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REMARKS ON TIME-OF-FALL FUZING

J. K. Hale - 5126

ABSTRACT

One of the components of a clock fuzing system for a free-fall weapon must calculate the time for the weapon to fall to some point in space. In this memorandum this point is referred to as the prescribed burst point. If t represents time passing during a delivery maneuver, and the time of fall T of the weapon to the prescribed burst point varies continuously with t, then the derivative $\frac{dT}{dt}$ is called the setting rate and is the rate at which the computer must change the clock setting if the clock is to be correctly set at all times during the delivery maneuver. The purpose of this memorandum is to give a prescribed burst point which leads to small setting rates during an entire maneuver.

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REMARKS ON TIME-OF-FALL FUZING

INTRODUCTION

One of the components of a clock fuzing system for a free-fall weapon must calculate the time it takes the weapon to fall to some point in space. In this memorandum this point shall be referred to as the prescribed burst point (PBP). This point is arbitrary except for the requirement that it approaches the desired burst point (DBP) as the path of the projectile approaches a path which passes through the DBP.

Two methods which have been used for specifying the PBP are:

1. The vertical distance of the PBP from the release point is equal to the vertical distance of the DBP from the release point, or
2. The horizontal distance of the PBP from the release point is equal to the horizontal distance of the DBP from the release point.

Let t represent time passing during the delivery maneuver, and let the time of fall T of the weapon to the PBP vary continuously with t. The derivative $\frac{dT}{dt}$ is called the setting rate and is the rate at which the computer must change the clock setting if the clock is to be correctly set at all times during the delivery maneuver. One difficulty involved in a clock fuzing system is to specify a PBP in such a way that, during the entire maneuver, the setting rate $\frac{dT}{dt}$ is not too "large." The rate $\frac{dT}{dt}$ is not too large if it is less than the rate at which the clock can be set with the required accuracy. For some tactical delivery systems, this difficulty has not been overcome up to the present time. For instance, in Case I above for a lob bombing maneuver, the setting rates are too large for small angles, and in Case 2 for the same maneuver, the rates are too large for large angles.

The purpose of this memorandum is to give a PBP for which the setting rate is satisfactory for a variety of maneuvers. In particular, the lob bombing maneuver is discussed in some detail, and it is seen that the PBP described here leads to small setting rates during the entire maneuver.

GENERAL FORMULA

Let t denote time passing during the aircraft maneuver, and let $T = T(t)$ denote the time of fall of the weapon. Let the desired burst point be the origin of an (x, z) rectangular coordinate system with vertical z-axis directed upward and horizontal x-axis directed "toward" the approaching aircraft. (See Fig. 1.) At any time t, the CG of the aircraft is at $[x(t), z(t)]$ moving with velocity $V(t) > 0$ along a path having a tangent line which makes an angle with the positive x-axis of $180 - \phi(t)$, for any angle ϕ.

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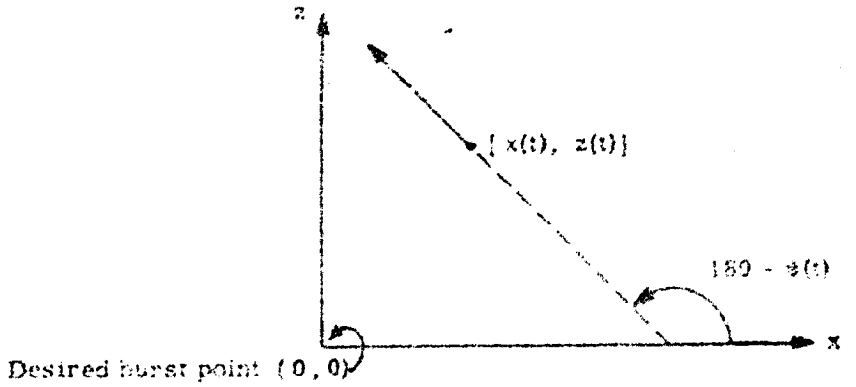

Fig. 1 -- Position and direction of aircraft at time t

In the (x, z) system at time t, the weapon has co-ordinates (X, Z) given by

$$\left.\begin{aligned} X &= x(t) - V\cos\phi \cdot T(t), \\ Z &= z(t) + V\sin\phi \cdot T(t) - \frac{1}{2} g T^2(t). \end{aligned}\right\} \tag{1}$$

In the discussion of the prescribed burst points in the introduction two methods of specifying the PBP are given. It is clear that for the first case, all these points lie on the x-axis, and for the second, all points lie on the z-axis. In the present discussion, depending on the value of ϕ, the prescribed burst points may be on any line passing through the origin. In fact, the PBP is chosen as any point on the path of the weapon at which

$$X \cos\phi = -kZ \tag{2}$$

for any value of $k > 0$. The time required for the weapon to fall to such a point is given by

$$\frac{kg}{2} T^2 - V(k \sin\phi - \cos^2\phi)T - \{kz(t) + x(t)\cos\phi\} = 0,$$

or

$$T = \frac{V}{kg}\left\{(k \sin\phi - \cos^2\phi) + \sqrt{(k \sin\phi - \cos^2\phi)^2 + \frac{2kg}{V^2}\left[kz(t) + x(t)\cos\phi\right]}\right\}. \tag{3}$$

Notice that in Equation 3 only the positive square root is considered. This choice results because the desired burst point is the one which involves the longer time of flight. It should be emphasized that there is always a solution of Equation 3 provided that the path of the weapon enters the region above and to the left of the line $x = -kz$ for $\phi \leq 90°$ and the region above and to right of the line $x = kz$ for $\phi \geq 90°$.

In the following pages, the setting rate and time of fall are discussed for particular types of maneuvers.

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LEVEL BOMBING

Let t_1 denote the time for the aircraft to arrive at a point directly over the target. If it is assumed for level bombing that $dV/dt = 0$, $z(t) = z_o$ (a constant greater than or equal to zero), then $x(t) = V \cdot (t_1 - t)$ and $\phi = 0$ for all values of t. Equation 3 then becomes

$$T(t) = \frac{V}{kg}\left[-1 + \sqrt{1 + \frac{2k^2g}{V^2} z_o + \frac{2kg}{V}(t_1 - t)}\right],$$

and

$$\frac{dT}{dt} = \frac{-1}{\sqrt{1 + \frac{2k^2g}{V^2} z_o + \frac{2kg}{V}(t_1 - t)}} .$$

Thus,

$$\left|\frac{dT}{dt}\right| = \frac{1}{\sqrt{1 + \frac{2kg}{V^2}\left[kz_o + V(t_1 - t)\right]}} \leq 1 \text{ sec/sec}$$

for all values of $t \geq 0$.

LOB AND TOSS BOMBING

It is assumed that the path of the airplane is the circular path shown in Fig. 2, where $x_o = x(0)$, $z_o = z(0)$, ϕ is the same angle as before, the acceleration of the airplane is ng, and the velocity V is constant. It follows that $\phi = \frac{ngt}{V} - \phi_o$ and

$$\left.\begin{aligned} z(t) &= z_o + \frac{V^2}{ng}\cos\phi_o - \frac{V^2}{ng}\cos\phi , \\ x(t) &= x_o + \frac{V^2}{ng}\sin\phi_o - \frac{V^2}{ng}\sin\phi . \end{aligned}\right\} \qquad (4)$$

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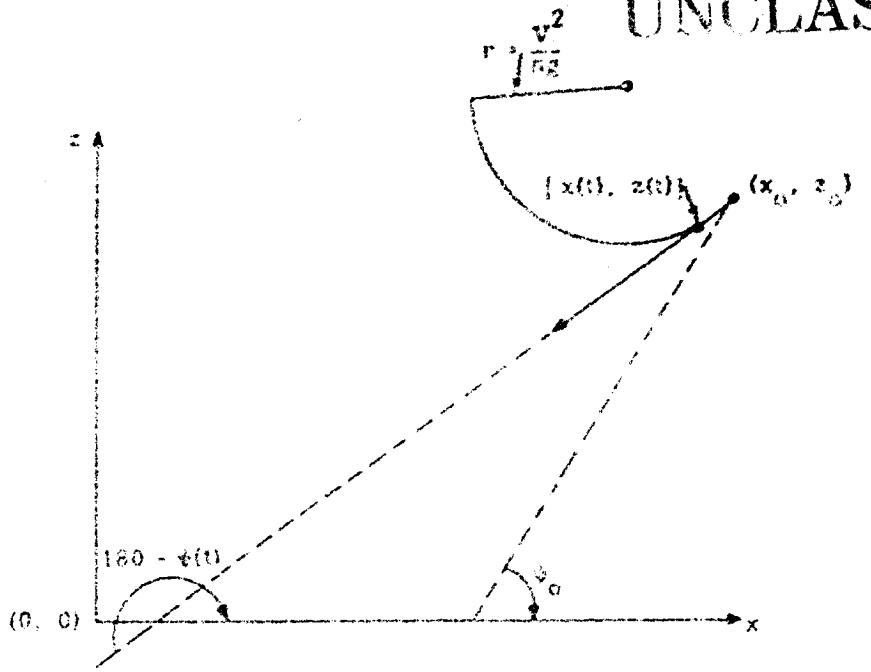

Fig. 2 -- Parameters for a lob or toss bombing maneuver

If Equation 4 is substituted in Equation 3 the result is

$$T = T(t) = \frac{V}{kg}\left[(k \sin\phi - \cos^2\phi) + \sqrt{(k \sin\phi - \cos^2\phi)^2 + A + 2B\cos\phi - \frac{k}{n}\sin 2\phi}\right]. \quad (5)$$

where

$$A = \frac{2k^2 g}{V^2}\left(z_o + \frac{V^2}{ng}\cos\phi_o\right), \text{ and}$$

$$B = \frac{k}{n}\left(\frac{ngx_o}{V^2} + \sin\phi_o\right).$$

Furthermore,

$$\frac{dT}{dt} = \frac{n}{k}\left[(k\cos\phi + \sin 2\phi) + \frac{(k\sin\phi - \cos^2\phi)(k\cos\phi + \sin 2\phi) - B\sin\phi - \frac{k}{n}\cos 2\phi}{\sqrt{(k\sin\phi - \cos^2\phi)^2 + A + 2B\cos\phi - \frac{k}{n}\sin 2\phi}}\right]. \quad (6)$$

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For certain values of k, the function $\frac{dT}{dt}$ is continuous and has a maximum which is a function of k. An ideal situation would be to choose that value of k which would give the least maximum. Because of the complexity of the function $\frac{dT}{dt}$, this would involve drawing many families of curves and approximating the best value of k. Since this is a laborious task, only a very special case (k = 1) is considered in the following section.

SETTING RATE FOR LOB BOMBING

This section considers the setting rates for the lob bombing maneuver, that is, a maneuver in which the airplane flies the path shown in Fig. 2 with $\psi_o = 0$. Also, the PBP is chosen as any point on the path of the projectile at which $x \cos \psi = -z$.

For any particular maneuver, there are two points on the path of the aircraft (i.e., two values of ψ) such that, if the weapon were released at either of these points, the path of the weapon would pass through the desired burst point. Any point having this property shall be called a <u>release point</u>. In the following paragraph, the formula for the setting rate at a release point is derived.

At a release point, the numbers x_o, z_o, ψ, and T must satisfy the equations X = 0 and Z = 0. If T is eliminated and these equations are solved for $x_o = x_o(\psi, z_o)$, it follows that

$$x_o(\psi, z_o) = \frac{V^2}{ng}\left[\sin\psi + n\cos\psi\left(\sin\psi - \sqrt{\sin^2\psi - \frac{2}{n}\cos\psi + \frac{2}{n} - \frac{2gz_o}{V^2}}\right)\right]. \tag{7}$$

Equation 7 gives the horizontal distance from the target at which the lob maneuver should begin in order to have a release angle ψ for a given vertical distance from the target. Only the positive square root is chosen here because the negative square root leads to values of x_o for which the weapon would pass through the DBP at a point where the weapon is moving upward. Finally, $\left(\frac{dT}{dt}\right)_{RP}$ (the setting rate at a release point) is given by Equation 6 with x_o replaced by Equation 7.

Figure 3 is a plot of $\left(\frac{dT}{dt}\right)_{RP}$ versus ψ for different values of z_o and V = 1000 ft/sec, g = 32 ft/sec^2, n = 4, and k = 1. Figure 4 is a graph of $x_o(\psi, z_o)$ as a function of ψ for the same values of z_o, V, g, n, and k, $0 \le \psi \le 90^\circ$. Figures 5 and 6 show similar results for $\psi \ge 90^\circ$.

Figure 3 shows that the setting rate at a release point is always finite, but for certain values of the parameters, the setting rate over the entire maneuver may not be so well behaved. For k = 1, V = 1000 ft/sec, g = 32 ft/sec^2, n = 4, and $\psi_o = 0$, Fig. 7 gives the values of the

parameters x_o, z_o, and ϕ for which dT/dt is infinite. For any set of values of x_o, z_o, and ϕ which do not lie on these curves, dT/dt is continuous during the entire maneuver. Figures 8 and 9 indicate the setting rate and time of flight for five typical maneuvers. For each of these maneuvers, the release point is obtained from Fig. 4.

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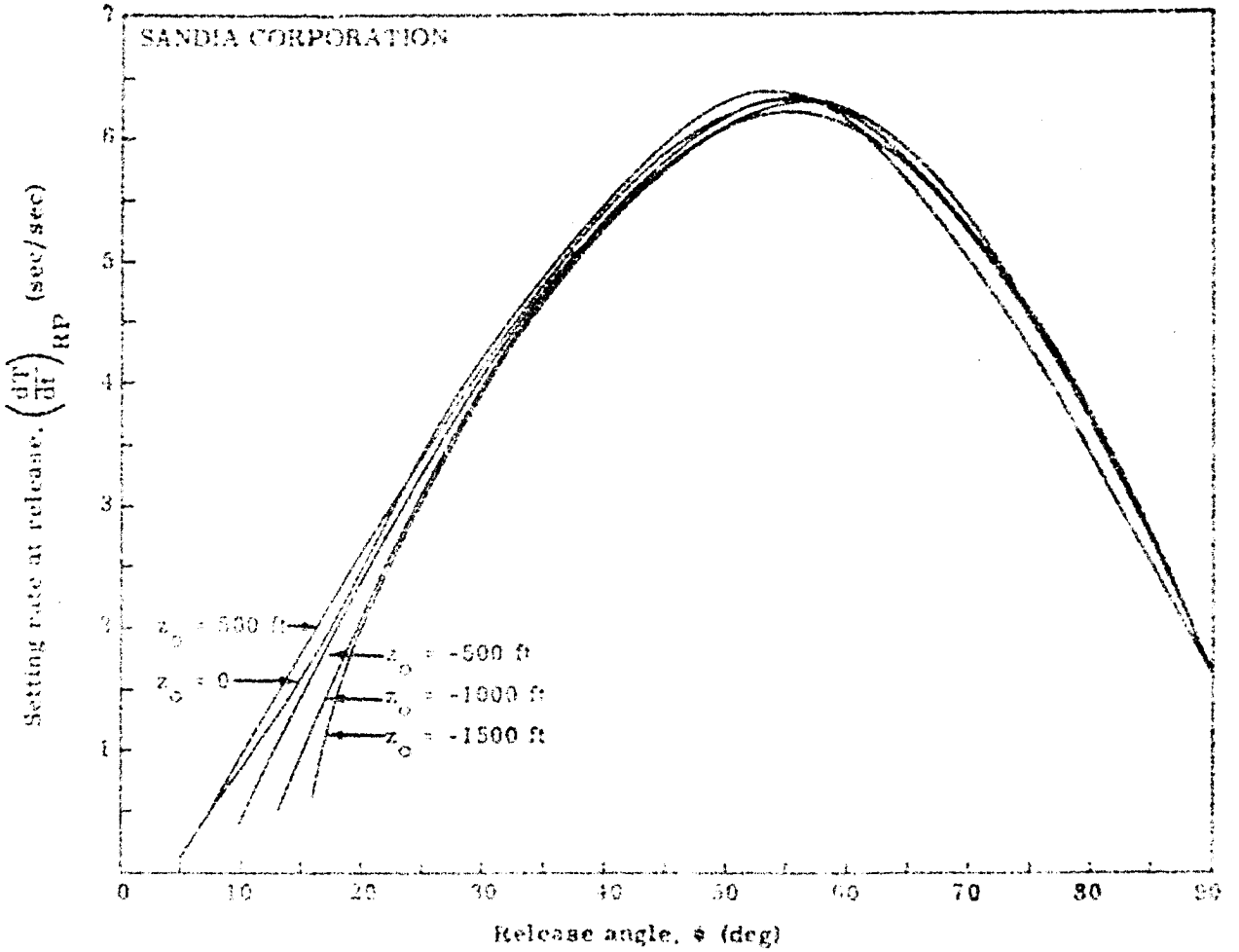

Fig. 3 -- Setting rates for the LABS maneuver for any release point as a function of the release angle ($\leq 90^{\circ}$) for various values of the difference, z_o, between the altitude at pull-up and the desired burst point for V = 1000 ft/sec and an acceleration of 4 g's

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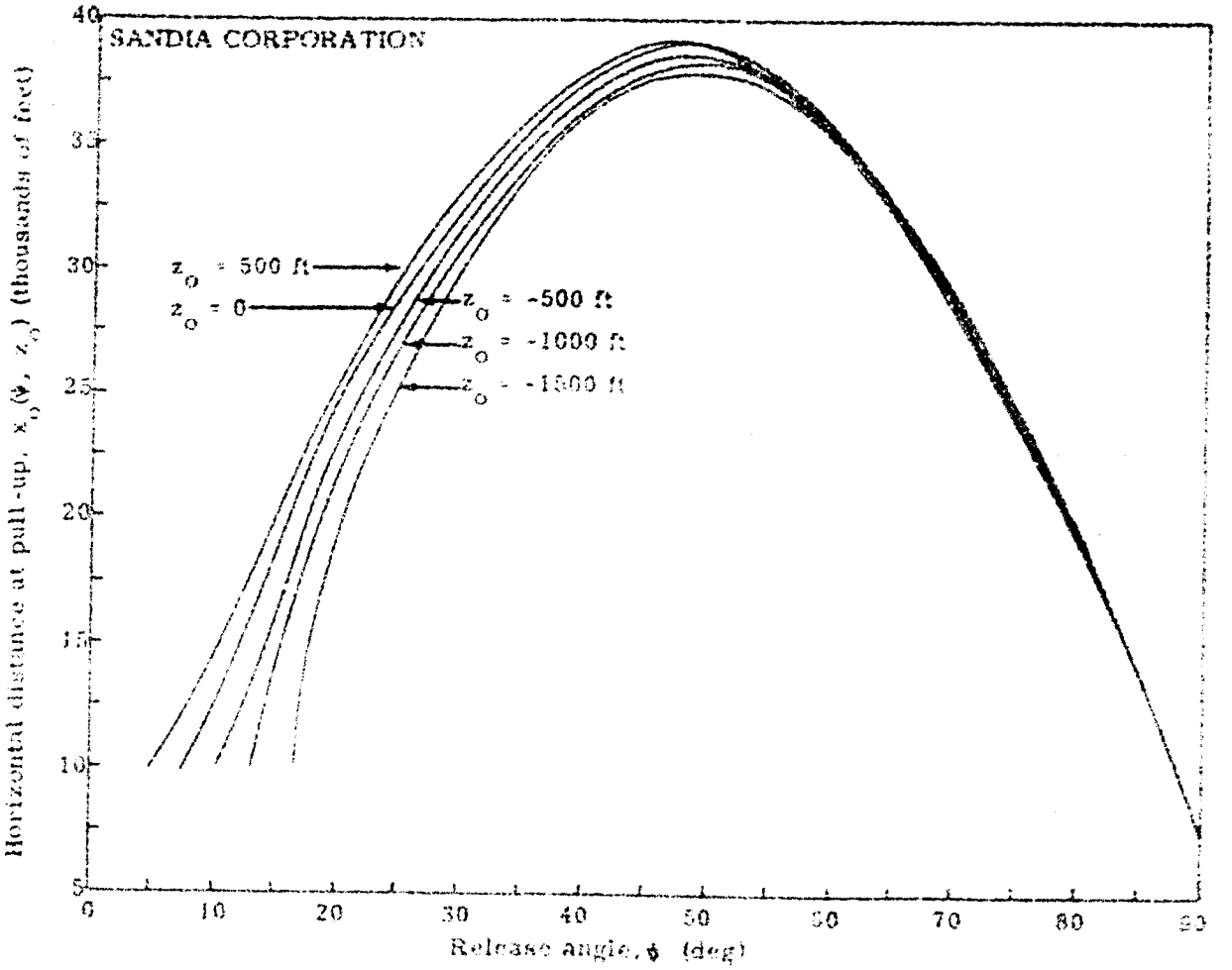

Fig. 4 -- Horizontal distance at pull-up for the LABS maneuver as a function of release angle ($\leq 90^\circ$) for various values of the difference, z_o, between the altitude at pull-up and the desired burst point for V = 1000 ft/sec and an acceleration of 4 g's

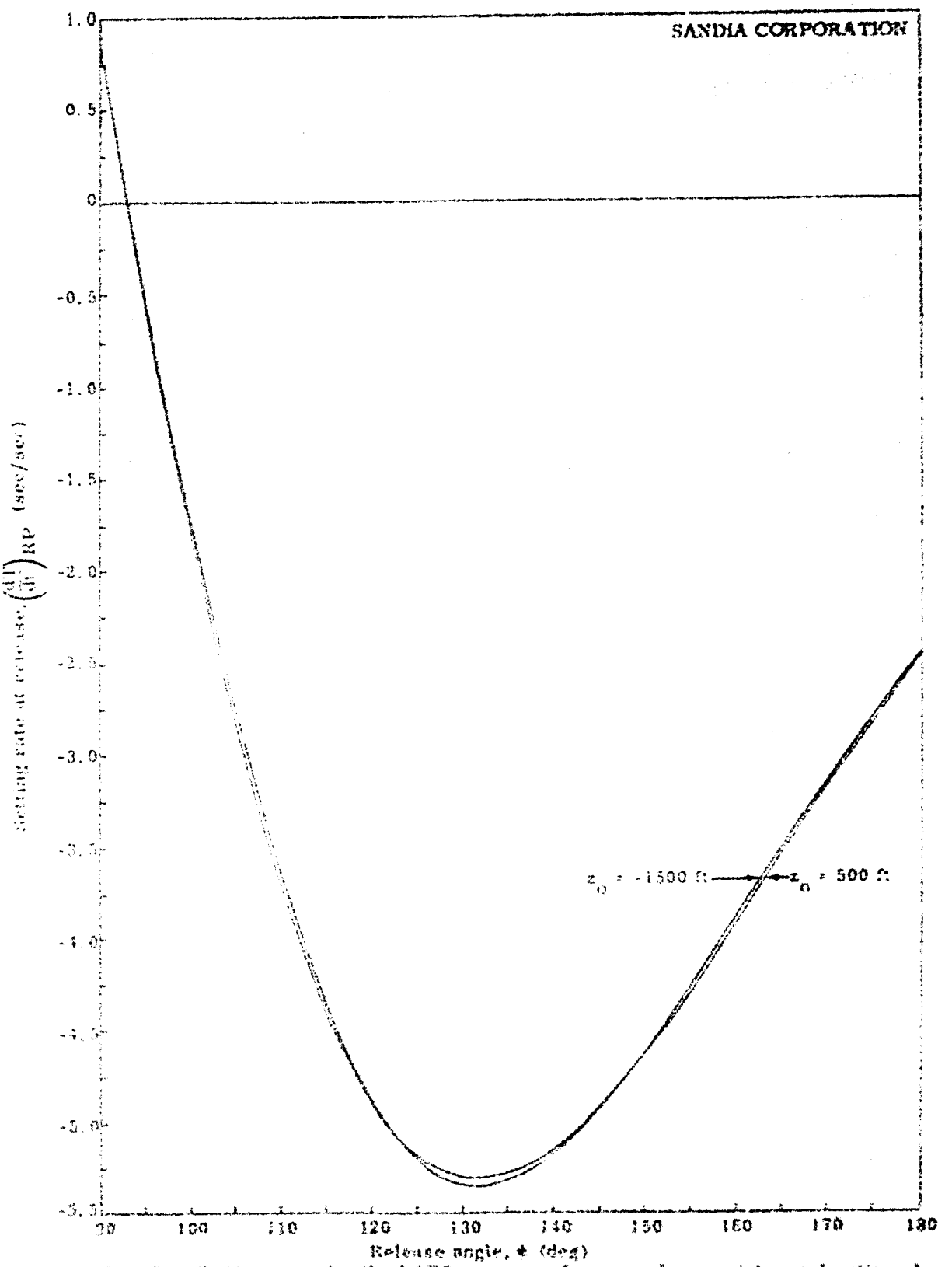

Fig. 5 -- Setting rates for the LABS maneuver for any release point as a function of the release angle ($\geq 90°$) for various values of the difference, z_o, between the altitude at pull-up and the desired burst point for V = 1000 ft/sec and an acceleration of 4 g's

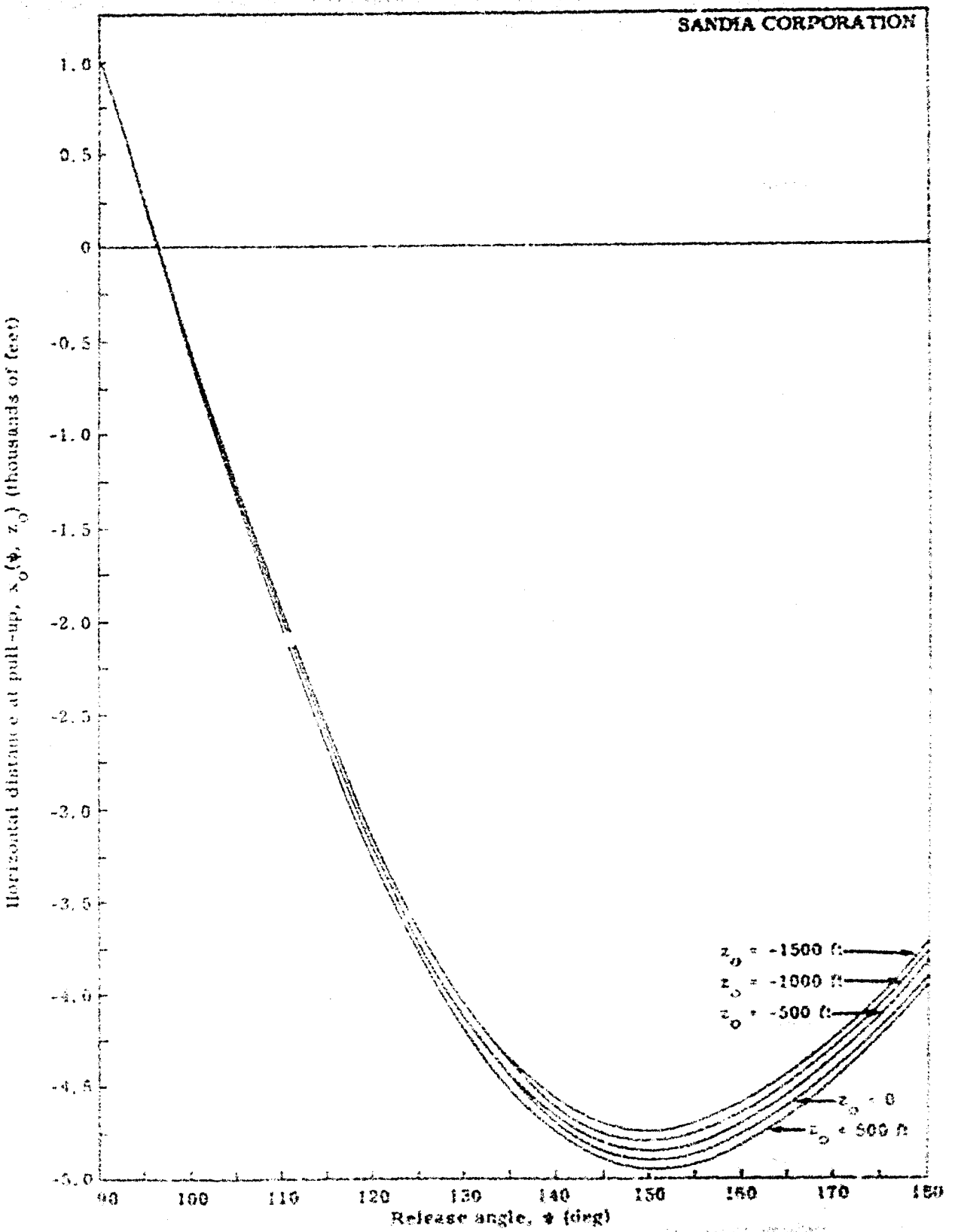

Fig. 6 -- Horizontal distance at pull-up for the LABS maneuver as a function of release angle ($\geq 90°$) for various values of the difference, z_0, between the altitude at pull-up and the desired burst point for V = 1300 ft/sec and an acceleration of 4 g's

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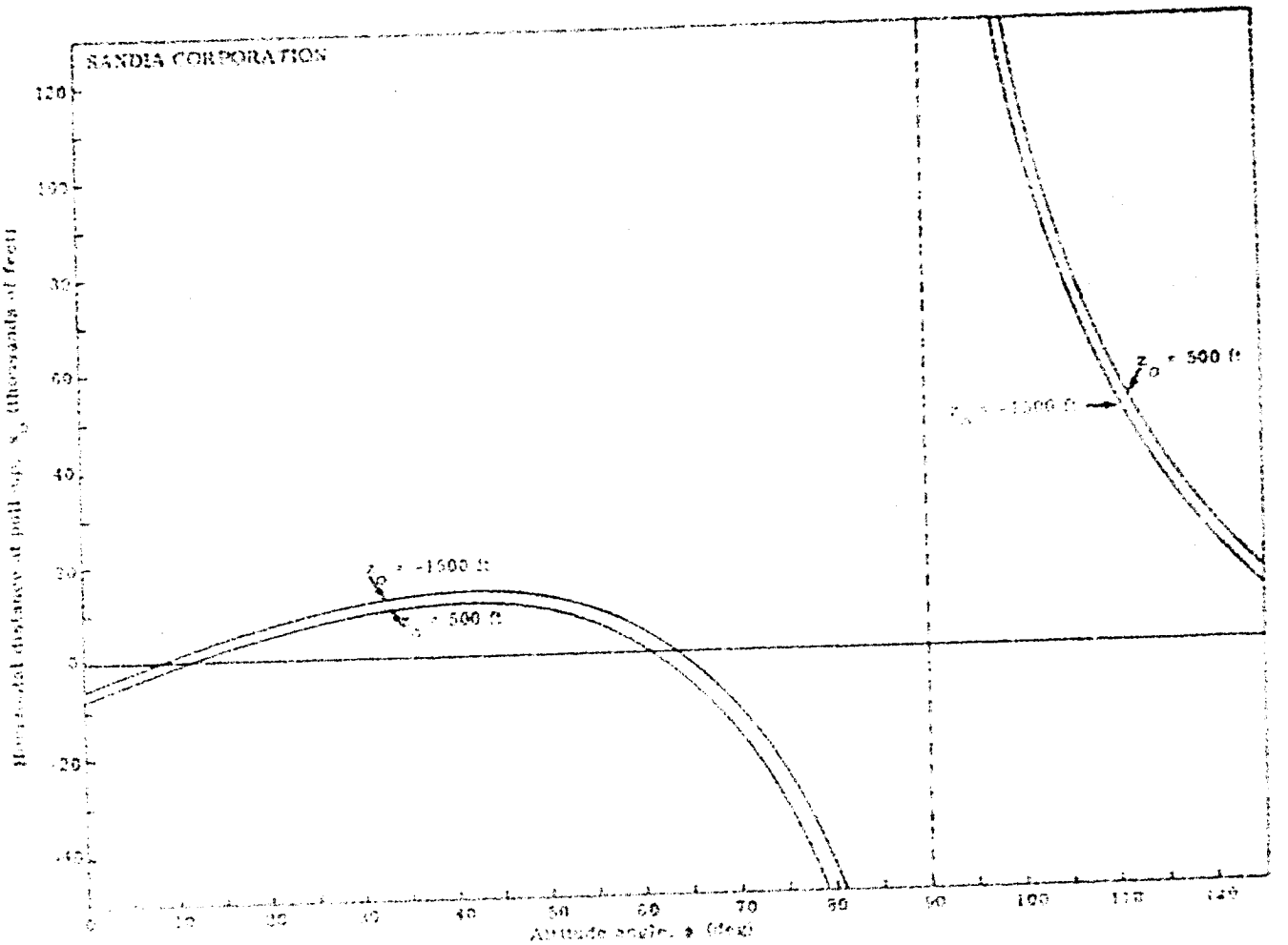

Fig. 7 — Values of parameters x_o, z_o, and φ (for the LABS maneuver) for which the setting rate is infinite

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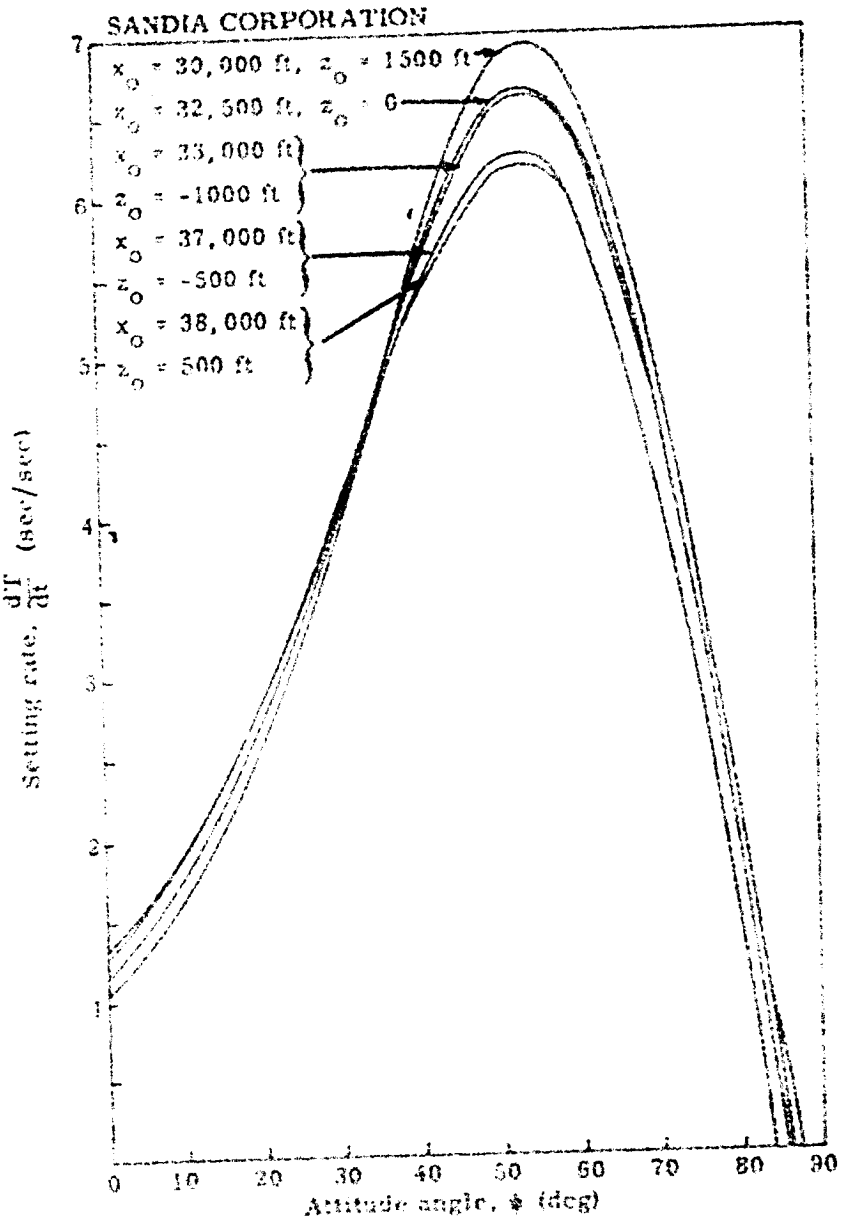

Fig. 8 -- Setting rate as a function of attitude angle for various LABS maneuvers for V = 1000 ft/sec and an acceleration of 4 g's

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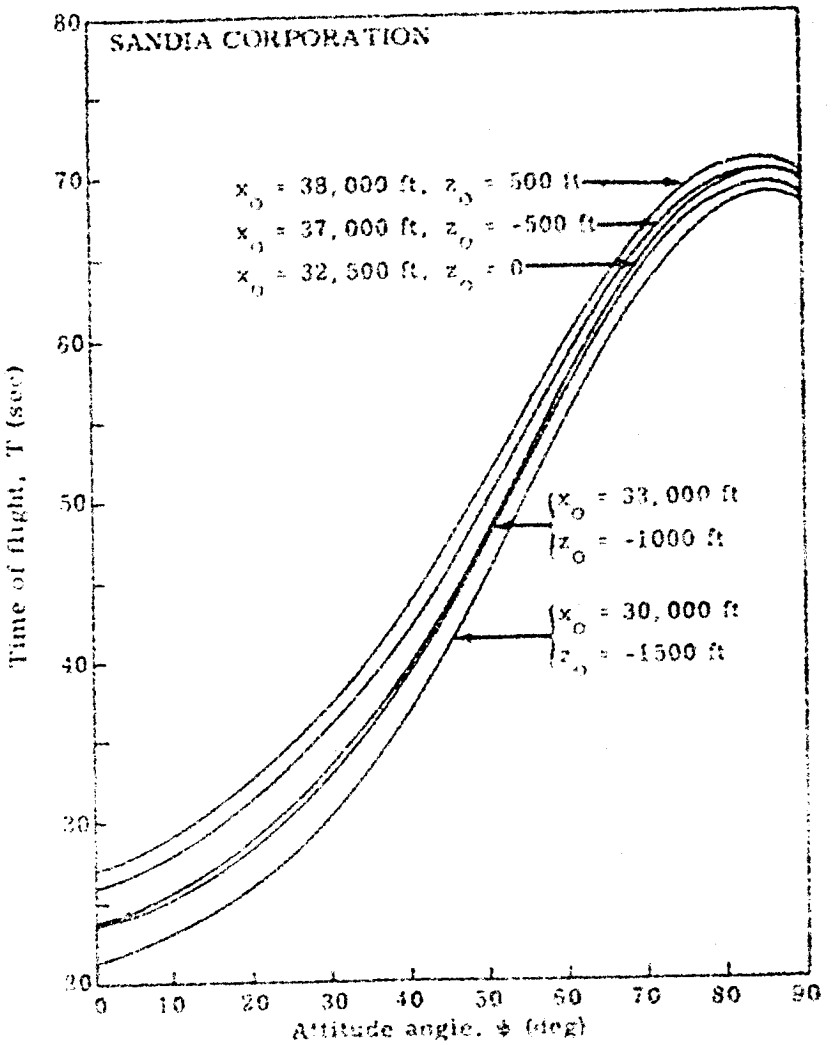

Fig. 8 -- Time of flight as a function of attitude angle for various LABS maneuvers for V = 1000 ft/sec and an acceleration of 4 g's

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