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PREDICTION OF THE REDUCTION IN BOMB RANGE CAUSED BY RELEASE OSCILLATION

E. T. Clark

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ABSTRACT

An approximate method is presented for calculating the range reduction caused by the release oscillation of a bomb. With this method the release oscillation causing any particular range error may be calculated. Appendices are included demonstrating the application of this method to the Mk 5 and Mk 6. It is evident that measurement of drag coefficient at large angles of attack will be required for these configurations, before the most accurate prediction of range reduction can be made using the technique outlined in this memorandum.

Since the amplitudes of release oscillations are functions of the carrier as well as the bomb and ambient release conditions, it is apparent that neglect of release oscillation in reduction of range bombing introduces a significant lack of repeatability which should not be attributed to the bomb alone.

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Case No. 409.0

August 10, 1953

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INTRODUCTION

Various bomb-carrier combinations have demonstrated large amplitude bomb oscillations following high speed release. These release characteristics may be undesirable for three reasons:

1. The accelerations caused by large amplitude oscillations may damage bomb components.

2. Excessive delivery errors may arise from these large amplitude oscillations.

3. In extreme cases, the weapon may hit the carrier.

The first and third of these undesirable characteristics are not considered in this report. The second is studied in a general manner, yielding a method which can be applied to any particular bomb whose aerodynamic characteristics are known functions of Mach number and angle of attack. Following are the definitions of the quantities used in this memorandum.

NOMENCLATURE

ρ = free stream density - slugs/ft^3

V = free stream velocity - ft/sec

q = free stream dynamic pressure - lbs/ft^2 = $1/2\ \rho V^2$, or

= pitching velocity - rad/sec

q_c = free stream dynamic pressure at release

M = Mach No. = V/a

R_e = Reynolds No. = $\rho V d/\mu$

μ = viscosity of air in free stream - $\frac{\text{lbs sec}}{\text{ft}^2}$

a = speed of sound in free stream - ft/sec

I = body transverse moment of inertia - slug ft^2

m = body mass - slugs

d = body diameter - feet

S = body cross sectional area - feet^2

= $\pi d^2/4$

l = body length - feet

$T_{1/2}$ = time to damp to one-half amplitude - seconds

$= \frac{-.693}{\beta}$

$T_{1/10}$ = time to damp to one-tenth amplitude - seconds

$= \frac{-2.3}{\beta}$

P = period of a motion - seconds/cycle

$= \frac{2\pi}{\omega}$

β = damping rate of a motion - radians per second

$$\cong \frac{-\pi}{16} \frac{\rho V d^2}{m} C_{L_\alpha} + \frac{\pi}{32} \frac{\rho V d^4}{I} (C_{M_q} + C_{M_{\dot{\alpha}}})$$

ω = frequency of a motion - rad/sec

$$\cong \left(\frac{-m\rho V^2 d^3 C_{M_\alpha}}{8 I} \right)^{1/2}$$

e = base of natural logarithms

t = time - seconds; equal to zero at release

t_{m_k} = time from release to the midpoint of the kth half cycle - seconds

$= (2k-1) \frac{\pi}{2\omega}$

θ = angle between the mean trajectory tangent during a half cycle and the horizontal - radians.

α = angle of attack - used in radian measure in all equations in this report.

α_o = initial half amplitude - radians

α_{o_k} = the amplitude to which α has damped at the midpoint of the kth half cycle - radians

$$= \alpha_o e^{\beta t_{m_k}} = \alpha_o e^{(2k-1)\frac{\beta\pi}{2\omega}}$$

$\dot{\alpha}$ = rate of change of angle of attack - rad/sec

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L = the number of cycles required to damp to one-half amplitude

N = the number of half cycles through which the motion goes as it damps to one-tenth amplitude

= [illegible]

k = summation index

.O.F. = time of fall - seconds

R = range reduction - feet

C_D = drag coefficient

$= \frac{\text{drag}}{qS}$

C_{D_0} = drag coefficient at $\alpha = 0$

C_{D_l} = drag coefficient based on length

$= \frac{\text{drag}}{q\,d\,l}$

C_L = lift coefficient

= lift/qS

C_{L_α} = slope of the lift coefficient curve as α tends to zero - per radian

C_M = moment coefficient

= moment/ $q\,S\,d$

C_{M_α} = slope of the moment coefficient curve as α tends to zero - per radian

C_{M_q} = slope of the moment coefficient curve as the pitching velocity tends to zero

$$\left.\frac{\partial C_M}{\partial \frac{q\,d}{2V}}\right|_{q \to 0}$$

$C_{M_{\overset{\circ}{\alpha}}}$ = slope of the moment coefficient curve as $\overset{\circ}{\alpha}$ tends to zero

$$= \left.\frac{\partial C_M}{\partial \frac{\overset{\circ}{\alpha}\,d}{2V}}\right|_{\overset{\circ}{\alpha} \to 0}$$

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DISCUSSION OF THEORY

The reduction in range caused by release oscillation can be predicted theoretically as outlined below.

The oscillatory motion of a bomb following its release from any carrier can be approximately represented by the following relation, when proper values of α_o, β and ω are used.*

$$\alpha = \alpha_o e^{\beta t} \sin \omega t$$

This motion damps to 10 per cent of its initial value in time $T_{1/10}$. It will be assumed that practically no additional retardation is caused after the motion has damped to 1/10 amplitude. From the above expression $T_{1/10}$ is calculated to be

$$T_{1/10} = \frac{\log_e 1/10}{\beta} \cong \frac{-2.3}{\beta}$$

During the time $T_{1/10}$, the motion goes through a number of half cycles given by

$$N = \frac{T_{1/10}}{P/2} = \frac{-2.3\omega}{\pi\beta}$$

A retardation increment arises due to the increased drag during each of these N half cycles. For the kth half cycle we can approximate+ the range reduction caused by the retardation increment in the following manner.

Δ Range$_k$ $\cong$ (decrement of horizontal velocity caused by the retardation increment associated with the kth half cycle) multiplied by (the time during which this decrement in horizontal velocity exists)

To evaluate the range reduction caused by the complete oscillation we can sum, over the N half cycles, the reduction in range caused by each individual half cycle.

---

*Two approximations are involved here. First it is assumed that β and ω are constant for $0 \leq \alpha \leq \alpha_o$, which is reasonably correct for small amplitudes but becomes generally less valid for large amplitudes. Secondly, the equation for α neglects transverse C.G. motion due to lift forces which exist for $\alpha \neq o$. This second approximation should cause no significant inaccuracy.

+In a vacuum the stated technique would be exact, since then the relation of range = (horizontal release velocity) X (Time of Fall) is a true equality.

Now

(the decrement in horizontal velocity caused by the retardation increment associated with the kth half cycle)

$$= \text{(mean retarding acceleration during the } k^{th} \text{ half cycle)} \times \frac{P}{2} \times \cos \theta_k ;$$

and

(the time during which the decrement in horizontal velocity exists)

$$= \text{T.O.F.} - t_{m_k} . \quad \text{(See Figure 1.)}$$

Also

(mean retarding acceleration during the kth half cycle)

$$= \frac{\text{Mean drag caused by } \alpha \neq 0}{m}$$

$$= \frac{\frac{2}{P}\int_0^{P/2} \left[C_D(\alpha) - C_{D_0}\right] q\, S\, dt}{m}$$

where

$$\alpha = \alpha_{c_k} \sin \omega t$$

and

$$\alpha_{c_k} = \alpha_0 e^{\beta t_{m_k}}$$

Thus the effect on range of the retardation increment caused by the kth half cycle can be written as

$$\Delta R_k = \left[\frac{1}{m}\int_0^{P/2} \left[C_D(\alpha) - C_{D_0}\right] q\, S\, dt\right] \left[\text{T.O.F.} - t_{m_k}\right] \cos \theta_k$$

The range reduction caused by the oscillatory motion following release as it damps to 1/10 of its initial amplitude can be evaluated as

$$R = \sum_{k=1}^{k \leq N} \Delta R_k$$

The previous expression for N will generally not yield an integer value. However, in the following portion of the report N will be taken as the integer closest to the number calculated from

$$N = \frac{-2.3\omega}{\pi\beta}$$

For any particular bomb at a given Mach number the variation of C_D with α may be represented quite accurately by a power series in α. For relatively small amplitudes, say .35 radians or less, a parabolic variation is sufficiently accurate.

In general, we assume the C_D at a particular Mach number can be represented as follows:

$$C_D = C_{D_o} + d_1\alpha + d_2\alpha^2 + d_3\alpha^3 + d_4\alpha^4 + d_5\alpha^5.$$

Using this expression and setting

$$\alpha = \alpha_{o_k} \sin \omega t = \alpha_o e^{(2k-1)\frac{\beta\pi}{2\omega}} \sin \omega t$$

we evaluate* the quantity

$$\frac{1}{m}\int_0^{P/2} \left[C_D(\alpha) - C_{D_o}\right] q\, S\, dt$$

as

$$\frac{q_k S}{m\omega}\left[2d_1\alpha_o e^{(2k-1)\frac{\beta\pi}{2\omega}} + \frac{\pi}{2}d_2\alpha_o^2 e^{(2k-1)\frac{\beta\pi}{\omega}} + \frac{4}{3}d_3\alpha_o^2 e^{(2k-1)\frac{3\beta\pi}{2\omega}} + \frac{3\pi}{8}d_4\alpha_o^4 e^{(2k-1)\frac{2\beta\pi}{\omega}} + \frac{16}{15}d_5\alpha_o^5 e^{(2k-1)\frac{5\beta\pi}{2\omega}}\right]$$

* Evaluation of this integral is verified in APPENDIX III. The dynamic pressure, q, has been taken as constant during this half cycle.

The expression for range reduction now becomes

$$R = \frac{3}{m\omega} \sum_{k=1}^{k=N} q_k \left[T.O.F. - t_{m_k}\right] \cos \theta_k \left[2d_1 a_o e^{(2k-1)\frac{\beta\pi}{2\omega}}\right.$$

$$+ \frac{\pi}{2} d_2 a_o^2 e^{(2k-1)\frac{\beta\pi}{\omega}} + \frac{4}{3} d_3 a_o^3 e^{(2k-1)\frac{3\beta\pi}{2\omega}}$$

$$\left. + \frac{3\pi}{8} d_4 a_o^4 e^{(2k-1)\frac{2\beta\pi}{\omega}} + \frac{16}{15} d_5 a_o^5 e^{(2k-1)\frac{5\beta\pi}{2\omega}}\right]$$

For cases where the damping is slow so that q varies appreciably and $\cos \theta_k$ departs significantly from unity during the time $T_{1/10}$ use of the preceding equation for R is recommended.

In many instances it will be sufficiently accurate to use the value of q at release and $\cos \theta_k = 1$ during the time $T_{1/10}$.

Making these additional approximations we can write that the range reduction is represented by the following relation.

$$R = \frac{3q_o}{m\omega} \sum_{k=1}^{k=N} \left[T.O.F. - t_{m_k}\right] \left[2d_1 a_o e^{(2k-1)\frac{\beta\pi}{2\omega}} + \frac{\pi}{2} d_2 a_o^2 e^{(2k-1)\frac{\beta\pi}{\omega}}\right.$$

$$\left. + \frac{4}{3} d_3 a_o^3 e^{(2k-1)\frac{3\beta\pi}{2\omega}} + \frac{3\pi}{8} d_4 a_o^4 e^{(2k-1)\frac{2\beta\pi}{\omega}} + \frac{16}{15} d_5 a_o^5 e^{(2k-1)\frac{5\beta\pi}{2\omega}}\right]$$

For release from a given altitude at a given Mach number either of these relations reduce to

$$R = r_1 a_o + r_2 a_o^2 + r_3 a_o^3 + r_4 a_o^4 + r_5 a_o^5$$

where the constants r_1, r_2, ... r_5 are defined by comparison with the applicable equation for R.

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From this equation it is apparent that the effect on range of any release oscillation may be approximated, such that an a priori range correction may be applied for release from any carrier, if the release oscillation from this carrier is predictable.*

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* Ref. Sym.: 1921-3-(90) shows that for a given bomb a linear relation between release oscillation amplitude and a parameter $\frac{g/I}{\sqrt{1-M^2}}$ should exist for each carrier. Thus release oscillation amplitudes may be reasonably well predicted.

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August 10, 1953

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APPENDIX I

MK 5 RANGE REDUCTION

Figure I-1 presents a plot of R/α_0^2 versus Mach number for the Mk 5 Bomb for release altitudes of 10,000 to 40,000 feet. From this plot the range reduction may be calculated with fair accuracy for α_0 not exceeding an estimated .35 radians. (Note the limited extent of available experimental data indicated in Figure I-6.)

The peaks at Mach .8 in Figure I-1 arise from a more rapid increase in C_D with angle of attack at Mach .8 than at either Mach .7 or Mach .9. (See Figure I-6.) If this peculiarity has arisen from a flow separation phenomenon at the low wind-tunnel test Reynolds numbers, it is probable that the calculations at Mach .8 are less applicable to full scale than at other Mach numbers.

Figure I-2 presents range reduction versus amplitude of release for release altitudes of 10,000 to 40,000 feet and Mach .8. To obtain calculated range reduction applicable for large α_0, a 5th degree polynomial was used to represent C_D vs. α. This polynomial representation was inaccurate for small α_0. Therefore, Figure I-2 consists of a re-plot of Figure I-1 data for small α_0 faired smoothly into the plot of the range reduction applicable for large α_0. This procedure yielded range reduction values which are approximately correct for all α_0 less than 1.3 radians.

Also shown in Figure I-2 are four "experimental" range reduction figures obtained as the difference between the measured range of four drop tested units and the calculated range of these units for zero angle of attack.

It appears that the experimental range reduction values are larger than those calculated. This is ascribed in part to the range reduction caused by small bomb trim which is exhibited by most Mk 5 units, but which was neglected in obtaining the experimental range reduction figures.*

Figures I-3, 4, 5, and 6 present the Mach variations of $C_{L\alpha}$, $C_{M\alpha}$, $C_{M_q} + C_{M_{\dot{\alpha}}}$, and C_D versus α which were used in the calculations of range reduction.

The value of C_D in Figure I-6 for Mach .8 at $\alpha = 1.58$ radians (90°) was obtained by assuming that the Mk 5 at this angle of attack has practically the same drag as a right circular cylinder with fineness ratio 3. C_D is known+ for a two-dimensional cylinder in incompressible low-Reynolds number flow. Extrapolation of

---

*These "experimental" results would also show better agreement if they were all reduced to a basis of the standard moment of inertia, weight, and dynamic pressure versus release Mach and altitude used in the calculations of this report. Since these data are not of primary importance to this report, this laborious refinement is deemed unnecessary.

+"Fluid Mechanics", Dodge and Thompson, p 343.

these data from $R_e = 10^6$ to $R_e = 20 \times 10^6$, and application of a factor of .65 to account for a fineness ratio of about 3 yields $C_{D_1} = .38$. Transforming this value to C_D as usually defined resulted in $C_D = 1.40$.

NACA standard atmosphere was used to relate density and Mach number to release altitude and speed. The significant physical characteristics used in the calculation are

$$m = 5[illegible] \text{ slugs}$$

$$I = 340 \text{ slug-ft}^2$$

$$l = 3.[illegible]5 \text{ ft.}$$

APPENDIX II

MK 6 RANGE REDUCTION

Figure II-1 presents a plot of R/α_0^2 versus Mach number for the Mk 6 Bomb for release altitudes of 10,000 to 40,000 feet, from which the range reduction due to an oscillation of initial amplitude less than .35 radians may be calculated.

It is of interest to compare Figures I-1 and II-1, which represent R/α_0^2 for the Mk 5 and Mk 6 respectively. For Mach numbers less than .8 the range reduction under like release conditions and corresponding to a particular value of α_0 is larger for the Mk 6 than for the Mk 5.*

At Mach .8 the opposite is true, due primarily to the marked increase in the Mk 5 drag due to angle of attack, which was mentioned in APPENDIX I.

Figures II-2-5 present the Mach variations of C_{L_α}, C_{M_α}, $C_{M_q} + C_{M_{\dot{\alpha}}}$, and C_D versus α used in the calculations.

Physical characteristics used in these calculations are

$$m = 263 \text{ slugs}$$

$$I = 1188 \text{ slug-ft}^2$$

$$d = 5 \text{ ft.}$$

*Of these two weapons the Mk 5 release problem is however most severe, since its low moment of inertia compared to the Mk 6 leads to appreciably higher values of α_0 for similar release conditions.

APPENDIX III

Evaluation of $\frac{1}{m}\int_0^{P/2}\left[C_D(a) - C_{D_o}\right] q\, s\, dt.$

$$C_D(a) - C_{D_o} = d_1 a + d_2 a^2 + d_3 a^3 + d_4 a^4 + d_5 a^5$$

where

$$a = a_{o_k} \sin \omega t$$

and

$$a_{o_k} = a_o e^{(2k-1)\frac{\beta\pi}{2\omega}}$$

Take q as constant,

$$q = q_k.$$

Therefore

$$\frac{q_k s}{m}\int_0^{P/2}\left[C_D(a) - C_{D_o}\right] dt = \frac{q_k s}{m}\int_0^{\pi/\omega}\Big[d_1 a_{o_k} \sin \omega t$$

$$+ d_2\left(a_{o_k}\right)^2 \sin^2 \omega t + d_3\left(a_{o_k}\right)^3 \sin^3 \omega t + d_4\left(a_{o_k}\right)^4 \sin^4 \omega t$$

$$+ d_5\left(a_{o_k}\right)^5 \sin^5 \omega t\Big] dt$$

$$= \frac{q_k s}{\omega m}\Big[d_1 a_{o_k}(-\cos \omega t) + d_2 a_{o_k}^2\left(-1/2 \cos \omega t \sin \omega t + \frac{\omega t}{2}\right)$$

$$+ d_3 a_{o_k}^3\left(-1/3 \cos \omega t\left\{\sin^2 \omega t + 2\right\}\right)$$

$$+ d_4 a_{o_k}^4\left(\frac{-\sin^3 \omega t \cos \omega t}{4} + \frac{3}{8}\left\{-1/2 \cos \omega t \sin \omega t + \frac{\omega t}{2}\right\}\right)$$

$$+ d_5 a_{o_k}^5\left(\frac{-\sin^4 \omega t \cos \omega t}{5} + \frac{4}{5}\left\{-1/3 \cos \omega t\left(\sin^2 \omega t + 2\right)\right\}\right)\Big]_0^{\pi/\omega}$$

$$= \frac{q_k s}{\omega m}\left[2d_1 a_{o_k} + \frac{\pi}{2} d_2 (a_{o_k})^2 + \frac{4}{3} d_3 (a_{o_k})^3 + \frac{3\pi}{8} d_4 (a_{o_k})^4 + \frac{16}{15} d_5 (a_{o_k})^5\right]$$

Substituting for a_{0_k} we obtain the relation given in the report.

$$\frac{1}{\pi}\int_0^{2\pi}\left[C_D(\alpha) - C_{D_0}\right] q\, d\alpha$$

$$= \frac{q_k \omega}{\pi\omega}\left[2d_1 a_0 e^{(2k-1)\frac{\beta\pi}{2\omega}} + \frac{\pi}{2} d_2 a_0^2 e^{(2k-1)\frac{\beta\pi}{\omega}}\right.$$

$$+ \frac{\pi}{3} d_3 a_0^3 e^{(2k-1)\frac{3\beta\pi}{2\omega}} + \frac{3\pi}{8} d_4 a_0^4 e^{(2k-1)\frac{2\beta\pi}{\omega}}$$

$$\left. + \frac{16}{15} d_5 a_0^5 e^{(2k-1)\frac{5\beta\pi}{2\omega}}\right]$$

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10 X 10 to the ½ inch, 5th lines accented.
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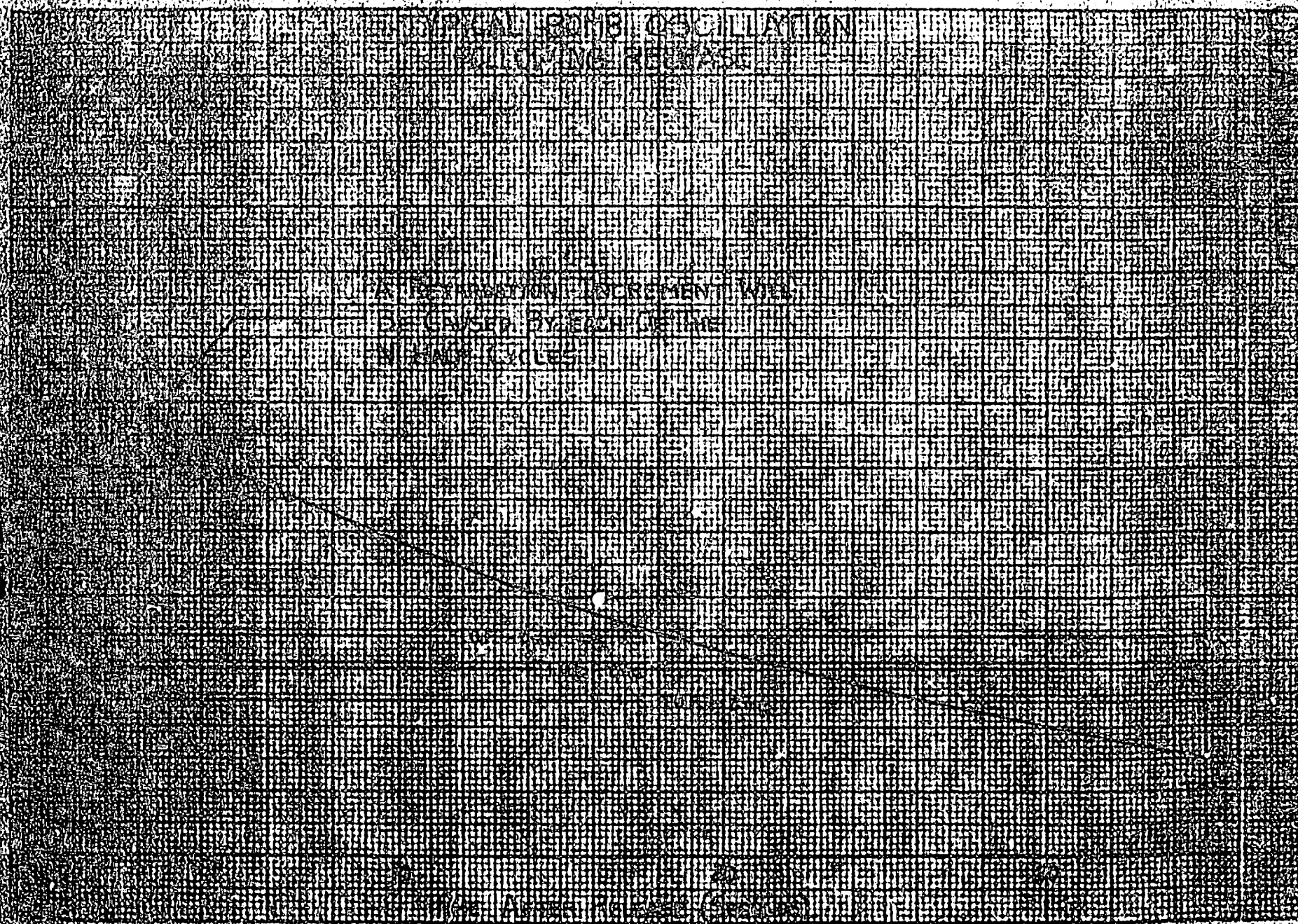

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FIGURE 4

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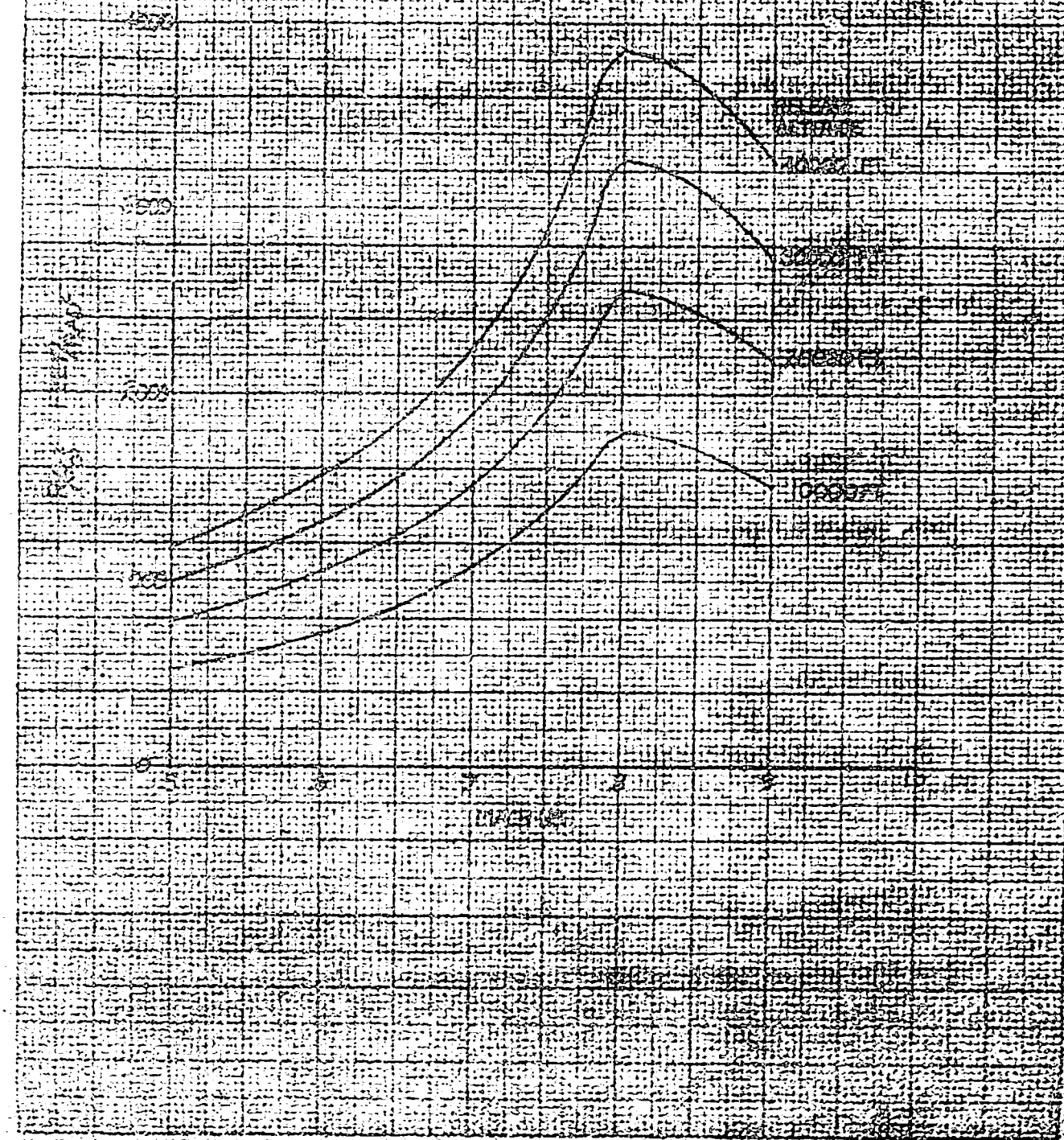

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FIG. I-2

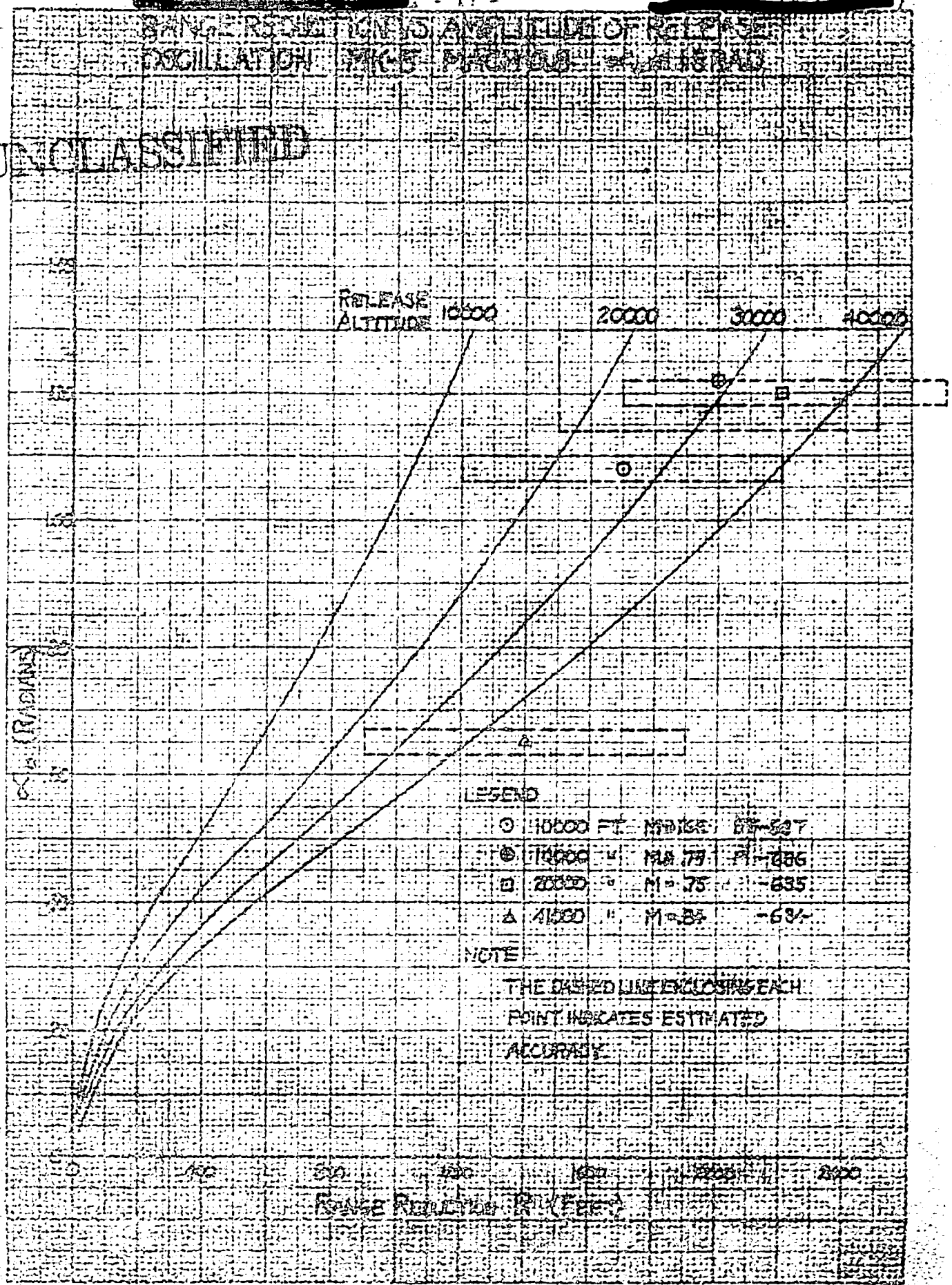

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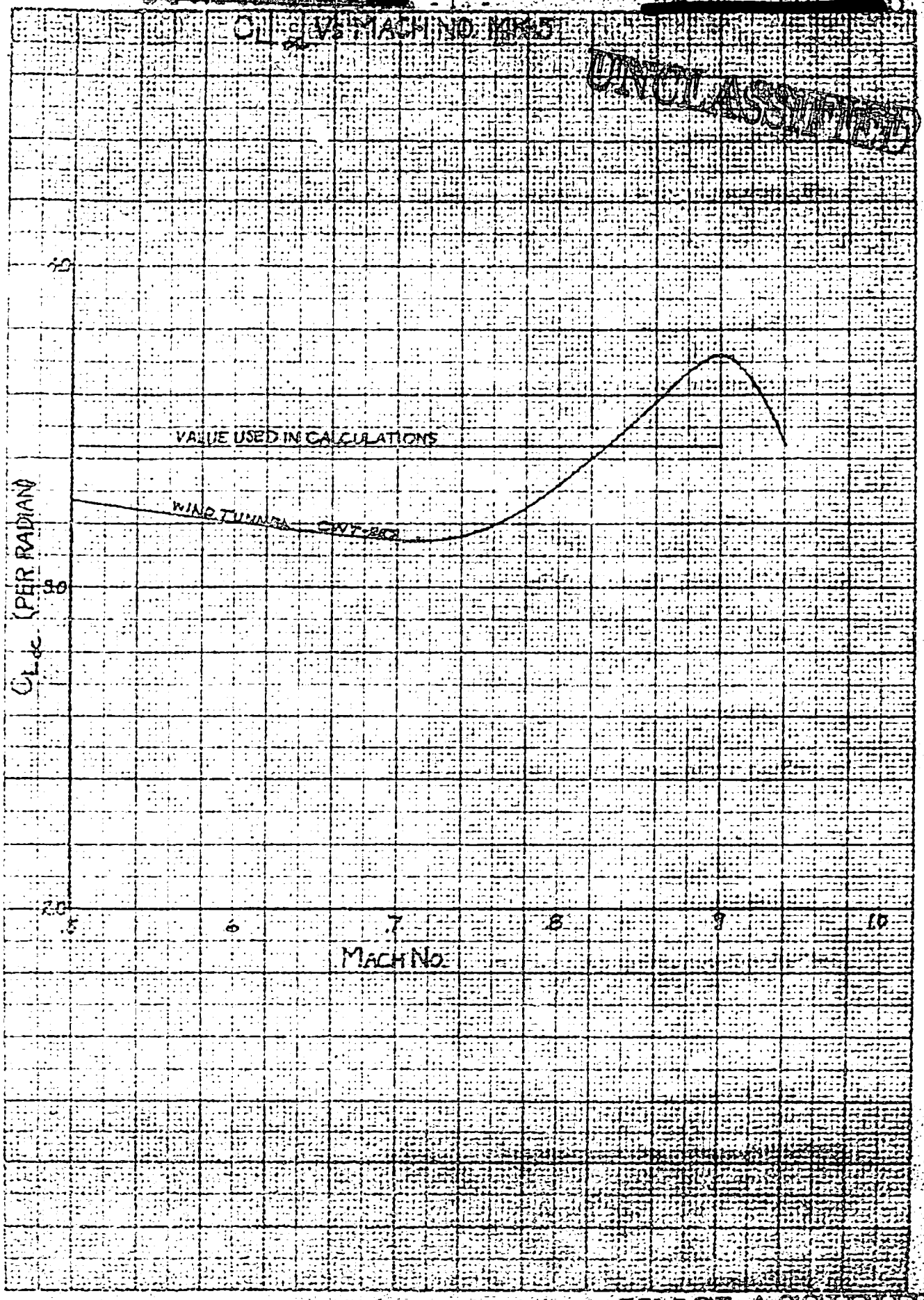
Fig. 1-3



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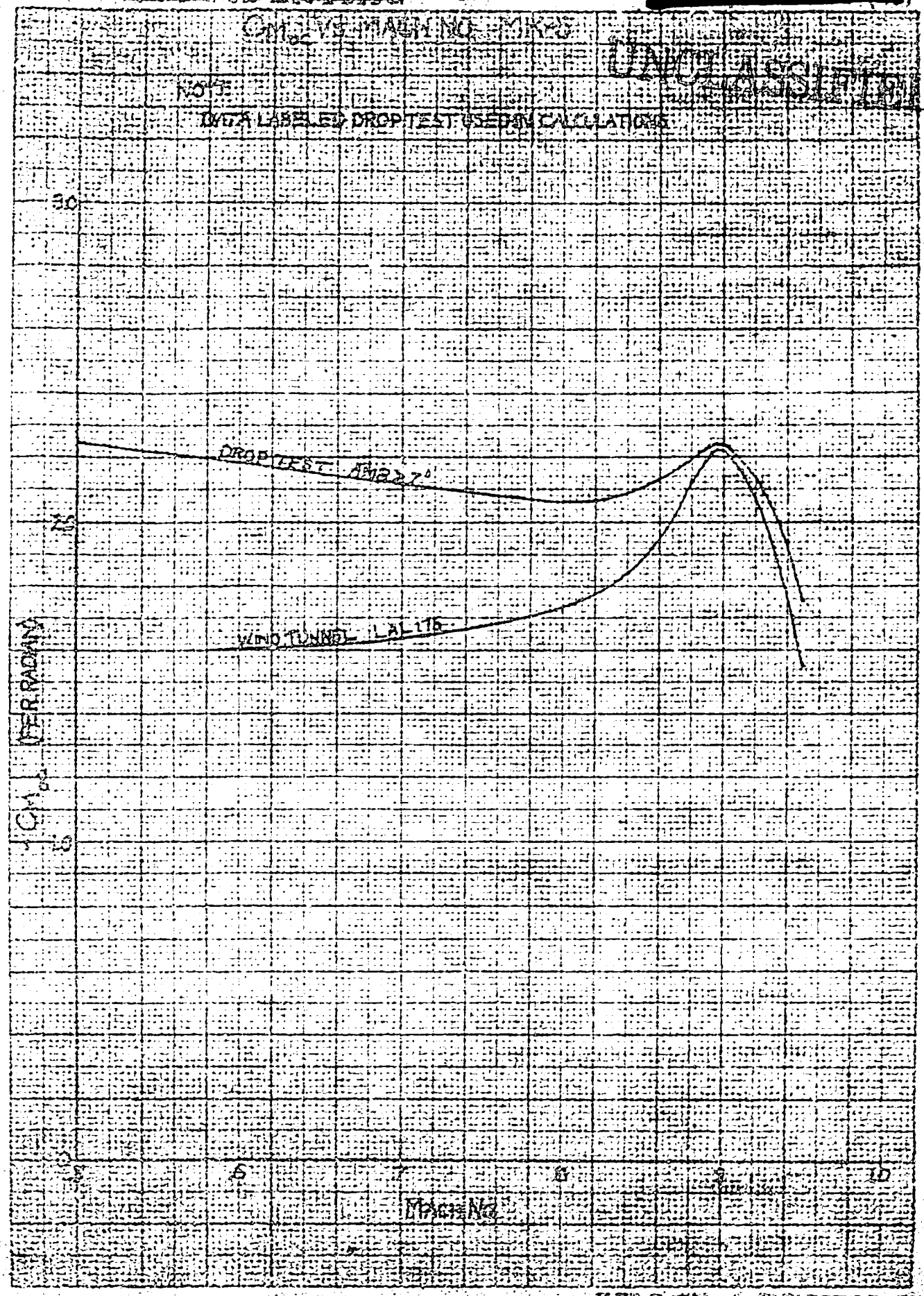

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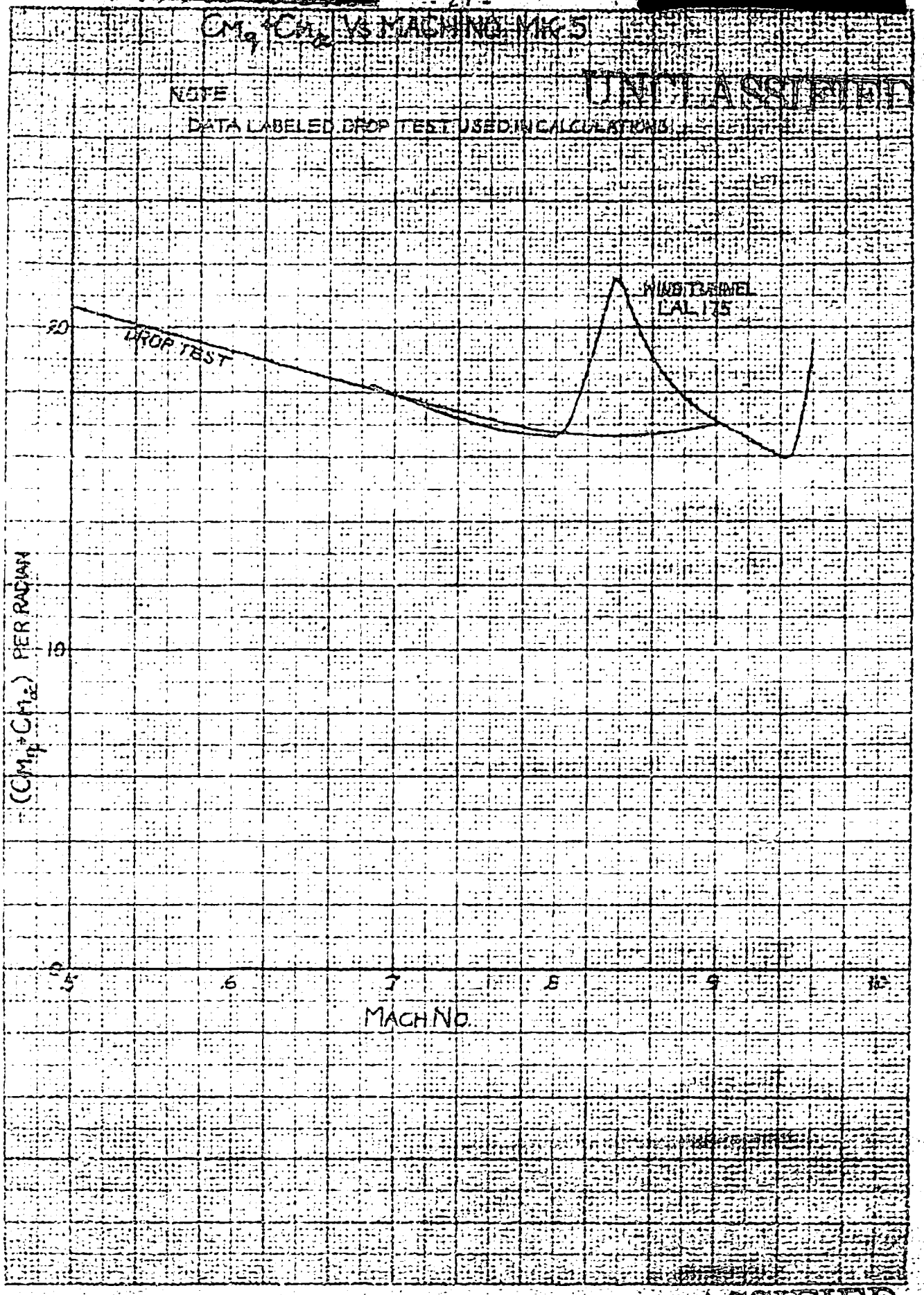
FIG. I-5


C_{M_q} & C_{M_α} VS MACH NO. MK 5
NOTE
DATA LABELED DROP TEST USED IN CALCULATIONS
UNCLASSIFIED
WIND TUNNEL
LAL 175
DROP TEST
20
10
0
$(C_{M_q} + C_{M_\alpha})$ PER RADIAN
.5
.6
.7
.8
.9
1.0
MACH NO.

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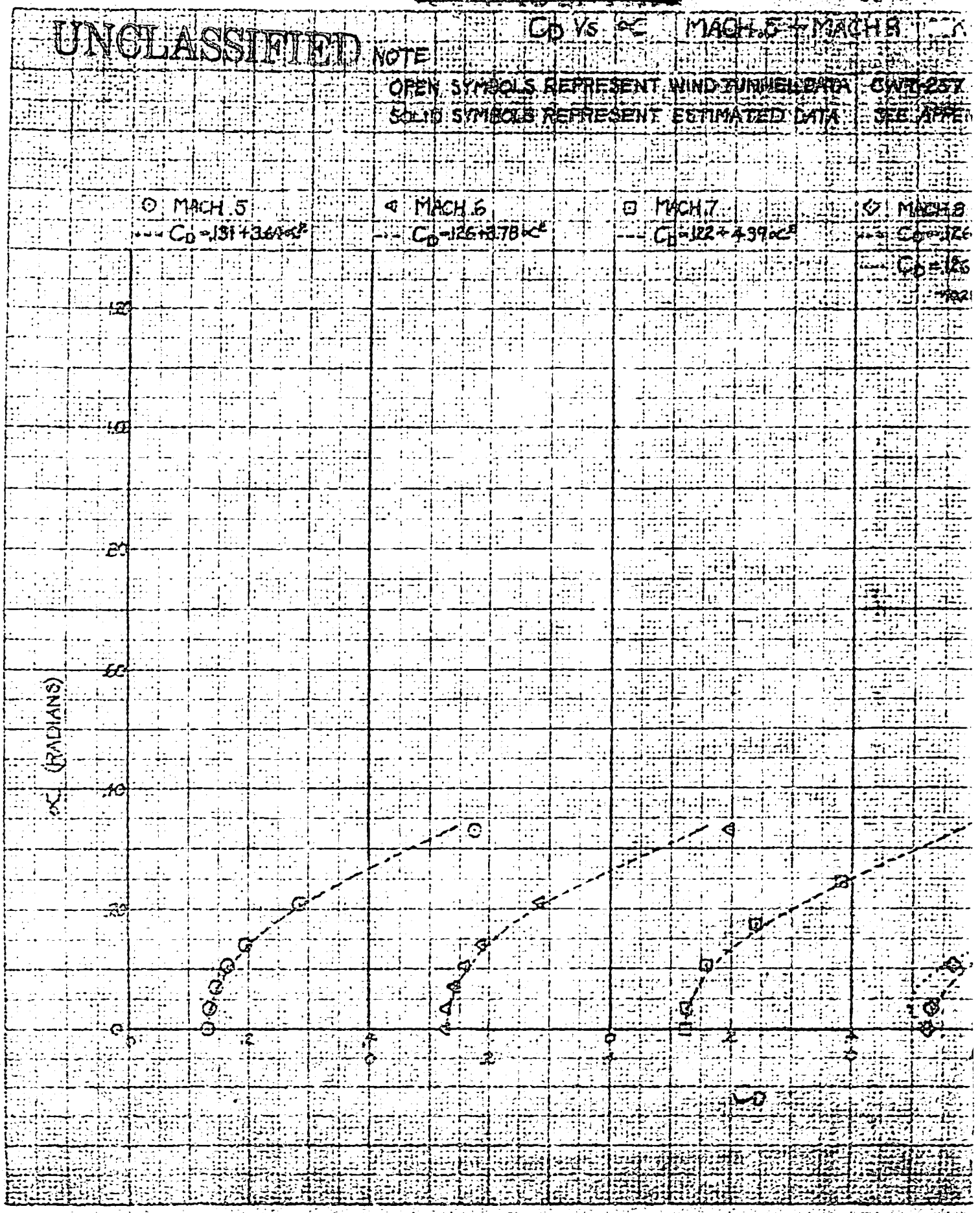

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FIG. 1-6

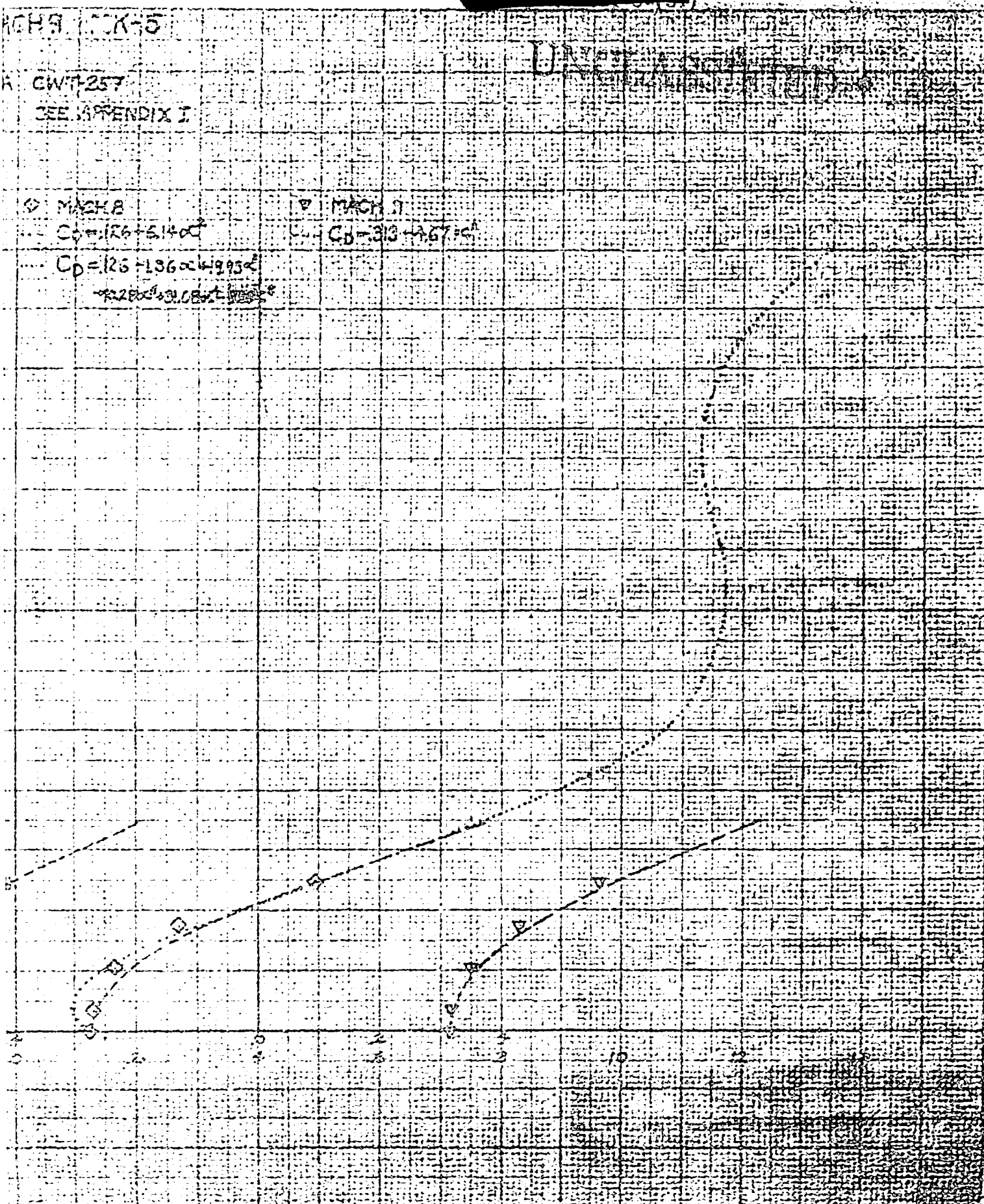

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FIG. II-1

RELATION OF MK 6 RANGE REDUCTION TO INITIAL AMPLITUDE OF RELEASE OSCILLATION FOR DIFFERENT RELEASE ALTITUDES AND MACH NOS. VALID FOR $\alpha_0 = .35$ RADIANS

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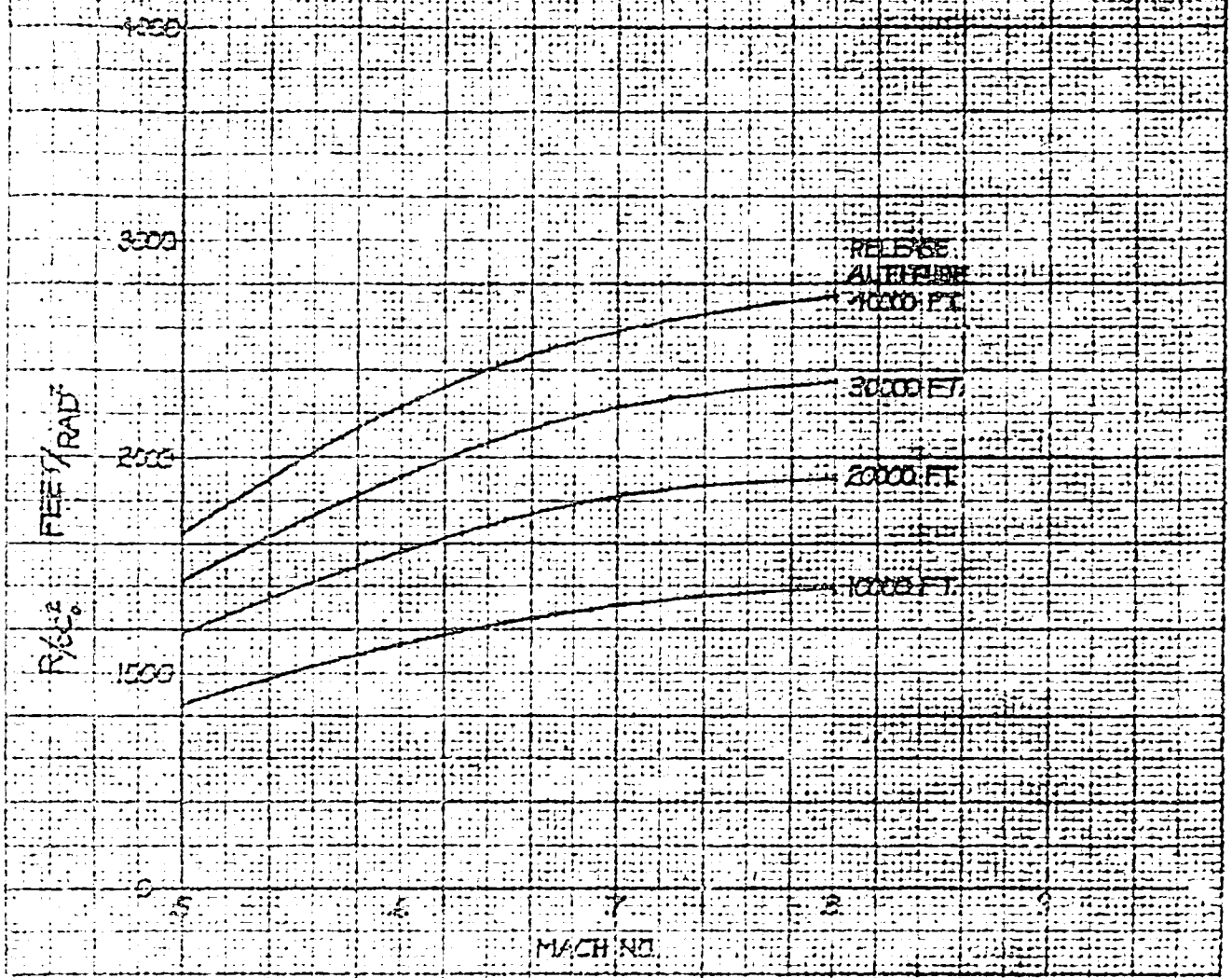

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Fig. II-2

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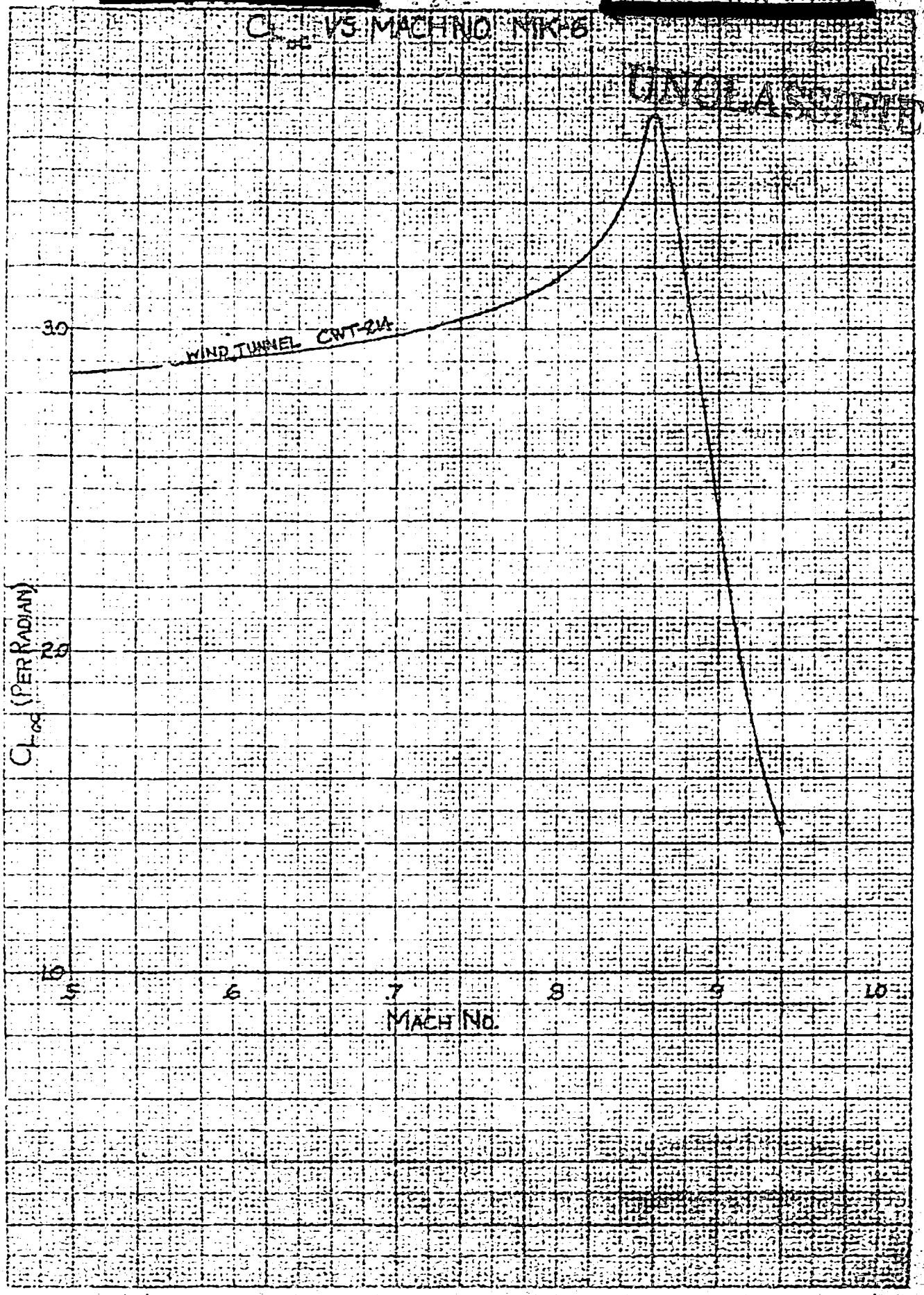

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FIG II-3

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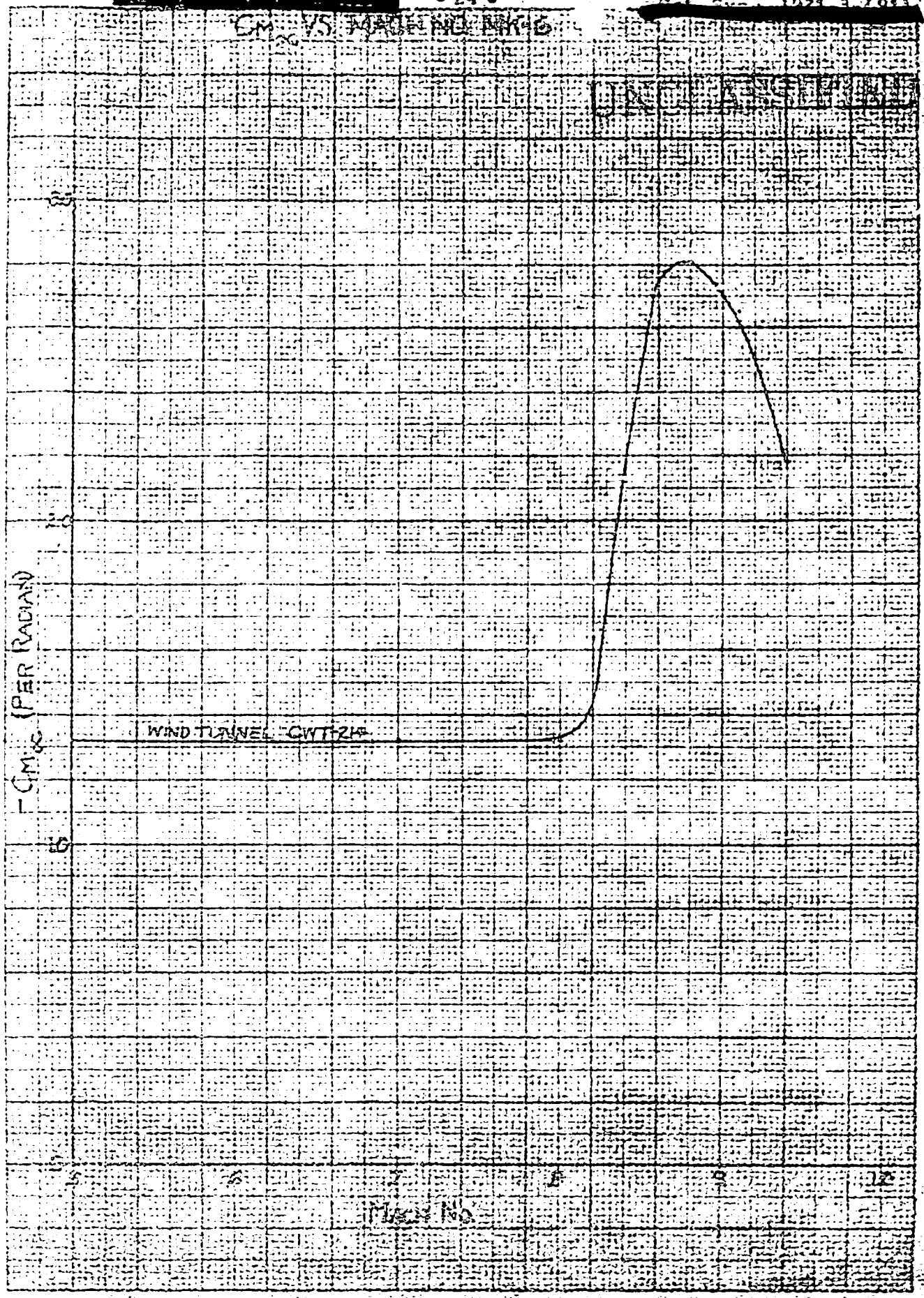

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FIG. II-4

$C_{M_q} + C_{M_{\dot{\alpha}}}$ VS MACH NO. FIG 5

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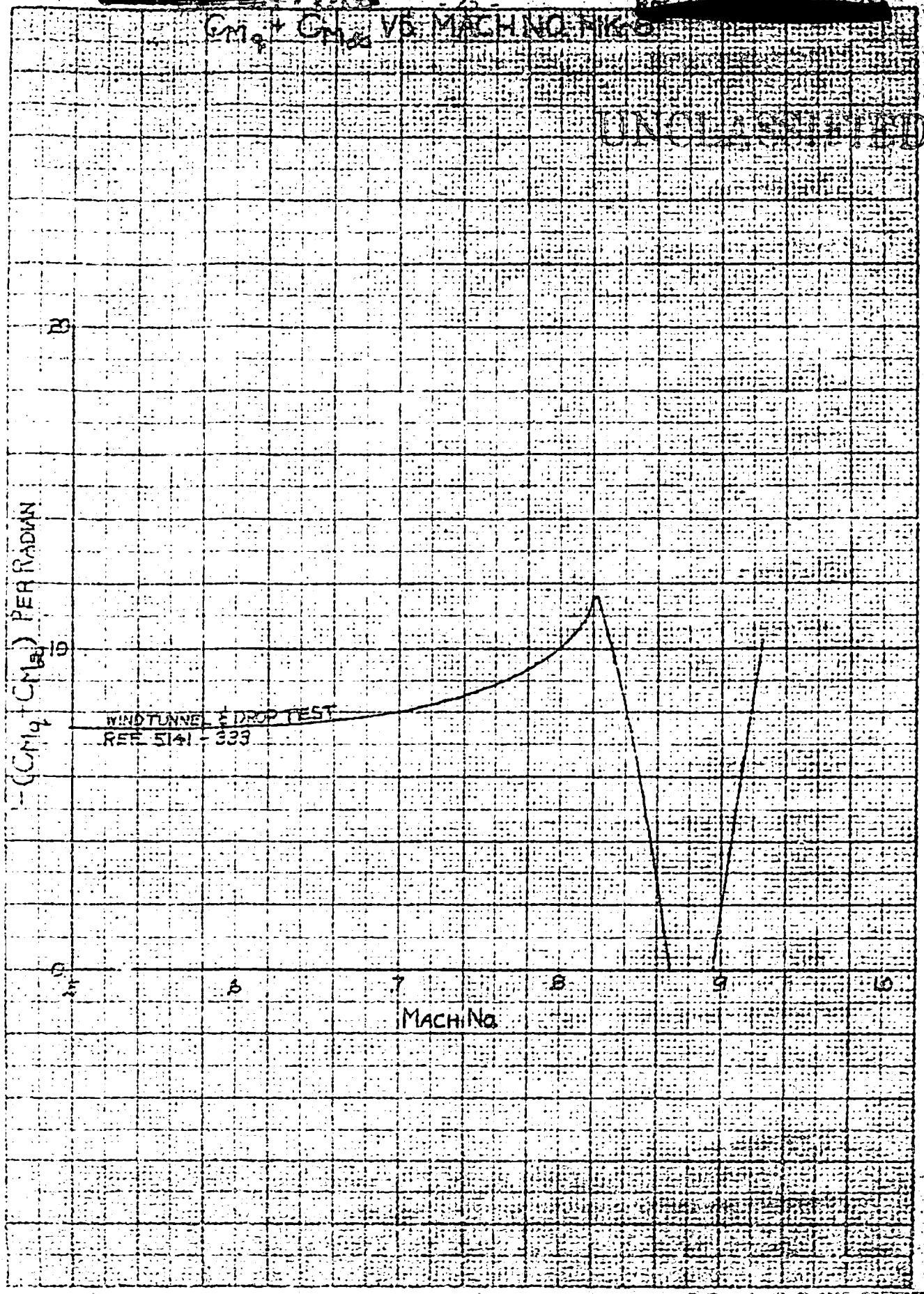

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