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Technical Memorandum

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THE FLOW OF CHARGE FROM AN INFINITE PLATE

Eugene J. McGuire, 5233

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THE FLOW OF CHARGE FROM AN INFINITE PLATE

Eugene J. McGuire, 5233

Sandia Laboratory, Albuquerque, New Mexico

ABSTRACT

One simple scheme to measure an x-ray spectrum involves the measurement of the velocity of electrons emerging from a target struck by the x-ray beam. We have examined the dynamics of electron flow from an infinite plate to determine the development of the current pulse in space and time. In addition, we develop a rough criterion, in terms of total current and emission velocity, as to whether the infinite parallel plate is a good analogue of an experiment with a finite emission area.

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THE FLOW OF CHARGE FROM AN INFINITE PLATE

I. INTRODUCTION

We consider a pulse of current density of magnitude J_o, duration τ, with initial electron velocity v_o, emerging from an infinite ungrounded plate. A back-of-the-envelope calculation can be performed as follows:

The surface charge density at time t is $J_o t$ for $t \leq \tau$ and $J_o \tau$ for $t \geq \tau$. An electron in the pulse sees a force of magnitude $\frac{J_o te}{\varepsilon_o}$ and its velocity is

$$v = -\frac{J_o e t^2}{2M_e \varepsilon_o} + v_o \text{ for } t \leq \tau$$

and

$$v = -\frac{J_o e \tau t}{M_e \varepsilon_o} + v_o + \frac{J_o e \tau^2}{2M_e \varepsilon_o} \text{ for } t \geq \tau$$

It is clear that at some time $v = 0$; the pulse turns around and returns to the plate. Thus, the current pulse can reach a position x (depending on J_o, v_o and τ) and go no farther.

An immediate counter to this back-of-the-envelope calculation is that if the pulse is distributed in time at the emitter, it will be distributed in space at any time. Attraction to the plate will remove some of the charge, reducing the surface charge density and consequently the attractive force. One may have here a feedback process which, while it reduces the total charge passing any point x to a value less than $J_o \tau$,

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still allows a major fraction of the charge to reach distances greater than that allowed in the aforesaid back-of-the-envelope calculation.

We shall show that the latter contention is correct; that, in fact, when the feed back mechanism is relevant the current seen at any reasonable distance from the plate is independent of J_o!

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II. EQUATIONS GOVERNING CURRENT FLOW

Because we are dealing with an emitted current and a return current, we consider a two current problem. We have two velocities and two types of charge (i.e., electrons moving away from the emitter and electrons moving towards the emitter). The physical boundary conditions apply to the emitted particles (i.e., j_1, v_1 and ρ_1). For the return particles (j_2, v_2 and ρ_2) we use the condition that the return parameters (j_2,ρ_2) are zero until $j_1(x,t) \leq 0$. Then

$$j_2(x,t) = j_2(x,t) + j_1(x,t)$$

$$j_1(x,t) = 0$$

$$\rho_2(x,t) = \rho_2(x,t) + \rho_1(x,t)$$

$$\rho_1(x,t) = 0 \qquad (1)$$

We do not allow returning electrons to turn around and rejoin the emitted stream. This cannot occur as the force on the electron is always attractive toward the plate (emitter).

The equation governing the electron streams are (defining the charge on the electron as positive to avoid minus signs),

$$\frac{\partial E(x,t)}{\partial x} = \frac{e}{\epsilon_o} (\rho_1(x,t) + \rho_2(x,t)) \qquad (2)$$

$$\frac{\partial v_i(x,t)}{\partial t} + v_i(x,t) \frac{\partial v_i(x,t)}{\partial x} = \frac{e}{M_e} E(x,t) \qquad (3)$$

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$$e \frac{\partial \rho_i(x,t)}{\partial t} + \frac{\partial j_i(x,t)}{\partial x} = 0 \tag{4}$$

We use scaled units $x = x' \ (v_o \ \tau)$, $v_i = v_i' \ (v_o)$, $t = t'\tau, \rho_i = \rho_i' \rho_o$, $j_i = j_i' \ J_0 = j_i' \ (e \ \rho_o v_o)$, and $E' = \frac{\epsilon_o E}{J_o \tau}$. Then Equations (2-4) become

$$\frac{\partial E'(x',t)}{\partial x'} = \rho_1'(x',t') + \rho_2'(x',t') \tag{2a}$$

$$\frac{\partial v_i'(x',t')}{\partial t'} + v_i'(x',t') \frac{\partial}{\partial x'} v_i'(x',t') = C \ E'(x',t') \tag{3a}$$

$$\frac{\partial \rho_i'(x',t')}{\partial t'} + \frac{\partial j_i(c',t')}{\partial x'} = 0 \tag{4a}$$

where

$$C = \frac{e}{M_e \epsilon_o} \ \frac{J_o \tau^2}{v_o}$$

Using $\epsilon_o = 10^{-9}/36\pi$ farads/met, $M_e = 9.0 \times 10^{-31}$ Kg and $e = 1.6 \times 10^{-19}$ coul, we find $C = 2.0 \times 10^{22} \ J_o \ \tau^2/v_o$. Using $J_o = \alpha \times 10^6$ amp/met^2, $\tau = \gamma \times 10^{-9}$ sec and $v_o = \beta c = \beta \times 3 \times 10^8$ met/sec we find $C = 67 \ . \ \alpha \ \gamma^2/\beta$.

Equation (3a) can be modified by using $v_i = j_i/\rho_i$; it becomes

$$\frac{\partial j_i}{\partial t} - \frac{j_i}{\rho_i} \frac{\partial \rho_i}{\partial t} = \rho_i \ C \ E - j_i \frac{\partial}{\partial x} \ (j_i/\rho_i) \ ;$$

using Equation (4a)

$$\frac{\partial j_i}{\partial t} = \rho_i \ C \ E - \frac{j_i}{\rho_i} \frac{\partial j_i}{\partial x} - j_i \frac{\partial}{\partial x} (j_i/\rho_i)$$

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The final two terms on the right-hand side can be combined to give

$$\frac{\partial j_i}{\partial t} = \rho_i \, C \, E - \frac{\partial}{\partial x} \left(j_i^{\,2}/\rho_i \right) \tag{5}$$

Equations (2a), (4a) and (5) were integrated by a finite difference method. The boundary conditions were

$$j_1(0,t) = \rho_1(0,t) = 1 \qquad 0 \le t \le 1$$

$$j_1(0,t) = \rho_1(0,t) = 0 \qquad t > 1,\ t < 0$$

When the value of $j_1(x,t) < 0$ we used Equation 1 to bring j_2 and ρ_2 into the problem.

The equations were solved for $C = 10^K$ ($K = 0, 1, 2, 3$). An estimate of the grid spacing ($\Delta x, \Delta t$) for the finite difference integration can be made from the observation (after the fact) that the electrons which turn back do so at the trailing edge of the pulse (i.e., near the plate). Near the plate we can use the back-of-the-envelope estimate for the force on an electron. In scaled units $F = C\ j\ t$, where $j = j_1(0,t)$, so that $v = v_o - j\ t^2 C/2$, where $v_o = v_1(0,t)$, and $x = t(v_o - Cjt^2/6)$. An electron will turn around at

$$t = \sqrt{\frac{2v_o}{Cj}} = \sqrt{\frac{2}{C}} \quad \text{and} \quad x = \frac{2}{3}\, v_o \sqrt{\frac{2v_o}{Cj}} = \frac{2}{3}\sqrt{\frac{2}{C}}$$

For $C = 1000$ these criteria lead to $x \approx .03$ $t \approx .045$. We require that at least ten steps occur before turnaround, leading to $\Delta x \le .003$. We further require $\Delta t \le \Delta x$ for stability.

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However, in numerically integrating the equations by the finite difference scheme we found $\Delta x = .003$, $\Delta t = .003$ and $\Delta x = .010$, $\Delta t = .008$ to produce instabilities; while $\Delta x = .010$, $\Delta t = .005$ produced stable results. The results reported in the next section were obtained using the latter spacings. As a result of choosing $\Delta t = 1/2\ \Delta x$ we find that disturbances propagate too rapidly, e.g. the completion of the first step $t = \Delta t$, but some charge appears at $x = \Delta x = 2\ \Delta t$. This has some effect on the final results, but probably does not modify the general conclusions.

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III. RESULTS

A difficulty with this problem is that there is no unique significant quantity to measure (for two parallel plates one can measure anode current for various cathode emissions). Current density as a function of time at different positions seemed to be a reasonable quantity of interest. In Fig. 1, we show the current density as a function of time at $x = .50$, and in Fig. 2 at $x = .99$. Fig. 1 seems to indicate that there are two different regions, i.e., $C \leq 1$ and $C \geq 10$. For the former the pulse seen at $x = .5$ is similar to the emitted pulse, though slightly smeared out. However, for $C \geq 10$, the pulse is sharply peaked, in no way resembling the emitted pulse, and in addition is greatly reduced in magnitude.

With regard to magnitude, for $C \geq 10$, we observe that $C\, j_o^+(x) = D(x)$; where $D(x)$ is independent of C and where j^+ refers to current moving away from the plate, j^- to return current density, and j_o^+ to the peak current.

But examining Fig. 2, we see that the division into two regimes is not as clear as in Fig. 1. For $C = 1$ the current distribution has become less flat and begins to approach the higher C current distributions. Again for $C \geq 10$ we observe

$$C j_o^+(x) = D(x)$$

In Fig. 3, we plot j_o^+ vs x for $C = 100$ and find

$$j_o^+(x) = A/x^2 \text{ for } x \geq .5 \ .$$

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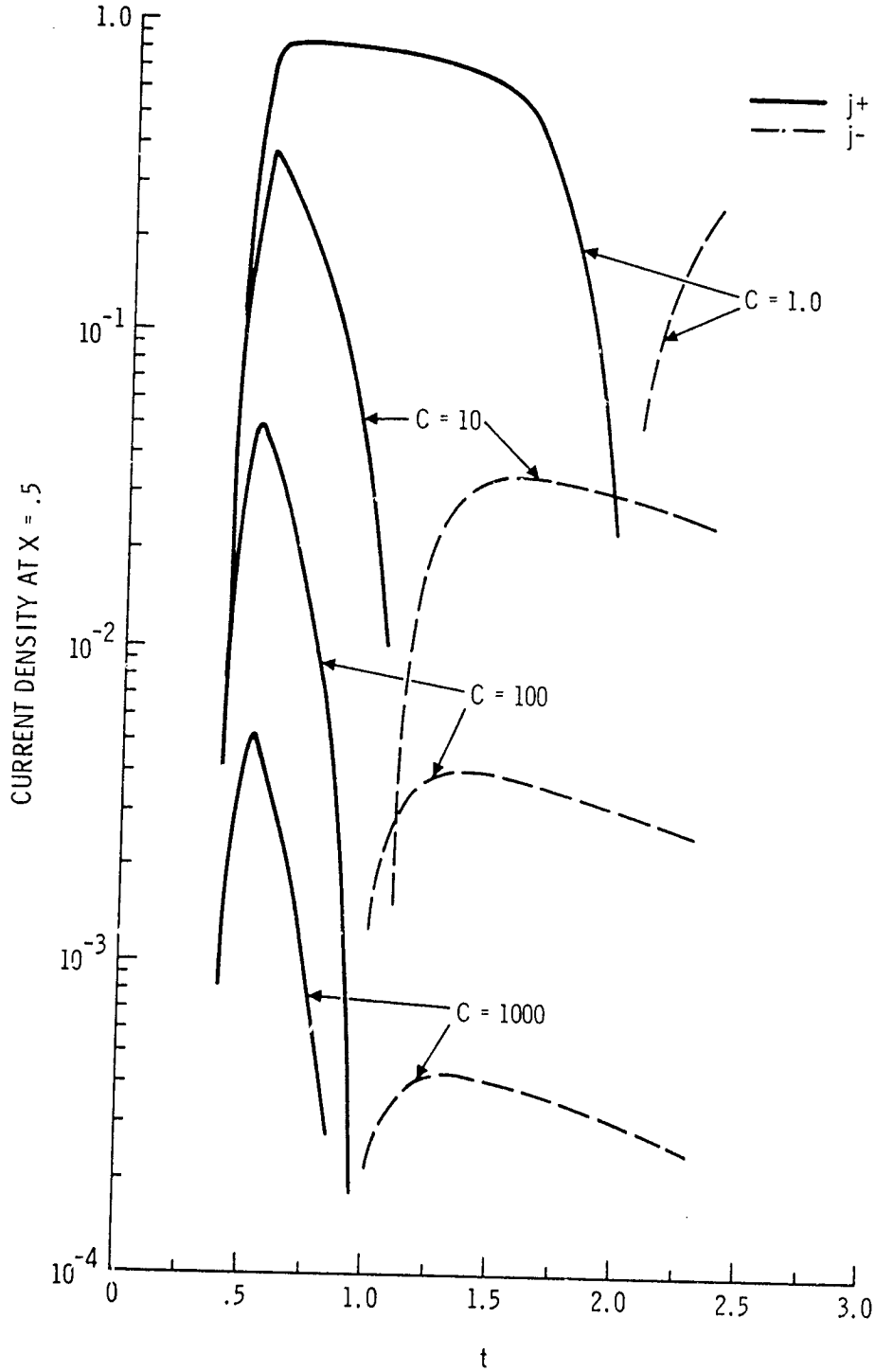

Figure 1
Current versus time at x = .5.

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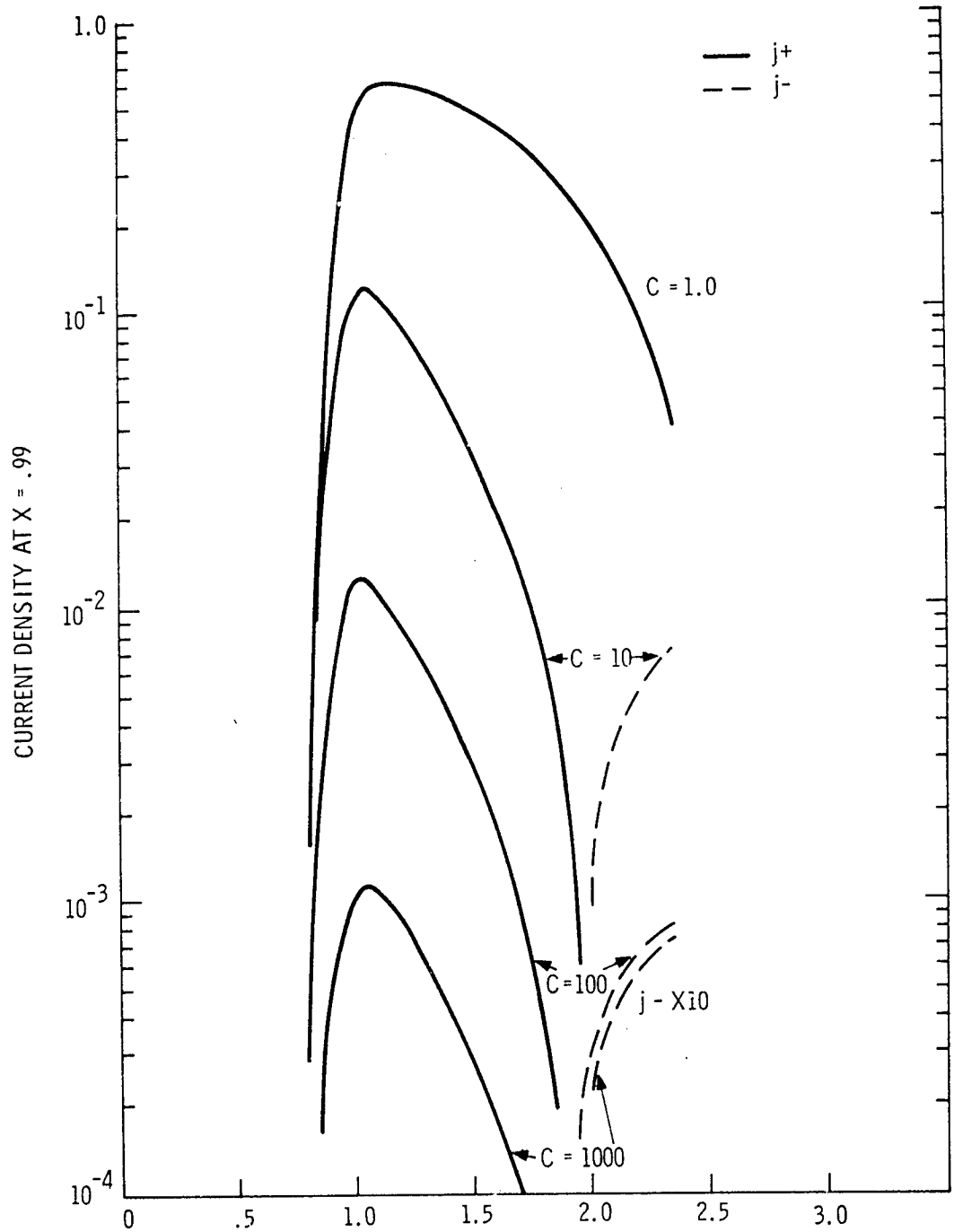

Figure 2
Current versus time at x = .99.

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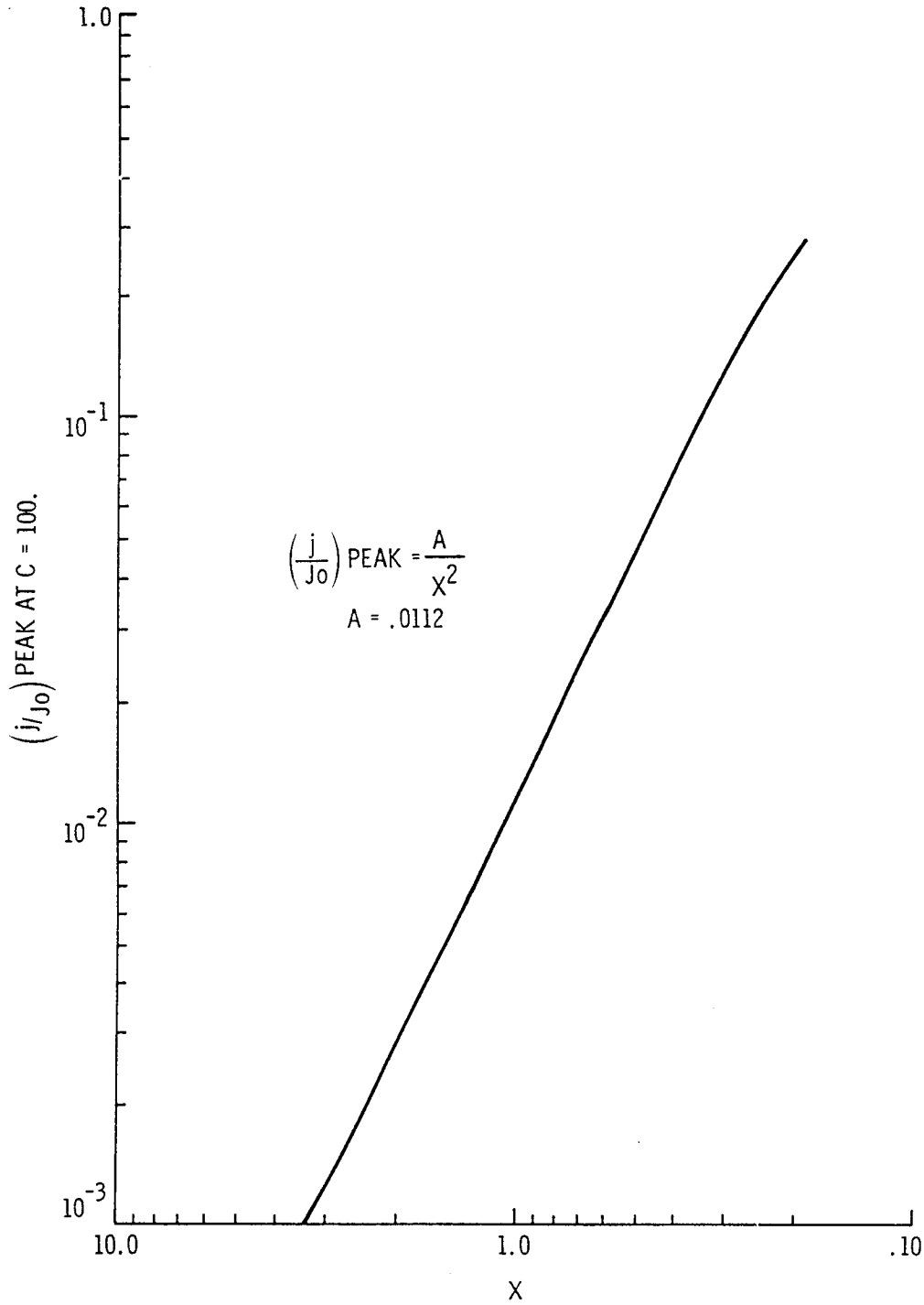

Figure 3
Peak current as a function of position.

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Further A = .0112 so that

$$100 \cdot \frac{.0112}{x^2} \cdot D(x) = F/x^2$$

Thus

$$F = 1.12 \text{ and } j_o^+(x) = \frac{1.12}{Cx^2}$$

Using $j_i' = j_i/J_o$, $C = 2 \times 10^{22}\ J_o\ \tau^2/v_o$, $x' = x/(v_o\tau)$ and $v_o = \beta \times 3 \times 10^8$ met/sec, the relation $j'^+_o(x) = \frac{1.12}{C(x')^2}$ becomes

$$J_o^+ \text{ (amp/met}^2) = \frac{1.5 \times 10^3\ \beta^3}{x^2} \quad \text{(x in meters) or}$$

$$J_o^+ \text{ (amp/cm}^2) = \frac{1.5 \times 10^3\ \beta^3}{x^2} \quad \text{(x in cm)} \qquad (6)$$

Thus, we have the result that the maximum current one can have at distances $x \geq v_o\ \tau$ for emitted currents J_o is independent of both J_o and τ and depends on the cube of the emitted velocity!

This is similar to the Childs-Langmuir Law where for parallel plates with voltage V and spacing d, $J_{CL} \sim V^{3/2}/d^2$. However at the anode, the electron velocity $v_{anode} \sim V^{1/2}$ so $J_{CL} \sim (v_{anode})^3/d^2$. The physics of the two situations is of course very different as in the Childs-Langmuir case the limiting current density is uniform between the plates.

This is treated in greater detail in the Appendix.

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IV. VELOCITY DISTRIBUTION

Since we have both j and ρ at x,t we can find $v(t)$ at fixed x. In Fig. 4 we show $v(t)$ at $x = .5$ for $.4 \leq t \leq .9$. Again, we point out that the finite charge density at $x = .5$ for $.4 \leq t \leq .5$ is due to the numerical integration grid, and is not real.

From Fig. 4 it is clear that there is a considerable velocity spread even when monoenergetic electrons are emitted.

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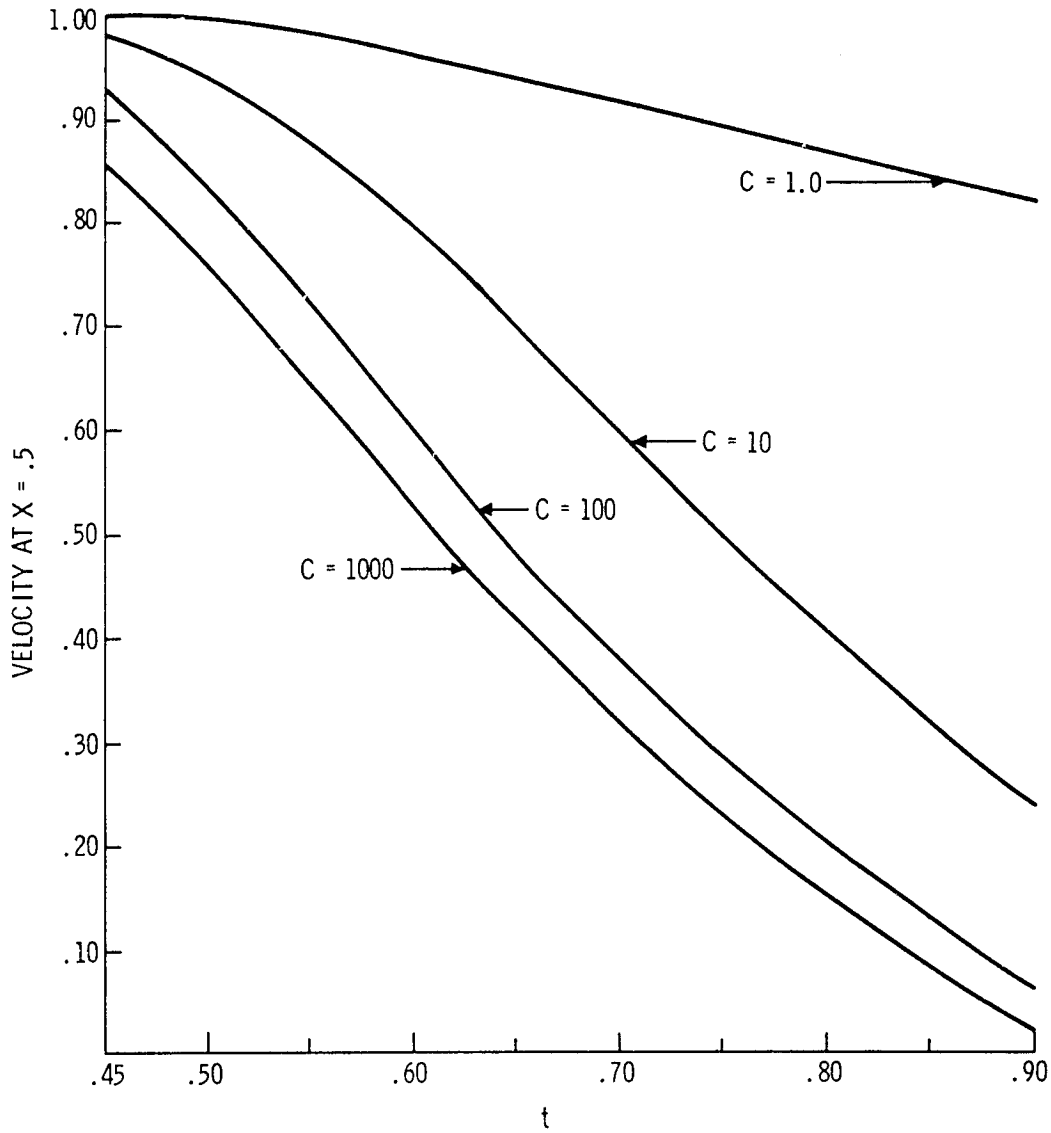

Figure 4
Velocity as a function of time at x = .5.

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V. TWO GROUPS AND FOUR STREAMS

When electrons are emitted with different velocities from the plate, one's intuition indicates that the fastest electrons will travel farthest and turn around last. To examine this point we added another emitted current such that for the fast group $j_1 = 1$, $v_1 = 1$ and $\rho_1 = 1$ at the cathode while for the slow group $j_3 = 1$, $v_3 = 1/2$ and $\rho_3 = 2$ at the cathode. The grid spacing was modified to $\Delta x = .010$, $\Delta t = .0025$. No further stability examination was made.

The conclusion reached in this examination are so drastic there seems no reasonable way of presenting it graphically. In Fig. 5 we plot j_1 and j_3 vs time at $x = .5$ for $C = 1.0$. However for $C = 10.0$ $j_3 = 0$ for all times ≥ 1 at $x = .5$, i.e., the electrons emitted with $v_3 = 1/2$ never reach the position $x = .5$.

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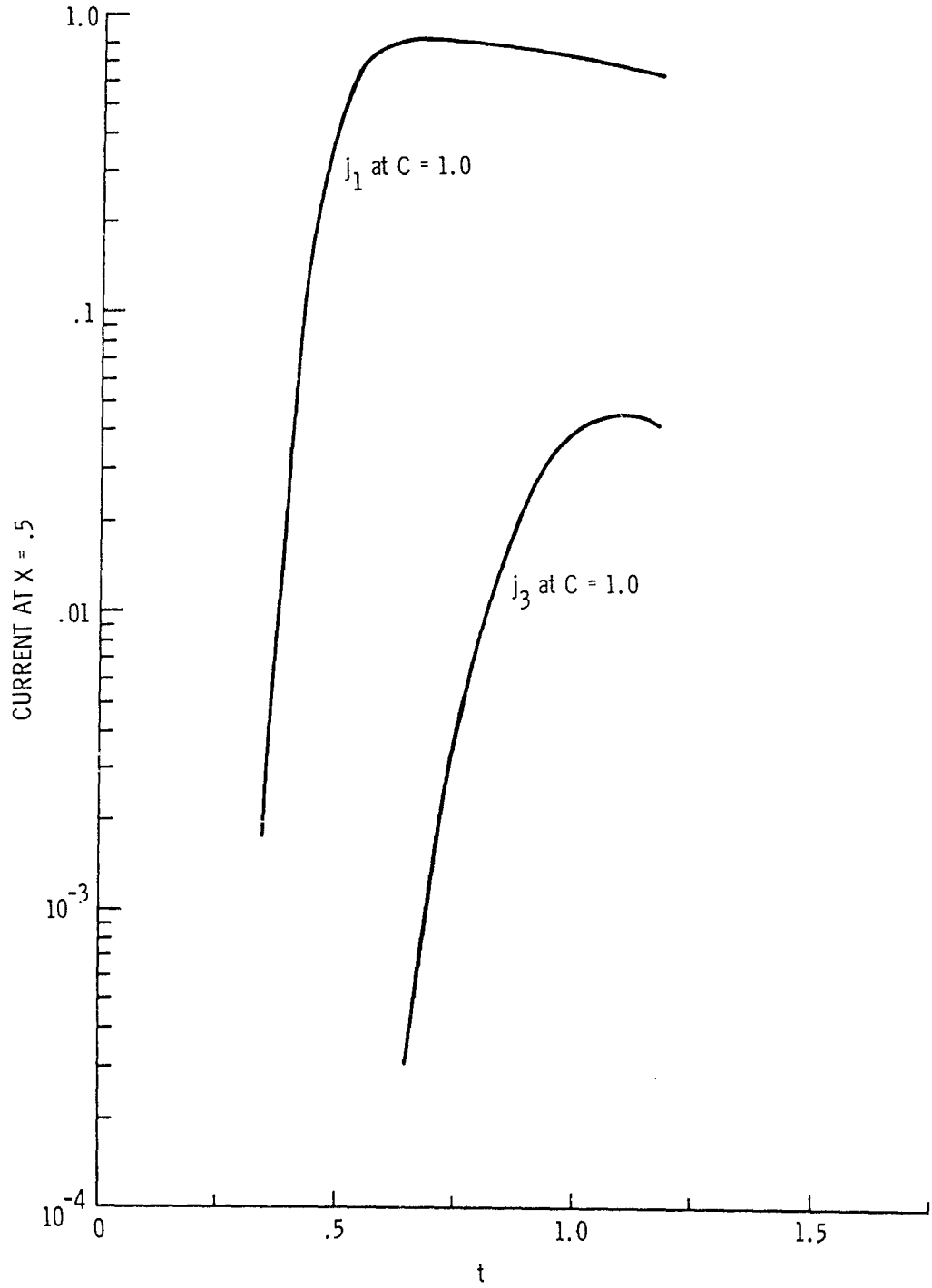

Figure 5
Current versus time at x = .5.

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VI. RELEVANCE TO EXPERIMENT

Physical experiments on large current pulses normally have a natural length, e.g., beam diameter. The current emitting area can be thought of as a disk of radius a. Fields should then involve terms of the form $1/(X^2 + a^2)$. The parallel plate argument is relevant to distances away from the plate equal to or less than a. The drastic effects noted in preceeding sections should not be relevant if at a the space charge limited peak current is greater than the emitted current: that is if

$$J_o^+ = \frac{1.5 \times 10^3}{a^2} \beta^3 > J_o \quad .$$

This can be rewritten as

$$\pi J_o a^2 = I < 1.5 \times \pi \times 10^3 \times \beta^3 \qquad (7)$$

This is now a restriction on total current, not current density. For electron kinetic energy between 10 and 100 keV, $\beta = .3$ is a reasonable mean. Then the parallel plate disaster should not occur if $I < 120$ amps.

The equations which we solved were not relativistic but we argue that up to 100 keV the neglect of relativistic effects should not be significant.

We have indicated where the parallel plate argument breaks down. We have not said what happens when it does break down. This would require an investigation of the problem of emission from a finite disk

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(which we hope to do). The parallel plate case should be relevant to the disk when the current drawn exceeds 120 amps.

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VII. CONCLUSIONS

The results presented here indicate that the dynamics of electron emission from an infinite plate is not a completely sterile problem (i.e., we found a current density limit similar to the Childs-Langmuir Law). However, when the infinite parallel plate is relevant to an experiment one should not expect any significant information from the experiment. In this context, conclusions drawn from a discussion of an infinite parallel plate are relevant if $I/\beta^3 > 5 \times 10^3$ amps, where I is the total current and β is the ratio of electron emission velocity to the velocity of light.

Precisely what one should expect when the infinite parallel plate is not relevant cannot be determined from this investigation. We plan to continue this study by examining current flow from a finite emitter.

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APPENDIX I

The Childs-Langmuir Law for finite emission velocity.

We wish to derive the space charge limited current between parallel plates when there is a potential V_o across the plates and when the electrons are emitted from the cathode with velocity v_o.

Physically the current between the plates will build up until at some point the repulsive space charge potential is just sufficient to bring an electron to rest. On the cathode side of the zero velocity position the electric field is decelerating and on the anode side it is accelerating. The governing equations are

$$(1) \quad \partial^2 V/\partial x^2 = \frac{e\rho}{\epsilon_o} \qquad (2) \quad M\, v \frac{\partial v}{\partial x} = e \frac{\partial V}{\partial x} \qquad (3) \quad j = e\rho v = J_o$$

From (3) we have $\rho = J_o/ev$ and from (2)

$$\frac{1}{2} mv^2 = e\,V + \frac{1}{2} mv_o^{\,2} \,. \qquad (4)$$

The latter equation is the energy equation. We note that the anode velocity v_a is given by

$$v_a = \sqrt{\frac{2eV_o}{m} + v_o^{\,2}} \,.$$

Substituting into (1) we have

$$\frac{\partial^2 V}{\partial x^2} = \frac{J_o}{\epsilon_o} \frac{1}{\sqrt{\frac{2eV}{m} + v_o^{\,2}}} \qquad (5)$$

Multiplying through by $\partial V/\partial x$, and integrating leads to

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$$\left(\frac{\partial V}{\partial x}\right)^2 = \frac{2\ J_o\ m}{\epsilon_o e} \sqrt{\frac{2eV(x)}{m} + v_o^{\ 2}} + C\ , \qquad (6)$$

where C is an arbitrary constan' . At the point where $v(x) = 0$, the expression under the square root is zero. But here the electric field is also zero since it is at this point that it changes sign. Therefore, $C = 0$ and

$$\left(\frac{\partial V}{\partial x}\right) = \pm \left(\frac{2\ J_o\ m}{\epsilon_o\ e}\right)^{\frac{1}{2}} \left(\frac{2eV(x)}{m} + v_o^{\ 2}\right)^{\frac{1}{4}} . \qquad (7)$$

The point at which $v = 0$ is defined as $x = x_o$. Then for $x < x_o$ we take the minus sign and for $x > x_o$ we take the plus sign.

The above equation can be integrated to give

$$\left(\frac{2eV}{m} + v_o^{\ 2}\right)^{3/4} \left(\frac{2m}{3e}\right) = - \left(\frac{2\ J_o\ m}{\epsilon_o\ e}\right)^{\frac{1}{2}} x + C_1 \qquad x < x_o\ ,$$

$$\left(\frac{2eV}{m} + v_o^{\ 2}\right)^{3/4} \left(\frac{2m}{3e}\right) = \left(\frac{2\ J_o\ m}{\epsilon_o\ e}\right)^{\frac{1}{2}} x + C_2 \qquad x > x_o\ . \qquad (8)$$

Inserting the boundary conditions: at $x = 0$, $V = 0$, and at $x = d$, $V = V_o$ leads to

$$\frac{2m}{3e}\left[\left(\frac{2eV}{m} + v_o^{\ 2}\right)^{3/4} - v_o^{\ 3/2}\right] = -\left(\frac{2J_o}{\epsilon_o}\frac{m}{e}\right)^{\frac{1}{2}} x \qquad x < x_o$$

$$\frac{2m}{3e}\left[\left(\frac{2eV}{m} + v_o^{\ 2}\right)^{3/4} - \left(\frac{2eV_o}{m} + v_o^{\ 2}\right)^{3/4}\right] = \left(\frac{2J_o}{\epsilon_o e}\right)^{\frac{1}{2}} (x-d) \quad x > x_o\ . \qquad (9)$$

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Thus the peak current obtained from a single infinite plane emitter at position x is close to the space charge limited current flowing between grounded plates with separation $d = x$, when the emission velocity is the same for both cases.

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