7}$$

for $0 < r < r_m$

Differentiating (7) with respect to "r" gives

$$\frac{dF(r)}{dr} = \frac{2\, r_m^2\, e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}}{(D_o)_m\, r^3} \left[1 + \frac{r^2}{2\sigma^2} - e^{\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}\right]$$

$$= - \frac{2\, r_m^2\, e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}}{(D_o)_m\, r^3} \sum_{n=2}^{\infty} \frac{r^{2n}}{2^n\, \sigma^{2n}\, \lfloor n}$$

which is negative for all positive values of "r" and "σ".

Since $F(r_m) = 1$ and $\frac{dF(r)}{dr} < 0$, then $F(r) > 1$ for $0 < r < r_m$ and the theorem is proved.

In the foregoing it has been assumed that the bombing error has a circular Gaussian distribution and that it is independent of the fuzing distribution. Since the above theorem holds for all values of "σ" it is reasonable to believe (although no formal proof has been devised) that it also holds for delivery systems having different standard deviations for range and azimuth. Now if the weapon does not approach the target from the vertical direction, the effect of the fuzing distribution is to increase the range error slightly without affecting the azimuth error. Hence, it is believed that the theorem holds for cases more general than those indicated in the hypotheses.

IV COMPARISON OF CIRCULAR AND INFINITE AREA TARGET

The expected overlap of the lethal area of a weapon and a circular target with the IGZ at its center is[1]

$$S(r) = \frac{\pi}{\sigma^2}\left\{R^2\, p(r) + r^2\left[1 - p(r)\right] - e^{-\frac{1}{2\sigma^2}(r^2+R^2)}\left[r^2 I_o\left(\frac{Rr}{\sigma^2}\right) + Rr\, I_1\left(\frac{Rr}{\sigma^2}\right)\right]\right\}, \quad (8)$$

where

$$p(r) = e^{-\frac{1}{2\sigma^2}(r^2+R^2)} \sum_{n=1}^{\infty}\left(\frac{r}{R}\right)^n I_n\left(\frac{Rr}{\sigma^2}\right) \quad (9)$$

and

$$I_n(x) = \sum_{m=o}^{\infty} \frac{\left(\frac{1}{2}x\right)^{n+2m}}{\lfloor m \; \lfloor n+m.} \quad (10)$$

The expected damage to a circular area target of radius "R" with a fuzing distribution is

$$D_c = \int_{-\infty}^{\infty} S(r)P(h)\, dh. \quad (11)$$

For optimum fuzing the expected damage becomes $(D_c)_m = S(r_m)$. (12)

It will now be shown that for a given fuzing distribution P(h), and overpressure curve r(h), that the loss in damage due to fuzing error is always greater for an infinite area target than for a circular area target, regardless of the CEP and the radius of the target.

Stated mathematically in terms of (1), (2), (3), (11) and (12) we must show that

$$1 - \frac{D_c}{(D_c)_m} < 1 - \frac{D_\infty}{(D_\infty)_m}$$

which is equivalent to showing that

$$\frac{D_c}{(D_c)_m} > \frac{D\infty}{(D\infty)_m}. \quad (13)$$

We will follow the same procedure used in the previous section. Since the integrand is always positive for (2) and (11) a sufficient condition for (13) to be true is that

$$\frac{S(r)}{S(r_m)} > \frac{r^2}{r_m^2} \quad \text{or that}$$

$$F(r) \equiv \frac{r_m^2}{S(r_m)} \cdot \frac{S(r)}{r^2} > 1 \qquad (14)$$

for $0 < r < r_m$. As $F(r_m) = 1$, the inequality (14) may be established by showing that

$$\frac{dF(r)}{dr} < 0 \quad \text{for } 0 < r < r_m.$$

Differentiating (14) with respect to r gives

$$\frac{dF(r)}{dr} = \frac{r_m^2}{S(r_m) r^3} \left[r \frac{dS(r)}{dr} - 2S(r) \right]. \qquad (15)$$

By making use of (8) and (9), (15) may be written in the form

$$\frac{dF(r)}{dr} = - \frac{2\pi\, r_m^2 R^2}{r^3\, \sigma 2\, S(r_m)} e^{-\frac{1}{2\sigma^2}(r^2+R^2)} \sum_{n=2}^{\infty} \left(\frac{r}{R}\right)^n I_n\left(\frac{Rr}{\sigma^2}\right). \qquad (16)$$

From (16) we see that $\frac{dF(r)}{dr} < 0$

for all positive values of R, r, and σ, hence the theorem is proved. If "R" approaches infinity. the derivative of F(r) approaches zero and the circular target case approaches the infinite area case, as it should.

Here again it has been assumed that the bombing error has a circular Gaussian distribution and that "σ" is independent of the fuzing distribution. It is believed that these are unnecessary restrictions for the same reasons as given for point targets at the end of the previous section.

V COMPARISON OF OTHER TARGETS WITH INFINITE AREA TARGET

Less definite conclusions may be reached in comparing targets other than the point and circular with the infinite area target. In fact, it is easy to conceive of a target-IGZ arrangement for which the loss due to fuzing would be greater than that for an infinite target, although it is not one a military planner would be likely to select. Such a case might be a large, narrow, circular ring target with the IGZ at the center. Assume that $\sigma = 0$ and the lethal radius with optimum fuzing is just sufficient for the entire target to be destroyed. With a fuzing distribution, the expected lethal radius may be reduced to an extent that the loss due to fuzing will be close to 100%. This is an extreme case cited to illustrate the point.

Let us consider first a complex target and focus our attention on one element of the target which is at a distance "ρ" from the IGZ. The probability that this element

will be covered by the lethal area of the weapon is[1]

$$p(\rho, r) = e^{-\frac{1}{2\sigma^2}(r^2+\rho^2)} \sum_{n=1}^{\infty} \left(\frac{r}{\rho}\right)^n I_n\left(\frac{\rho r}{\sigma^2}\right) \tag{17}$$

The expected damage to the target element with a fuzing distribution is

$$D_e = \int_{-\infty}^{\infty} p(\rho, r) \quad P(h) \; dh. \tag{18}$$

For optimum fuzing the expected damage becomes

$$\left(D_e\right)_m = p(\rho, r_m) \tag{19}$$

We will now attempt to determine how large "ρ" may be for the loss in damage due to fuzing error for the target element to be less than that for an infinite area target for the same fuzing distribution and overpressure curve.

Proceeding in the same manner as previously, we desire to determine the minimum restriction possible on "ρ" such that

$$1 - \frac{D_e}{\left(D_e\right)_m} < 1 - \frac{D_\infty}{(D_\infty)_m},$$

or what

is equivalent

$$\frac{D_e}{(D_e)_m} > \frac{D_\infty}{(D_\infty)_m}. \tag{20}$$

A sufficient but not necessary condition for (20) to hold is, from (2), (3), (18), and (19), that

$$\frac{p(\rho, r)}{p(\rho, r_m)} > \frac{r^2}{r^2_m} \quad \text{or that}$$

$$F(r) \equiv \frac{r^2_m}{p(\rho, r_m)} \cdot \frac{p(\rho, r)}{r^2} > 1 \tag{21}$$

for $0 < r < r_m$.

Differentiating (21) with respect to "r" gives

$$\frac{dF(r)}{dr} = \frac{r_m^2}{p(\rho, r_m)\, r^3} \left[r \frac{dp(\rho, r)}{dr} - 2\, p(\rho, r) \right] \tag{22}$$

Making use of (17), we may evaluate (22) to give

$$\frac{dF(r)}{dr} = \frac{2r_m^2}{p(\rho, r_m)\, r^3} \left[\frac{r^2}{2\sigma^2} e^{-\frac{1}{2\sigma^2}(r^2+\rho^2)} I_o\left(\frac{\rho r}{\sigma^2}\right) - p(\rho, r) \right] \tag{23}$$

Since $F(r_m)=1$, a sufficient but not necessary condition for (20) to be true is from (23) that

$$p(\rho, r) \geq \frac{r^2}{2\sigma^2} e^{-\frac{1}{2\sigma^2}(r^2+\rho^2)} I_o\left(\frac{\rho r}{\sigma^2}\right). \tag{24}$$

From the equation of (24), ρ/σ may be plotted as a function r/σ as shown in Fig. 1. The inequality of (24) then corresponds to ρ/σ being below the curve of Fig. 1.

We see from Fig. 1 that the condition $\rho < \sqrt{2}\,\sigma$ applies for any range of values of "r". However, if the fuzing distribution and overpressure curve is such that "r" has a negligible probability of approaching zero, a less restrictive condition may be placed on "ρ". For example, if the conditions are such that "r" only has values in excess of 2.6 σ, then "ρ" may have values up to 2.3 σ.

We conclude from the foregoing that for a given fuzing distribution and overpressure curve that the loss in damage due to fuzing error is greater for an infinite area target than any complex or area target for which the maximum value of "ρ" satisfies the inequality (24) for the range of values of "r" associated with the fuzing distribution. Of course for most targets "ρ" could be much larger than indicated by the rule just given as the greater loss suffered by elements near the edge of the target will be compensated for by the reduced loss suffered by target elements near the IGZ. In fact, for an area target it is safe to say that if there are no large holes in the vicinity of the IGZ and the periphery is, in general, concave toward the IGZ, the rule would still apply. The same may be safely said for any complex target for which the outer boundary is, in general, concave toward the IGZ and the target element density is substantially uniform or at least does not increase appreciably with increasing distance from the IGZ. However, target IGZ combinations which do not fulfill the conditions just mentioned would, no doubt, be ruled out by considerations other than fuzing accuracy.

VI CONCLUSIONS

It has been shown analytically that from a blast damage standpoint for an air burst weapon a point, circular, and other targets with certain restrictions impose a less stringent requirement on the accuracy of the fuzing system than does an infinite area target. It appears that practically all targets which would be selected for

attack, on the basis of other considerations, would meet these restrictions.

The analysis presented here has been based on the "cookie cutter" concept of damage. This assumes that damage is 100% for target elements within the lethal radius of GZ and that no damage occurs at greater distances. A more realistic approach, but more difficult to calculate, is to assume that expected damage is a continuous monotonically decreasing function of distance from ground zero. One such function which, with proper selection of "k", gives results closely approximating observed values is

$$D(\rho) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\rho-\rho_0}{k\rho_0}} e^{-\frac{1}{2}y^2} dy, \qquad (25)$$

where ρ_0 is the weapon radius (value of ρ for which $D(\rho)$ is 0.5). It may be shown that loss in damage due to fuzing error is exactly the same for an infinite area target whether calculated by the "cookie cutter" method or by use of (25). Other targets which have been calculated both ways show a greater loss by the "cookie cutter" method. It appears that this is true for most targets with a few exceptions (e.g. point targets for which the expected damage is very close to one). In view of the foregoing it may be said that the conclusions previously reached on the basis of the "cookie cutter" type calculations will in general hold for the more realistic calculations employing equation (25).

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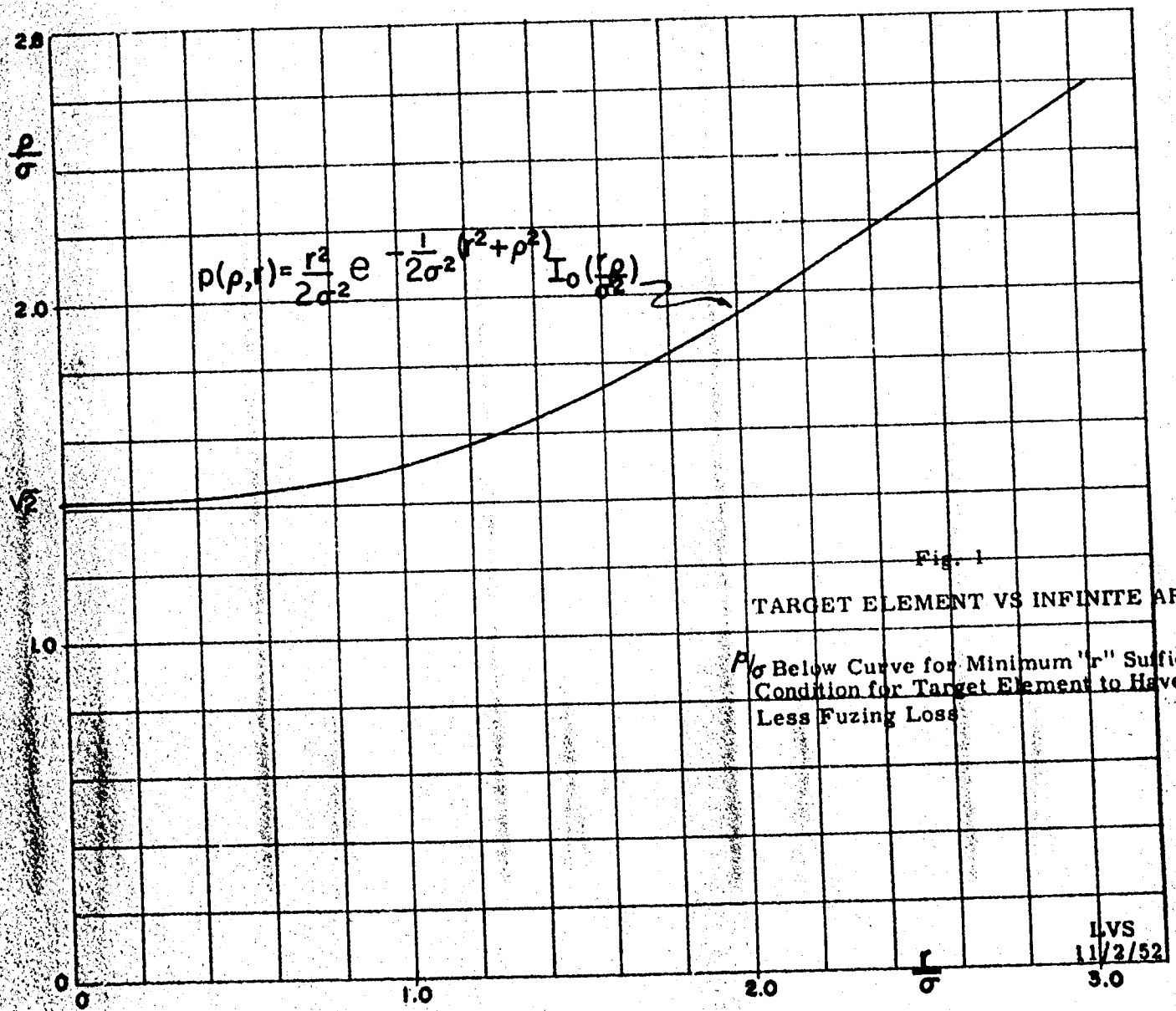

Fig. 1

TARGET ELEMENT VS INFINITE AREA

ρ/σ Below Curve for Minimum "r" Sufficient Condition for Target Element to Have Less Fuzing Loss

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