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A COMPARATIVE STUDY OF THE REACTIVITY EFFECTS OF UNCERTAINTY IN ABSOLUTE DELAYED-NEUTRON YIELD IN NEUTRON MULTIPLYING SYSTEMS

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70 Keywords/Descriptors :REACTOR KINETICS; REACTIVITY; DELAYED NEUTRONS; DELAYED NEUTRON FRACTIONS; REACTOR CORES; SUBCRITICAL ASSEMBLIES; DIMPLE REACTOR; ZEBRA REACTOR; FAST FISSION; THERMAL FISSION; ISOTOPE RATIO; URANIUM 235; URANIUM 238; URANIUM 233; PLUTONIUM 239; THORIUM 232

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A Comparative Study of the Reactivity Effects of Uncertainty in Absolute Delayed-Neutron Yield in Neutron Multiplying Systems

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Abstract—*Estimates of reactivity (isotopic and total) and corresponding uncertainties in reactivity resulting from uncertainty in absolute delayed neutron yield have been made for a fast reactor core ZEBRA and a thermal reactor core DIMPLE using the period-reactivity relation and the calculated values of total effective delayed neutron fraction. Computations are made for four delayed neutron yield evaluations (Keepin, Tomlinson, Tuttle '75, Tuttle '79) and selected measurements for three reactor periods. Isotopic reactivity is found the lowest for Keepin yield and the highest for that of Tuttle (1975) in both the cores, and is most sensitive to ^{239}Pu data in Zebra and to ^{235}U in Dimple. For small reactivity changes, negligible differences among the data sets are found. In Zebra, uncertainty in isotopic reactivity is the highest for ^{239}Pu isotope with Keepin set giving the highest uncertainty in total reactivity and Tuttle sets the lowest, the values lying between $\simeq \pm 1.4 - 4\%$. In Dimple core, ^{235}U scores much above ^{238}U with Keepin set in the lead. But the lowest uncertainty comes from Tomlinson set for ^{235}U isotope and from Tuttle sets for ^{238}U isotope. Total uncertainty in overall reactivity in Dimple is highest for Keepin set ($\pm 4.4\%$) and lowest for Tomlinson set ($\pm 1.9\%$). The measured yields consistently give higher uncertainties in reactivity. For both fast and thermal fissions, for all the isotopes and for all the periods, maximum six-group contribution to reactivity comes from ^{233}U isotope and the second group of delayed neutrons is the largest contributor to reactivity in all cases. The present study is of importance to the interpretation of reactor kinetic experiments and indicates the need for reduction of uncertainties in absolute or total DN yields.*

I. INTRODUCTION

Of the diverse uses that delayed neutron (DN) information have in nuclear science and technology,[1,2] the most major and demanding are in the measurement and interpretation of fission reactor reactivity effects. These reactivity effects arise from changes in material content or temperature of reactors and include control rod worths, reactivity and feedback coefficients (temperature coefficients, void coefficients, Doppler coefficient, deformation reactivity, burn-up reactivity, shut-down reactivity margin, danger coefficient,etc). In nuclear fission reactors, there are two scales of reactivity: the theoretical and the experimental. While theoretical reactivity $\frac{\Delta k}{k}$ is calculated in absolute units as the difference of two eigenvalues,[3] experimental reactivity ρ is measured by analysing the time behaviour of reactor neutrons using DN parameters as represented by fractional group yields (a_i) or group yields (β_i) and decay constants (λ_i).[4] The two scales are related to each other by the effective delay fraction or beta-effective (β_{eff}):[5]

$$\rho = \frac{1}{\beta_{eff}}\left(\frac{\delta k}{k}\right)$$

and [1,6]

$$\beta = \frac{\overline{\nu}_d}{\overline{\nu}} = \sum_i \beta_i \qquad \beta_{eff} = \overline{\gamma}\beta = \sum_i \gamma_i \beta_i = \sum_i \beta_{ieff}$$

$$\beta_i = \frac{\overline{\nu}_{di}}{\overline{\nu}} \qquad a_i = \frac{\overline{\nu}_{di}}{\overline{\nu}_d} = \frac{\beta_i}{\beta} \qquad \sum_i a_i = 1$$

$$\beta^q_{ieff} = \frac{\text{Delayed neutron worth}}{\text{Total worth of all prompt and delayed neutrons}}$$
$$= \frac{\int [\sum_{g'} \chi^{di}_{g'} \phi^*_{g'}] \ \nu^q_{di} \ [\sum_g N^q (\sigma^q_f \phi)_g] \, dV}{\int \sum_q [(\sum_{g'} \chi^p_{g'} \phi^*_{g'}) \ (\sum_g N^g (\nu_p \sigma_f)^q_g \phi_g) + \sum_i (\sum_{g'} \chi^{di}_{g'} \phi^*_{g'}) \ \nu^q_{di} \ (\sum_g N^q (\sigma^q_f \phi)_g)] \, dV}$$

Here β refers to total DN fraction, $\overline{\nu}_d$ to the absolute yield of delayed neutrons, $\overline{\nu}$ to total fission neutron multiplicity, q to fissile isotopes, i to DN groups, g and g' to neutron energy groups and integrals are over reactor volume V. β_i's are the delayed neutron fractions of the precursor groups, i.e., the fractions of the total yield of delayed neutrons in fission $\overline{\nu}$ that leads to a neutron in the ith group. $\overline{\gamma}$ and γ_i denote the neutron effectiveness factors and represent the effectiveness of a delayed neutron with respect to a prompt neutron in causing fission. Data required for the calculations of β_{ieff} are: the absolute or total delayed neutron yield per fission (ν^q_{di}, assumed constant), the real and adjoint neutron fluxes (ϕ, ϕ^*), the delayed and the prompt neutron energy spectra (χ^{di}, χ^p), fission cross-sections (σ_f), atomic density (N) and the prompt fission multiplicity, i.e., the number of prompt neutrons per fission ($\overline{\nu}_p$). The major source of uncertainty in β_{eff} is in absolute delayed neutron yield and the most stringent requirements for knowledge of β_{eff} are in the interpretation of critical experiments for control reactivities, sample reactivity worths and other reactor parameters.[7]

In this report, a comparative analysis of the isotopic and total reactivity effects of beta-effective and of uncertainties in reactivity (isotopic and total) due to uncertainties in absolute DN yield is performed for the zero-power fast reactor ZEBRA[8,9] and the zero-power thermal reactor DIMPLE.[10]

II. ABSOLUTE DELAYED NEUTRON YIELD

This is the number of delayed neutrons produced per fission of a fissile nuclide and is the one experimental information that is immediately available and from which the delayed neutron fraction (β), the group yield (β_i) and the fractional group yield or group fraction (a_i)

are determined. The connection between these parameters has already been displayed in Sec. I. Accuracies required on DN yield depend on specific reactor applications and are deduced from global accuracy required on β_{eff}.[11] For a target accuracy of $\pm 3\%$ on β_{eff}, desired accuracies on $\overline{\nu}_d$ are $\pm 1.5 - 2\%$ for critical experiment operation and interpretation, $\pm 4\%$ for power reactor (fast and thermal) operation and $\pm 9\%$ for reactor design, dynamics and safety studies. The desired percentage accuracies on DN yield for the fissionable isotopes of significance in power reactor development and for the interpretation of fast-spectrum critical experiments in particular are: ± 1.5 (^{233}U, ^{235}U, ^{238}U, ^{239}Pu), ± 3 (^{232}Th), ± 5 (^{240}Pu, ^{241}Pu) and ± 20 (^{242}Pu, ^{241}Am).

Tables I and II respectively list the comparative results of $\overline{\nu}_d$ from evaluations and selected measurements for fast and thermal fissions. Due to the large number of $\overline{\nu}_d$ measurements, particularly for some of the fissionable nuclides, only a selection of these is listed here. The ^{242}Pu values quoted under Keepin are those of Krick and Evans.[12] All values in the table are average. Besant et al [13] indicated lower values for the more common isotopes, while Walker and Weaver[14] proposed the values of $1.65 \pm 10\%$ for thermal and $1.57 \pm 10\%$ for fast fission of ^{235}U. The variation of yields between isotopes are expected in terms of the systematics of nuclear fission and have been well correlated with models of the fission process. Recent work[15,16] has demonstrated that DN yield and many nuclear properties are strongly correlated to the values of 2Z-N of nuclei/compound nuclei where Z is the atomic number and N is the neutron number. These correlations can serve to predict the DN yields of nuclides having no experimental values. Differences in the yield of ^{235}U and ^{239}Pu are notable. Same is with ^{233}U and ^{235}U nuclides. The above comparisons of $\overline{\nu}_d$ shows that for fast fission, the uncertainties (error margins) lie in the range of about $2 - 8\%$ for ^{232}Th, $3 - 10\%$ for ^{233}U, $1 - 10\%$ for ^{235}U, $1 - 6\%$ for ^{238}U, $2 - 9\%$ for ^{239}Pu, $8 - 10\%$ for ^{240}Pu, $7 - 14\%$ for ^{241}Pu, and $11 - 31\%$ for ^{242}Pu. For thermal fission, the uncertainties are $3 - 7\%$ (^{233}U), $2 - 5\%$ (^{235}U), $3 - 8\%$ (^{239}Pu) and $7 - 14\%$ (^{241}Pu). Note that the uncertainties are larger than the desired accuracies for all the isotopes and the experimental uncertainties are generally of the same order or higher than the evaluations.

In view of the present emphasis on ^{233}U fuel breeding cycle, the data for ^{233}U and ^{232}Th are inadequate. Though ^{235}U and ^{239}Pu data seem to be within the design accuracies needed for reactors involving these fuels, for the Pu higher isotopes, the DN yields are unsatisfactory. For instance, the measured yield for ^{242}Pu has an uncertainty of 31% which need to be improved considerably. These call for additional measurements and more consistent evaluated results particularly for reducing the uncertainty in the yields to reach the $\pm 1.5\%$ accuracy. A slight readjustment of the absolute yields for ^{238}U and ^{239}Pu might be possible taking the integral experiment results into account. The independency of the absolute yields with respect to neutron energy between 1 keV and 4 MeV seems to be clearly established. But, the yield values for all interesting materials are almost non-existent above 7 MeV except in the 14 MeV region. Complementary information are therefore required between 4 and 14 MeV, especially for fast breeders with relatively hard spectra. The present interest in the utilization of high level nuclear waste by separation into individual actinides and fission products, and the possibility of using higher actinides as reactor fuels have led to generation of DN data and data for fission product yields for higher actinides (^{237}Np, ^{242}Am, ^{243}Am, ^{243}Cm, ^{245}Cm).[17,18]

TABLE I

Comparison of Absolute Delayed Neutron Yield From Fast Fission (n/100 Fissions)

Fission nuclide	Keepin (Ref.25)	Tomlinson (Ref.24)	Tuttle '75 (Ref.25)	Tuttle '79 (Ref.32)	Select Measurements(Ref.1)
^{232}Th	4.96 ± 0.30 (6%)	5.2 ± 0.4 (7.7%)	5.47 ± 0.12 (2.2%)	5.31 ± 0.23 (4.2%)	4.96 ± 0.30 (6.05%)
^{233}U	0.70 ± 0.06 (8.6%)	0.69 ± 0.02 (2.9%)	0.73 ± 0.02 (2.7%)	0.74 ± 0.04 (4.9%)	0.78 ± 0.08 (10.3%)
^{235}U	1.65 ± 0.08 (4.5%)	1.65 ± 0.03 (1.8%)	1.71 ± 0.02 (1.3%)	1.67 ± 0.04 (2.1%)	1.71 ± 0.17 (9.9%)
^{238}U	4.12 ± 0.25 (6.1%)	4.40 ± 0.21 (4.8%)	4.51 ± 0.06 (1.3%)	4.39 ± 0.10 (2.3%)	4.12 ± 0.25 (6.1%)
^{239}Pu	0.63 ± 0.04 (7.1%)	0.64 ± 0.02 (3.1%)	0.66 ± 0.01 (2.0%)	$0.63 \pm .016$ (2.5%)	0.65 ± 0.06 (9.2%)
^{240}Pu	0.88 ± 0.09 (10.2%)	0.88 ± 0.09 (10.25%)	0.96 ± 0.11 (11.5%)	0.95 ± 0.08 (8.4%)	0.88 ± 0.09 (10.2%)
^{241}Pu	1.54 ± 0.22 (14.3%)	1.59 ± 0.16 (10.0%)	1.63 ± 0.16 (9.8%)	1.52 ± 0.11 (7.3%)	–
^{242}Pu	01.5 ± 0.50 (33%)	1.6 ± 0.5 (31.2%)	2.28 ± 0.25 (11.0%)	2.21 ± 0.26 (11.8%)	1.6 ± 0.5 (31.25%)

TABLE II

Comparison of Absolute Delayed Neutron Yield From Thermal Fission (n/100 Fissions)

Fission nuclide	Keepin (Ref.25)	Tomlinson (Ref.24)	Tuttle '75 (Ref.25)	Tuttle '79 (Ref.32)	Select Measurements(Ref.1)
^{233}U	0.66 ± 0.045 (6.8%)	0.69 ± 0.02 (2.9%)	$0.664 \pm .018$ (2.8%)	0.667 ± 0.029 (4.3%)	0.66 ± 0.04 (6.06%)
^{235}U	1.58 ± 0.075 (4.7%)	1.65 ± 0.03 (1.8%)	$1.654 \pm .042$ (2.5%)	1.621 ± 00.05 (3.1%)	1.58 ± 0.07 (4.4%)
^{239}Pu	0.61 ± 0.045 (7.4%)	0.64 ± 0.02 (3.1%)	$0.624 \pm .024$ (3.9%)	0.628 ± 0.038 (6.0%)	0.61 ± 0.05 (8.2%)
^{241}Pu	1.54 ± 00.22 (14.3%)	1.59 ± 0.16 (10.0%)	01.56 ± 0.16 (10.2%)	1.52 ± 0.11 (7.3%)	1.57 ± 0.15 (9.6%)

III. DELAYED NEUTRON GROUP CONSTANTS

In reactor calculations, delayed neutron data are represented by a sum of six exponential terms with time constants λ_i and fractional yields a_i together with associated energy spectra for the six groups. This classical six-group structure of delayed neutron emission is a mathematical one having originated as six-term, twelve-parameter optimum least-squares fit to experimentally measured aggregate data.[19,20] For a single fissionable isotope, the optimum (best) fit was achieved using six of the pseudo-precursors in the sense that five (or fewer) gave a poorer predictive fit (unsatisfactory convergence and large errors) and seven (or more) periods led to invariably large weighted variance of the fit. Hence, the use of six groups even though majority of these groups contain more than one physical precursors and the group constants represent composite values. Quite sometime ago, Besant et al[13] suggested an approach taking a more direct physical representation, but the simple six-group temporal representation of delayed neutrons has come to stay and is very well established.

Tables III and IV quote the details of fast- and thermal-fission DN group data from Keepin[21] (excluding ^{241}Pu and ^{242}Pu). The uncertainties in the data are least-squares propagated errors with greater statistical precision for the fast fission measurements. The ^{241}Pu thermal data are as reported by Cox[22] and taken from Lewins.[23] The sixth-group absolute yield for thermal fission and the six-group absolute yields for fast fission of ^{241}Pu together with the error margins were calculated as $a_i\beta\overline{\nu}$ with $\overline{\nu} = 2.934 \pm 0.012$ and $\overline{\nu} = 2.99 \pm 0.06$ respectively.[23] For ^{242}Pu isotope, the fractional group yields and decay constants are as esimated by Tomlinson[24] with Tuttle's (1975)[25] uncertainties. The absolute group yields (Y_i) for ^{242}Pu are calculated using the relation

$$Y_i = a_i \overline{\nu}_d$$

and the corresponding uncertainties from

$$\Delta Y_i = Y_i \left(\frac{\Delta a_i}{a_i} + \frac{\Delta \overline{\nu}_d}{\overline{\nu}_d}\right).$$

The total fractional yield β given for each isotope are from Lewins[23] except for ^{242}Pu for which β is calculated from $\frac{\overline{\nu}_d}{\overline{\nu}}$ and the error on β using the formula[26]

$$\Delta\beta = \frac{\overline{\nu}_d\left(\frac{\Delta\overline{\nu}_d}{\overline{\nu}_d} - \frac{\Delta\overline{\nu}}{\overline{\nu}}\right)}{\overline{\nu} + \Delta\overline{\nu}}$$

with $\overline{\nu} = 2.18 \pm 0.0921$.[27]

It is noted from the tables that the group decay constants λ_i do not depend very much either on the fissile isotope or on the energy of the neutron causing fission (fast or thermal). The small variation in λ_i between isotopes, however, underlies the fact that the six-groups are not explicit species with a unique physical λ_i. Half-lives $(=\frac{0.693}{\lambda_i})$ are seen to span a range from about 0.17 sec to 56 sec and mean times $(=\frac{1}{\lambda_i})$ from a few to some 80 sec. Quoted standard deviations for the total fractional yield β are as obtained from the usual combination of assumed independent deviations of the $\beta\overline{\nu}$ and $\overline{\nu}$ data. The fractional group yield or yield repartition a_i values for each isotope, depend more on the precursor group i than on the isotope. Since the internal six-group data are highly correlated by the fitting

TABLE III

Delayed-Neutron Half-Lives, Decay Constants, and Yields From Fast (Fission-Spectrum) Fission

Isotope	Group index, i	Half-life T_i, sec	Decay constant λ_i, sec^{-1}	Relative group yield, $a_i \equiv \beta_i/\beta$	Absolute group yield, %
^{232}Th	1	56.03 ± 0.95	0.0124 ± 0.0002	0.034 ± 0.002	0.169 ± 0.012
$\beta =$	2	20.75 ± 0.66	0.0334 ± 0.0011	0.150 ± 0.005	0.744 ± 0.037
0.0223	3	5.74 ± 0.24	0.121 ± 0.005	0.155 ± 0.021	0.769 ± 0.108
± 0.0014	4	2.16 ± 0.08	0.321 ± 0.011	0.446 ± 0.015	2.212 ± 0.110
	5	$0.571 \pm .042$	1.21 ± 0.090	0.172 ± 0.013	0.853 ± 0.073
	6	$0.211 \pm .019$	3.29 ± 0.297	0.043 ± 0.006	0.213 ± 0.031
^{233}U	1	55.11 ± 1.86	0.0126 ± 0.0004	0.086 ± 0.003	0.060 ± 0.003
$\beta =$	2	20.74 ± 0.86	0.0334 ± 0.0014	0.274 ± 0.005	0.192 ± 0.009
0.00266	3	5.30 ± 0.19	0.131 ± 0.005	0.227 ± 0.035	0.159 ± 0.025
$\pm .00007$	4	2.29 ± 0.18	0.302 ± 0.024	0.317 ± 0.011	0.222 ± 0.012
	5	$0.546 \pm .108$	1.27 ± 0.266	0.073 ± 0.014	0.051 ± 0.010
	6	$0.221 \pm .042$	3.13 ± 0.675	0.023 ± 0.007	0.016 ± 0.005
^{235}U	1	54.51 ± 0.94	0.0127 ± 0.0002	0.038 ± 0.003	0.063 ± 0.005
$\beta =$	2	21.84 ± 0.54	0.0317 ± 0.0008	0.213 ± 0.005	0.351 ± 0.011
0.00660	3	6.00 ± 0.17	0.115 ± 0.003	0.188 ± 0.016	0.310 ± 0.028
$\pm .00013$	4	2.23 ± 0.06	0.311 ± 0.008	0.407 ± 0.007	0.672 ± 0.023
	5	$0.496 \pm .029$	1.40 ± 0.081	0.128 ± 0.008	0.211 ± 0.015
	6	$0.179 \pm .107$	3.87 ± 0.369	0.026 ± 0.003	0.043 ± 0.005
^{238}U	1	52.38 ± 1.29	0.0132 ± 0.0003	0.013 ± 0.001	0.054 ± 0.005
$\beta =$	2	21.58 ± 0.39	0.0321 ± 0.0006	0.137 ± 0.002	0.564 ± 0.025
0.0161	3	5.00 ± 0.19	0.139 ± 0.005	0.162 ± 0.020	0.667 ± 0.087
$\pm .00062$	4	1.93 ± 0.07	0.358 ± 0.014	0.388 ± 0.012	1.599 ± 0.081
	5	$0.490 \pm .023$	1.41 ± 0.067	0.225 ± 0.013	0.927 ± 0.060
	6	$0.172 \pm .009$	4.02 ± 0.214	0.075 ± 0.005	0.309 ± 0.024

TABLE III (Continued)

Isotope	Group index, i	Half-life T_i, sec	Decay constant λ_i, sec^{-1}	Relative group yield, $a_i \equiv \beta_i/\beta$	Absolute group yield, %
^{239}Pu	1	53.75 ± 0.95	0.0129 ± 0.0002	0.038 ± 0.003	0.024 ± 0.002
$\beta =$	2	22.29 ± 0.36	0.0311 ± 0.0005	0.280 ± 0.004	0.176 ± 0.009
0.00212	3	5.19 ± 0.12	0.134 ± 0.003	0.216 ± 0.018	0.136 ± 0.013
$\pm .00010$	4	2.09 ± 0.08	0.331 ± 0.012	0.328 ± 0.010	0.207 ± 0.012
	5	$0.549 \pm .049$	1.26 ± 0.115	0.103 ± 0.009	0.065 ± 0.007
	6	$0.216 \pm .017$	3.21 ± 0.255	0.035 ± 0.005	0.022 ± 0.003
^{240}Pu	1	53.56 ± 1.21	0.0129 ± 0.0004	0.028 ± 0.003	0.028 ± 0.003
$\beta =$	2	22.14 ± 0.38	0.0313 ± 0.0005	0.273 ± 0.004	0.238 ± 0.016
0.00289	3	5.14 ± 0.42	0.135 ± 0.011	0.192 ± 0.053	0.162 ± 0.044
$\pm .00035$	4	2.08 ± 0.19	0.333 ± 0.031	0.350 ± 0.020	0.315 ± 0.027
	5	$0.511 \pm .077$	1.36 ± 0.205	0.128 ± 0.018	0.119 ± 0.018
	6	$0.172 \pm .033$	4.04 ± 0.782	0.029 ± 0.006	0.024 ± 0.005
^{241}Pu	1	$54.0 \pm 001.$	0.0128 ± 0.0002	0.010 ± 0.003	0.016 ± 0.007
$\beta =$	2	$23.2 \pm .5$	0.0299 ± 0.0006	0.229 ± 0.006	0.372 ± 0.055
0.00544	3	5.6 ± 0.6	0.124 ± 0.013	0.173 ± 0.025	0.281 ± 0.075
$\pm .00055$	4	1.97 ± 0.11	0.352 ± 0.018	0.390 ± 0.050	0.634 ± 0.016
	5	0.43 ± 0.04	1.61 ± 0.15	0.182 ± 0.019	0.296 ± 0.067
	6	0.20 ± 0.07	3.47 ± 01.7	0.016 ± 0.005	0.026 ± 0.011
^{242}Pu	1	53.75 ± 01.25	0.0129 ± 0.0003	0.004 ± 0.001	0.006 ± 0.004
$\beta =$	2	23.54 ± 01.04	0.0295 ± 0.0013	0.195 ± 0.032	0.312 ± 0.149
0.00734	3	5.315 ± 0.365	0.131 ± 0.009	0.162 ± 0.048	0.259 ± 0.158
± 0.0019	4	2.06 ± 0.12	0.338 ± 0.020	0.411 ± 0.153	0.658 ± 0.450
	5	0.50 ± 0.03	1.39 ± 0.09	0.218 ± 0.087	0.349 ± 0.248
	6	0.192 ± 0.024	3.65 ± 0.44	0.010 ± 0.003	0.016 ± 00.01

TABLE IV

Delayed-Neutron Half-Lives, Decay Constants, and Yields From Thermal Fission

Isotope	Group index, i	Half-life, sec T_i, sec	Decay constant λ_i, sec^{-1}	Relative group yield, $a_i \equiv \beta_i/\beta$	Absolute group yield, %
^{233}U	1	55.00 ± 0.54	0.0126 ± 0.0003	0.086 ± 0.003	0.057 ± 0.003
$\beta =$	2	20.57 ± 0.38	0.0337 ± 0.0006	0.299 ± 0.004	0.197 ± 0.009
0.00281	3	5.00 ± 0.21	0.139 ± 0.006	0.252 ± 0.040	0.166 ± 0.027
$\pm .00005$	4	2.13 ± 0.20	0.325 ± 0.030	0.278 ± 0.020	0.184 ± 0.016
	5	$0.615 \pm .242$	1.13 ± 0.40	0.051 ± 0.024	0.034 ± 0.016
	6	$0.277 \pm .047$	2.50 ± 0.42	0.034 ± 0.014	0.022 ± 0.009
^{235}U	1	55.72 ± 1.28	0.0124 ± 0.0003	0.033 ± 0.003	0.052 ± 0.005
$\beta =$	2	22.72 ± 0.71	0.0305 ± 0.0010	0.219 ± 0.009	0.346 ± 0.018
0.00700	3	$6.22 \pm .23$	0.111 ± 0.004	0.196 ± 0.022	0.310 ± 0.036
$\pm .00008$	4	$2.30 \pm .09$	0.301 ± 0.011	0.395 ± 0.011	0.624 ± 0.026
	5	$0.610 \pm .083$	1.14 ± 0.15	0.115 ± 0.009	0.182 ± 0.015
	6	$0.230 \pm .025$	3.01 ± 0.29	0.042 ± 0.008	0.066 ± 0.008
^{239}Pu	1	54.28 ± 2.34	0.0128 ± 0.0005	0.035 ± 0.009	0.021 ± 0.006
$\beta =$	2	23.04 ± 1.67	0.0301 ± 0.0022	0.298 ± 0.035	0.182 ± 0.023
0.00227	3	$5.60 \pm .40$	0.124 ± 0.009	0.211 ± 0.048	0.129 ± 0.030
$\pm .00004$	4	$2.13 \pm .24$	0.325 ± 0.036	0.326 ± 0.033	0.199 ± 0.022
	5	$0.618 \pm .213$	1.12 ± 0.39	0.086 ± 0.029	0.052 ± 0.018
	6	$0.257 \pm .045$	2.69 ± 0.48	0.044 ± 0.016	0.027 ± 0.010
^{241}Pu	1	$54.0 \pm 01.$	0.0128 ± 0.0002	0.010 ± 0.003	$.0154 \pm 0.004$
$\beta =$	2	23.2 ± 0.5	0.0299 ± 0.0006	0.229 ± 0.006	$.365 \pm 0.01$
0.00545	3	05.6 ± 0.6	0.124 ± 0.013	0.173 ± 0.025	$.275 \pm 0.04$
$\pm .00054$	4	1.97 ± 0.1	0.352 ± 0.018	0.390 ± 0.050	$.620 \pm 0.08$
	5	$0.43 \pm .04$	1.61 ± 0.15	0.182 ± 0.019	$.290 \pm 0.03$
	6	$0.20 \pm .07$	3.47 ± 01.7	0.016 ± 0.005	$.026 \pm 0.01$

procedure, for standard deviations within isotopes, it is better to use a_i and λ_i than β_i and λ_i. It has been found that the average accuracy of the fit of λ_i's and a_i's is about 0.2% with a maximum deviation of 1% and the resulting error in reactor kinetics parameters calculated using a six-group representation is very small.[28] In particular, beta-effective is insensitive to modifications in a_i for a particular isotope. A recent analysis of kinetic experiments at the PROTEUS facility has shown[29] that Brady and England's[30] six-group DN parameters (which have been incorporated into the ENDF/B-VI and JEF-2.2 evaluations) yield reactivity consistently lower than those obtained using Keepin's dataset by more than 10% and β_{eff} values higher than the current values by 10-20%. Since there is no definite experimental evidence that the ENDF/B-VI data are superior, Williams (the author) concludes that it is better to continue with the ENDF/B-V and JEF-1.1 DN evaluations.

IV. METHOD OF CALCULATION

For small reactivity perturbations in stationery-fuelled critical reactor systems, the reactivity ρ is determined from the very famous period-reactivity relation or the "in-hour" equation[31] in point model, the basis of reactor kinetics as

$$\rho = \frac{l_p}{Tk} + \sum_{i,q} \frac{\beta^q_{ieff}}{1 + \lambda^q_i T}$$

where the asymptotic or persistent or steady reactor period T is measured after the transient components have died and the neutron flux has assumed an exponential form with the time constant T. In practice, the term involving the effective prompt neutron life time l_p is negligible, and the above relation takes the form

$$\rho = \sum_{i,q} \frac{\beta^q_{ieff}}{1 + \lambda^q_i T} \tag{1}$$

Eq.(1) shows that the kinetic behaviour during period measurements is essentially determined by the delayed-neutron periods and the relative abundances or group yields. If we ignore the slight difference between the spectra of the delayed groups, we can write

$$\beta^q_{ieff} = \beta^q_{eff} a^q_i$$

and Eq.(1) becomes

$$\rho = \sum_q \beta^q_{eff} \sum_i \frac{a^q_i}{1 + \lambda^q_i T} = \sum_q \rho(q) \tag{2}$$

$$\rho(q) = \beta^q_{eff} \sum_i \frac{a^q_i}{1 + \lambda^q_i T} = \frac{\overline{\nu}^q_d}{\overline{\nu}^q_p} \sum_i \frac{a^q_i}{1 + \lambda^q_i T} \tag{3}$$

Since $\beta^q_{eff} = \frac{\overline{\nu}^q_d}{\overline{\nu}^q_p}$, taking differentials, the uncertainty in isotopic reactivity resulting from uncertainty in isotopic delayed-neutron yield is obtained from Eq.(3) as

$$\Delta\rho(q) = \left[\beta^q_{eff}\sum_i \frac{a^q_i}{1+\lambda^q_i T}\right]\frac{\Delta\overline{\nu}^q_d}{\overline{\nu}^q_d} \tag{4}$$

The total uncertainty or error in global (overall) reactivity is calculated from

$$\Delta\rho = \left[\sum_q (\Delta\rho(q))^2\right]^{1/2} \tag{5}$$

Eqs. (2) through (5) serve as the basic equations for calculating reactivity and the corresponding uncertainty in reactivity. For this, the isotopic contributions to beta-effective (β^q_{eff}) are first calculated from the known β_{eff} values. These contributions are typical of power reactors when the fuel is fresh. Then the delayed neutron contributions to reactivity for each isotope $[a^q_i/(1+\lambda^q_i T)]$ are calculated using Keepin's six-group DN parameters a_i's and λ_i's (Tables III and IV). Product of these two quantities [Eq.(3)] gives the isotopic contribution to reactivity $\rho(q)$. Finally, Eq.(4) is used to calculate the uncertainties in reactivity for uncertainties of the absolute DN yields from Keepin,[25] Tomlinson,[24] Tuttle (1975),[25] Tuttle (1979)[32] and for some measured values.[1] The global reactivity is obtained by summing isotopic reactivity for all the isotopes $[\rho = \sum_q \rho(q)]$ and the total uncertainty in overall global reactivity from Eq.(5). Computations are repeated for representative reactor periods of 20 s, 100 s and 300 s. To calculate reactivity and the reactivity uncertainties for the measured yields, the mean of the isotopic contributions to β_{eff} and the mean of reactivities from the four evaluations are taken. Beta-effective values were calculated from the flux and adjoint fluxes using the delayed χ's of Batchelor and Hyder and the a_i, λ_i values of Keepin.[33]

The reactor cores investigated are those of the Zero Energy Breeder Reactor Assembly ZEBRA and the zero-power experimental reactor DIMPLE. Zebra was designed to serve the dual purpose of making basic neutron physics measurements on large dilute systems and for assembling close geometrical mock-ups of actual designs of power reactors. It consisted of natural uranium and plutonium fuel with U/Pu ratio of 3.8 in the inner core and of 3.0 in the outer core. The plutonium isotopic composition was: $^{239}Pu = 77.7\%$, $^{240}Pu = 18.6\%$, $^{241}Pu = 3.2\%$ and $^{242}Pu = 0.5\%$. The reflector was natural uranium ($\simeq 15000$ kg). The reactor was uncooled, had a thermal power of 1 kW and a maximum fast neutron flux of 2.0E+10 n/cm^2-s. Dimple had a small core containing 3% enriched UO_2 pins in a light water moderator and reflector. The zero energy assembly consisted of a large aluminium primary vessel in which a wide range of experimental cores could be assembled. The reactor was controlled by varying the height of the light water moderator allowing study of the experiments without the introduction of perturbing control media.

V. RESULTS AND DISCUSSIONS

Tables V and VI show the calculated isotopic contributions to β_{eff} for four absolute DN yields for the Zebra (fast) and the Dimple (thermal) assemblies respectively. It is seen that the two largest contributions come from ^{238}U and ^{239}Pu for the Zebra core and they are very close. For all the nuclides, Keepin yield gives the lowest β_{eff}, where as Tuttle (1975) yield gives the highest. Only for ^{242}Pu nuclide, all values are equal. The two Tuttle yields give values which are close to each other within $\sim 3\%$. In the case of Dimple assembly, the behaviour is similar except that 81% of the total contribution to β_{eff} comes from ^{235}U isotope and the two Tuttle yields differ by only $\sim 2\%$. The influence of fractional yields and half-lives for the six DN groups is shown in Tables VII, VIII and IX. The six-group contributions are seen to be the lowest for ^{238}U isotope and the highest for ^{233}U isotope for all the periods for fast fission. While ^{239}Pu and ^{233}U isotopes are nearly double that of ^{238}U, the other isotopes are about 1.5 times higher. For thermal fission, the lowest contribution comes from ^{241}Pu and the highest again from ^{233}U. ^{239}Pu isotope is about $25 - 35\%$ and ^{233}U isotope about $40 - 60\%$ higher than ^{241}Pu; but ^{235}U is only $12 - 15\%$ higher. For both fast and thermal fissions (Tables VIII and IX), group 2 (half-life ~ 20 sec) is seen to be the most significant in calculating the reactivity for all the nuclides, its contribution varying from about $40 - 70\%$. Tables X and XI show that for both the Zebra and Dimple cores, Keepin yield gives the lowest reactivity and that of Tuttle (1975) the highest, the other two sets lying in between. The two Tuttle sets give values which are close to Tomlinson's. Reactivity in Zebra fast core is seen to be more sensitive to ^{239}Pu data than for ^{238}U, even if their contributuions to β_{eff} are very close. In the Dimple core, about 86% of the contribution to reactivity comes from ^{235}U isotope, possibly due to its large contribution (81%) to β_{eff}. Table XII gives the spread in reactivity (difference between maximum and minimum reactivity for a given period) among different sets of yield data. It is seen that the spread is maximum for ^{239}Pu (2.75 pcm for T=20 s) in the fast core and for ^{235}U (9.9 pcm for T=20 s) in the thermal core and decreases with increase in reactor period in both the cores. Thus for large periods or small reactivity perturbations of a few cents, differences among the various data sets are negligible. But for larger reactivity changes (more than about 30 cents), the differences are not insignificant. As regards the effect of absolute yield uncertainties on isotopic and total reactivity (Tables X, XI, XIII and XIV), they increase with decrease in reactor period for all the nuclides in both the cores. In Zebra core, the uncertainty in isotopic reactivity is highest for ^{239}Pu and among the yield evaluations, Keepin set gives the highest uncertainty in reactivity and Tuttle sets the lowest. In the Dimple core, the uncertainty in reactivity due to ^{235}U isotope is about 3-13 times larger than due to ^{238}U isotope. Keepin set gives the highest uncertainty in reactivity; but lowest uncertainty is from Tomlinson set for ^{235}U isotope and the Tuttle sets for ^{238}U isotope. The total uncertainties in overall reactivity follow a similar trend in the Zebra core. But in the Dimple core, the lowest is that of Tomlinson. When translated into percentages, the aforesaid total uncertainties lie between $\pm 1.4 - 4.3\%$ in the Zebra core and $\pm 1.9 - 4.4\%$ in the Dimple core for all the data sets. It can, therefore, be said that for both the cores and for all the data sets, the total uncertainty in total reactivity lies in the range of $\pm 2 - 4\%$. While the values from the Tomlinson set are about half of those from the Keepin set for both the cores, those from the Tuttle ('75) set is three times smaller for Zebra core, two times smaller for Dimple core and two times smaller for the Tuttle ('79) set. It is noted that the percentage uncertainties in total reactivity are independent of reactor period, where as in absolute units they are not. Because the error margins are on the higher side for the measured yields, they give the highest error in isotopic (except for ^{240}Pu isotope) as well as in total reactivity ($\sim \pm 5\%$).

VI. CONCLUSIONS

Reactivity and uncertainty in reactivity due to uncertainty in absolute DN yield have been estimated for a fast and a thermal reactor core using four different data sets and measured values of yields. The considerations which have gone into the present study are valid for the analysis and interpretation of kinetic experiments (e.g., power-history method[4]) in other reactor systems. In the specific application considered here, it is found that the reactivity contribution of ^{239}Pu isotope is $\sim$ 60% higher than ^{238}U isotope even if their beta-effective values differ by only $\sim$ 2%. This would mean that if two isotopes possess same/similar β_{eff} values, it does not automatically follow that their reactivity effects would be same/similar. The overall error in overall reactivity is found to be in the range of about $\pm 2-5\%$ for all the data sets for both the cores and they are independent of reactor period for a given data set. Kinetic measurements show that the total error in reactivity is about $\pm 5\%$ (including the calculational uncertainty of $\pm 3\%$) to which the uncertainty in total DN yield contributes about $\pm 3\%$. Therefore, there is incentive to reduce the uncertainties on total DN yield which will partially reduce the overall uncertainty in reactivity. In view of the key role that DN yield data play in the analysis of safety-related parameters (e.g., in the prediction of the fission reactor reactivity scale, β_{eff}) and in the burning of minor actinides in reactors, there is a need for a more complete and accurate knowledge on absolute DN yield.[34] This is possible by conducting additional measurements and by having more consistent evaluated results. If the desired accuracy is not achieved for the absolute yields, improvement in the accuracy of other DN data may be of little benefit.

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TABLE V

Calculated Isotopic Contributions to β_{eff} for Four Sets of Absolute Delayed Neutron Yield (ZEBRA Fast Assembly)

Fission	Isotopic Contribution to Beta-effective (pcm)			
nuclide	Keepin	Tomlinson	Tuttle '75	Tuttle '79
^{235}U	018.6	019.5	019.8	019.1
^{238}U	137.2	144.0	146.0	141.3
^{239}Pu	139.7	146.6	148.7	143.9
^{240}Pu	010.8	011.4	011.5	011.2
^{241}Pu	018.0	018.9	019.2	018.5
^{242}Pu	000.4	000.4	000.4	000.4
Total	324.7	340.8	345.6	334.4

TABLE VI

Calculated Isotopic Contributions to β_{eff} for Four Sets of Absolute Delayed Neutron Yield (DIMPLE Thermal Assembly)

Fission	Isotopic Contribution to Beta-effective (pcm)			
nuclide	Keepin	Tomlinson	Tuttle '75	Tuttle '79
^{235}U	621.3	655.3	656.1	642.3
^{238}U	145.7	153.7	153.9	150.7
Total	767.0	809.0	810.0	793.0

TABLE VII

Typical Calculation of $\sum_{i=1}^{6} \frac{a_i}{1+\lambda_i T}$ for T=20 s (^{235}U Isotope, Fast Fission)

Group	Fast Fission			Thermal Fission		
index,i	a_i	$\lambda_i(\text{sec}^{-1})$	$\frac{a_i}{1+\lambda_i T}$	a_i	$\lambda_i(\text{sec}^{-1})$	$\frac{a_i}{1+\lambda_i T}$
1	0.038	1.27E-2	0.03030	0.033	1.24E-2	0.02644
2	0.213	3.17E-2	0.13035	0.219	3.05E-2	0.13602
3	0.188	1.15E-1	0.05697	0.196	1.11E-1	0.06087
4	0.407	3.11E-1	0.05637	0.395	3.01E-1	0.05627
5	0.128	1.40E 0	0.00441	0.115	1.14E 0	0.00483
6	0.026	3.87E 0	0.00033	0.042	3.01E 0	0.00069
Total			0.27873			0.28512

TABLE VIII

Calculated Six-Group Delayed Neutron Contribution to Reactivity for Three Reactor Periods (Fast Fission)

Fission	$\sum_{i=1}^{6} \frac{a_i}{1+\lambda_i T}$			Group 2 Contribution		
nuclide	T=20 s	100 s	300 s	T=20 s	100 s	300 s
^{232}Th	0.230	0.077	0.030	0.090	0.035	0.014
^{233}U	0.344	0.128	0.052	0.164	0.063	0.025
^{235}U	0.279	0.097	0.038	0.130	0.051	0.020
^{238}U	0.193	0.061	0.023	0.083	0.033	0.013
^{239}Pu	0.305	0.110	0.040	0.173	0.068	0.027
^{240}Pu	0.293	0.103	0.035	0.168	0.066	0.026
^{241}Pu	0.255	0.087	0.034	0.143	0.057	0.023
^{242}Pu	0.231	0.076	0.029	0.123	0.049	0.020

TABLE IX

Calculated Six-Group Delayed Neutron Contributionto Reactivity for Three Reactor Periods (Thermal Fission)

Fission	$\sum_{i=1}^{6} \frac{a_i}{1+\lambda_i T}$			Group 2 Contribution		
nuclide	T=20 s	100 s	300 s	T=20 s	100 s	300 s
^{233}U	0.354	0.132	0.054	0.179	0.068	0.027
^{235}U	0.285	0.099	0.039	0.136	0.054	0.022
^{239}Pu	0.322	0.116	0.046	0.186	0.074	0.03
^{241}Pu	0.255	0.087	0.034	0.143	0.057	0.023

TABLE X

Calculated Isotopic Contributions to Reactivity for Three Reactor Periods and Corresponding Uncertainties in Reactivity Arising from Yield Uncertainties for Four Evaluations and Few Select Measurements (ZEBRA Assembly)

Isotope[1]	Reactivity, $\rho(q)$ (pcm)			Uncertainty in DN Yield	Uncertainty in Reactivity, $\Delta\rho(q)$ (pcm)		
	T=20 s	100 s	300 s	(%)	T=20 s	100 s	300 s
^{235}U	5.18	1.79	0.71	5	0.26	0.09	0.036
	5.44	1.88	0.74	2	0.11	0.04	0.015
	5.52	1.91	0.75	1.3	0.07	0.03	0.001
	5.32	1.84	0.73	2	0.11	0.04	0.015
	5.36	1.86	0.73	10	0.54	0.19	0.073
^{238}U	26.44	8.40	3.21	6	1.59	0.50	0.19
	27.76	8.81	3.37	5	1.39	0.44	0.17
	28.14	8.94	3.42	1.3	0.37	0.12	0.05
	27.24	8.65	3.31	2	0.54	0.17	0.07
	27.40	8.52	3.28	6	1.64	0.51	0.20
^{239}Pu	42.66	15.41	5.66	7	2.99	1.08	0.40
	44.77	16.17	5.94	3	1.34	0.49	0.18
	45.41	16.40	6.02	2	0.91	0.33	0.12
	43.95	15.87	5.83	3	1.32	0.48	0.18
	44.20	15.96	5.86	9	4.00	1.44	0.53
^{240}Pu	3.16	1.11	0.37	10	0.32	0.11	0.04
	3.34	1.17	0.40	10	0.33	0.12	0.04
	3.36	1.18	0.40	11.5	0.39	0.14	0.05
	3.28	1.15	0.39	8	0.26	0.09	0.03
	3.29	1.16	0.39	10	0.33	0.12	0.04
^{241}Pu	4.59	1.56	0.60	14	0.64	0.22	0.08
	4.82	1.64	0.63	10	0.48	0.16	0.06
	4.90	1.66	0.65	10	0.49	0.17	0.07
	4.72	1.60	0.62	7	0.33	0.11	0.04
	4.76	1.62	0.63	-	-	-	-
^{242}Pu	0.09	0.03	0.01	33	0.028	0.01	0.003
	0.09	0.03	0.01	31	0.028	0.01	0.003
	0.09	0.03	0.01	11	0.001	.003	0.001
	0.09	0.03	0.01	12	0.011	.004	0.001
	0.09	0.03	0.01	31	0.028	0.01	0.003

[1]For the items in the first column, the first row is for Keepin yield, the second for Tomlinson's, third for Tuttle (1975), fourth for Tuttle (1979) and the fifth for Measured values.

TABLE XI

Calculated Isotopic Contributions to Reactivity for Three Reactor Periods and Corresponding Uncertainties in Reactivity Arising from Yield Uncertainties for Four Evaluations and Few Select Measurements (DIMPLE Assembly)

Isotope[2]	Reactivity, $\rho(q)$ (pcm)			Uncertainty in DN Yield (%)	Uncertainty in Reactivity, $\Delta\rho(q)$ (pcm)		
	T=20 s	100 s	300 s		T=20 s	100 s	300 s
^{235}U	177.06	61.38	24.23	5	8.85	3.07	1.21
	186.76	64.74	25.56	2	3.74	1.30	0.51
	186.99	64.82	25.59	2.5	4.68	1.62	0.64
	183.06	63.46	25.05	3	5.49	1.90	0.75
	183.47	63.60	25.11	5	9.17	3.18	1.25
^{238}U	28.09	8.92	3.41	6	1.69	0.54	0.20
	29.63	9.41	3.60	5	1.48	0.47	0.18
	29.66	9.42	3.60	1.3	0.39	0.12	0.05
	29.04	9.22	3.53	2	0.58	0.18	0.07
	29.11	9.24	3.53	6	1.75	0.55	0.21

TABLE XII

Spread in Isotopic Reactivity(pcm) (Maximum to Minimum)

Isotope	ZEBRA Fast Core			DIMPLE Thermal Core		
	T=20 s	100 s	300 s	T=20 s	100 s	300 s
^{235}U	0.34	0.12	0.04	9.93	3.44	1.36
^{238}U	1.70	0.54	0.21	1.57	0.50	0.19
^{239}Pu	2.75	0.99	0.36	-	-	-
^{240}Pu	0.20	0.07	0.03	-	-	-
^{241}Pu	0.31	0.10	0.05	-	-	-
^{242}Pu	0.00	0.00	0.00	-	-	-
Total	5.30	1.82	0.69	11.50	3.94	1.55

[2]For the items in the first column, the first row is for Keepin yield, the second for Tomlinson's, third for Tuttle (1975), fourth for Tuttle (1979) and the fifth for Measured values.

TABLE XIII

Total Reactivity and Total Uncertainty in Overall Reactivity (ZEBRA Fast Assembly)

Data Set	Total Reactivity, $\rho = \sum_q \rho(q)$ (pcm)			Total Uncertainty in Overall Reactivity,$\Delta\rho$ (pcm)			Total Uncertainty in Overall Reactivity,$\Delta\rho$ (%)		
	T=20 s	100 s	300 s	T=20 s	100 s	300 s	T=20 s	100 s	300 s
Keepin	82.12	28.30	10.56	3.47	1.22	0.45	4.2	4.3	4.3
Tomlinson	86.28	29.70	11.42	2.02	0.69	0.26	2.3	2.3	2.3
Tuttle '75	87.42	30.12	11.25	1.17	0.41	0.15	1.4	1.4	1.4
Tuttle '79	84.60	29.14	10.89	1.49	0.53	0.20	1.8	1.8	1.8
Measurements	85.09	29.13	10.89	4.37	1.54	0.57	5.1	5.3	5.3

TABLE XIV

Total Reactivity and Total Uncertainty in Overall Reactivity (DIMPLE Thermal Assembly)

Data Set	Total Reactivity, $\rho = \sum_q \rho(q)$ (pcm)			Total Uncertainty in Overall Reactivity,$\Delta\rho$ (pcm)			Total Uncertainty in Overall Reactivity,$\Delta\rho$ (%)		
	T=20 s	100 s	300 s	T=20 s	100 s	300 s	T=20 s	100 s	300 s
Keepin	205.15	70.30	27.64	9.01	3.12	1.23	4.4	4.4	4.4
Tomlinson	216.39	74.15	29.15	4.02	1.38	0.54	1.9	1.9	1.9
Tuttle '75	216.65	74.24	29.19	4.69	1.63	0.64	2.2	2.2	2.2
Tuttle '79	212.11	72.68	28.58	5.53	1.91	0.76	2.6	2.6	2.6
Measurements	212.57	72.84	28.64	9.34	3.23	1.27	4.4	4.4	4.4

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