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FALL OF SMALL PARTICLES IN THE UPPER ATMOSPHERE

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ABSTRACT

The altitude history of labeled debris of Hardtack Orange shot is calculated. This calculation, which ignores vertical atmospheric transport, indicates that particulate debris should soon fall to an altitude which can be reached by existing rocket samples systems. After this initial rapid descent the fall rate decreases so the material remains at an intermediate altitude (100 to 250 kft) for a protracted period.

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FALL OF SMALL PARTICLES IN THE UPPER ATMOSPHERE

Sandia Corporation and other agencies have become interested in sampling via rocket the debris from a recent labeled shot (Hardtack Orange). Debris from this shot rose a considerable distance above the burst point. We desire to know whether the debris has fallen from this ultimate height and if it has fallen, the altitude history. This calculation explores the height time behavior for various particle sizes in an atmosphere free of vertical movement and having a density which is an exponential function of altitude.

Particle diameters of interest are much less than the mean free path of air molecules at these high altitudes. The appropriate force law then is:

$$F = -1.35 \frac{4\pi}{3} a^2 \rho \bar{v} V \qquad (1)$$

where F is the retarding force, a is the particle radius, ρ is air density, $\bar{v}$ is the average air molecule velocity, and V is the relative velocity of the particle.[1] Equating this force to the gravitational attraction ($4/3\pi a^3 \delta g$), we may derive the fall velocity:

$$V = \frac{a\delta g}{1.35\rho\bar{v}} \qquad (2)$$

where δ is the particle specific gravity and g is the gravitational acceleration.

The altitude history of a particle may be determined by integrating this velocity with respect to time. The average molecular speed $\bar{v}$ is a weak function of altitude; we approximate by assuming a constant value for it as well as the gravitational constant. On the other hand, air density varies rapidly with altitude. Considering downward as positive, we specify the dependence as:

$$\rho = \rho_o e^{\frac{s}{\lambda}} \qquad (3)$$

where ρ_o is the air density at the initial height, s is the distance below this initial height, and λ is the e-fold distance (~25 kft). Substituting Equation 3 in Equation 2 and integrating with respect to time to determine the particle distance below the initial height, we have

$$\int_o^s e^{\frac{s}{\lambda}} ds = \frac{a\delta g}{1.35\rho_o \bar{v}} \int_o^t dt$$

or

$$\lambda (e^{\frac{s}{\lambda}} - 1) = V_o t \tag{4}$$

or

$$s = \lambda \ln \left(1 + \frac{V_o t}{\lambda}\right)$$

where V_o is the initial fall velocity.

The altitude histories of various particle sizes have been computed using Equation 4 and the parameters listed in Table 1. Initial heights of 300 and 350 kft have been assumed. Results are displayed on Figure 1 for particles having a specific gravity of five. Results may be scaled by noting V_o is proportional to the product of particle radius and specific gravity.

It will be noticed the particles fall rapidly from the initial height until they arrive at a particular altitude which depends on the particle diameter. Thereafter, the fall rate is less and equal to that of any other particle at its characteristic altitude. The altitude separation of particle sizes rapidly becomes constant and amounts to $\lambda \ln \frac{a_1}{a_2}$. This trend may be demonstrated

formally from Equation 4. Using this same equation, one may also show the eventual particle altitude history is independent of the initial height.

The smallest particle considered is 10^{-3} microns in diameter and should possess about 100 atoms. This seems about the smallest particle for which Equation 1 is valid ($\approx$10 percent error). For yet smaller particles the local density gradient effect should be taken into account and the behavior is better described by diffusion. The downward progress is then slow and incomplete.

Considering the other end of the particle spectrum, at 66 kft the air molecule mean free path is 1 micron, the same as the diameter of our largest particle. About and below that altitude, Equation 1 is not valid for this large particle, rather Stokes' law is more nearly suitable. We approximate for the curve in Figure 1 by assuming Equation 1 holds above 66 kft and Stokes' law below. Such a procedure overestimates the fall rate above 66 kft and underestimates it below. Details of the intermediate behavior are covered in texts.[1]

While this analysis is not exact at the particle-diameter range extremes, a meteorological difficulty makes further refinement pointless. We have no basis for estimating the vertical transport by atmospheric flows. Neglecting such transport probably incurs a greater error than those brought about by our analysis approximations.

Conclusions

Figure 1 gives the altitude history for particles of various diameters assumed to have a specific gravity of five at initial heights of 300 and 350 kft. This figure, based on an analysis which assumes no vertical atmospheric motion, demonstrates that particles at first fall rapidly until a height characteristic of the particle is reached. Then the fall is gradually slowed and is the same as that of any other particle at its altitude. The final height-time history is after, at most, a few weeks, independent of initial height. Results may be scaled as the altitude history is the same for particles whose product of radius and specific gravity are equal.

If atmospheric vertical transport is not of overwhelming significance, the situation for high-altitude rocket sampling of debris seems favorable over a considerable time interval. Within a week the smallest particle should have fallen enough to be reached by existing sampler rockets (300 kft). After this initial rapid drop the fall rate is so slow that the particles remain available for a long period. For example, at 200 days the altitude from 65 to 225 kft encompasses the particle-diameter range from 1 to 10^{-3} microns.

TABLE I

Parameter	Value
Air density (300 kft)	5×10^{-9} gm/cm^3
Average air molecule velocity	4.7×10^{4} cm/sec
Air viscosity	1.8×10^{-4} poise
Gravitational constant	980 cm/sec^2

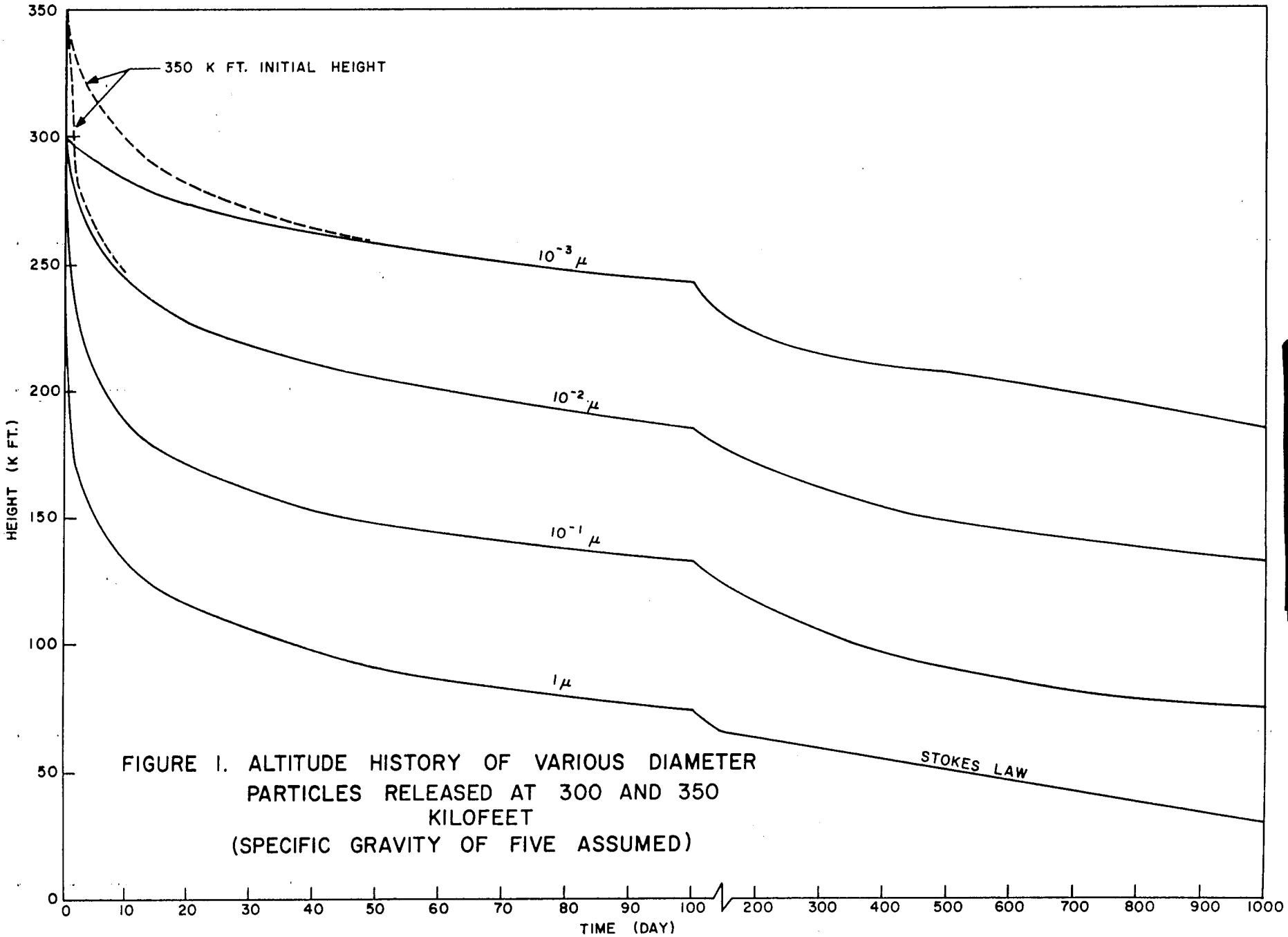

FIGURE I. ALTITUDE HISTORY OF VARIOUS DIAMETER PARTICLES RELEASED AT 300 AND 350 KILOFEET (SPECIFIC GRAVITY OF FIVE ASSUMED)

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LIST OF REFERENCES

Kennard, E. H., Kinetic Theory of Gases, McGraw-Hill Book Company, 1938.