SC 1004-A (3-55)

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1032

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TABLE OF CONTENTS

	Page No.
INTRODUCTION	4
A SIMPLIFIED EXAMPLE	4
A MORE GENERAL EXAMPLE	6
DUDDERS.	8
GENERAL FORMULATION.	9

LIST OF TABLES

Table 1 - Probability of Going from Initial Condition to Final Condition with One Signal	5
Table 2 - Probability of Going from Initial Condition with One Signal (Generalized Example).	7

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RAMDOM DETONATION AND DUDDERS

by

D. C. Kleinecke, 8124-2

INTRODUCTION

In the course of the never-ending attempt to reduce the probability of a premature atomic detonation a great deal of words have been spent on what might be called "the psychoteur" (defined as a psychotic, saboteur, or plain old-fashioned idiot). Prematures caused by a psychoteur are not component malfunctions, but nevertheless Sandia Corporation should make every attempt to reduce the probability of such prematures. This memorandum shows one way in which the probability of detonation by a psychoteur can be estimated; presumably this will make it possible to study ways in which the probability can be reduced.

The theoretical psychoteur operates as follows: somehow he comes across an atomic warhead and attempts to detonate it. He is assumed to have all the equipment (power supplies, leads, etc.) he needs to apply signals at the warhead input. However, he is assumed to be completely ignorant of the detailed warhead design. Hence, he takes two leads with the proper voltages and begins applying them, at random, to pairs of pins at the input. He continues to do this repeatedly until the warhead detonates.

The psychoteur is not to be confused with the genuine saboteur who approaches the warhead armed with (at least) some knowledge of detailed design and circumvents the built-in safety features (for example, by cutting into the warhead and bypassing safety devices). The genuine saboteur cannot be frustrated by details of design and is not, therefore, a design concern of Sandia Corporation. However, the history of disasters and near-disasters throughout the world shows that the psychoteur does indeed exist and in great enough numbers that he cannot be ignored.

A SIMPLIFIED EXAMPLE

As a simple example of the psychoteur in action consider an idealized warhead which is fired in two steps. If signals are applied at regular intervals the warhead must first be armed and then be fired; if the firing circuit is pulsed in an unarmed condition the warhead is dudded; if the signal immediately following an arming signal does not fire the warhead the arming is lost. There are many possible combinations of applied signals, and when pairs of pins are selected to be pulsed, "A" equals the probability that the pair arms, and "B" equals the probability that the pair fires (or duds; no pair both arms and fires). The probability of neither firing nor arming is $C = 1 - A - B$.

After a signal has been applied, the warhead is in one of four possible conditions; passive, armed, dudded, or fired. The passive condition is the normal, unmodified condition. The condition of the warhead is academic after it has been fired, but it is convenient to consider a fired warhead as simply another condition and to think of a ghostly psychoteur continuing to probe

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with vaporized leads. Starting from any one of these four conditions, or states, there is a definite probability of getting to another state after one more signal. These probabilities are given in Table 1.

Initial Condition	Final Condition			
	Passive	Armed	Dudded	Fired
Passive	C	A	B	0
Armed	C	A	0	B
Dudded	0	0	1	0
Fired	0	0	0	1

TABLE 1
Probability of Going from Initial Condition to Final Condition with One Signal

The first two columns of Table 1 have been filled in accordance with the original assumptions; when the warhead has been dudded or fired it stays dudded or fired regardless of what the psychoteur may do, and so the probabilities are 1 (equals certainty) of staying in the initial condition.

Now suppose the psychoteur is allowed to probe forever into the warhead. Clearly, the warhead is ultimately either dudded or fired; the question is: what is the probability of firing instead of dudding?

The probability of firing after an infinite number of signals is fairly easily computed. Let P(A) be probability of firing if the warhead starts in an armed condition; P(B) the probability of firing if the warhead starts fired; P(C) if the warhead starts passive; and P(D) if the warhead starts dudded. Then P(A) equals the sum of probabilities of getting to each condition after the first signal (beginning from the armed condition) times the probability of firing beginning at that condition; and the same is true for P(B), P(C), and P(D). In symbols:

$$P(A) = CP(C) + AP(A) + BP(B) \quad ,$$
$$P(B) = P(B) \quad ,$$
$$P(C) = CP(C) + AP(A) + BP(D) \quad ,$$
$$P(D) = P(D) \quad .$$

Obviously P(B) = 1 and P(D) = 0, so that:

$$(1-A)\,P(A) = CP(C) + B \quad , \qquad (1)$$
$$(1-C)\,P(C) = AP(A) \quad .$$

These two equations are quickly solved, so that:

$$P(C) = \frac{AB}{1-A-C} = A \quad . \qquad (2)$$

This result shows that the probability of ultimately firing the warhead (assuming it was originally passive) is equal to the probability of arming it.

It is possible to use the same sort of reasoning to find the probability of firing in a finite number of signals. Let P(A,N) be the probability of firing in N signals or less if the warhead is originally armed, and define P(B,N), P(C,N), and P(D,N) in the same way. Then, as before:

$$P(A,N) = CP(C,N-1) + AP(A,N-1) + B \quad ,$$

$$P(C,N) = CP(C,N-1) + AP(A,N-1) \quad .$$

It is convenient to use the generating functions

$$P(A;X) = \sum_{N=0} P(A,N)\, X^N$$

$$P(C;X) = \sum_{N=0} P(C,N)\, X^N$$

to solve these equations. Since P(A,0) = 0 and P(C,0) = 1

$$P(A;X) = CXP(C;X) + AXP(A,X) + BX/(1-X) \quad ,$$

$$P(C;X) = CXP(C;X) + AXP(A,X) \quad ;$$

or, in terms more like equation 1:

$$(1-AX)P(A,X) = CXP(C;X) + BX/(1-X) \quad ,$$

$$(1-CX)P(C;X) = AXP(A,X) \quad .$$

So that

$$P(C;X) = \frac{ABX}{1-(A+C)X} \cdot \frac{X}{1-X}$$

$$= \frac{A}{A+C}\left(B - \sum_{N=0} (A+C)^N X^N + (A+C) \sum_{N=0} X^N\right) \quad ;$$

and

$$P(C,N) = A\left[1-(A+C)^{N-1}\right] \quad . \quad N > 0$$

This expression for P(C,N) naturally goes to A, the value previously obtained for P(C), as N goes to infinity. The rate at which P(C,N) approaches P(C) is fairly slow if the probability, B = 1-A-C, of hitting the firing pins is low. For example if B = 0.01, then 458 repetitions are required to reduce the difference between P(C) and P(C,N) to less than 1%. However, a psychoteur, once he gets an opportunity to begin his dirty work, could easily have time for several hundred repetitions. It can be concluded from this that the dependence of firing probability on the number of repetitions can be ignored and the more easily obtained values for an infinite number of repetitions can be used to estimate the firing probability.

A MORE GENERAL EXAMPLE

The calculations in the preceeding example can be extended to more general examples. Suppose, now, that the psychoteur's operations are the same as in the last example except that the warhead behaves in a more general fashion. If the warhead is initially passive, the probabilities are the same as before: A for arming, B for dudding, C for remaining passive. If the warhead

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is initially armed it stays armed through K succeeding signals (K = 1 in the preceeding example), and the probabilities of rearming and firing are D and E (which need not be D=A and E=B as in the preceeding example). In addition, there is a probability F that the armed warhead will be dudded rather than fired (F=0 in the preceeding example).

In order to handle the fact that the warhead stays armed through K repetitions, additional armed conditions A1, A2, A3...., AK are introduced which depend on how long the warhead has been armed. Then the analogue of Table 1 is Table 2, which applies to this more general example (G=1-D-E-F).

Initial Condition		Final Condition								
		Passive	Armed						Dudded	Fired
			A1	A2	A3	. . .		AK		
Passive		C	A	0	0	. . .		0	B	0
Armed	A1	0	D	G	0	. . .		0	F	E
	A2	0	D	0	G	. . .		0	F	E
	A3	0	D	0	0	. . .		0	F	E
	. . .		.	.	.	. . .		.	.	
	AK	G	D	0	0	. . .		0	F	E
Dudded		0	0	0	0	. . .		0	1	0
Fired		0	0	0	0	. . .		0	0	1

TABLE 2

Probability of Going from Initial Condition to Final Condition with one signal (generalized example)

As before, let P(B), P(C), and P(D) be the probabilities of ultimately firing if the warhead starts in a fired, passive, and dudded condition; let P(A1), P(A2), ... P(AK) be the probabilities if the warhead starts in the various armed conditions. Then the analogues to equation (1) are:

$$
\begin{aligned}
(1-C)\ P(C) &= AP(A1), \qquad (3)\\
(1-D)\ P(A1) &= GP(A2) + E \quad ,\\
P(A2) &= DP(A1) + GP(A3) + E \quad ,\\
P(A3) &= DP(A1) + GP(A4) + E \quad ,\\
&\cdots\cdots\cdots\\
P(AK) &= DP(A1) + GP(C) + E \quad .
\end{aligned}
$$

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The solution to equations (3) is

$$P(C) = \frac{AES}{(A+B)\ (1-DS) - G^K A}, \tag{4}$$

where $S = 1 + G + \ldots + G^{K-1} = (1-G^K) / (1-G)$. If $K = 1$, $F = 0$, $D = A$, $E = B$ and $G = C$, then solution (4) reduces to solution (2) of the preceeding example.

An especially interesting case of formula (4) arises when $K = 1$, $F = 0$, $D = (1-P)A$, $E = P + (1-P)B$, and $G = (1-P)C$, which occurs when the first example is modified by assuming that an armed warhead has a probability P of spontaneously firing before the next signal is applied. In this case, solution (4) becomes

$$P(C) = \frac{P + B(1-P)}{B + AP} A \quad . \tag{5}$$

This formula was used by G. Davis (8124-2) in Sandia Technical Memorandum SCTM 61-58-81, "A Study of Proposed XW-45 Safing Methods and Resultant Reliability Implications".

DUDDERS

The probability P(C) of a psychoteur ultimately firing the warhead can be reduced by introducing dudders into the warhead. These dudders might be squibs which blow apart the detonator cables (and a monitoring line) so that the warhead cannot fire. The dudders are activated by introducing signals on various non-functional pairs of pins.

The presence of dudders naturally increases the failure probability of the warhead, but this may be acceptable when the psychoteur premature probability is reduced. Probably it is not desirable to allow the psychoteur to easily detect the fact that he has dudded the warhead, since he may continue his attempts to detonate and be caught in the act. However, some kind of dudding monitor line is useful to the user who cannot be sure the warhead has not been accessible at some time to a psychoteur who dudded it.

Without going into the hardware implications of building and installing dudders, the effectiveness of dudders can be studied by changing values of B and F in equation (4). Incidentally, the smallest possible value for P(C) is obviously obtained by letting $C = D = G = 0$, which corresponds to dudding the warhead unless the first two signals are exactly right; in this case $P(C) = AE$.

It may be possible to build dudders which dud an armed warhead but not an unarmed warhead, but this possibility will not be considered. Instead, suppose there are a certain number of pairs of pins which dud whenever a signal is applied across them. If it is assumed that dudders operate before any spontaneous firing of an armed warhead, that $K = 1$, and that there is a probability P of spontaneous firing of an armed warhead, then $B = R + S$, $D = (1-P)A$, $E = P + (1-P)R$, $F = (1-P)S$, and $G = (1-P)C$, where R is the probability of applying a signal to a pair of firing pins, and S the probability of applying a signal to a dudder.

Then formula (4) becomes

$$P(C) = \frac{P + R(1-P)}{R + S + AP} A \quad ,$$

which differs from equation (5) (except for notation, since R corresponds to B) only in the S term in the denominator. Since R+AP is the sum of the probability of getting the firing pins and the probability of getting the arming pins followed by a spontaneous detonation, it is probably quite small. Then the S term dominates in the denominator, and fairly modest probabilities of getting dudders result in large reductions in P(C). For example, if R+AP=0.01, then S=0.09 reduces P(C) to one tenth of its original value.

Several other ways have been proposed to prevent the psychoteur from firing the weapon; for example, environmental switches. Each of these other systems has a definite failure probability. When these other systems fail the dudders still provide some safety against the psychoteur. It must be decided whether the increased failure probability with dudders is worth the additional factor in premature safety.

GENERAL FORMULATION

The psychoteur firing probabilities can be computed in much more complicated situations than those envisioned in either of the first two examples. The general methods apply, moreover, not only to the psychoteur but also to other modes of failure and premature detonation caused by repeated applications of a single effect; for example, the failure probability of a bomb swinging free in a bomb bay and pounding against the side of the bay.

It is necessary to define a series of possible warhead conditions as in the examples and to fill out the analogue of Tables 1 and 2 for these conditions. Suppose the conditions defined are C1, C2, C3,.....CM and the entry in the table corresponding to any initial condition CI going to final condition CJ is TIJ. The equations corresponding to equations (1) and (3) are then

$$P(CI) = \sum_J TIJ \cdot P(CJ) \quad ,$$

where P(CI) is the probability of ultimately firing beginning in condition CI. This equation is written more conveniently in matrix and vector notation as

$$P = TP \quad . \tag{6}$$

This equation is not in itself enough for solving for the vector P; however, if the components for firing from the fired condition and the dudded condition are set equal to 1 and 0, respectively, the equations, if properly formulated, should be solvable.

If the probabilities after finite numbers of repetitions are desired, the proper equations are

$$P(CI,N) = \sum_J TIJ \cdot P(CJ,N-1) \quad ,$$

where P(CI,N) is the probability of firing in N steps or less, beginning in condition CI. In matrix notation:

$$P(N) = TP(N-1) \quad ,$$

where P(N) is the vector with components P(CI,N). Then

$$P(N) = T^N P(0) \quad , \tag{7}$$

and exact values are most easily obtained by diagonalizing T so that powers of T are conveniently calculated.

These processes can be illustrated by the problems involving the simple situation presented in Table 3.

Initial Condition	Final Condition		
	Passive	Dudded	Fired
Passive	C	B	A
Dudded	0	1	0
Fired	0	0	1

Table 3 Simple Example

Equation (6) has the form,

$$\begin{pmatrix} P(1) \\ P(2) \\ P(3) \end{pmatrix} = \begin{pmatrix} C & B & A \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P(1) \\ P(2) \\ P(3) \end{pmatrix}$$

$$= \begin{pmatrix} CP(1) + BP(2) + AP(3) \\ P(2) \\ P(3) \end{pmatrix}$$

and since P(2) =0 and P(3) = 1,

$$P(1) = CP(1) + A \quad .$$

Hence

$$P(1) = \frac{A}{1 - C} \quad . \tag{8}$$

Equation (7) has the form $P(N) = T^N P(0)$, where T is the matrix derived from Table 3. Suppose UTU^- is a diagonal matrix; then

$UP(N) = Q(N) = (UTU^-)^N Q(0)$. This is a standard mathematical problem discussed in any book on matrix theory. One solution is

$$U = \begin{pmatrix} 1-C & -B-A \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ;$$

then

$$UTU^- = \begin{pmatrix} C & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

Now P(0) is the probability of reaching fired condition in zero repetitions, so that

$$P(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} , \qquad Q(0) = UP(0) = \begin{pmatrix} -A \\ 0 \\ 1 \end{pmatrix} .$$

Then

$$Q(N) = \begin{pmatrix} -C^N A \\ 0 \\ 1 \end{pmatrix} , \qquad P(N) = U^- Q(N) = \begin{pmatrix} \frac{A}{1-C} (1-C^N) \\ 0 \\ 1 \end{pmatrix} ,$$

so that the probability of firing in N or less repetitions starting in a passive condition is

$$P(N) = \frac{A}{1-C} (1-C^N) ,$$

which clearly goes into expression (8) when N goes to infinity.

These results might have been obtained more easily by other methods, but they illustrate the method of solving the general problem. In a complex situation where many warhead conditions are defined, it will probably be necessary to use a computing machine to solve the problems. The manipulations involved are well known machine problems, and solutions should be readily available for use.

The reader who wishes to see this general formulation treated in its usual technical form may be interested in the discussion in Chapter IV of Sandia Corporation Monograph SCR-11, "Lectures on Game Theory, Markov Chains and Related Topics" by G. L. Thompson.

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