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OFFICE MEMORANDUM

712269

TO : Distribution

DATE: November 14, 1972

FROM : Eric Rodgers E.R.

MES-200

SUBJECT : A SUGGESTED METHOD FOR OBTAINING A VERTICAL VELOCITY DISTRIBUTION FOR AN OSCILLATING PION BEAM

SYMBOL : MP-3

In a recent memorandum[1] it was stated that edge effects can lead to undesirable dose distributions if pion Bragg peaks are oscillated vertically through a tumor volume or if the patient is moved horizontally during treatment. A method was suggested for elimination of the undesirable effects for horizontal motion, but complete elimination of edge effects for vertical motion is impossible. There is an edge effect for vertical motion associated with the Bragg peak and a more extensive effect associated with the plateau. In a recent memorandum[2], Liska suggested a variable velocity for the peak through the tumor volume as a means of elimination of what we refer to here as the plateau edge effect. The purpose of the present memorandum is to describe an analytical method for finding a vertical velocity distribution that gives a constant dose throughout a treatment volume to within less than 1%.

The suggested method corrects the plateau edge effect but does not eliminate the peak edge effect. Instead of elimination of the peak effect, the method places it outside the tumor volume. Figure I shows that each oscillation begins and ends with the peak just outside the treatment volume. This causes the dose drop off to be not as sharp as one might like but it is probably not serious. It means that under ideal conditions, the dose at the lower edge of the tumor drops to essentially zero in a distance equal to one peak width which is about 1.5 to 2.0 cm. This is much better than is possible with photon radiation.

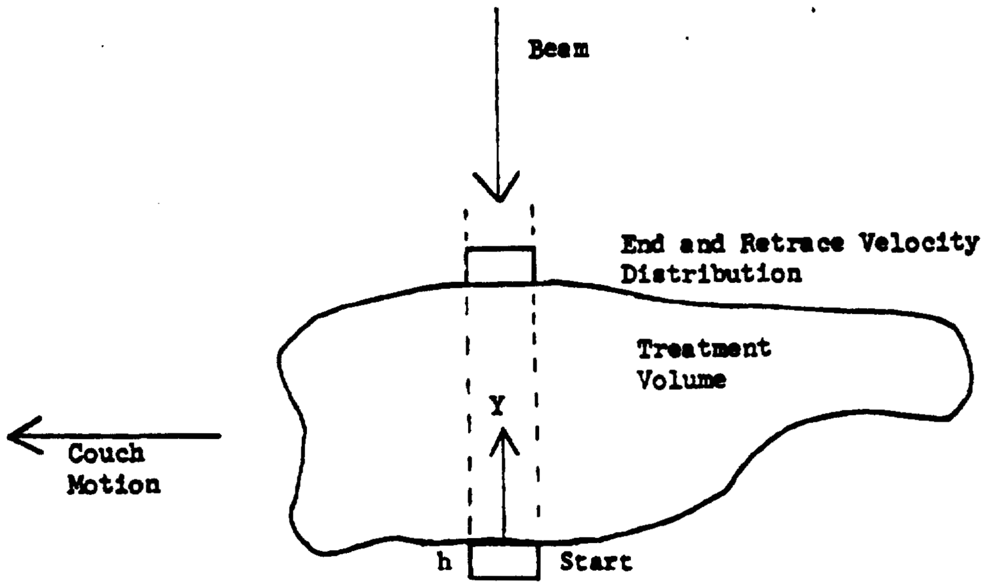

Figure I

In the discussion that follows, it is assumed that the following parameters are known:

h = peak width

r = ratio of peak dose to plateau dose with RBE values included

K = average peak dose rate including RBE

$\bar{v}_1$ = average velocity of peak across bottom boundary of treatment volume. This is arbitrary and determines the dose delivered per upward sweep of the peak.

With these assumptions, the dose per upward journey of the peak through any point in the treatment volume is

$$\frac{Kh}{\bar{v}} + \frac{KT}{r} = \frac{Kh}{\bar{v}_1} \qquad (1)$$

where $\bar{v}$ is the average velocity of the peak through the point in question and T is the total elapsed time of the motion from the lowest position to the position where the top of the peak has just reached the point in question. We can solve (1) for $\bar{v}$ and get

$$\bar{v} = \frac{rh}{\left(\frac{rh}{\bar{v}_1} - T\right)} \qquad (2)$$

We now compute the velocities attained as the peak moves upward through successive peak widths. Thus for the first peak width, $T = 0$, and $\bar{v} = \bar{v}_1$.

Expression (2) gives only the average velocities. For the treatment to be of any value, we must know actual velocities. The average velocity $\bar{v}_1$ through the first peak width does not determine a unique velocity distribution. To get started, we differentiate (2) to get an acceleration for the motion across the first peak width. We get

$$a_1 = \frac{rh}{\left(\frac{rh}{\bar{v}_1} - T\right)^2} = \frac{\bar{v}_1^{\,2}}{rh} \text{ with } T = 0 \qquad (3)$$

We now use

$$h = v_1 t_1 + 1/2 a_1 t_1^{\,2} \qquad (4)$$

to get the initial velocity v_1 for the motion across the first peak width h. Here t_1 is the time for the top of the peak to cross the first peak width. It is gotten from

$$t_1 = \frac{h}{\bar{v}_1} \qquad (5)$$

We now proceed to the second peak width and compute the initial velocity from

$$v_2 = v_1 + a_1T_1 \qquad (6)$$

the average velocity $\bar{v}_2$ from (2),

the time for crossing from

$$t_2 = \frac{h}{\bar{v}_2}$$

and a_2 from

$$h = v_2t_2 + 1/2a_2t_2^2 \quad .$$

We repeat this process for the remainder of the upward journey.

We shall now go through the procedure by working out a numerical example for an assumed set of parameters. We assume

$h = 1.5$ cm

$r = 6$

$K = 50$ rads/sec

$\bar{v}_1 = 10$ cm/sec

The last assumption means that the dose is 7.5 rads per upward journey. We compute t_1,

$$t_1 = \frac{1.5}{10} = 0.15 \text{ sec}$$

and

$$a_1 = \frac{\bar{v}_1^2}{rh} = \frac{100}{9} = 11.1111 \text{ cm/sec}^2.$$

We get v_1 from

$$1.5 = 0.15\ v_1 + 1/2 \times 11.1111 \times 0.0225 \quad \text{or}$$

$$v_1 = 9.16667 \text{ cm/sec} \ .$$

Now we consider the second peak width. Its initial velocity v_2 is

$$v_2 = 9.16667 + 11.1111 \times 0.15$$

$$= 10.83333 \text{ cm/sec}$$

$$\bar{v}_2 = \left(\frac{rh}{rm/v_1 - T}\right) = \frac{9}{0.9 - 0.15} = 12 \text{ cm/sec}$$

$$t_2 = \frac{1.5}{12} = 0.125 \text{ sec.}$$

To get a_2 we use

$$1.5 = v_2 t_2 + 1/2 a_2 t_2^2$$

$$1.5 = 10.83333 \times 0.125 + 1/2 \times a_2 \times (0.125)^2 \text{ or}$$

$$a_2 = 18.66504 \text{ cm/sec}^2$$

We repeat this process to complete the distribution as far as is desired. Table I gives the results for a total vertical motion of 16.5 cm. This would enable a volume 15 cm thick to be treated. In the table, the column headings have the following meanings:

n = ordinal number of peak width from the bottom

y_n = distance from bottom of tumor through nth peak width.

T_n = elapsed time from bottom of tumor through nth peak width

t_n = time for top edge of peak to pass through the nth peak width

v_n = velocity of peak as its top edge reaches the bottom edge of the nth peak width. It is also the peak velocity as the edge reaches the top of the n - 1 th peak width

$\bar{v}_n$ = average velocity of top of peak through the nth peak width

a_n = acceleration of peak during passage of its top through the nth peak width.

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Table I

n	Y_n	T_n sec	t_n sec	v_n cm/sec	$\bar{v}_n$ cm/sec	a_n cm/sec
1	1.5	0.15	0.15	9.16667	10	11.1111
2	3.0	0.275	0.125	10.8333	12	18.6650
3	4.5	0.379167	0.104167	13.1665	14.4	23.6866
4	6.0	0.465972	0.086805	15.6338	17.28	37.9346
5	7.5	0.538310	0.072338	18.9267	20.7360	50.0294
6	9.0	0.598592	0.060282	22.5458	24.8832	77.5432
7	10.5	0.648827	0.050235	27.2202	29.8599	105.0650
8	12.0	0.690689	0.041862	32.4982	35.8319	159.2810
9	13.5	0.725574	0.034885	39.1660	42.9982	219.7180
10	15.0	0.754812	0.029238	46.8308	51.5978	305.9214
11	16.5	0.779010	0.024198	55.7754	61.9886	513.5781

For any value of y the peak dose per sweep is $\frac{Kh}{\bar{v}}$ and the plateau dose is $\frac{K}{r}$ T. Values of peak dose, plateau dose and total dose per sweep are given in Table II for the values of y that are indicated.

Table II

y_1 cm	Peak dose per sweep (rads)	Plateau dose per sweep (rads)	Total dose per sweep (rads)
0	7.5000	0.0000	7.5000
1.5	6.2500	1.2500	7.5000
3.0	5.2083	2.2917	7.5000
4.5	4.3403	3.1597	7.5000
6.0	3.6169	3.8831	7.5000
7.5	3.0141	4.4859	7.5000
9.0	2.5117	4.9887	7.5000
10.5	2.0931	5.4069	7.5000
12.0	1.7443	5.7557	7.5000
13.5	1.4535	6.0465	7.5000
15.0	1.2099	6.2901	7.5000

Some remarks on the method

1. As shown in the tables, the method described gives the total elapsed time and velocities attained to place exactly the same dose at peak width intervals as the beam moves upward. The method also gives exactly the elapsed time, the velocity attained and the exact dose for every value of y. As an example, let us take y = 10 cm. This value extends 1 cm into peak width region for n = 7. We use

$1 = v_7 t + 1/2 a_7 t^2$ to get the time taken for this 1 cm.

Putting in values from Table I, we get

$t = 0.034447$

We add this to T_6 to get T for 10 cm. The result is

0.633039

We multiply this by $\frac{50}{6}$ and get plateau dose = 5.27533 rad.

Points with y = 10 cm will get peak dose while the top edge of the peak moves from y = 10 cm to y = 11.5 cm. We find the time for this as the time to go from 10 cm to 10.5 cm plus the time to go from 10.5 cm to 11.5 cm. The first of these is simply

$t_7 - 0.034447 = 0.015788$ sec

We get the time from 10.5 to 11.5 from

$1 = v_8 t + 1/2 a_8 t^2$

The result is 0.028746 sec. The sum of these t values

= 0.044534 sec

The peak dose is 50 x 0.04454 = 2.22670 rad.
The total dose is 7.5020 rads.
This differs insignificantly from the values of 7.5000 given in Table II. This is true in general for y values intermediate between those shown. As other examples, for y = 2.5 cm, the total dose is 7.4874 rad, for y = 5.0 cm, the total is 7.4936 rad, and for y = 8.5 cm, the total is 7.4971 rad. These values all differ by a small fraction of one percent from the 7.5 rad per sweep for the y values in the tables.

2. As seen in Table II, the contribution of the peak decreases as the peak moves upward (downward in the table). This means that most of the dose for large values of y comes from lower LET radiation. This is unfavorable for OER. This can mainly be overcome by giving half the dose from an opposite port. If this is impossible, the OER of peak and plateau might be included in determining the ratio r.

3. The solid curve in Fig. 2 shows velocity as a function of depth for the data in the table. The curve actually is a series of straight line segments across successive peak widths. This follows from the fact that the peak crosses each successive peak width with constant acceleration. If it should be desirable to have a smooth curve template for computer control, a very small rounding of corners between peak widths could produce such a curve with negligible effect on the dose rate.

4. The 50 rads/sec assumed for K is of no special significance. It could follow from a dose rate of 1000 rad/min in the peak with an RBE of 3. The actual value to be used will have to be determined after many measurements of dose rate and RBE value.

The value 1.5 cm that was used for h appears reasonable on the basis of present information. It might seem that the method described would be quite sensitive to the value assumed for h. Actually this is not true. For a given pion beam of fixed cross section, the average dose rate in the peak is inversely proportional to h which causes r also to be inversely proportional to h. To show that the velocity distribution is not very sensitive to the value used for h, the method has been used to work out a distribution for h = 2.0 cm. Since 2.0 = 4/3 of 1.5, we change r and K to 3/4 of their value for h = 1.5. The value of $\bar{v}_1$ is chosen to give the same dose rate of 7.5 rads per sweep as with h = 1.5 cm. The resulting velocity distribution is shown as the dotted curve in Fig. 2.

5. The upward motion of the beam may be stopped at any point and if the velocity distribution be reversed for the downward journey, the downward dose will be the same as the upward dose. It is not true that the

downward motion may be stopped anywhere. For a uniform dose, the peak must return to its lowest point. It then must move upward so that its initial velocity at the tumor edge and its upward velocity distribution from there are always the same.

As the patient moves as indicated in Fig. 1, the treatment thickness will in general vary as shown. The oscillation should be computer controlled so that the lower stopping region follows the contour, and each oscillation should start off to give the same velocity distribution in the treatment value. Since the method gives equal doses per oscillation, the table speed will have to vary with thickness of treatment value.

6. Throughout the discussion, dose rates have been expressed as rads, when actually they are rads times RBE. From 1931 to 1959 it was common to use "rem" as a unit for rads time RBE. With the introduction of the terms "quality factor" and "dose equivalent" in 1959 to be used in radiation protection, the "rem" went with "dose equivalent" so that radiation therapy is without an acceptable name for RBE x rads.

7. Mechanical devices that can vary an absorber thickness to cause the Bragg peak to move in a desired velocity pattern have been designed and prototypes are under construction.[3] Velocities and accelerations that are needed should be easily attainable.

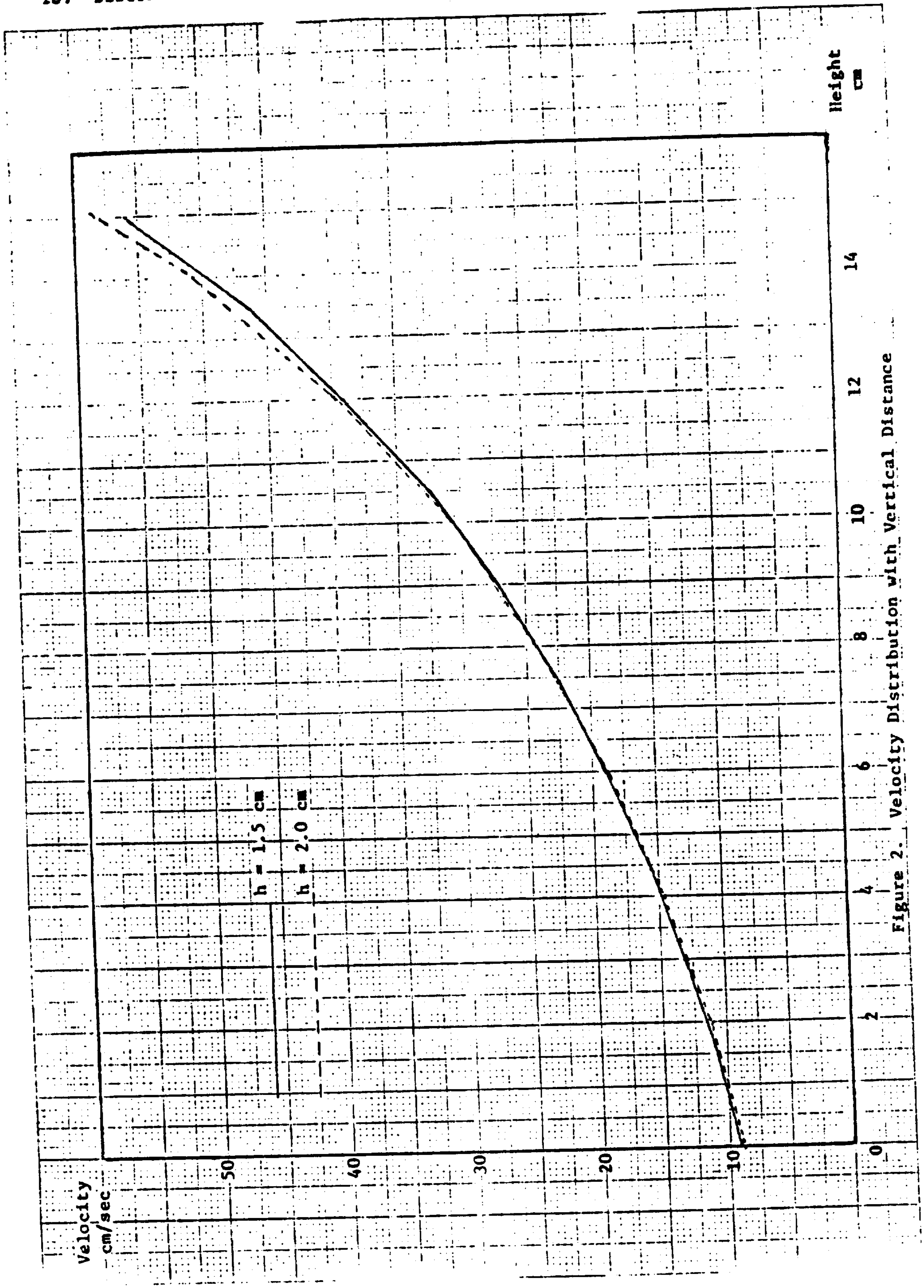

Figure 2. Velocity Distribution with Vertical Distance

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3. Donald J. Liska and Jerry D Wallace, "Biomed Pion Energy Control Actuators," Los Alamos Scientific Laboratory, Unpublished Report, September 1, 1972.

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