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Lepton Flavor changing processes and CP violation in the 331 model

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Abstract

By extending the electroweak gauge group to $SU(3)_L \times U(1)_Y$, the 331 model incorporates dilepton gauge bosons Y which do not respect individual lepton family number. We point out that, in addition to family diagonal couplings such as Y–e–e that change lepton family number by two units, dileptons may also have family non-diagonal couplings such as Y–μ–e. The latter coupling violates lepton family number by a single unit and manifests itself via lepton flavor changing decays such as $\mu \to 3e$ and $\mu \to e\gamma$. The family non-diagonal interaction can be CP violating and typically generates extremely large leptonic electric dipole moments. We demonstrate a natural mechanism for eliminating both single unit lepton flavor violation and large leptonic CP violation. Although we focus on the 331 model, our results are applicable to other dilepton models as well, including SU(15) grand unification.

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I. INTRODUCTION

While the standard model (SM) is extremely successful and is consistent with known experimental data, it nevertheless leaves some questions unexplained. Among these questions is the issue of why there are exactly three families of quarks and leptons. The 331 model gives a natural answer to this family replication question and furthermore gives some indication as to why the top quark is so heavy.

In the 331 model, the $SU(2)_L \times U(1)_Y$ electroweak gauge group of the SM is extended to $SU(3)_L \times U(1)_X$ [1,2]. Unlike the SM, where anomalies cancel family by family, anomalies in the 331 model only cancel when all *three* families are taken together. This is accomplished by choosing one of the families, which we take as the third one, to transform differently under the 331 gauge group. A different third family conveniently allows a heavy top, but also introduces tree level flavor changing neutral currents (FCNC).

Since the 331 model reduces to the standard electroweak theory, tree level FCNC is restricted to interactions not present in the SM. In the gauge sector, only the new neutral gauge boson Z' has a flavor changing coupling to the ordinary quarks [1,3]. Because the leptons are treated democratically, they do not suffer FCNC (ignoring possible flavor changing neutral Higgs interactions). In the SM, the absence of FCNC and massless neutrinos is sufficient to show that individual lepton flavors are conserved. While both conditions are true in the minimal 331 model, it turns out that lepton flavor is no longer conserved. Lepton flavor violation occurs through the interactions of the dilepton gauge bosons Y^+ and Y^{++} which both carry two units of lepton number. Since dileptons do not carry lepton family information, only the total lepton number, $L \equiv L_e + L_\mu + L_\tau$, is conserved (in the absence of anomalies).

It is well known that dilepton interactions may violate individual lepton family number by two, for instance in the process $e^-e^- \rightarrow Y^{--} \rightarrow \mu^-\mu^-$, yielding spectacular signatures for dilepton models [4]. However, little attention has been placed on the possibility of single unit lepton flavor violation in these models. Experimentally, the non-observation of such decays as $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ put strong constraints on $\Delta L_i = \pm 1$ processes. In this paper, we examine the leptonic sector of the 331 model in detail and study the dilepton contributions to lepton flavor violation. While lepton flavor violation universally occurs in the presence of massive neutrinos, such contributions are often extremely small due to a GIM cancellation. We show that, even with massless neutrinos, the 331 model allows possibly large lepton flavor violation mediated by dilepton exchange.

Unlike the SM, dilepton exchange may also contribute to large CP violation in the leptonic sector. This occurs because additional phases are present in the mixing matrix describing the lepton couplings to the dilepton gauge bosons. These phases remain even with massless neutrinos, and cannot be rotated away. We examine the possibility of detecting such CP violation by calculating the dilepton contributions to leptonic electric dipole moments (EDM). Our results show that dilepton mediated leptonic CP violation may be extremely large, and is closely related to lepton flavor violation.

Another source of CP violation in the 331 model is that coming from the Higgs sector. Since the minimal 331 model requires four Higgs multiplets, there are many possibilities for Higgs sector CP violation. In order to examine such scenarios, we present a detailed discussion of the minimal 331 Higgs sector and show how it reduces to a three Higgs doublet SM with additional $SU(2)_L$ singlet and triplet scalars carrying lepton number. While a three Higgs doublet model gives a natural framework for spontaneous CP violation [5–7], we note that both tree level flavor changing neutral Higgs (FCNH) [8,9] and the additional singlet and triplet scalars [10] present additional mechanisms for CP violation in the 331 model.

In order for the 331 model to be consistent with stringent experimental bounds on lepton flavor violation and lepton EDMs, we find that the family non-diagonal dilepton couplings must be very small. We show that a natural solution is to simply set them to zero (at least at tree level) which may be accomplished by restricting the lepton Yukawa couplings by an appropriate discrete symmetry. An interesting feature of our analysis is that, while the details are specific to the 331 model, the general results hold for any model incorporating dilepton gauge bosons such as SU(15) grand unification [11–14].

In the next section we present a quick review of the 331 model and its particle content. In Sec. 3, we examine the breaking of the 331 model to the SM and show how CP violation may arise in the reduced Higgs sector. In Sec. 4, we show how $\Delta L_i = \pm 1$ lepton flavor violation occurs and study the related leptonic CP violation. We present our conclusions in Sec. 5. Details on the diagonalization of the charged lepton mass matrix are given in an appendix.

II. A REVIEW OF THE 331 MODEL

Construction of the 331 model was first presented in Refs. [1,2] and subsequently expanded upon in Refs. [3,15]. In this section, we present a brief review of the model. Since the original papers have used a variety of different notations, this review also serves to set up the conventions used in this paper.

A. Fermion representations

Since each lepton family has three helicity states (assuming massless neutrinos), they fall naturally into $SU(3)_L$ anti-triplets [16]

$$\psi_i = \begin{pmatrix} \ell_i^- \\ -\nu_i \\ \ell_i^+ \end{pmatrix}_L , \tag{1}$$

where $i = 1,2,3$ is a family index. We choose the standard embedding of SU(2) in SU(3) (given by $T^a = \frac{1}{2}\lambda^a$ for triplets where λ^a are the usual Gell-Mann matrices) so that the first two components of (1) corresponds to the ordinary electroweak doublet. As a result, we find that the hypercharge is given by $Y/2 = \sqrt{3}T^8 + X$ where leptons have vanishing X charge, $X = 0$. Our choice of hypercharge corresponds to twice the average electric charge of $SU(2)_L$ representations, *i.e.* $Q = T^3 + Y/2$. Thus each lepton family is in the $(\mathbf{1},\mathbf{3}^*)_0$ representation of $SU(3)_c\times SU(3)_L\times U(1)_X$. A result of this embedding is that there are *no* new leptons in the 331 model.

Note that upon reduction to SU(2), both SU(3) triplets and anti-triplets decompose into a doublet and a singlet. Since SU(2) is pseudo-real, there is no distinction between these two cases. However, in order to get rid of some unimportant phases, we multiply standard SU(2) doublets by $i\tau^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ before embedding them into SU(3) anti-triplets. This is the origin of the minus sign in Eq. (1).

While all three lepton families are treated identically, anomaly cancellation requires that one of the three quark families transform differently from the other two [1,2]. In particular, cancelling the pure $SU(3)_L$ anomaly requires the same number of triplets as anti-triplets. Since there are three lepton anti-triplets and three quark colors, we find that anomaly cancellation requires that two families of quarks transform as triplets, $(\mathbf{3},\mathbf{3})_{-1/3}$, whereas the third transforms as an anti-triplet, $(\mathbf{3},\mathbf{3}^*)_{2/3}$. All left handed anti-particles are put in as singlets in the usual manner, $(\mathbf{3}^*,\mathbf{1})_{-2/3,1/3,4/3}$ for the first two families and $(\mathbf{3}^*,\mathbf{1})_{-5/3,-2/3,1/3}$ for the third. We will not elaborate any further on the quarks.

B. The gauge sector

When the electroweak gauge group is extended to $SU(3)_L\times U(1)_X$, we find 5 new gauge bosons beyond the SM. We denote the $SU(3)_L$ gauge bosons by W_μ^a ($a = 1\dots 8$) with $a = 1,2,3$ forming the $SU(2)_L$ subgroup of $SU(3)_L$. The $U(1)_X$ gauge boson is given by X_μ. We define the two gauge couplings, g and g_X according to

$$D_\mu = \partial_\mu - igT^a W_\mu^a - ig_X \frac{X}{\sqrt{6}} X_\mu \ , \tag{2}$$

with the conventional non-abelian normalization $\mathrm{Tr}T^aT^b = \frac{1}{2}\delta^{ab}$ in the fundamental (triplet) representation. The factor $1/\sqrt{6}$ was chosen [2,3] so that for triplets $X/\sqrt{6} \equiv T^9 X$ with $\mathrm{Tr}T^9T^9 = \frac{1}{2}$.

From above, we have found the hypercharge to be given by $Y/2 = \sqrt{3}T^8 + X = \sqrt{3}T^8 + \sqrt{6}T^9 X$. As a result, when 331 is broken to the SM, we find the gauge matching conditions

$$\frac{1}{g'^2} = \frac{3}{g^2} + \frac{6}{g_X^2} \ , \tag{3}$$

where the $U(1)_Y$ coupling constant g' is given by $\tan\theta_W = g'/g$. The consequences of this relation will be explored in the next section where the reduction to the SM is carried out in more detail.

Since $\mathbf{8}_0 \to \mathbf{3}_0 + \mathbf{2}_3 + \mathbf{2}_{-3} + \mathbf{1}_0$ under $SU(3)_L\times U(1)_X \to SU(2)_L\times U(1)_Y$, the new gauge bosons form a complex $SU(2)_L$ doublet of dileptons, (Y^{++}, Y^+) with hypercharge 3 and a singlet, W^8. This new U(1) gauge boson W^8 mixes with the $U(1)_X$ gauge boson X to give the hypercharge boson B and a new Z'.

C. Higgs fields

At first glance, only two Higgs representations are necessary for symmetry breaking, one to break 331 to the SM and the other to play the role of the SM Higgs. However, the Yukawa couplings are restricted by $SU(3)_L$ gauge invariance. In order to give realistic masses to all the particles, there must be a minimum of four Higgs in the 331 model [17]. These four multiplets are the three triplets, Φ, ϕ and ϕ' in representations $(\mathbf{1},\mathbf{3})_1$, $(\mathbf{1},\mathbf{3})_0$ and $(\mathbf{1},\mathbf{3})_{-1}$ respectively, and a sextet $(\mathbf{1},\mathbf{6})_0$ denoted H.

$SU(3)_L\times U(1)_X$ is broken to $SU(2)_L\times U(1)_Y$ when Φ acquires a VEV, giving masses to the Y and Z' gauge bosons and the new quarks. At this stage of symmetry breaking, the other three Higgs fields decompose into $SU(2)_L\times U(1)_Y$ representations as $\mathbf{3}_0 \to \mathbf{2}_1+\mathbf{1}_{-2}$, $\mathbf{3}_{-1} \to \mathbf{2}_{-1}+\mathbf{1}_{-4}$ and $\mathbf{6}_0 \to \mathbf{3}_2+\mathbf{2}_{-1}+\mathbf{1}_{-4}$. Taking this decomposition into account, we may write the Higgs fields explicitly.in terms of $SU(2)_L$ component fields as

$$\Phi = \begin{pmatrix} \Phi_Y \\ \varphi^0 \end{pmatrix} \qquad \phi = \begin{pmatrix} \Phi_1 \\ \Delta^- \end{pmatrix} \qquad \phi' = \begin{pmatrix} \tilde{\Phi}_2 \\ \rho^{--} \end{pmatrix} , \tag{4}$$

and

$$H = \begin{pmatrix} T & \tilde{\Phi}_3/\sqrt{2} \\ \tilde{\Phi}_3^T/\sqrt{2} & \eta^{--} \end{pmatrix} . \tag{5}$$

In the above, $\Phi_Y = (\Phi_Y^{++}, \Phi_Y^{+})$ is the Goldstone boson doublet "eaten" by the dileptons. $\Phi_i = (\phi_i^+, \phi_i^0)$ $(i = 1,2,3)$ are three standard model Higgs doublets where $\tilde{\Phi}_i = i\tau^2\Phi_i^*$, and T is an $SU(2)_L$ triplet,

$$T = \begin{pmatrix} T^{++} & T^{+}/\sqrt{2} \\ T^{+}/\sqrt{2} & T^0 \end{pmatrix} . \tag{6}$$

As a result, the scalars give rise to a three Higgs doublet SM with an additional $SU(2)_L$ triplet and charged singlets.

D. Lepton number assignment

Because both the charged lepton and its anti-particle are in the same multiplet, the assignment of lepton number is not entirely obvious. Starting with $L(\ell^-) = L(\nu) = 1$ and $L(\ell^+) = -1$, we find that the dilepton doublet (Y^{++}, Y^{+}) carries lepton number $L = -2$. Lepton numbers for the scalars may be assigned by inspection of the Yukawa couplings. We find that Φ_Y and T carry lepton number $L = -2$ and Δ^-, ρ^{--} and η^{--} have $L = 2$. φ^0 and the SM Higgs doublets carry no lepton number as expected. This assignment is consistent with the scalars giving rise to the longitudinal components of the dilepton gauge bosons, even after $SU(2)_L$ breaking.

Given the above assignment of lepton number, the only place where it may be explicitly violated is in the scalar potential. This may be done either via soft (dimension three) or hard (dimension four) terms. In addition, the triplet T (with $L = -2$) has a neutral component which may acquire a VEV and spontaneously break lepton number. These possibilities may be classified as follows:

- *no explicit L violation and* $\langle T\rangle = 0$: This is the minimal 331 model where total lepton number is conserved. However, because of the presence of dilepton gauge bosons, individual lepton family number may be violated. The parameters of the Higgs potential may be chosen so that there is a stable minimum which maintains $\langle T\rangle = 0$ [15,18].

- *no explicit L violation but* $\langle T\rangle \neq 0$: In this case, lepton number is spontaneously broken, thus leading to a triplet Majoron model [19]. This case is ruled out experimentally by Z lineshape measurements.

- *explicit L violation in the Higgs potential*: This case has been discussed in [18,20] in the context of neutrinoless double beta decay and Majorana neutrino masses. In general, when L is violated explicitly, it induces a non-zero triplet VEV $\langle T\rangle$ unless some fine tuning is imposed.

III. REDUCTION TO THE STANDARD MODEL

The Higgs VEVs are arranged to first break SU(3)$_L\times$ U(1)$_X$ to the SM and then to break the SM. This symmetry breaking hierarchy may be represented as

$$SU(3)_L\times U(1)_X \xrightarrow{\langle\Phi\rangle} SU(2)_L\times U(1)_Y \xrightarrow{\langle\phi\rangle,\langle\phi'\rangle,\langle H\rangle} U(1)_Q\ . \tag{7}$$

In this section, we consider the first stage of symmetry breaking and examine the reduction of the 331 model to SU(2)$_L\times$ U(1)$_Y$.

A. 331 symmetry breaking and gauge matching conditions

When 331 is broken to the SM, the neutral gauge bosons W^8_μ and X_μ mix to give the Z'_μ and hypercharge B_μ bosons. In analogy with the SM, we find

$$\begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{331} & \sin\theta_{331} \\ -\sin\theta_{331} & \cos\theta_{331} \end{pmatrix} \begin{pmatrix} W^8_\mu \\ X_\mu \end{pmatrix}\ , \tag{8}$$

where $\tan\theta_{331} = \sqrt{2}g/g_X$. The hypercharge coupling constant g' is given from the gauge matching conditions (3) by

$$g' = \frac{1}{\sqrt{3}} g\cos\theta_{331} = \frac{1}{\sqrt{6}} g_X \sin\theta_{331}\ . \tag{9}$$

Since SU(3)$_L\times$ U(1)$_X$ is semi-simple, with two coupling constants, g and g_X, the Weinberg angle is not fixed as it would be for unification into a simple group. However, the unknown coupling g_X or equivalently θ_{331} may be determined in terms of θ_W. We find $\cos\theta_{331} = \sqrt{3}\tan\theta_W$, which gives

$$\alpha_X \equiv \frac{g_X^2}{4\pi} = \alpha\frac{6}{1-4\sin^2\theta_W}\ . \tag{10}$$

This shows the interesting property that $\sin^2\theta_W < 1/4$ with $\sin^2\theta_W \approx 1/4$ corresponding to strong coupling for the U(1)$_X$ [2,3]. Although this is a tree level result, it remains valid when the running of the coupling constants is taken into account. Since $\sin^2\theta_W(M_Z) = .233$ is already close to $1/4$ and runs towards larger values as the scale is increased, this restriction gives an absolute upper limit on the 331 breaking scale, $\mu \lesssim 3$ TeV.

Since this upper limit corresponds to infinite α_X, more realistic limits may be set by requiring the validity of perturbation theory. Note, however, that even at the Z-pole, we find a large $\alpha_X \approx 0.7$ corresponding to $\sin^2\theta_{331} \approx 0.09$. Since α_X is large, it quickly runs to a Landau pole at around 3 TeV regardless of the 331 scale and indicates that a more complete theory may be necessary where the U(1)$_X$ is embedded in a non-abelian group.

At this first stage of symmetry breaking, both dileptons and the Z' gain masses. Assuming the $\mathrm{SU}(2)_L$ subgroup remains unbroken, both members of the dilepton doublet (Y^{++}, Y^{+}) gain identical masses. Generalizing to arbitrary Higgs representations for the moment, we find

$$\begin{aligned} M_Y^2 &= \frac{g^2}{2}\sum_i (C_2(R_i) - X_i^2/3)|\langle\chi_i\rangle|^2 c_i \\ M_{Z'}^2 &= \frac{2g^2}{3\sin^2\theta_{331}}\sum_i X_i^2|\langle\chi_i\rangle|^2 , \end{aligned} \tag{11}$$

where R_i and X_i denote the $\mathrm{SU}(3)_L$ representation and $\mathrm{U}(1)_X$ charge of the Higgs χ_i. $c_i = 1$ for complex representations and $1/2$ for real ($X_i = 0$) ones. $C_2(R)$ is the quadratic Casimir of SU(3) in representation R, $T^a T^a = C_2(R)I$.

From (11), we may define a generalization of the ρ parameter,

$$\rho_{331} \equiv \frac{M_Y^2}{M_{Z'}^2 \sin^2\theta_{331}} = \frac{3}{4}\frac{\sum_i (C_2(R_i) - X_i^2/3)|\langle\chi_i\rangle|^2 c_i}{\sum_i X_i^2|\langle\chi_i\rangle|^2} . \tag{12}$$

If there are more than one 331 breaking Higgs present, then their X charges must be chosen so as to preserve a common unbroken $\mathrm{SU}(2)_L$ subgroup. For an SU(3) representation labeled by (p,q), this may be done by picking $X = p - q$. Using $C_2(p,q) = \frac{1}{3}(p^2 + q^2 + pq) + (p+q)$ in the standard normalization, we find

$$\rho_{331} = \frac{3}{4}\frac{\sum_{(p,q)}(p + q + pq)|\langle\chi_{(p,q)}\rangle|^2 c_i}{\sum_{(p,q)}(p-q)^2|\langle\chi_{(p,q)}\rangle|^2} . \tag{13}$$

In the minimal 331 model, this symmetry breaking is accomplished by the triplet Higgs Φ with $X = 1$ (*i.e.* (p,q)=(1,0)). Defining the 331 breaking VEV by $\langle\Phi\rangle = u/\sqrt{2}$, we find $M_Y = \frac{g}{2}u$ and $\rho_{331} = 3/4$. Since $\sin^2\theta_{331} \lesssim 0.09$, the definition of ρ_{331} tells us that the Z' must be considerably heavier than the dileptons, $M_{Z'} \gtrsim 3.9 M_Y$. Demanding that $\alpha_X(M_{Z'}) < 2\pi$ gives the upper limit $M_{Z'} < 2.2$ TeV, and hence $M_Y < 430\left(\sqrt{4\rho_{331}/3}\right)$ GeV for the masses of the new gauge bosons[21].

Lower bounds on the dilepton mass have be studied in [13,22–24]. The best current lower bound comes from polarized muon decay [24] which is especially sensitive to a non-standard charged-current interaction [25]. At 90% C.L., we find $M_Y > 300$ GeV [21] with a corresponding limit $M_{Z'} > 1.4\left(\sqrt{3/4\rho_{331}}\right)$ TeV on the Z' mass.

The imposition of both lower and upper limits on the scale of 331 physics is very constraining. Although larger values of ρ_{331} coming from a non-minimal Higgs sector would relax these bounds [21], the range of new physics is still limited to within about one order of magnitude above the Z-pole. As a result this model has the positive feature that it is easily testable.

B. Reduction of the Higgs sector

We now focus on the minimal Higgs sector, given by the three $\mathrm{SU}(3)_L$ triplets, (4), and the $\mathrm{SU}(3)_L$ sextet, (5). The most general scalar potential involving these fields is given by

$$V(\Phi,\phi,\phi',H) = V^{(2)} + V^{(3)} + V^{(4a)} + \cdots + V^{(4e)} \,, \tag{14}$$

where

$$\begin{aligned}
V^{(2)} =& \mu_1^2\Phi^\dagger\Phi + \mu_2^2\phi^\dagger\phi + \mu_3^2\phi'^\dagger\phi' + \mu_4^2 \mathrm{Tr}\, H^\dagger H \\
V^{(3)} =& \alpha_1\Phi\phi\phi' + \alpha_2(\Phi^T H^\dagger \phi') + \alpha_3(\phi^T H^\dagger \phi) + \alpha_4 HHH + \mathrm{H.C.} \\
V^{(4a)} =& a_1(\Phi^\dagger\Phi)^2 + a_2(\phi^\dagger\phi)^2 + a_3(\phi'^\dagger\phi')^2 + a_4(\Phi^\dagger\Phi)(\phi^\dagger\phi) \\
&+ a_5(\Phi^\dagger\Phi)(\phi'^\dagger\phi') + a_6(\phi^\dagger\phi)(\phi'^\dagger\phi') \\
&+ a_7(\Phi^\dagger\phi)(\phi^\dagger\Phi) + a_8(\Phi^\dagger\phi')(\phi'^\dagger\Phi) + a_9(\phi^\dagger\phi')(\phi'^\dagger\phi) + [a_{10}(\Phi^\dagger\phi)(\phi'^\dagger\phi) + \mathrm{H.C.}] \\
V^{(4b)} =& b_1\Phi^\dagger H\Phi\phi + b_2\phi'^\dagger H\phi'\phi + b_3\phi^\dagger H\Phi\phi' + \mathrm{h.c.} \\
V^{(4c)} =& c_1\phi\phi HH + c_2\Phi\phi' HH + \mathrm{H.C.} \\
V^{(4d)} =& d_1(\Phi^\dagger\Phi)\mathrm{Tr}\, H^\dagger H + d_2(\Phi^\dagger HH^\dagger\Phi) + d_3(\phi^\dagger\phi)\mathrm{Tr}\, H^\dagger H + d_4(\phi^\dagger HH^\dagger\phi) \\
&+ d_5(\phi'^\dagger\phi')\mathrm{Tr}\, H^\dagger H + d_6(\phi'^\dagger HH^\dagger\phi') \\
V^{(4e)} =& e_1(\mathrm{Tr}\, H^\dagger H)^2 + e_2\mathrm{Tr}\, H^\dagger HH^\dagger H \,.
\end{aligned} \tag{15}$$

The quartic terms, $V^{(4a)},\ldots,V^{(4e)}$, have been broken up according to the SU(3) representation contents, $(\mathbf{3}\times\mathbf{3}\times\mathbf{3}^*\times\mathbf{3}^*)$, $(\mathbf{3}\times\mathbf{3}\times\mathbf{3}^*\times\mathbf{6})$, $(\mathbf{3}\times\mathbf{3}\times\mathbf{6}\times\mathbf{6})$, $(\mathbf{3}\times\mathbf{3}^*\times\mathbf{6}\times\mathbf{6}^*)$ and $(\mathbf{6}\times\mathbf{6}\times\mathbf{6}^*\times\mathbf{6}^*)$ respectively.

According to the previously worked out lepton number assignment, the terms α_3, α_4, a_{10}, b_3 and c_2 violate lepton number explicitly. Soft lepton number violation may be accomplished by setting α_3, $\alpha_4 \neq 0$ [18,20]. Since we are presently interested in the minimal 331 model where lepton number is not violated, we instead take $\alpha_3 = \alpha_4 = a_{10} = b_3 = c_2 = 0$. In addition, the remaining parameters must be chosen so that the $SU(2)_L$ triplet T does not develop a VEV and hence break lepton number spontaneously. As we have discussed in the previous section, this theory is not a complete theory. Thus lepton number conservation may be a consequence of physics beyond the 331 model.

The first stage of symmetry breaking is governed by the triplet Φ with potential

$$\begin{aligned}
V &= \mu_1^2\Phi^\dagger\Phi + a_1(\Phi^\dagger\Phi)^2 + \cdots \\
&= a_1(\Phi^\dagger\Phi - u^2/2)^2 + \cdots \,,
\end{aligned} \tag{16}$$

where $\langle\Phi\rangle = u/\sqrt{2} = \sqrt{-\mu_1^2/2a_1}$ (with u chosen to be real). Of the original six real degrees of freedom, five become the longitudinal modes of the dileptons and the Z', leaving the physical heavy $SU(2)_L$ singlet $\sqrt{2}\mathrm{Re}\,\varphi^0$ with mass $M^2 = -2\mu_1^2 = 2a_1u^2$. The singlets Δ^- and ρ^{--} also become heavy with masses $M_{\Delta^-}^2 = a_7u^2/2$ and $M_{\rho^{--}}^2 = a_8u^2/2$.

The decomposition of the sextet H is a bit trickier. Due to the term d_2, we expect the masses to obey $M_T^2 < M_{\Phi_3}^2 < M_{\eta^{--}}^2$, equally spaced with $\Delta M^2 = d_2u^2/4$. In this case, the $SU(2)_L$ triplet is naturally light, with Φ_3 and η^{--} heavy. However, this is unappealing since H was introduced in the first place so the charged leptons may get their masses from $\langle\Phi_3\rangle$. Thus we need to set $d_2 \approx 0$, with the consequence that both T and η^{--} may be light [26].

After 331 breaking, the resulting scalars take the form of a three Higgs doublet model with the additional light fields T and η^{--}. For the three Higgs doublets only, we find the tree level reduced potential

$$\begin{aligned} V_{3HD}(\Phi_i) = & \sum_i m_i^2(\Phi_i^\dagger\Phi_i) + \sum_{i<j}[m_{ij}^2(\Phi_i^\dagger\Phi_j) + \text{H.C.}] \\ & + \sum_{i\le j}\lambda_{ij}(\Phi_i^\dagger\Phi_i)(\Phi_j^\dagger\Phi_j) + \sum_{i<j}\lambda'_{ij}(\Phi_i^\dagger\Phi_j)(\Phi_j^\dagger\Phi_i) \\ & + [\lambda_{1313}(\Phi_1^\dagger\Phi_3)(\Phi_1^\dagger\Phi_3) + \lambda_{1223}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_3) + \text{H.C.}]\ . \end{aligned} \tag{17}$$

A completely general three Higgs doublet potential includes additional possible terms in the last line. However, since the model was originally $\mathrm{SU}(3)_L\times\mathrm{U}(1)_X$ invariant, only the ones explicitly shown here are present at tree level. The coefficients are given by

$$\begin{aligned} m_1^2&=\mu_2^2 + a_4u^2/2 & m_{12}^2&=-\alpha_1^*u/\sqrt{2} \\ m_2^2&=\mu_3^2 + a_5u^2/2 & m_{13}^2&=-b_1^*u^2/2\sqrt{2} \\ m_3^2&=\mu_4^2 + d_1u^2/2 & m_{23}^2&=\alpha_2u/2 \\ \lambda_{11}&=a_2 - a_4^2/2a_1 & \lambda_{1313}&=-c_1^* \\ \lambda_{22}&=a_3 - a_5^2/2a_1 & \lambda_{1223}&=b_2^*/\sqrt{2} \\ \lambda_{33}&=e_1 + e_2/2 - d_1^2/2a_1 & & \\ \lambda_{12}&=a_8 + a_9 - a_4a_5/2a_1 & \lambda'_{12}&=-a_9 \\ \lambda_{13}&=d_3 + d_4/2 - a_4d_1/2a_1 & \lambda'_{13}&=-d_4/2 \\ \lambda_{23}&=d_5 - a_5d_1/2a_1 & \lambda'_{23}&=d_6/2\ . \end{aligned} \tag{18}$$

In performing the dimensional reduction, we have assumed $\alpha_1, \alpha_2 \sim v^2/u$ and $b_1 \sim v^2/u^2$ are small where v is an $\mathrm{SU}(2)_L$ breaking VEV. This assumption is necessary to ensure $m_{ij}^2 \sim v^2$ and hence to preserve the symmetry breaking hierarchy.

Three Higgs doublet models have been studied previously, usually in the context of the Weinberg model of CP violation [5–7]. However, in this case V_{3HD} is *not* invariant under $\Phi_i \to -\Phi_i$ which is often imposed to enforce natural flavor conservation (NFC)[27]. Although it is possible to eliminate the m_{ij}^2 terms by a unitary rotation of the Φ_i's, doing so would complicate the equations by introducing additional quartic couplings and would also affect the Yukawa couplings. Thus we find it more convenient to leave these off-diagonal terms in V_{3HD}.

In the absence of NFC there may be large FCNH processes. Since the Φ_i are remnants of the original $\mathrm{SU}(3)_L\times\mathrm{U}(1)_X$ invariant fields, their couplings are restricted over that of a generic $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ three Higgs doublet model. However, we find that these additional constraints are insufficient to implement NFC. In the quark sector, this should come as no surprise because the third family is explicitly different, resulting in both Z' mediated FCNC in the gauge sector and FCNH in the scalar sector. In the leptonic sector, both Φ_1 and Φ_3 may couple to leptons, resulting in FCNH and lepton flavor violation. However, since the leptons are treated identically, it is possible to impose an additional discrete symmetry that allows only a single Higgs to couple to the leptons. This possibility is explored further in the next section.

Because T and η^{--} carry lepton number, they do not mix with the three doublets (in the absence of lepton number violation). Analysis of the scalar potential indicates that a stable minimum with $\langle T \rangle = 0$ can be found for large regions of parameter space [15,18]. As long as T does not pick up a VEV, both T and η^{--} have no effect on symmetry breaking of the SM. This allows us to ignore these additional scalars and only focus on the three Higgs doublets of the 331 model.

C. Higgs sector CP violation

There are several options for CP violation in the 331 model. With complex Yukawa couplings, hard CP violation occurs through the CKM phase. In addition to the ordinary CKM coupling of the W charged current to quarks, the 331 model also has dilepton charged current couplings. This leads to new mixing angles as well as additional CP violating phases in both the leptonic and hadronic sector. This is perhaps the most straightforward generalization of CP violation in the SM. However, the additional phases may lead to novel effects such as large lepton EDMs which are otherwise undetectably small in the SM.

CP violation may also occur in the extended Higgs sector [8,5]. For three Higgs doublets, CP violation may be either explicit (complex m_{ij}^2, λ_{1313} and λ_{1223} in V_{3HD}) or spontaneous. In both cases, CP violation occurs through charged and neutral Higgs exchange. The original motivation for introducing three doublets to the SM was to obtain CP violation in the scalar sector without FCNH. On the other hand, the 331 model *has* FCNH but requires three doublets for mass generation. In this case, CP violation from tree level FCNH cannot be ignored [8,9]. In addition, since the new triplet and singlet T and η^{--} couple to leptons, they may also contribute to leptonic CP violation as discussed in Ref. [10].

D. Standard Model breaking

When m_i^2, $m_{ij}^2 < 0$ in V_{3HD}, the three Higgs doublets pick up (possibly complex) VEVs $\langle \Phi_i \rangle = v_i/\sqrt{2}$ and breaks SU(2)$_L\times$ U(1)$_Y$. The resulting physical scalars are four charged Higgs, $H_{1,2}^{\pm}$, and five neutral ones $h_{1,\ldots,5}^0$. The physical states $H_{1,2}^+$ and the Goldstone mode are related to the original ϕ_i^+ via a 3×3 unitary matrix with a single physical CP violating angle (distinct from the usual CKM angle) [28]. CP violation in the neutral Higgs sector manifests itself in the mixing of the CP even and CP odd scalars.

While the other light scalars T and η^{--} have no effect on symmetry breaking, they acquire masses related to the VEVs v_i. Because SU(2)$_L$ is broken, the triplet will become split in mass and T^{++} and η^{++} will mix. This second stage of symmetry breaking will also have an effect on the SU(3)$_L$ particles. In particular, the dilepton doublet will become split in mass and the Z and Z' will mix. Expressions for all tree level gauge boson masses and Z-Z' mixing have been given in [3]. Because of the symmetry breaking hierarchy, these effects may be considered as perturbations to the results where SU(2)$_L$ remains unbroken. However, in the 331 model, this must often be treated with care since the two scales are within an order of magnitude of each other.

IV. LEPTON FLAVOR VIOLATION AND CP VIOLATION

We now turn to the leptonic sector of the 331 model. Since the leptons are in the $\mathbf{3}_0^*$ representation of SU(3)$_L\times$ U(1)$_X$, the lepton bi-linear $\psi\psi$ transforms as $\mathbf{3}_0^* \times \mathbf{3}_0^* =$

$3_0 + 6_0^*$. Thus leptons may have gauge invariant Yukawa couplings to the triplet ϕ and sextet H. We write the Yukawa interaction as

$$-\mathcal{L} = \frac{1}{\sqrt{2}}\overline{\psi_i'^{\alpha}} h_s^{ij} \psi_j'^{\beta c} H^*_{\alpha\beta} - \frac{1}{2}\overline{\psi_i'^{\alpha}} h_a^{ij} \psi_j'^{\beta c} \phi^{\gamma} \epsilon_{\alpha\beta\gamma} + \text{H.C.} \,, \tag{19}$$

where the primes denote weak eigenstates. Here, i, j are family indices and $\alpha, \beta, \gamma = 1, 2, 3$ are SU(3) group indices. From the symmetry properties of (19), the Yukawa coupling matrix h_s is symmetric and h_a is antisymmetric. The above factors have been chosen so the charged lepton mass matrix will take on a simple form and differs from the convention used in [20].

We may rewrite Eq. (19) in terms of $SU(2)_L$ component fields. Using the definitions of (4) and (5), the Yukawa interactions may be written

$$\begin{aligned}-\mathcal{L} = &\overline{L'_L}[h_s\Phi_3 + h_a\Phi_1]e'_R \\ &+\frac{1}{\sqrt{2}}\overline{L'_L}h_s\tilde{T}L'^c_L - \frac{1}{2}\overline{L'_L}h_a(i\tau^2)L'^c_L\Delta^- + \frac{1}{\sqrt{2}}\overline{e'^c_R}h_s e'_R\eta^{++} + \text{H.C.} \,,\end{aligned} \tag{20}$$

where the family indices have been suppressed and $L_L = (\nu, \ell^-)_L$ is the SM lepton doublet. The first line gives a two Higgs doublet SM interaction and the second line gives the interaction with new 331 scalars. While Δ^- is heavy, T and η^{++} may be light, and resemble the scalars introduced in Ref. [10] for generating leptonic CP violation [29]. As we noted before, this model does not satisfy the requirements for NFC and hence violates lepton family number via FCNH. However, unlike a general two Higgs doublet model with arbitrary Yukawa couplings, $SU(3)_L$ gauge invariance restricts the form of h_s and h_a. This has important consequences as shown below.

A. Lepton masses and mixing

When the SM is broken by the Higgs doublet VEVs $\langle\Phi_i\rangle = v_i/\sqrt{2}$, the charged leptons get a mass matrix $M_\ell = (h_s v_3 + h_a v_1)/\sqrt{2}$. Since h_s (h_a) is (anti-)symmetric, M_ℓ is an arbitrary complex 3×3 matrix. We diagonalize this matrix by a bi-unitary transformation $E_L^\dagger M_\ell E_R = \text{diag}(m_e, m_\mu, m_\tau)$. As a result, physical (mass) eigenstates are related to the weak eigenstates according to

$$e'_L = E_L e_L \qquad e'_R = E_R e_R \qquad \nu'_L = F_L \nu_L \,, \tag{21}$$

where we also introduce a unitary transformation for the neutrinos.

In terms of the physical basis, the W and dilepton charged currents become

$$\begin{aligned} J_+^\mu &= \bar{\nu}\gamma^\mu\gamma_L[F_L^\dagger E_L]e = \bar{\nu}\gamma^\mu\gamma_L V_W e \\ J_{Y^+}^\mu &= \overline{e^c}\gamma^\mu\gamma_L[E_R^T F_L]\nu = \overline{e^c}\gamma^\mu\gamma_L V_Y V_W^\dagger \nu \\ J_{Y^{++}}^\mu &= -\overline{e^c}\gamma^\mu\gamma_L[E_R^T E_L]e = -\overline{e^c}\gamma^\mu\gamma_L V_Y e \,, \end{aligned} \tag{22}$$

where $V_W = F_L^\dagger E_L$ and $V_Y = E_R^T E_L$ are unitary mixing matrices in the leptonic sector. Thus we find that in addition to a possible leptonic CKM mixing coming from massive neutrinos, lepton family number may also be violated in the interaction with dileptons. Note that the current $J_{Y^{++}}$ in (22) may be rewritten as $J^\mu_{Y^{++}} =$

$-\frac{1}{2}\overline{e^c}\gamma^\mu(V_Y\gamma_L - V_Y^T\gamma_R)e$, showing that the doubly charged dilepton has both left- and right-handed couplings and that the family diagonal coupling is purely axialvector.

If the neutrinos are massless, then we may pick $F_L = E_L$, or equivalently $V_W = 1$. In this case, the ordinary W charged current is family diagonal, and the dilepton interaction is determined completely by V_Y. In general, a 3×3 unitary matrix is fixed by three angles and six phases. Unlike the normal CKM case, because V_Y is determined entirely from the charged lepton sector, we may only rotate away 3 phases, corresponding to $E_{L,R} \to E_{L,R}K$ (where K is a diagonal matrix of phases) which preserves the reality of the diagonal charged lepton masses. As a result, V_Y depends on a total of six real parameters: three angles and three phases.

If the triplet T gets a VEV, then the neutrinos pick up a Majorana mass $M_\nu = \sqrt{2}h_s\langle T\rangle$. Neutrino masses may also arise by adding right-handed neutrino states. In both cases, F_L must then be chosen to diagonalize the neutrino mass matrix. For Majorana neutrinos, M_ν is symmetric and we can find F_L such that $F_L^\dagger M_\nu F_L^*$ is diagonal. In general, diagonalization may be more complicated.

With massive neutrinos, V_W describes mixing in the ordinary leptonic sector. The number of possible CP violating phases depends on the nature of the neutrinos. For Majorana neutrinos, if V_Y is fixed as above to have three angles and three phases, then there is no more freedom to rotate away any phase because of the Majorana nature of the neutrinos. Hence, there are three angles and six phases in V_W. On the other hand, we may choose to rotate away three phases in V_W by redefining the charged lepton phases, leaving V_W with three angles and three phases and V_Y with three angles and six phases. In both cases, there are a total of nine possible CP violating phases. Physically, there should be no difference between these cases, so we may choose to distribute the phases among the various charged currents in the most convenient manner. For Dirac neutrinos, we may remove three additional phases, leaving a total of six CP violating phases. A possible distribution of phases is one in V_W and five in V_Y, so that V_W has the usual form for the Dirac case.

While nine, or even six, CP violation phases may seem like a lot, in many specific 331 models of neutrino mass, the neutrino mass matrices are related to the charged lepton mass matrices, and hence lead to relations among the mixing angles and phases. Thus the number of independent phases may be no larger than three, the minimum coming from the doubly charged dilepton current. In particular, for Majorana neutrinos that get masses from the *same* Yukawa couplings h_s and h_a, there is no additional freedom, and the matrix V_W may be specified in terms of the six parameters of V_Y, although the exact relation is usually rather complicated [20].

B. Dilepton mediated rare lepton decays

Even with massless neutrinos, the doubly charged dilepton may have family non-diagonal interactions because of the new mixing given by V_Y. As a result, lepton flavor violating processes such as $\mu \to 3e$ and $\mu \to e\gamma$ may occur. In addition, the phases in V_Y lead to leptonic CP violation which may be observed by detecting a triple product correlation in $\mu \to 3e$ [30] decay or by measuring non-zero lepton EDMs. Since these exotic decays have not been seen, this leads to strong constraints on the allowed mixing coming from V_Y.

The decay $\mu \to 3e$ proceeds via tree level dilepton exchange as shown in Fig. 1.

Ignoring final state particle masses, we find

$$\frac{\mathrm{BR}(\mu \to 3e)}{\mathrm{BR}(\mu \to e\overline{\nu_e}\nu_\mu)} = \left(\frac{M_W}{M_Y}\right)^4 |V_Y^{11}|^2(|V_Y^{12}|^2 + |V_Y^{21}|^2) , \tag{23}$$

and similar expressions for the processes $\tau \to 3\mu$, $\tau^- \to \mu^+e^-e^-$, and $\tau^- \to e^+\mu^-\mu^-$ with the appropriate replacement of the family indices. For $\tau^- \to e^-\mu^+\mu^-$ and $\tau^- \to \mu^-e^+e^-$, the family diagonal coupling $|V_Y^{11}|^2$ must be replaced by the appropriate off-diagonal coupling $|V_Y^{i3}|^2 + |V_Y^{3i}|^2$ with $i = 2, 1$ respectively. The present experimental limits are [31]

$$\begin{aligned} \mathrm{BR}(\mu \to 3e) &< 1.0 \times 10^{-12} \\ \mathrm{BR}(\tau \to 3\ell) &< 3.4 \times 10^{-5} , \end{aligned} \tag{24}$$

(at 90% C.L.), where ℓ denotes either μ or e. The constraints for the various $\tau \to 3\ell$ channels are given in [31] and are all less than the order of 10^{-5}. Clearly the experimental bounds are not as well determined for τ decay as it is for μ decay. This allows for relatively large e–τ and μ–τ mixing, with important consequences for the electron and muon EDM.

A standard method for suppressing flavor changing processes is to make the exchanged particle very heavy. However, in the present case there is an upper limit on the dilepton mass, $M_Y < 430$ GeV (in the minimal case where $\rho_{331} = 3/4$). As a result, we can restrict the mixing allowed by V_Y. Assuming the lepton families are almost diagonal, $V_Y \approx 1$, we may write $V_Y^{ij} = \delta^{ij} + 2\alpha^{ij}e^{i\theta_{ij}}$ in the small mixing approximation where $\alpha^{ij} = -\alpha^{ji}$ are the three mixing angles and $\theta_{ij} = -\theta_{ji}$ the three CP violating phases of V_Y. In the appendix, we show how α and θ may be related to the original Yukawa couplings h_s and h_a of (19). In terms of this parametrization, the experimental bounds (24) give the limits

$$\begin{aligned} |\alpha^{12}| &< 1.0 \times 10^{-5} \\ |\alpha^{13}| &< 0.096 \\ |\alpha^{23}| &< 0.096 , \end{aligned} \tag{25}$$

justifying the small mixing approximation, at least for the first two families.

Curiously, there is a second choice for V_Y consistent with the above limits. In this case, Y^{--} has a mostly off-diagonal coupling to the first two families, $Y^{--} \to e^-\mu^-$, or, in terms of the mixing matrix, $|V_Y^{12}| \approx |V_Y^{21}| \approx 1$. The other components are restricted by

$$\begin{aligned} |V_Y^{11}|^2 &< 4.1 \times 10^{-10} \\ |V_Y^{13}|^2 + |V_Y^{31}|^2 &< 0.062 \\ |V_Y^{23}|^2 + |V_Y^{32}|^2 &< 0.062 , \end{aligned} \tag{26}$$

and $|V_Y^{22}|^2 \lesssim 10^{-3}$ from unitarity of V_Y. This large mixing case corresponds to $\alpha^{12} \approx \pi/4$, and occurs in the limit when the diagonal Yukawa couplings are identical, $h_s^1 = h_s^2$, with the result that $m_e, m_\mu = (h_s^1|v_3| \pm y^{12}|v_1|)/\sqrt{2}$. The third family has the standard diagonal form, $m_\tau = h_s^3|v_3|/\sqrt{2}$.

It is easy to show that these two cases are the only possible solutions consistent with (24). Furthermore, these limits on V_Y are independent of any neutrino masses and mixing. However, the second case may be marginally ruled out from an analysis of transverse electron polarization in muon decay, as we indicate below. On the theoretical side, as well, there appears to be no principle which would enforce the necessary equality between h^1_s and h^2_s. Thus the second case will not be further investigated.

Lepton flavor violating processes of the form $\mu \to e\gamma$ may also occur via either W^-, Y^- or Y^{--} exchange at one-loop. For both singly charged cases, a neutrino is running in the loop, and hence the amplitude vanishes for massless neutrinos. For massive neutrinos, the GIM cancellation is not perfect, but nevertheless leads to a large suppression of the amplitude. On the other hand, since the Y^{--} has both right- and left-handed couplings, it leads to a large contribution to $\mu \to e\gamma$ as shown in Fig. 2.

Assuming the intermediate charged leptons are light, $m_i \ll M_Y$, the one loop diagrams lead to transition magnetic and electric dipole moments

$$\mu_{12}, d_{12} = \frac{3eG_F}{4\sqrt{2}\pi^2}\left(\frac{M_W}{M_Y}\right)^2 \sum_i (V_Y^{1i}V_Y^{i2*} \pm V_Y^{i1}V_Y^{2i*})m_i \ , \tag{27}$$

resulting in a decay width of

$$\Gamma_{\mu\to e\gamma} = \frac{m_\mu^3}{8\pi}(|\mu_{12}|^2 + |d_{12}|^2) \ , \tag{28}$$

(ignoring the electron mass). Since $\alpha^{12} \ll 1$, the intermediate state τ dominates, leading to a branching ratio

$$\mathrm{BR}(\mu \to e\gamma) = \frac{54\alpha}{\pi}\left(\frac{M_W}{M_Y}\right)^4 \left(\frac{m_\tau}{m_\mu}\right)^2 (|V_Y^{13}|^2|V_Y^{32}|^2 + |V_Y^{31}|^2|V_Y^{23}|^2) \ . \tag{29}$$

Compared to $\mu \to 3e$ decay, Eq. (23), the loop factor α/π is compensated for by the larger phase space and the heavy τ. Using the upper limit on M_Y and the experimental limit $\mathrm{BR}(\mu \to e\gamma)_{\mathrm{expt}} < 4.9 \times 10^{-11}$ [31], we find

$$|\alpha^{13}\alpha^{23}| < 5.9 \times 10^{-6} \ , \tag{30}$$

a combined limit much stronger than the individual ones of Eq. (25).

C. Lepton electric dipole moments

In addition to large transition dipole moments, one-loop diagrams similar to those of Fig. 2 may lead to large EDMs. The electron EDM is calculated to be

$$d_e = \frac{3eG_F}{2\sqrt{2}\pi^2}\left(\frac{M_W}{M_Y}\right)^2 \sum_i \mathrm{Im}\,(V_Y^{1i}V_Y^{i1*})m_i \tag{31}$$

$$\approx -\frac{3\sqrt{2}eG_F}{\pi^2}\left(\frac{M_W}{M_Y}\right)^2 \sum_i m_i|\alpha^{1i}|^2 \sin 2\theta_{1i} \ , \tag{32}$$

and similarly for d_μ and d_τ. We observe that Y^{--} mediated CP violation occurs *only* through lepton flavor changing interactions. Putting in numbers, we estimate

$$d_e \approx 8.5|\alpha^{13}|^2 \sin 2\theta_{13} \times 10^{-21} e \text{ cm} \tag{33}$$
$$d_\mu \approx 8.5|\alpha^{23}|^2 \sin 2\theta_{23} \times 10^{-21} e \text{ cm} , \tag{34}$$

where terms proportional to $|\alpha^{12}|^2$ ($< 10^{-10}$ from Eq. (25)) have been ignored. The estimate for d_e is extremely large compared to the experimental limit $|d_e| < 1.9 \times 10^{-26} e$ cm [32] but depends on undetermined e–τ mixing parameters.

An interesting consequence of having only off-diagonal CP violating interactions is the inverse relation $d_\mu/d_\tau \approx -m_\tau/m_\mu$. While any observed EDM would indicate physics beyond the SM (which predicts unobservably small lepton EDMs [33]), this relation may be of use in verifying the 331 model of CP violation.

In principle, CP violation may also show up in ordinary muon decay due to interference between the W^- and Y^- induced amplitudes. In the presence of lepton flavor violation, the unobserved final state neutrinos may be in any family. Nevertheless, this is easily taken into account [34], and does not affect the investigation of polarized muon decay in Ref. [24]. For non-diagonal V_Y, the muon decay transverse polarization parameters β and β' [35] become non-zero,

$$\left.\begin{matrix}\beta\\ \beta'\end{matrix}\right\} = -8\left(\frac{M_W}{M_Y}\right)^2 \left\{\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\} (V_Y^{12} V_Y^{21*})$$
$$\approx 32\left(\frac{M_W}{M_Y}\right)^2 |\alpha^{12}|^2 \left\{\begin{matrix}\cos 2\theta_{12}\\ \sin 2\theta_{12}\end{matrix}\right\} . \tag{35}$$

In practice, this indication of CP violation in muon decay is unobservable, as it is proportional to the very small μ–e mixing. We predict $\beta'/A \lesssim 10^{-11}$ where $A = 16(1 + (M_W/M_Y)^4) \approx 16$ normalizes the decay rate. This is some eight orders of magnitude below current experimental limits [36]. On the other hand, had there been large mixing, as in (26), we would have found $|\beta/A|$, $|\beta'/A| \sim \frac{1}{2}(M_W/M_Y)^2 \geq 0.017$ which is ruled out by experiment at 90% C.L.

So far we have only considered lepton flavor changing processes mediated by dilepton gauge bosons. In general, scalar exchange will also contribute to both lepton flavor violation and CP violation. However, since the lepton Yukawa couplings are very small, these superweak interactions are often negligible compared to the dilepton interaction. Only in the absence of lepton flavor violation will the scalar sector play an important role in CP violation.

D. Elimination of lepton flavor violation

In order to suppress lepton flavor violation, the dilepton mixing angles α must be very small. From the appendix, we see that this means the anti-symmetric Yukawa coupling needs to be very small, $h_a|v_1| \ll h_s|v_3|$. We now have a naturalness problem since the limits on μ–e transitions require h_a to be about five orders of magnitude less than h_s (which is already small to accommodate the observed lepton masses). One solution to this problem is to simply set $h_a = 0$ which can be enforced by a discrete symmetry $\phi \to -\phi$ (along with an appropriate transformation of the quark fields).

This discrete symmetry actually serves two purposes. It prevents the doubly charged dilepton from having family non-diagonal couplings and prevents FCNH by allowing only a single Higgs multiplet (the sextet) to couple to the leptons. With massless neutrinos, this symmetry prevents $\Delta L_i = \pm 1$ lepton flavor violation (although $\Delta L_i = \pm 2$ would still be allowed).

Since dilepton mediated CP violation occurs through $\Delta L_i = \pm 1$ interactions, it is also eliminated by this discrete symmetry, leaving CP violation to the scalar sector. With massless neutrinos in the three Higgs doublet model, CP violation only occurs through mixing of the CP even and odd neutral Higgs. Because the Yukawa couplings are proportional to the charged lepton masses, $h_s \sim m_\ell/M_W$, the one-loop contribution to the lepton EDM is proportional to the cube of the lepton mass,

$$d_\ell \simeq \frac{e\sqrt{2}G_F m_\ell^3}{8\pi^2 M^2} \ln\left(\frac{m_\ell}{M}\right)^2 \delta \,, \tag{36}$$

where M and δ are the effective scalar mass and mixing.

Another source of CP violation, briefly touched upon above, is the mixing of the 331 scalars T^{++} and η^{++}. Since the unmixed scalars couple to leptons of different chirality, large CP violating effects are proportional to the amount of singlet-triplet mixing as well as their mass splitting. The one-loop EDM induced by T^{++}–η^{++} mixing is again proportional to m_ℓ^3, giving the same estimate, Eq. (36), but this time reduced by a factor $\delta M^2/M^2$ where δM^2 is the singlet–triplet mass splitting.

While both scalar one-loop contributions to the electron EDM are proportional to the electron mass cubed and hence very small, two-loop contributions have been shown to be important [37] and can lead to a fairly large electron EDM, albeit still smaller than the dilepton loop result (34). The two-loop contribution also dominates for the muon EDM, but the τ is sufficiently heavy that the one-loop contribution may be more important in that case. Assuming large CP violation in the scalar sector and a typical scalar mass of 100 GeV leads to the order of magnitude estimates $d_e \sim 10^{-27}$, $d_\mu \sim 10^{-25}$ and $d_\tau \sim 10^{-23}$ e cm. This prediction is similar to that of other flavor conserving scalar models of CP violation [5,37,38].

V. CONCLUSION

We have seen that in the general 331 model the leptons gain mass via symmetric and anti-symmetric couplings to two Higgs doublets. This leads to the possibility of both FCNH and lepton flavor violation mediated by dilepton exchange. In addition to neutrino mixing, there are nine physical parameters in the leptonic sector: three masses m_i, three mixing angles α^{ij} and three CP violating phases θ_{ij}. These, in turn, may be related to the Yukawa couplings h_s (three real parameters in the diagonal basis) and h_a (three complex parameters).

Lepton family mixing may be described by these three angles α^{ij} and three additional angles β^{ij} that diagonalize the neutrino mass matrix. For small mixing, the mixing angles for the W^-, Y^- and Y^{--} charged currents are given by $\alpha^{ij} - \beta^{ij}$, $\alpha^{ij} + \beta^{ij}$ and $2\alpha^{ij}$ respectively. For massless neutrinos we are free to choose $\beta^{ij} = \alpha^{ij}$ which ensures the W^- charged current respects lepton family. In this case, family mixing is given by $2\alpha^{ij}$ for both dilepton currents.

CP violation may occur in the gauge sector, but for massless neutrinos would only show up in the off-diagonal dilepton couplings; whenever the CP violating phase θ_{ij} shows up, α^{ij} must also be present. Thus CP violation and lepton flavor violation are closely related, giving the unusual prediction for the EDMs $d_\mu/d_\tau \approx -m_\tau/m_\mu$. Additional CP violation may be present in the scalar sector, and need not be related to lepton flavor violation. The scalar contributions are only important when $\alpha^{ij} \approx 0$ and arise through a combination of a three Higgs doublet model [5] and through the mixing of T^{++} and η^{++} [10].

Experimentally, the non-observation of lepton flavor violation puts strong restrictions on the mixing angles α^{ij}. The simplest way of accommodating this is to postulate a discrete symmetry which prevents ϕ from coupling to the leptons, thus setting $h_a = 0$. This gives rise to a purely symmetric mass matrix and vanishing α^{ij} (eliminating dilepton mediated CP violation as well).

Since all leptons are embedded in a single $SU(3)_L$ representation, most models of Majorana neutrino mass give rise to simple relations between charged lepton and neutrino masses and mixing [20]. In particular, when $h_a = 0$ all mixing vanishes, $\alpha^{ij} = \beta^{ij} = 0$, so the 331 model allows the interesting possibility of neutrino masses with no mixing.

Although our focus has been on the 331 model, the results are easily generalized to encompass all models with dilepton gauge bosons resulting from an SU(3) generalization of the standard electroweak theory. In particular, the SU(15) grand unified theory [11–14] also leads to lepton flavor non-conservation via dilepton exchange. This point seems to have been missed in earlier analyses.

Similar to the 331 model, leptons in SU(15) get symmetric and anti-symmetric contributions to their mass matrices, this time from Higgs in the **120** and **105** of SU(15) respectively [12]. Thus the 331 results for lepton masses and mixing, including CP violation governed by dilepton exchange, are equally applicable to SU(15) theory. One crucial difference, however, is that dileptons in SU(15) may be very heavy, leading to a natural suppression of rare lepton processes. Indeed, much of the appeal of the 331 model is that the new physics it predicts is guaranteed to be below a few TeV, well within the reach of future colliders. We look forward to both direct and indirect tests that will soon conclusively decide the fate of this model.

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Appendix A. Diagonalizing the charged lepton mass matrix

In this appendix, we examine the new leptonic mixing matrix V_Y and show how it may be related to the lepton Yukawa couplings of Eq. (19). In particular we find a convenient way of determining the three angles and three phases of V_Y in terms of h_s and h_a.

The unitary matrix V_Y comes from diagonalization of the charged lepton mass matrix, $M_\ell = (h_s v_3 + h_a v_1)/\sqrt{2}$ where h_s (h_a) is (anti-)symmetric. In general, the VEVs v_1 and v_3 may be complex, leading to CP violation in the scalar sector. However, these phases can always be absorbed into the Yukawa matrices. Thus we assume this has already been done, and that v_1 and v_3 are both real and positive.

Starting with the lepton Yukawa interaction (19), we may perform the $SU(3)_L$ invariant transformation in family space, $\psi' \to U\psi'$ where U is a unitary matrix. This has no effect on the gauge interactions, but replaces the Yukawa couplings h_i by $U h_i U^T$ in (19). Since h_s is symmetric, we can always find a matrix U such that $U h_s U^T$ is real and diagonal. As a result, this freedom allows us to pick $h_s = \mathrm{diag}(h_s^1, h_s^2, h_s^3)$ where h_s^i is real and positive without any loss of generality. This immediately reduces the number of parameters of h_s from six complex entries to three real ones.

When h_s is chosen in this form, $U h_a U^T$ remains antisymmetric and has three complex entries, h_a^{12}, h_a^{13}, and h_a^{23}. In terms of real parameters, this may be written as $h_a^{ij} = y^{ij} e^{i\delta_{ij}}$, $(i \neq j)$ where $y^{ij} = -y^{ji}$ and $\delta^{ij} = \delta^{ji}$. In this special form, there are now only nine real Yukawa parameters which are completely determined in terms of the three physical charged lepton masses, three mixing angles and three physical phases of V_Y. In this way, the remaining nine real degrees of freedom present in a general complex 3×3 mass matrix have been absorbed in the three angles and six phases of the unobservable unitary matrix U.

Even with h_s in this restricted form, diagonalization of M_ℓ is non-trivial. However, in order to suppress lepton flavor violation, it is natural to assume that the antisymmetric contribution to M_ℓ is small, $h_a|v_1| \lll h_s|v_3|$, so that the mass matrix is almost diagonal. In this limit, and to first order in h_a, we find the charged lepton masses arise only from the symmetric Yukawa coupling, $m_i = h_s^i|v_3|/\sqrt{2}$. The unitary matrices that diagonalize M_ℓ are given by $E_L^{ij} \approx E_R^{ji} \approx \delta^{ij} + \alpha^{ij} e^{i\theta_{ij}}$ where the three angles $\alpha^{ij} = -\alpha^{ji} \lll 1$ and three phases $\theta_{ij} = -\theta_{ji}$ are given by

$$
\begin{aligned}
\alpha^{ij} &\approx \frac{|v_1|}{|v_3|} \frac{y^{ij}}{(h_s^i)^2 - (h_s^j)^2} \sqrt{(h_s^i)^2 + (h_s^j)^2 - 2h_s^i h_s^j \cos 2\delta_{ij}} \\
\tan\theta_{ij} &\approx -\frac{h_s^i + h_s^j}{h_s^i - h_s^j} \tan\delta_{ij} \, . \qquad \text{(A1)}
\end{aligned}
$$

If all the Yukawa couplings are real ($\delta_{ij} = 0$) so there is no explicit CP violation, then the mixing angles have the simple form $\alpha^{ij} \approx y^{ij}|v_1|/(h_s^i + h_s^j)|v_3|$.

Since $V_Y = E_R^T E_L$, in this small mixing limit it has the form $V_Y^{ij} = \delta^{ij} + 2\alpha^{ij} e^{i\theta_{ij}}$ and is approximately diagonal. The physical picture that emerges is that the symmetric coupling, h_s, gives rise to the charged lepton masses, whereas the antisymmetric h_a determines both lepton mixing and (CKM-like) leptonic CP violation. An immediate consequence is that dilepton mediated lepton flavor violation and CP violation are intimately related. Both can be eliminated by demanding $h_a = 0$, as discussed in the main text.

VII. References

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Figure Captions

1. The lepton flavor violating process $\mu \to 3e$ via tree level dilepton exchange.

2. The one-loop diagrams leading to $\mu \to e\gamma$.

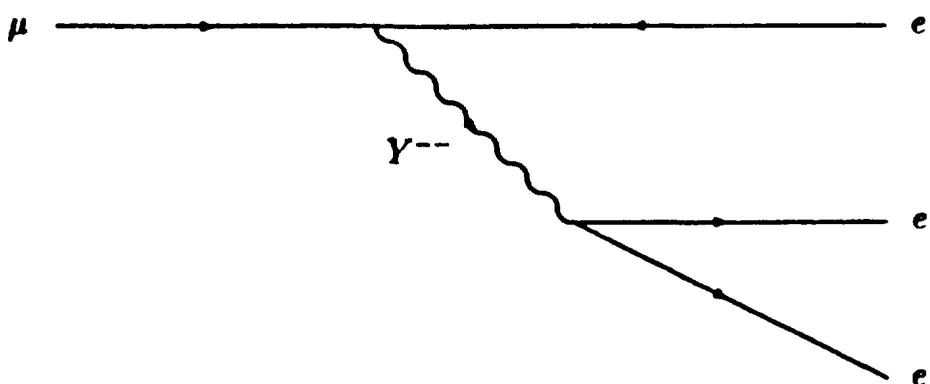

Fig. 1

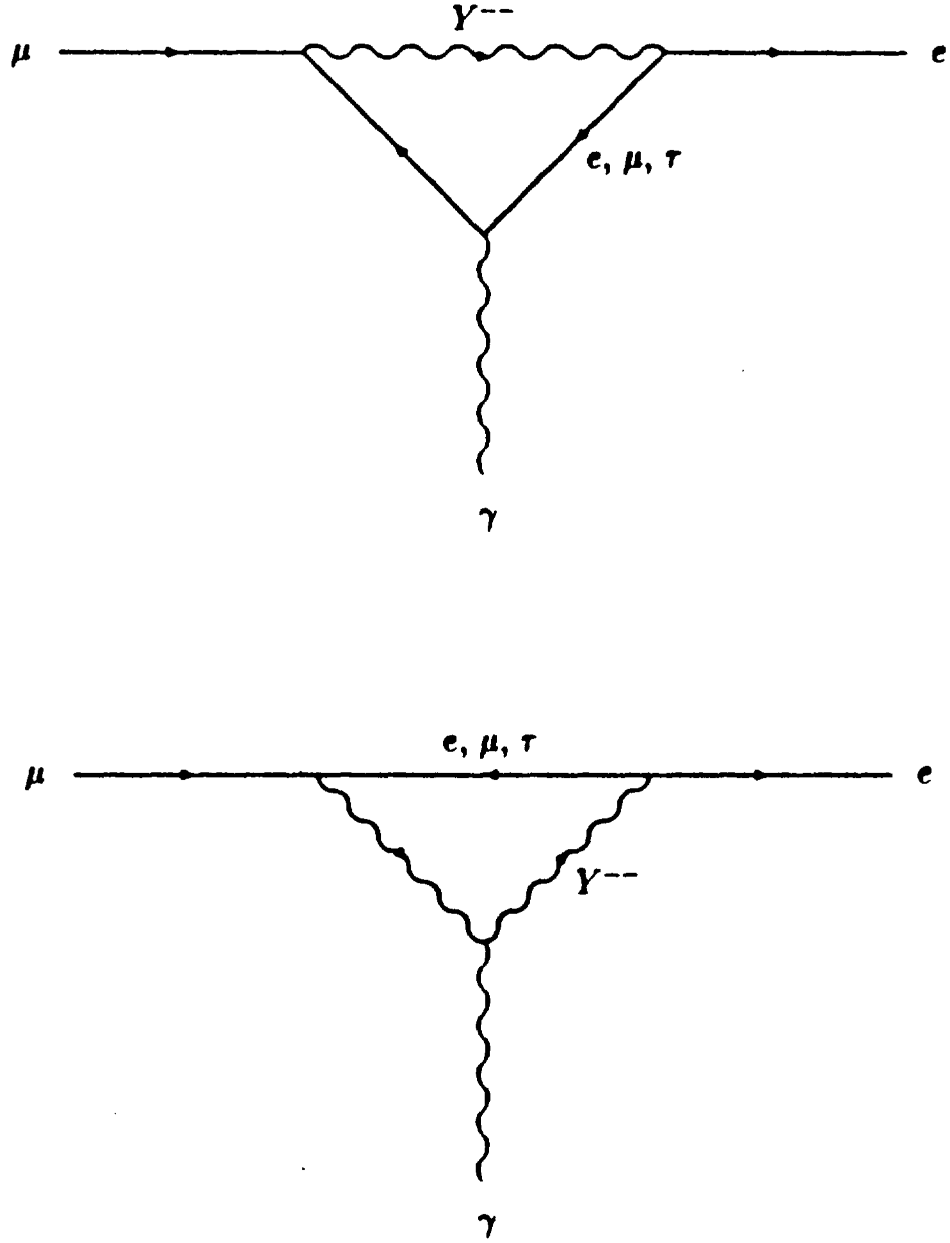

Fig. 2