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NUMERICAL ANALYSIS OF SINGLE-PHASE, NATURAL CIRCULATION IN A SIMPLE CLOSED LOOP

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RESUMEN

Las propiedades numéricas inherentes a los grandes programas para sistemas termo-hidráulicos pueden ser usualmente verificadas por medio de programas mas simples, que consideran casos de aplicación práctica restringida. Los autores proveyeron un ejemplo de esta metodología en su trabajo previo, en relación a la serie de códigos RELAP5. El problema considerado fue el del flujo por convección natural en una sola fase en un circuito cerrado, bajo condiciones inestables de flujo. Se derivaron las curvas de estabilidad neutral para el sistema. El efecto del número de nodos en el comportamiento esperado del sistema fue consecuentemente analizado. Los códigos ad hoc previamente desarrollados fueron refinados adicionalmente, para analizar el problema bajo dos aproximaciones: a) una forma nodal, basada en una aproximación por diferencias finitas y, b) una forma modal, basada en una descomposición modal de las ecuaciones de gobierno en series de Fourier. Se obtuvieron los parámetros del estado estacionario del sistema para las dos aproximaciones. El código nodal fue utilizado como aproximación standard y se usaron diferentes esquemas de precisión variable. Luego, el efecto del número de nodos fue determinado cuantitativamente. El código modal, con un número de modos entre 30 y 100, fue utilizado como una aproximación libre de difusión numérica. En este caso, la ecuación de la energía fue resuelta también considerando un término con difusión constante, que permitió simular parcialmente el efecto de la difusión numérica de la aproximación nodal. En esta forma, el análisis permitió efectuar el análisis modal incluyendo un valor promedio de la difusión que surgía del upwinding en la solución nodal. Los resultados muestran que la inclusión de esta difusión da cuenta razonablemente de la amortiguación de la solución, permitiendo una recuperación cualitativa del comportamiento nodal. La no-linealidad del sistema no permite la exacta coincidencia de los resultados obtenidos. Un análisis similar puede ser utilizado para evaluar el efecto de la discretización sobre la dinámica de sistemas termo-hidráulicos mas complejos.

ABSTRACT

The inherent numerical properties of large thermal-hydraulic system codes may be usually verified by means of simpler codes, dealing with selected cases of restricted practical application. The authors provided an example of such methodology in their previous work, in relation with the RELAP5 series of codes. The problem considered was the single-phase, natural circulation flow in a simple loop, under unstable flow conditions. Neutral stability curves were derived for the system. The effect of the number of nodes in the expected behavior of the system was consequently analyzed. The ad hoc codes previously developed have been further refined, to analyze the problem under two approaches: a) a nodal one, based on a finite-difference approximation and, b) a modal one, based in a modal decomposition of the governing equations in Fourier series. Theoretical values for the steady state parameters are obtained for both approximations. The nodal code was used as the standard approximation and different schemes of different order have been used. Then, the effect of the number of nodes in the damping of the system was quantitatively determined. The modal code, with the number of modes ranging from 30 to 100, was used as an approximation free of numerical diffusion. In this case, the energy equation was solved considering also a constant diffusion term, allowing a partial simulation of the numerical diffusion of the nodal approximation. In this way the analysis allowed the modal analysis to be pertormed including an average value of the diffusion arising from the upwinding in the nodal solution. Results show that the inclusion of this diffusion reasonably accounts for the damping of the solution, allowing a qualitative recovering of the nodal behavior. System non-linearity naturally precludes the exact coincidence of the results obtained. A similar analysis may be used to assess the effect of the number of nodes of a given discretization on the dynamics of more complex thermal-hydraulic systems.

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NUMERICAL ANALYSIS OF SINGLE-PHASE, NATURAL CIRCULATION IN A SIMPLE CLOSED LOOP

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ABSTRACT

The inherent numerical properties of large thermal-hydraulic system codes may be usually verified by means of simpler codes, dealing with selected cases of restricted practical application. The authors provided an example of such methodology in their previous work, in relation with the RELAP5 series of codes. The problem considered was the single-phase, natural circulation flow in a simple loop, under unstable flow conditions. Neutral stability curves were derived for the system and the effect of the number of nodes in the expected behavior of the system was consequently analyzed. The ad hoc codes previously developed have been further refined, to analyze the problem under two approaches: a) a nodal one, based on a finite-difference approximation and, b) a modal one, based in a modal decomposition of the governing equations in Fourier series. Theoretical values for the steady state parameters are obtained for both approximations. The nodal code was used as the standard approximation and different schemes of different order have been used. Then, the effect of the number of nodes in the damping of the system was quantitatively determined. The modal code, with the number of modes ranging from 30 to 100, was used as an approximation free of numerical diffusion. In this case, the energy equation was solved considering also a constant diffusion term, allowing a partial simulation of the numerical diffusion of the nodal approximation. In this way the analysis allowed the modal analysis to be performed including an average value of the diffusion arising from the upwinding in the nodal solution. Results show that the inclusion of this diffusion reasonably accounts for the damping of the solution, allowing a qualitative recovering of the nodal behavior. System non-linearity naturally precludes the exact coincidence of the results obtained. A similar analysis may be used to assess the effect of the number of nodes of a given discretization on the dynamics of more complex thermal-hydraulic systems.

I. INTRODUCTION

The theoretical results given by Pierre Welander [1] in a pioneering paper have been used by the authors in a previous paper [2] to test the capability of the RELAP5 series of codes [3] to predict instabilities in single-phase flow. These results were related to single-phase, natural circulation flow in a loop made of two parallel, adiabatic circular tubes with a point heat sink at the top and a point heat source at the bottom. A stability curve [1] may be defined for laminar flow and was extended to consider turbulent flow, now reported in [4]. Despite its restrictions, the problem of natural circulation in single-phase flows is quite common in many situations of interest in the nuclear industry. In [2], the analysis in [1] was generalised in order to keep strictly the same hypotheses of the original derivation. Then, an unstable flow condition was defined to check the effect of the nodalization on the appearance of oscillations. In this way it was possible to define the limits of applicability of a coarse nodalization using RELAP5. In this paper the results in [1] are revisited once again, to show the results of further developments of the *ad hoc* codes used to verify the effects of the nodes number in the stability maps. To comply with this purpose, finitedifferences schemes of various orders have been used. Also, a modal decomposition approach in terms of Fourier series was implemented. The results are presented in a non usual way, defining maps of departure from neutral stability. The sections to follow define the problem, some of the different schemes adopted, some trend plots and stability maps for the system under analysis.

II. THEORETICAL ANALYSIS

Fig. 1 shows the geometry of this simple hydraulic system, adapted from [1]. The vertical legs are adiabatic, smooth circular tubes of length L/2 and diameter D. The length of both the heat source and the heat sink is S. The cross section of the tubes is A. When S tends to zero, the heat transfer coefficient from/to the heat source/sink is increased to keep constant the total heat transferred. The reader is referred to [2] for a detailed description of the governing equations.

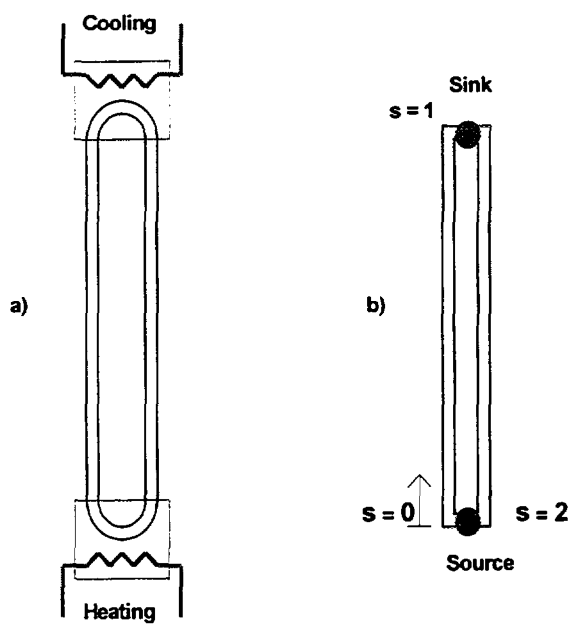

Figure 1. Sketch of the loop geometry.

The resulting dimensionless equations are:

$$\frac{dq}{dt} = \alpha \int_0^1 T \cdot ds - \varepsilon \cdot q^{\nu} \qquad (t > 0), \qquad (1)$$

for momentum and

$$\frac{\partial T}{\partial t} + q\,\frac{\partial T}{\partial s} = 0 \qquad (0 < s < 1, t > 0) \qquad (2)$$

for energy balance, in which α and ε measure the driving (buoyancy) forces and the friction in the loop respectively.

To obtain Equations (1) and (2) it was taken advantage of the observed anti-symmetry of temperature distribution along the loop, allowing for solving the problem only in the interval $s \in (0,1)$, provided that appropriate boundary conditions are specified for the dimensionless temperature in $s = 0^+$ and $s = 1^-$. Welander, solving the energy equations in the source and the sink for steady state conditions, gave these boundary conditions. This is justified by the small length of both, and assuming anti-symmetry of temperature distribution along the loop:

$$T(0^+,t) + T(1^-,t) = (1 + T(1^-,t))\left[1 - e^{-1/q}\right]; \; q \geq 0 \; (t > 0) \qquad (3)$$

$$T(0^+,t) + T(1^-,t) = (-1 + T(0^+,t))\left[1 - e^{-1/|q|}\right]; \; q < 0 \; (t > 0) \qquad (4)$$

Initial conditions are also needed for uniquely identifying the specific addressed transient evolution. These are in the form:

$$q(0) = q_0 \qquad (5)$$

$$T(s, 0) = T_0(s) \qquad (0 < s < 1) \qquad (6)$$

Stability of the positive flow fixed point can be studied, in similarity with the original treatment by Welander and current practice in this field, through linearisation of the equations by perturbation [1,4].

Figure 2 reports the neutral curve obtained for ν = 1.75 coherently with the adoption of the Blasius law for wall friction, also showing the location of a reference unstable case.

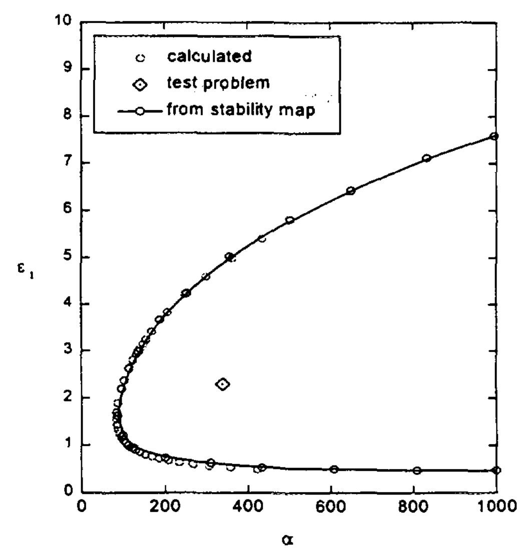

Figure 2 The neutral stability curve for the system of Fig. 1 [1], considering turbulent flow

Adopting Blasius friction law irrespectively of the flow rate does not account for the transition between laminar and turbulent flow, which would require the calculation of a Reynolds number in place of the above defined dimensionless flow rate. However, the reference value adopted for ν reflects a parametric choice that is useful in discussing the behaviour of system codes when applied to Welander's problem under turbulent flow conditions. Hence, this choice will be kept throughout the whole paper.

III. NUMERICAL RESULTS

In order to get a reference solution for the problem, free from the effects of truncation error arising from numerical discretization in the space co-ordinate, a modal expansion has been adopted for temperature along the thermosyphon loop. The following series is used for representing the temperature distribution along the loop:

$$T(s,t) = \sum_{k=0}^{\infty} S_{2k+1}(t)\cdot \sin[(2k+1)\pi s] + \sum_{k=0}^{\infty} C_{2k+1}(t)\cdot \cos[(2k+1)\pi s] \qquad (7)$$

It includes only odd numbered terms due to the above mentioned anti-symmetry. Since imposing the boundary conditions (5-6) would be difficult in the present case, the energy equation along the whole loop is solved introducing an appropriate mathematical representation for the source and the sink. The following form of the energy equation is therefore adopted:

$$\frac{\partial T}{\partial t} + q\frac{\partial T}{\partial s} = F_{mod}(q)[\,\delta(s)(1-T) + \delta(s-1)(-1-T)\,] \qquad (0 \le s \le 2) \qquad (8)$$

where $\delta(s)$ and $\delta(s-1)$ are Dirac's delta function centred on the location of the source (s=0) and the sink (s=1) respectively. The function $F_{mod}(q)$ is introduced to have perfect matching between the steady-state conditions calculated by the modal expansion and the ones obtained by the theoretical developments shown above (Ref. 5 gives a detailed analysis of the equivalent source definition and it was also used in [2]).

It can be seen that, due to a symmetrical treatment of the source and the sink in the modal expansion the source average fluid temperature is just the arithmetic mean of the inlet and outlet temperature. Since this is not the case in Welander's treatment, it is necessary to include a source-sink heat transfer multiplier having the following form:

$$F_{mod}(q) = \frac{1-e^{-1/q}}{1/q}\cdot\frac{2}{1+e^{-1/q}} \qquad (9)$$

It can be noted that for increasing dimensionless flow rate the value of the multiplier tends to 1.

If the effect of diffusion must be also accounted for (as it will be necessary in the following sections), a second order term is included in the energy balance equations, leading to:

$$\frac{\partial T}{\partial t} + q\frac{\partial T}{\partial s} = \mathcal{D}(q)\frac{\partial^2 T}{\partial s^2} + F_{mod}(q)\big(\delta(s)\cdot(1-T) + \delta(s-1)\cdot(-1-T)\big) \qquad (0 \le s \le 2) \qquad (10)$$

where $\mathcal{D}(q)$ is an appropriate diffusion coefficient, here assumed to be a function of the dimensionless flow rate.

Putting: $q = y_0$, $S_{2k+1}(t) = y_{2k+1}$ and $C_{2k+1}(t) = y_{2k+2}$ (k=0, 1, ...)

and applying the usual weighting in the integration domain, orthogonality of the trigonometric functions adopted in the expansion leads to the following system of ODEs

$$\dot{y}_0 = \alpha\sum_{l=0}^{\infty}\frac{2}{(2l+1)\pi}\cdot y_{2l+1} - \varepsilon y_0^{\nu}$$

$$\dot{y}_{2k+1} = (2k+1)\pi\, y_0\, y_{2k+2} - \mathcal{D}(y_0)\,(2k+1)^2\,\pi^2\, y_{2k+1}$$

$$\dot{y}_{2k+2} = -(2k+1)\pi\, y_0\, y_{2k+1} - \mathcal{D}(y_0)\,(2k+1)^2\,\pi^2\, y_{2k+2} + 2\,F_0(y_0)\left(1 - \sum_{l=0}^{\infty} y_{2l+2}\right)$$

$$(k = 0,1,\ldots,\infty) \qquad (11)$$

It may be noted that in the case of the rectangular loop with a point source and sink it is not possible by this expansion to reach a finite-dimensional set of equations governing the complete dynamics of the system. Anyway, a reliable truncated solution is feasible, with a reasonable computational effort, by considering a sufficient number of modes. Actually, the presence of the Dirac's delta function in the energy balance equation raises the problem of convergence of the series expansion to the exact solution, since $\delta(s)$ excites at the same extent all the modes. Nevertheless, as it is found that the series converges to the exact solution in steady state conditions, it is simply assumed that the same holds also for transient conditions. The ODEs system has been solved using a classical 4-th order Runge-Kutta method, adopting different numbers of modes. The results showed the expected projection of the chaotic attractor as obtained for an unstable case without diffusion (the conspicuos isolated point in Fig. 2)

Now the simplest scheme for the solution of the governing equations will be analysed (see [6] for more schemes and details). This is the forward-time, upwind-space differencing or FTUS. The algebraic equations expressing the energy balance are the following:

- $q \geq 0$

$$T_i^{n+1} = (1 - C)\, T_i^n + C\, T_{i-1}^n \qquad (i=2, \ldots, N-1)$$

$$T_N^{n+1} = \left(1 - C - \frac{\Delta t}{\Delta s} F_{nod}(q^n)\right) T_N^n + C T_{N-1}^n - \frac{\Delta t}{\Delta s} F_{nod}(q^n)$$

$$T_1^{n+1} = -T_N^{n+1} \tag{12}$$

- $q < 0$

$$T_i^{n+1} = (1 + C)\, T_i^n - C\, T_{i+1}^n \qquad (i=2, \ldots, N-1)$$

$$T_1^{n+1} = \left(1 + C - \frac{\Delta t}{\Delta s} F_{nod}(q^n)\right) T_1^n - C T_2^n + \frac{\Delta t}{\Delta s} F_{nod}(q^n) \tag{13}$$

$$T_N^{n+1} = -T_1^{n+1}$$

where C is the Courant number:

$$C = \frac{q \cdot \Delta t}{\Delta s} \tag{14}$$

and

$$\Delta s = \frac{1}{N-1} \tag{15}$$

with N being the number of nodes. The function $F_{nod}(q)$ is a source-sink heat transfer multiplier, similar to the one adopted for the nodal expansion, introduced in order to calculate the steady-state conditions in coincidence with the exact solution.

The momentum equation is discretised in time as follows:

$$q^{n+1} = q^n + \left(\frac{\alpha}{N-1} \sum_{i=1}^{N-1} \frac{T_i^n + T_{i+1}^n}{2} - \varepsilon\, (q^n)^{\nu} \right) \Delta t \tag{16}$$

The steady-state conditions calculated by the method are:

$$T_1^S = T_2^S = \ldots = T_{N-1}^S = -T_N^S = T_{leg}^S$$

$$T_{leg}^S = \frac{F_{nod}(q^S)}{2q^S + F_{nod}(q^S)} \tag{17}$$

where:

$$F_{nod}(q^S) = q^S \left(1 - e^{-1/q^S}\right) / e^{-1/q^S} \tag{18}$$

Thus, the result is:

$$(q^S)^{\nu+1} = \frac{\alpha}{2\nu} \frac{1 - e^{-(1/q^S)}}{q^{(1/q^S)}} \frac{2}{1 + e^{-(1/q^S)}} \frac{N-2}{N-1}$$

It can be easily shown that for $N \to \infty$ the above equation becomes coincident with the exact formulation given by Eqs. (1-3).

A different method has been adopted in the present paper to study the stability of the numerical solution of Welander's problem. This approach can be considered an extension of methods adopted for assessing stability of numerical schemes, in similarity with the usual techniques for linear stability analysis of PDEs. The main reasoning behind the methodology is shortly summarised in what follows. A finite-difference numerical method for a time-marching problem can been written as an algebraic ℵ-vector equation relating the ℵ values of the unknown function at the n-th and (n+1)-th time level ($\underline{y}^n$ and $\underline{y}^{n+1}$), grid parameters (in the present case, Δs and Δt) and physical parameters (α, ε). This algebraic equation represents the discretised form of the original PDEs together with the related boundary conditions. In our specific case for the above-described numerical methods it is:

$$\underline{F}(\underline{y}^n, \underline{y}^{n+1}, \Delta t, \Delta s, \alpha, \varepsilon) = 0 \tag{19}$$

It wil be now shown that:

- studying stability of steady-state solutions of a mathematical problem by numerical means is feasible and the effect of truncation error can be clearly pointed out;
- care must be taken in avoiding numerical instabilities or in recognising them in the obtained stability maps.

The vector function $\underline{F}$ is generally non-linear. Therefore, determining the steady state conditions (i.e., the fixed points) may require the iterative solution of the equation:

$$\underline{F}(\underline{y}^n = \underline{y}^S, \underline{y}^{n+1} = \underline{y}^S, \Delta t, \Delta s, \alpha, \varepsilon) = 0 \tag{20}$$

Once the fixed points have been determined, their stability can be studied through linearisation by perturbation. Then, considering small deviations from the selected fixed point

$$\underline{y}^n = \underline{y}^S + (\delta \underline{y})^n \qquad \underline{y}^{n+1} = \underline{y}^S + (\delta \underline{y})^{n+1} \tag{21}$$

and substituting into Eq. (19), second order terms can be neglected and Eq. (20) can be used to reach the following relationship between perturbations at the n-th and at the (n+1)-th time levels:

$$(\delta \underline{y})^{n+1} = -(\underline{\underline{J}}_S^{n+1})^{-1} \cdot \underline{\underline{J}}_S^n \cdot (\delta \underline{y})^n \tag{22}$$

where $\underline{\underline{J}}_S^n$ and $\underline{\underline{J}}_S^{n+1}$ denote the Jacobian matrices of $\underline{F}$ with respect to $\underline{y}^n$ and $\underline{y}^{n+1}$ respectively, calculated at the selected fixed point. It is clearly understood that the inverse of $\underline{\underline{J}}_S^{n+1}$ must exist for any meaningful time-marching numerical scheme; in particular, $\underline{\underline{J}}_S^{n+1}$ is equal to the identity matrix for explicit numerical methods and boundary conditions. It is then argued that stability can be discussed considering the eigenvalues of the matrix expressing the amplification of perturbations

$$\underline{\underline{A}} = -(\underline{\underline{J}}_S^{n+1})^{-1} \cdot \underline{\underline{J}}_S^{n} \qquad (23)$$

as results assuming exponential growth or decay of perturbation vectors. In particular, given the spectral radius of the matrix, $\rho(\underline{\underline{A}})$, it is useful to consider the quantity:

$$\Delta\rho = \rho(\underline{\underline{A}}) - 1 \qquad (24)$$

as a *margin in excess to neutral stability*, which takes negative values for stable conditions and positive values for unstable ones. This quantity can be therefore used to find neutral stability conditions and to set up stability maps. Then, it is here preferred to calculate $\Delta\rho$ throughout a selected $\alpha-\varepsilon$ rectangular domain, thus identifying with the aid of contour plots regions with a different degree of stability. This method is easier to implement in computer programs and has the advantage to provide a greater deal of information, at the price of a reasonable increase in computing effort.

Figure (3) reports the results obtained for the FTUS method with 30, 40, 50 and 100 nodes and $\Delta t=10^{-4}$. It can be noted that almost no unstable region is found within the addressed domain with 30 nodes, whereas increasing the detail of discretization unstable conditions are predicted for lower and lower values of α. This clearly explains the behaviour observed in [2] and shows the dramatic quantitative impact of truncation error on the prediction of stability.

Figure 4 illustrates the map for 100 nodes. It is interesting to compare the results obtained for the explicit upwind method with the results of the modal solution with a second order term simulating numerical diffusion. With this aim, the diffusion coefficient is defined as:

$$\vartheta(q) = \frac{|q|\,\Delta s}{2}\left(1 - \frac{|q|\,\Delta t}{\Delta s}\right) \qquad (25)$$

as resulting from the analysis of truncation error for the FTUS method. It may be shown that the predicted stability conditions are very similar for the nodal and the modal solution with equivalent dissipative effects and the agreement is improved by increasing the number of nodes. This confirms the overwhelming importance of the second order term alone in determining the overall truncation error effect on stability predictions. Figure 2 shows the linear stability curve obtained by the modal solution with no diffusion ($\vartheta(q)=0$). It shows, as expected, its close agreement with the stability curve obtained by the conventional linear stability analysis.

Finally, Figure 5 shows the stability maps for various first and 2nd order methods obtained with $\Delta t=10^{-4}$. It is clearly visible that these methods provide relatively very accurate predictions of the stability boundary. The changes observed in the maps increasing the number of nodes up to 100 are minimal, supporting the conclusion that in the present case the effect of truncation error on stability prediction is due almost exclusively to the second order *dissipative term. The low Courant number used makes the* FTUS results almost as diffusive as the ITUS ones.

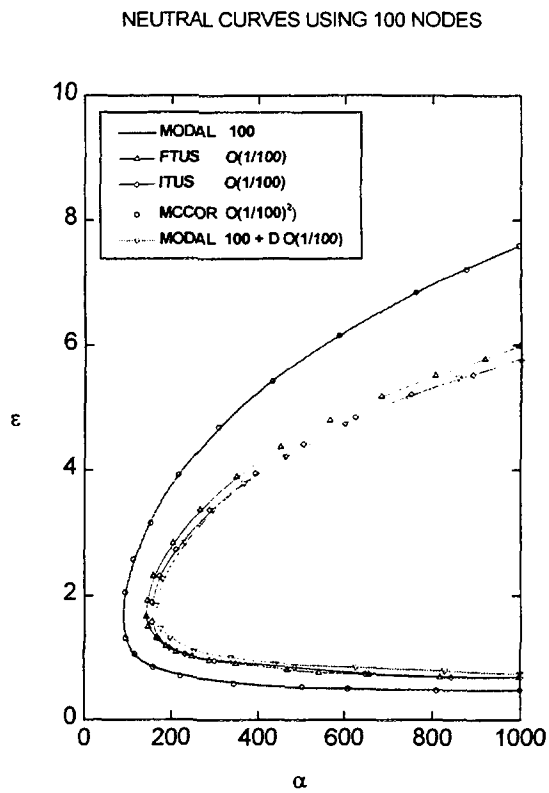

Figure 5 Comparison of the neutral curves obtained with different approximations (ITUS: implicit-time, Upwind-Space, MCCOR: Mac Cormack)

IV. CONCLUSIONS

The results obtained in the present work allowed assessing the ability of numerical methods in predicting stability in single-phase natural circulation. Although the considered specific problem is representative of a particular class of fluid-dynamic instabilities, a fundamental similarity exists with other stability phenomena considerably extending the validity of the obtained conclusions.

a) Though most of the observed qualitative trends were expected on the basis of previous knowledge about the properties of the numerical schemes considered, the results give quantitative information on nodalization effects.

b) The results shown support the use of higher order

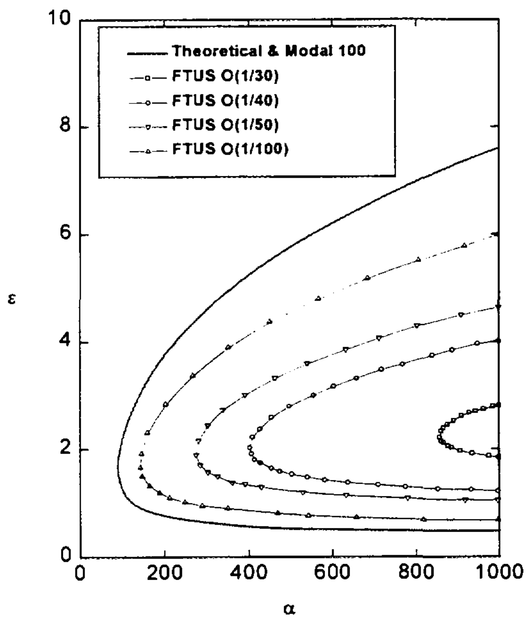

Figure 3 The effect of the number of nodes on the Neutral stability curve using the FTUS scheme.

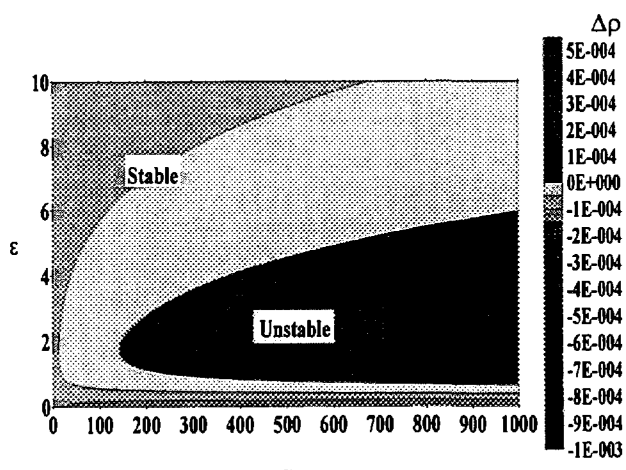

Figure 4 Stability map for the FTUS scheme, 100 nodes

numerical schemes in all the cases in which stability prediction is the relevant objective. However, feasibility of this choice should be demonstrated considering that a certain degree of diffusion is generally considered desirable in code applications since it helps increasing "robustness".

c) The methodology adopted in this work for setting up stability maps shows that a linear stability analysis based on numerical methods is feasible and, if the appropriate nodalization detail and/or higher order schemes are adopted, it can even result as reliable as the usual frequency-domain techniques.

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