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QUARK MATTER INSIDE NEUTRON STARS IN AN EFFECTIVE CHIRAL MODEL

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Abstract:

An effective chiral model which describes properties of a single baryon predicts that the quark matter relevant to neutron stars, close to the deconfinement density, is in a chirally broken phase. We find for the SU(2) model that pion-condensed up and down quark matter is preferred energetically at neutron star densities. It exhibits spin ordering and can possess a permanent magnetization. The equation of state of quark matter with chiral condensate is very well approximated by the bag model equation of state with suitably chosen parameters. We study quark cores inside neutron stars in this model using realistic nucleon equations of state. The biggest quark core corresponds to the second order phase transition to quark matter. Magnetic moment of the pion-condensed quark core is calculated.

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1. Introduction

Properties of neutron stars are determined by the equation of state of baryon matter at densities exceeding the nuclear saturation density $n_0 \approx 0.16\ fm^{-3}$. In this density range a phase transition to quark matter occurs which is expected to affect the neutron star structure. Recent studies of millisecond pulsars stimulated investigations of quark structure of neutron stars and even possibility of existence of quark stars [1]. Very recent data [2], strongly suggesting that γ-ray bursters are located at cosmological distances, make the collision of quark-rich stars a promising candidate for a model of this phenomenon [3].

In this paper we study implications for neutron stars of the phase transition from nucleon matter to quark matter described by an effective chiral model. This model is based on the assumption that two fundamental phase transitions predicted by QCD, the deconfinement transition and the chiral symmetry restoration, are separated in baryon density at low temperature. Deconfinement is assumed to occur at much lower densities than the chiral symmetry restoration. The effective chiral model predicts thus that at the deconfinement transition quarks are liberated from nucleons but the chiral symmetry remains spontaneously broken, as in the nucleon phase. This implies that the quark matter in neutron stars has broken chiral symmetry. Earlier studies of quark matter in neutron stars [4]-[6] based mainly on the bag model and perturbative QCD calculations, did not consider such a possibility.

The SU(2) chiral model for up and down quark matter was studied in detail in Refs.[7]-[9]. It predicts presence of a chiral field condensate in the ground state of quark matter. This phase is analogous to the pion condensate in nucleon matter, since the chiral field is composed of pions and sigma-mesons. Presence of the neutral pion condensate leads to a ferromagnetic ordering of quark spins which produces permanent magnetization of this phase. The ferromagnetic quark core if present in a neutron star can contribute to its magnetic moment.

This work is organized as follows. In Sect. 2 we briefly summarize properties of normal quark matter and quark matter with neutral and charged pion condensate in the SU(2) effective chiral model. In Sect. 3 we consider the equation of state of β-stable quark matter in neutron stars. In Sect.4 models of neutron stars with quark cores are constructed and magnetic moments of pion-condensed cores are calculated.

2. The SU(2) effective chiral model

The effective chiral model is described by the σ-model Lagrangian with quark and meson degrees of freedom,

$$\mathcal{L} = i\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - g\bar{\Psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\Psi - U(\sigma,\vec{\pi}) + \frac{1}{2}(\partial_{\mu}\sigma)^2 + \frac{1}{2}(\partial_{\mu}\vec{\pi})^2, \qquad (1)$$

where $\Psi = \binom{u}{d}$ is the isodoublet of quarks interacting with an isotriplet of pions $\vec{\pi}$ via pseudoscalar Yukawa coupling and with an isoscalar meson σ via a scalar Yukawa coupling. Both Yukawa interactions have the same coupling constant g. U is the potential which generates spontaneous breaking of the chiral symmetry,

$$U(\sigma,\vec{\pi}) = \frac{1}{4}\lambda^2(\sigma^2 + \vec{\pi}^2 - F_{\pi}^2)^2, \qquad (2)$$

where λ^2 is the meson coupling constant, $\lambda^2 = m_{\sigma}^2/2F_{\pi}^2$, m_{σ} is the mass of the σ-meson and $F_{\pi} = 93\ MeV$ is the pion decay constant.

The SU(2) effective chiral model is rather successful in reproducing properties of the lightest baryons [10]. The nucleon is described in terms of valence qnarks interacting with the soliton of the chiral field. The model is, in a sense, a relativistic generalization of the constituent quark model, which was very successful in hadron spectroscopy.

The effective quark mass is generated by the σ field: $m = g\sigma$. In the vacuum we have $m_0 = g\sigma_0$, where the vacuum expectation value of the σ field is $\sigma_0 = F_{\pi}$. It becomes energetically favourable at high densities that σ field vanishes and the spontaneously broken chiral symmetry is restored. This transition to the restored symmetry phase makes the quarks massless. It is assumed in this model that deconfinement phase transition occurs at a lower density than the chiral symmetry restoration. In this case the deconfinement phase transition leads to the chirally broken phase of quark matter with "constituent"-like masses generated dynamically by the quark interactions with the σ field.

The model parameters m_0 of the order of 500 MeV and m_σ around 1GeV give good fits to baryon properties [10]. However the fits are not sensitive to m_σ as long as it is large. In Ref. [8] we have studied properties of quark matter in this model for various values of these parameters.

One can distinguish two different phases of SU(2)-symmetric up and down quark matter with broken chiral symmetry: the normal quark matter with nonzero sigma field and vanishing pion field, and the pion-condensed quark matter with space-dependent pion and sigma fields [8].

2.1 Normal quark matter

In this case we neglect the pion fields and replace the sigma field by a constant expectation value $\sigma = \bar{\sigma}$. The quark mass $m = g\bar{\sigma}$ is generated through the interaction term in the Lagrangian, Eq.(1). The energy density of a uniform, quark matter is

$$nW = \frac{1}{(2\pi)^3}\gamma \int_0^{k_F} d^3k(k^2 + g^2\bar{\sigma}^2)^{1/2} + \frac{1}{4}\lambda^2(\bar{\sigma}^2 - F_\pi^2)^2, \tag{3}$$

where the first term is the energy density of the quark Fermi sea and the second one is the energy density of the σ field resulting from the potential term $U(\sigma, \vec{\pi})$, Eq.(2). For isospin-symmetric quark matter the spin-flavour-colour degeneracy $\gamma = N_f N_c \gamma_s$ is $\gamma = 12$. The energy density, Eq.(3), depends on the baryon density through the quark Fermi momentum $k_F = (6\pi^2 n_q/\gamma)^{1/3}$, where n_q is the quark density and the baryon density is $n_B = (1/3)n_q$. The ground state of the system corresponds to the value of $\bar{\sigma}$ which minimizes the energy W at a fixed value of the baryon density n_B. This gives $\bar{\sigma}$ as a function of baryon density, $\bar{\sigma} = \bar{\sigma}(n_B)$. Hence the quark mass is density dependent, $m = m(n_B)$. If we obtain in this procedure vanishing sigma field $\bar{\sigma} = 0$ at some baryon density n_r, the chiral symmetry will be restored. In such a case both the Lagrangian, Eq.(1), and the ground state have the same $SU(2)_L \times SU(2)_R$ symmetry.

In the chirally symmetric phase, with massless quarks and vanishing sigma field, the energy density is

$$nW = \frac{1}{(2\pi)^3}\gamma \int_0^{k_F} d^3k k + U(\sigma = 0, \vec{\pi} = 0). \tag{4}$$

This equation is equivalent to that for the MIT bag model with massless quarks. The last term in Eq.(4) plays the role of the bag constant,

$$U(\sigma = 0, \vec{\pi} = 0) \equiv B = \frac{1}{8}m_\sigma^2 F_\pi^2. \tag{5}$$

2.2 Quark matter with the pion condensate

If the pions appear in the ground state they will form a Bose-Einstein condensate. In the pion-condensed phase some components of the pion field acquire a nonzero expectation value. The following ansatz with non-vanishing expectation values of the neutral meson fields is assumed [11]

$$\sigma(\vec{r}) = \bar{\sigma}cos(\vec{q}\cdot\vec{r}), \quad \pi_3(\vec{r}) = \bar{\sigma}sin(\vec{q}\cdot\vec{r}), \quad \pi_1 = \pi_2 = 0. \tag{6}$$

The Dirac equation for quark fields with this ansatz has the form

$$(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m exp(i\gamma_5\tau_3\vec{q}\cdot\vec{r}))\phi(\vec{r}) = E\phi(\vec{r}), \tag{7}$$

where ϕ is the Dirac spinor and the effective quark mass is $m = g\bar{\sigma}$. This equation was solved in Ref.[8]. The quasiparticle spectrum is:

$$E_\pm(\vec{k}) = \{m^2 + \vec{k}^2 + \frac{1}{4}\vec{q}^2 \pm [m^2\vec{q}^2 + (\vec{q}\cdot\vec{k})^2]^{1/2}\}^{1/2}. \tag{8}$$

The ground state of quark matter is constructed by filling the Fermi seas for two branches of the spectrum (8) up to the quark Fermi energy E_F. The baryon density is

$$n_B = \frac{1}{3(2\pi)^3}\gamma\left[\int d^3k\Theta(E_F - E_-(\vec{k})) + \int d^3k\Theta(E_F - E_+(\vec{k}))\right], \tag{9}$$

where the spin-flavour-colour degeneracy for a single branch is $\gamma = 6$. The energy density of the system is

$$n_B W = \frac{1}{(2\pi)^3}\gamma\left[\int d^3kE_-(\vec{k})\Theta(E_F - E_-(\vec{k})) + \int d^3kE_+(\vec{k})\Theta(E_F - E_+(\vec{k}))\right]$$
$$+\frac{1}{2}\bar{\sigma}^2\vec{q}^2 + \frac{1}{4}\lambda^2(\bar{\sigma}^2 - F_\pi^2)^2. \tag{10}$$

It consists of the energy density of the occupied Fermi seas of E_- and E_+ quasiparticles and the energy density of the chiral field.

The integrals in Eqs.(9) and (10) can be evaluated analytically for the spectrum given by Eq.(8). The baryon densities n_B^- and n_B^+, corresponding to occupied Fermi seas of E_- and E_+ quasiparticles are, respectively,

$$n_B^\mp = \frac{\gamma}{3(2\pi)^2}\{(E_F^2 - m^2 - \frac{1}{4}q^2)k_z - \frac{1}{3}k_z^3$$
$$\pm\frac{1}{2}q(k_z\rho_z + m^2 ln|k_z + \rho_z|)\}\Big|_{k_{min}}^{k_{max}}, \tag{11}$$

where $\rho_z = \sqrt{m^2 + k_z^2}$. The integration limits in Eq.(11) are

$$k_{min} = \begin{cases} 0 & \text{for } m > \frac{1}{2}q \text{ and } E_F > m - \frac{1}{2}q \\ 0 & \text{for } m < \frac{1}{2}q \text{ and } E_F > \frac{1}{2}q - m \\ [(E_F - \frac{1}{2}q)^2 - m^2]^{1/2} & \text{for } m < \frac{1}{2}q \text{ and } E_F < \frac{1}{2}q - m \end{cases}, \tag{12}$$

and

$$k_{max} = [(E_F + \frac{1}{2}q)^2 - m^2]^{1/2} \tag{13}$$

for all cases in Eq.(12). The E_+ quark Fermi sea is occupied only if $E_F > \frac{1}{2}q + m$ and the integration limits in Eq.(11) are

$$k_{min} = 0, \quad k_{max} = [(E_F - \frac{1}{2}q)^2 - m^2]^{1/2}. \tag{14}$$

Energy densities of occupied E_- and E_+ Fermi seas are

$$n_B W_{\mp} = \frac{2\gamma}{3(2\pi)^2}\Big\{E_F^3 k_z - \frac{1}{4}[k_z(m^2+k_z^2)^{3/2} + \frac{3}{2}(m^2 k_z \rho_z + m^4 ln|k_z+\rho_z|)]$$

$$-\frac{3}{8}q^2[k_z\rho_z + m^2 ln|k_z+\rho_z|] \pm (\frac{3}{2}qm^2k_z + \frac{1}{2}qk_z^3 + \frac{1}{8}q^3k_z)\Big\}\Big|_{k_{min}}^{k_{max}}, \tag{15}$$

with the same integration limits as for $n_B^{\mp}$, Eq.(11).

The values of the variational parameters q and m in the ground state are determined by minimizing the grand potential density Ω with respect to $q = |\vec{q}|$ and $\bar{\sigma} = m/g$ at fixed baryon chemical potential. The grand potential density Ω at zero temperature is

$$\Omega(q, \bar{\sigma}) = n_B W - \mu n = -p, \tag{16}$$

where the energy density $n_B W$ and baryon density n_B are sums of contributions given, respectively, by Eqs.(15) and (11), $\mu = 3E_F$ is the baryon chemical potential and p is the pressure. One can notice that the ground state is determined by maximizing the pressure at fixed μ. Minimization of Ω gives the baryon density n_B, the energy per baryon W, the pressure p and the values of q and m as functions of baryon chemical potentials.

The results of this analysis are presented in detail in Ref.[8]. Here we only display in Fig.1 the energy per baryon as a function of baryon density for both normal and pion condensed quark matter in case of neutron-like quark matter. We show results for two limiting values of the parameter m_σ, which are $m_\sigma = 700 MeV$ and $m_\sigma = 1200 MeV$. The value of the quark mass is $m_0 = 500 MeV$. One can notice that the pion-condensed phase (C) is always of lower energy than the normal quark matter (N).

2.3 Magnetic properties of the pion-condensed quark matter

The quasiparticles diagonalizing the Dirac Hamiltonian are the up and down quarks with the spin polarized along the wave vector $\vec{q}$, as shown in Ref.[8]. Corresponding spin densities are

$$s_{u(d)} = \frac{1}{4}\langle\bar{\Psi}(1 \pm \tau_3)\gamma_0\Sigma_z\Psi\rangle. \tag{17}$$

The magnetization of the system is

$$M = g_q(\mu_u s_u + \mu_d s_d), \tag{18}$$

where $g_q = 2$ is the gyroscopic factor, μ_u and μ_d are the up and down quarks magnetons. We assume Dirac magnetic moments for quarks which give $\mu_u = -2\mu_d$. The quark matter with neutral pion condensate has a net magnetization. For symmetric up and down quark matter the total spin density is zero since the up and down quark Fermi seas are oppositely polarized, but the opposite electric charges of up and down quarks cause their magnetic moments to point in the same direction.

In Fig.2 we show the magnetization, Eq.(17), corresponding to spin densities (17) as a function of baryon density for neutron-like quark matter. It will be used to find the magnetic dipole moment of the neutron star core with pion-condensed quark matter.

3. Equation of state for quark matter in neutron stars

3.1 Equation of state of pion-condensed up and down quark matter

The equation of state of pion-condensed up and down quark matter is obtained from Eqs.(11) and (15). These give the baryon density and the energy density as functions of the baryon chemical potential μ. The pressure $p(\mu)$ is determined by minimizing the grand potential density Ω in Eq.(16).

We treat the σ-meson mass as a parameter of the model. Equations (11) and (15) give a one-parameter family of pion-condensed quark matter eqnations of state for a given value of g. We choose $g = 5$ since this value gives best fits of single baryon properties [10]. In Figs.3 and 4 we show respectively the baryon density and the pressure as functions of the baryon chemical potential for two values of m_σ: $m_\sigma = 700MeV$ and $m_\sigma = 1200MeV$. The curves in Figs.3 and 4 correspond to neutron-like quark matter with up quark fraction $x = 1/3$.

The analytical form of Eqs.(11) and (15) in the presence of the pion condensate is rather complicated. Numerically, however, the energy density $n_B W$ is very well approximated by a simple bag model formula for massless quarks:

$$\epsilon_{ud}(f_u, f_d; n_B) = \frac{3}{4}\pi^{2/3}(\sum_{i=u,d} f_i^{4/3})n_b^{4/3} + B, \tag{19}$$

where B is the bag constant and f_i are flavour concentrations, $f_i = n_i/n_B, i = u, d, s$. The corresponding pressure and baryon chemical potential is, respectively

$$P = \frac{1}{4}\pi^{2/3}(\sum_{i=u,d} f_i^{4/3})n_B^{4/3} - B \tag{20}$$

and

$$\mu = \pi^{2/3}(\sum_{i=u,d} f_i^{4/3})n_B^{1/3}. \tag{21}$$

The bag constant B has to be chosen appropriately for a given value of the scalar meson mass m_σ. In Fig.5 we show B as a function of m_σ as obtained by the least-squares fitting of the energy per baryon, Eq.(15), with the bag model formula (21) for 20 points. The quality of the fit is very good, with the maximum deviation below 0.2% and it can be safely used in astrophysical applications.

As a check of applicability of the bag model formulae to the pion-condensed quark matter in the effective chiral model we use Eq.(21) for the baryon chemical potential, which does not depend on the bag constant. In the effective chiral model B is a function of m_σ. Hence if one plots the baryon density as function of baryon chemical potential for two different values of m_σ, the two curves should coincide with one another. This is the case of Fig.3, where the curves correspond to $m_\sigma = 1200 MeV$ and $m_\sigma = 700 MeV$.

The agreement of the equation of state of pion-condensed quark matter corresponding to broken chiral symmetry and the chirally symmetric bag model equation of state indicates that restoration of chiral symmetry amounts to change in the bag constant. For $m_\sigma = 1200 MeV$ we find the value $B \approx 150 MeV fm^{-3}$ in case of the pion-condensed quark matter while the value corresponding to the chirally symmetric normal quark matter, Eq.(5), is $B = 200 MeV fm^{-3}$. The main reason of such a behaviour of the pion-condensed quark matter equation of state is the fact that the kinetic energy term in Eq.(10) for pion-condensed quark matter is essentially the same as for massless quarks. The coincidence of the pion-condensed quark matter equation of state, with broken chiral symmetry, and chirally symmetric equation of state for massless quarks is studied elsewhere [12].

The fact that the equation of state of pion-condensed quark matter is to a very good approximation reproduced by the bag model equation of state simplifies considerably further calculations, in particular construction of astrophysically relevant quark equation of state.

3.2 Quark matter in neutron stars

Quark matter in neutron stars is generally composed of up, down and strange quarks. It should be β-stable and electrically neutral. Charge neutrality condition is

$$n_e + \frac{1}{3}(n_d + n_s) = \frac{2}{3}n_u. \tag{22}$$

Here n_e is the electron density. It turns out that muons can be neglected since the electron chemical potential usually does not exceed the muon rest mass for baryon densities in neutron stars.

To account for the β-stability we have to add contributions to the energy density due to strange quarks and electrons. The strange quark energy density $\epsilon_s(n_s)$ is discussed below. The electron energy density is

$$\epsilon_e(n_e) = \frac{1}{4}3^{4/3}\pi^{2/3}(f_u - 1)^{4/3} n_B^{4/3}, \tag{23}$$

The β-stability requires that the total energy density of the system

$$\epsilon(f_u, f_d, f_s; n_B) = \epsilon_{ud}(n_u, n_d) + \epsilon_s(n_s) + \epsilon_e(n_e), \tag{24}$$

is stationary with respect to flavour concentrations. Since the flavour concentrations satisfy the sum rule

$$f_u + f_d + f_s = 3, \tag{25}$$

only two of them are independent variables.

The charge neutrality condition and β-stability requirement allow us to determine uniquely the flavour composition of quark matter at given baryon density n_B. The β-stability requirement leads to relations between quark and electron chemical potentials

$$\mu_d = \mu_u + \mu_e \tag{26}$$

and

$$\mu_d = \mu_s. \tag{27}$$

To determine the composition of quark matter we adopt the strange quark energy density in the form

$$\epsilon(f_s, n_B) = \frac{6}{(2\pi)^3} \int_{k<k_s} d^3k \sqrt{k^2 + m_{s,eff}^2}, \tag{28}$$

where $k_s = (\pi^2 f_s n_B)^{1/3}$ is the Fermi momentum of the strange quark Fermi sea.

For massless up and down quarks and neglecting electron rest mass from Eqs.(26) and (27) we find equations for flavour concentrations in the form

$$f_d^{1/3} = f_u^{1/3} + 3^{1/3}(f_u - 1)^{1/3}, \tag{29}$$

$$f_d^{1/3} = \sqrt{f_s^{2/3} + y^2}, \tag{30}$$

where $y = m_{s,eff}/(\pi^2 n_B)^{1/3}$. Using Eq.(25) one can solve Eqs.(26) and (27) for f_u, f_d and f_s. At lower densities when the chemical potential of the down quark is $\mu_d < m_{s,eff}$, there is no strange quark component in the quark matter, $f_s = 0$. The β-stable up and down quark matter is essentially neutron-like quark matter, $f_u \approx 1, f_d \approx 2$. The exact solution of Eq.(29) gives $f_u = 1.006, f_d = 1.994$.

When the chemical potential of the down quark exceeds the effective mass of the strange quark, $\mu_d > m_{s,eff}$, the strange quark Fermi sea starts to be populated. The threshold density is $n_B = m_{s,eff}^3/\pi^2 f_d$, where f_d is the above value of the equilibrium down flavour concentration. One should notice that solutions of Eqs.(29) and (30) depend only on one parameter y^2, whose value at the threshold is $y^2 = f_d^{2/3} = 1.58$. In Fig.6 the flavour concentrations are shown as functions of the parameter y^2. With decreasing y^2 concentration of strange flavour f_s increases, f_d decreases and f_u remains flat. In the limit $y^2 = 0$ all flavours have the same concentrations, $f_u = f_d = f_s = 1$.

Presence of strange quarks affects the equation of state of quark matter mainly through the flavour composition of β-stable quark matter. This fact is rather evident if we write the energy density of quark matter in the form

$$\epsilon_{uds}(f_u, f_d, f_s; n_B) = \frac{3}{4}\pi^{2/3} a n_B^{4/3} + B + \Delta\epsilon_s(f_s, n_B). \quad (31)$$

The first two terms represent the bag model energy density for three massless flavours, with

$$a = \sum_{i=u,d,s} f_i^{4/3} + 3^{1/3}(f_u - 1)^{4/3}. \quad (32)$$

The last term in this formula is the electron contribution, which can be neglected as f_u is always close to unity. The parameter a for β-stable quark matter is shown in Fig.7.

The correction $\Delta\epsilon_s(f_s, n_B)$ accounts for deviations of the strange quark energy density from the massless quark formula which are mainly due to higher effective mass of the strange quark,

$$\Delta\epsilon_s(f_s, n_B) = 3\pi^{2/3} n_B^{4/3} f_s^{4/3} \int_0^1 dz z^2 (\sqrt{z^2 + y^2 f_s^{-2/3}} - z) \equiv \frac{3}{4}\pi^{2/3} n_B^{4/3} \Delta a. \quad (33)$$

This expression shows that the correction Δa is small near the threshold, where f_s is small, as well as at higher densities, where y^2 is small. In Fig.8 we show $\Delta a/a$ as a function of y^2. The relative correction has a maximum value 0.092 for $y^2 = 0.7$.

To obtain actual values of $\Delta\epsilon_s(f_s, n_B)$ in the effective chiral model one should specify the effective mass of the strange quark, $m_{s,eff}$. In general this mass depends on density, $m_{s,eff}(n_B)$. It can be obtained by extending the effective chiral model to the SU(3) group. This is beyond the scope of this paper and will be studied elsewhere [12]. Here we shall only find some constraints which the parameter y^2 should satisfy for quark matter to be present inside neutron stars. These constraints are related to the fact that the quark equation of state, Eq.(31), allows for presence of sizable quark cores inside neutron stars only for values of the parameter a close to a specific value $a = a_t$ which depends on the nucleon equation of state, as discussed below.

4. Quark cores in neutron stars

The form (31) of the bag model equation of state is very useful for astrophysical considerations as it shows that there are two relevant parameters in this equation of state: The coefficient a of the kinetic term and the bag constant B. Both of them are subject to considerable uncertainties. In the effective chiral model considered here the parameter a is determined by the value of y^2.

In Ref.[13] we have studied presence of quark cores inside neutron stars for the bag model equation of state, Eq.(31), using realistic equations of state to describe the nucleon phase. We have found that the maximum quark core inside neutron star corresponds to a continuous phase transition from nucleon matter to quark matter. For a given equation of state such a continuous phase transition can occur only for a specified value of the parameter $a = a_t$, provided the deconfinement density has a proper value. The values of a_t, corresponding to the three equations of state from Ref.[13] are in the range $3.19 \leq a_t \leq 3.34$. Requirement that $a = a_t$ for a given nucleon equation of state is a necessary condition for presence of a sizable quark core inside neutron star. If the parameter a differs considerably from a_t, there are no astrophysically interesting quark cores inside neutron star, for any value of the bag constant.

In order to construct a phase transition from nucleon matter to quark matter in a neutron star we use three equations of state, AV14+UVII, UV14+UVII and UV14+TNI, derived by Wiringa, Fiks and Fabrocini [14]. The results discussed here were obtained by fitting with a polynomial the values of the energy per particle, E, of the β-stable matter given in Table V of Ref.[14]. This fit, which is given in the appendix, was then used to obtain the baryon chemical potential, μ_N, and the pressure, P_N, of the nucleon phase according to the formulae:

$$\mu_N = \frac{\partial(n_B E)}{\partial n_B} + m \tag{34}$$

and

$$P_N = (\mu_N - m - E)n_B. \tag{35}$$

Here m is the nucleon rest mass.

An important observation is that from Eq.(21) one can obtain the baryon density of the quark phase as a function of the baryon chemical potential, $n_B(\mu) \sim \mu^3$, which is independent of the bag constant B. For the nucleon phase one finds $n_B(\mu) \sim \mu^\alpha$ with α generally below unity, $\alpha < 1$, for any realistic nucleon equation of state [13]. The functions $n_B(\mu)$ for quark and nucleon phase have opposite curvatures. This allows us to find such a value a_t of the parameter a that $n_B^{(Q)}(\mu)$ for quark matter is tangent to $n_B^{(N)}(\mu)$ for nucleon matter. The value of the baryon chemical potential at the tangency point, μ_t, is distinguished, as the phase transition occurring at μ_t is continuous (second order). To construct a second order phase transition a suitable value of the bag constant has to be chosen [13]. In Table I parameters of the second order phase transitions are given, for the three nucleon equations of state.

The values of a_t for the three nucleon equations of state are in the range $3.19 \leq a_t \leq 3.34$. From Fig.7 where the parameter a is plotted as a function of y^2 one can find values of y^2 corresponding to the values of a_t in Table I. These values of y^2 are given in the Table I. We also give in Table I the values of the sigma meson mass corresponding to the values B_t of the bag constant for which the second order phase transition occurs.

We have constructed neutron star models corresponding to the second order phase transition to quark matter for the three nucleon equations of state. In this case the quark matter occupies the biggest fraction of the star. The neutron star models were obtained in a standard way by numerically integrating the Tolman-Oppenheimer-Volkoff equation (see e.g. Shapiro and Teukolsky [15]) for the equations of state with the phase transition from nucleon matter to quark matter described above. At subnuclear densities we used the equation of state derived by Baym, Bethe and Pethick [16]. In the crust we employed the equation of state due to Baym, Pethick and Sutherland [17] and, at low densities, the one due to Feynman, Metropolis and Teller [18].

The results are shown in Fig.9 where we show the neutron star mass as a function of the central density. Only those stars are considered which contain quark cores.

The quark core appears when the central baryon density n_b exceeds the deconfinement density n_t. For equations of state constructed in this paper quark cores exist inside neutron stars heavier than about $1M_\odot$.

Maximum masses M_{max} of neutron stars which contain quark cores in Fig.9 differ by about $0.1M_\odot$, i.e. they are much closer to each other than maximum masses of corresponding pure nucleon stars, with no phase transition to quark matter, which are $2.20M_\odot$, $2.12M_\odot$ and $1.84M_\odot$. One should also note that the values of M_{max} in Fig.9 exceed only slightly the lower observational limit of $1.55M_\odot$.

The quark core can contribute to the magnetic moment of a neutron star if the quark matter is magnetized. Using magnetization (18) we have calculated the magnetic moment of the quark core and its contribution to the polar magnetic field B_P of the star. In Fig.10 the magnetic field is shown as a function of the central baryon density n_b. The order of magnitude is $B_P \sim 10^{15}G$. This value exceeds magnetic fields observed in pulsars, which are typically $10^{12}G$. Very strongly magnetized neutron star, with magnetic field exceeding $10^{14}G$, was proposed recently as a model of repeating soft γ-ray bursters [19].

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Appendix

The energy per particle of neutron star matter for the three equations of state from Ref.[14] is fit with the following polynomials:
The AV14+UVII equation of state:

$$E(n_B) = 2.65511 + 76.744n_B - 183.611n_B^2 + 459.906n_B^3 - 122.832n_B^4, \qquad (36)$$

the UV14+UVII equation of state:

$$E(n_B) = 7.57891 - 1.23275n_B + 227.384n_B^2 - 146.596n_B^3 + 324.823n_B^4 - 120.355n_B^5 \quad (37)$$

and the UV14+TNI equation of state:

$$E(n_B) = 6.33041 - 28.1793n_B + 288.397n_B^2 - 65.2281n_B^3. \qquad (38)$$

In the above formulae n_B is in fm^{-3} and E is in MeV.

The sum of squares of residuals for 18 entries from Table V Ref.[14] is, respectively, $1.5927MeV^2$, $1.9629MeV^2$ and $1.2946MeV^2$.

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FIGURE CAPTIONS

Fig.1 Quark matter energy per unity of baryon number as a function of baryon density. Solid and dashed lines correspond, respectively, to $m_\sigma = 700 MeV$ and $m_\sigma = 1200 MeV$. N and C label, respectively, normal and pion-condensed quark matter.

Fig.2 Magnetization of pion-condensed quark matter versus baryon density. Solid and dashed lines correspond, respectively, to $m_\sigma = 700 MeV$ and $m_\sigma = 1200 MeV$.

Fig.3 Baryon density as a function of baryon chemical potential for pion-condensed quark matter. Solid and dashed lines correspond, respectively, to $m_\sigma = 700 MeV$ and $m_\sigma = 1200 MeV$.

Fig.4 Pressure as a function of baryon chemical potential for pion-condensed quark matter. Solid and dashed lines correspond, respectively, to $m_\sigma = 700 MeV$ and $m_\sigma = 1200 MeV$.

Fig. 5 Bag constant as a function of the σ meson mass.

Fig.6 Flavour concentrations of β-stable quark matter as functions of y^2.

Fig.7 The parameter a as a function of y^2.

Fig.8 The relative correction $\Delta a/a$ as a function of y^2.

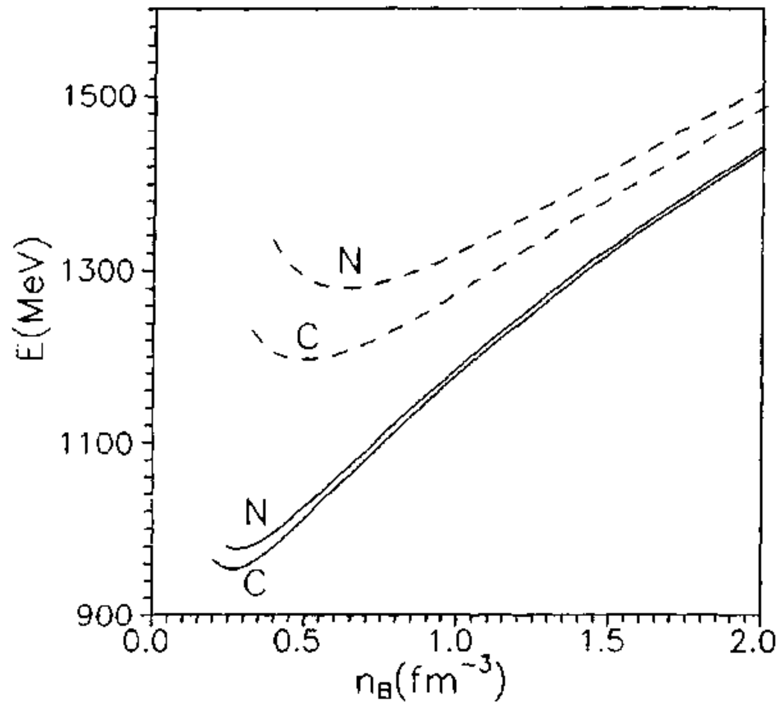
Fig.9 Neutron star mass M and quark core mass M_q as functions of the central baryon density. Solid, long-dashed and short-dashed lines correspond, respectively, to AV14+UVII, UV14+UVII and UV14+TNI equation of state.

Fig.10 Neutron star magnetic field due to magnetized quark core as a function of central baryon density. Solid, long-dashed and short-dashed lines correspond, respectively, to AV14+UVII, UV14+UVII and UV14+TNI equation of state.

TABLE I

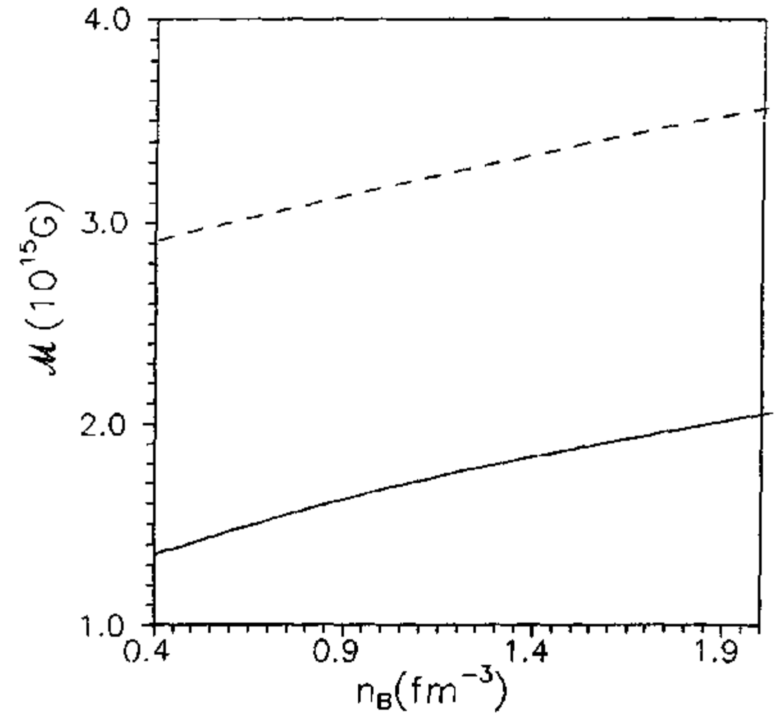
Parameters of quark equations of state for which continuous phase transition from nucleon matter to quark matter occurs. The columns contain respectively specification of the nucleon equation of state, the bag model parameter a_t, the baryon density n_t, the corresponding value of parameter y^2, the bag constant B_t and corresponding value of the sigma meson mass m_σ.

Equation of state:	a_t	$n_t[fm^{-3}]$	y^2	$B_t[MeV fm^{-3}]$	$m_\sigma[MeV]$
AV14+UVII	3.191	0.510	0.88	90.0	861
UV14+UVII	3.335	0.481	1.19	80.3	808
UV14+TNI	3.265	0.566	1.05	86.3	841



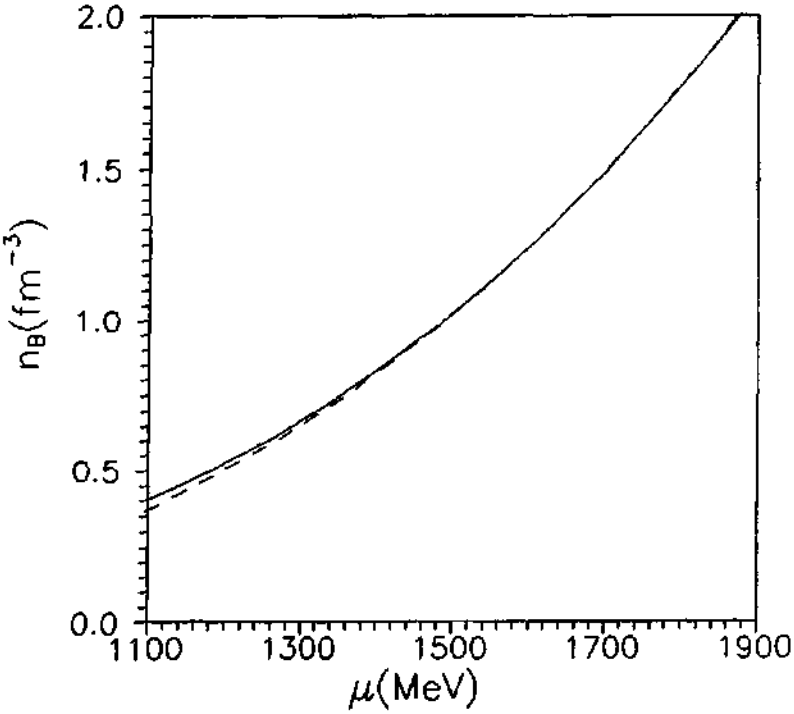
1500
1300
1100
900
E(MeV)
N
C
N
C
0.0
0.5
1.0
1.5
2.0
n_B(fm^-3)

Fig. 1



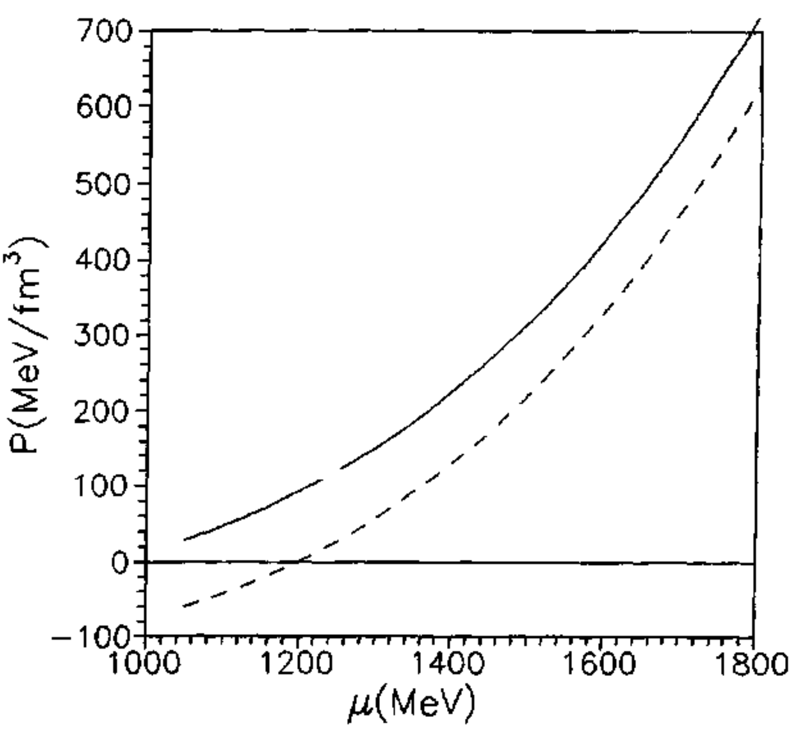
4.0
3.0
2.0
1.0
M (10^15 G)
0.4
0.9
1.4
1.9
n_B(fm^-3)

Fig. 2



2.0
1.5
1.0
0.5
0.0
$n_B(\mathrm{fm}^{-3})$
1100
1300
1500
1700
1900
μ(MeV)

Fig. 3



700
600
500
400
300
200
100
0
−100
$P(\mathrm{MeV/fm}^3)$
1000
1200
1400
1600
1800
μ(MeV)

Fig. 4

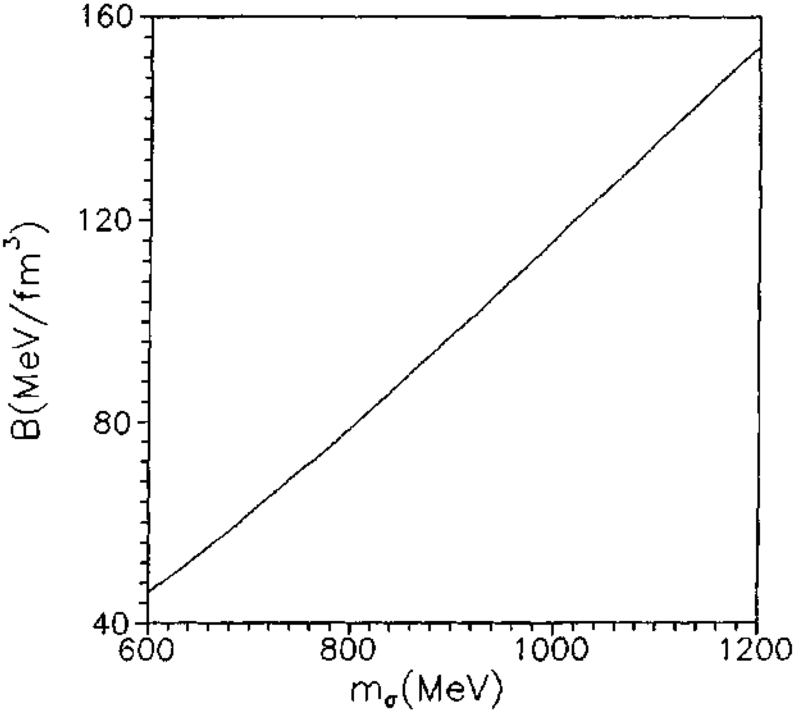

160
120
80
40
600
800
1000
1200
B(MeV/fm³)
m_σ(MeV)

Fig. 5

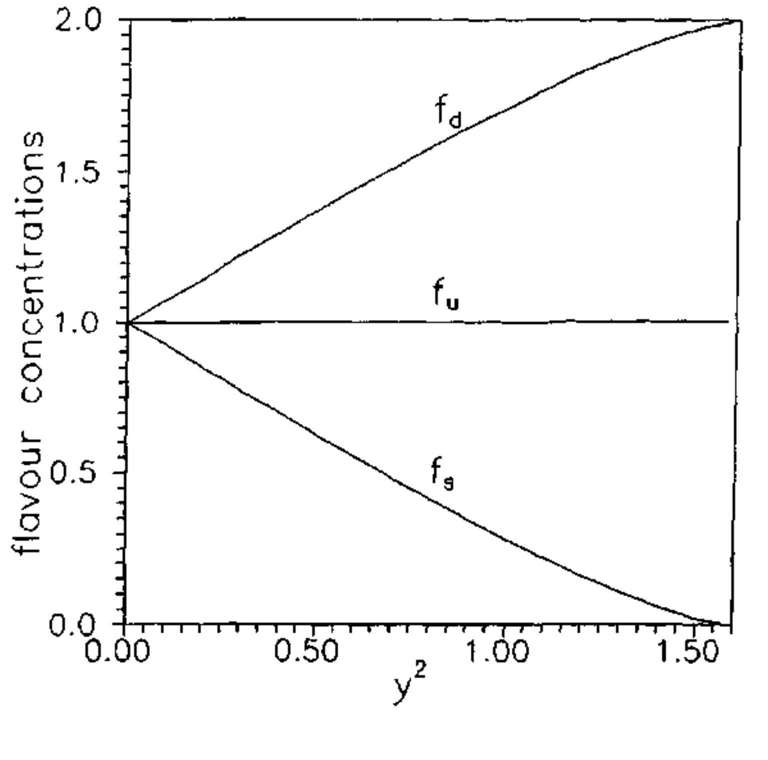

2.0
1.5
1.0
0.5
0.0
0.00
0.50
1.00
1.50
flavour concentrations
y²
f_d
f_u
f_s

Fig. 6

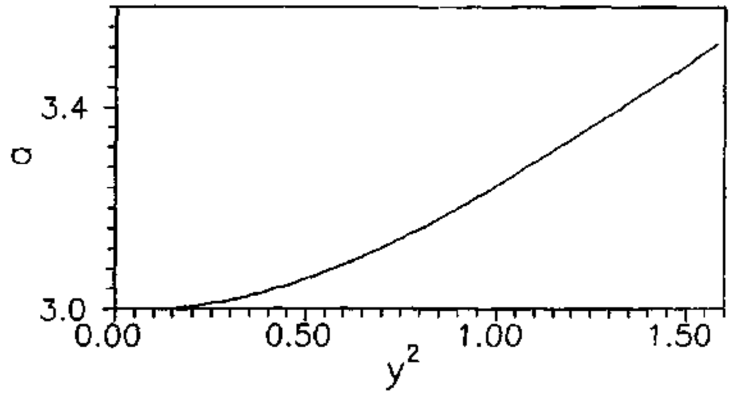


Fig. 7

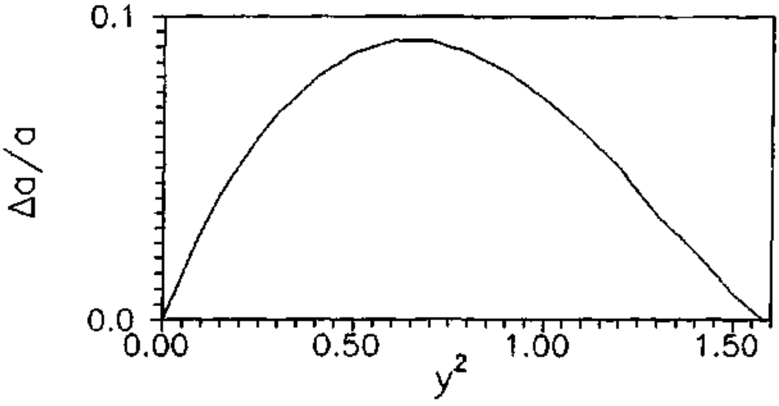


Fig. 8

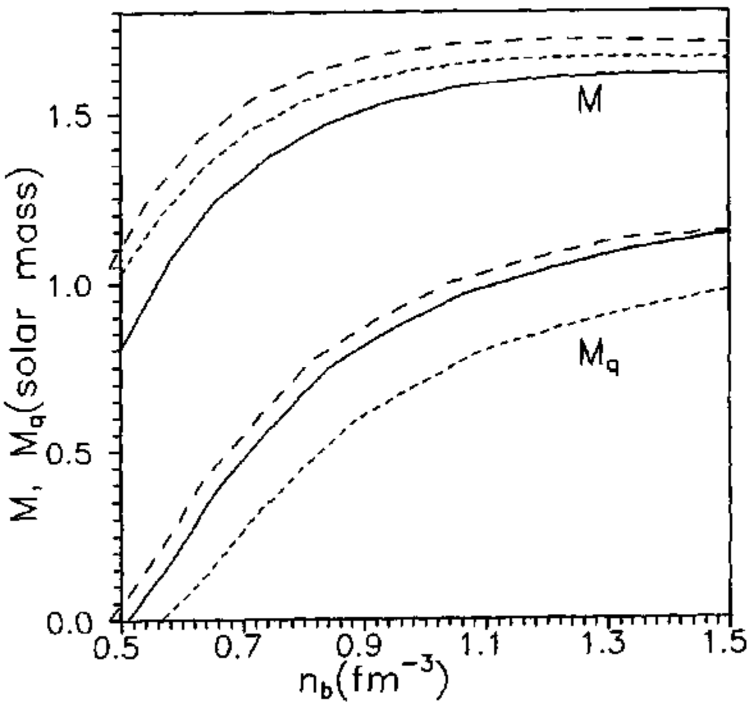

M, M_q(solar mass)
M
M_q
0.0
0.5
1.0
1.5
0.5
0.7
0.9
1.1
1.3
1.5
$n_b(fm^{-3})$

Fig. 9

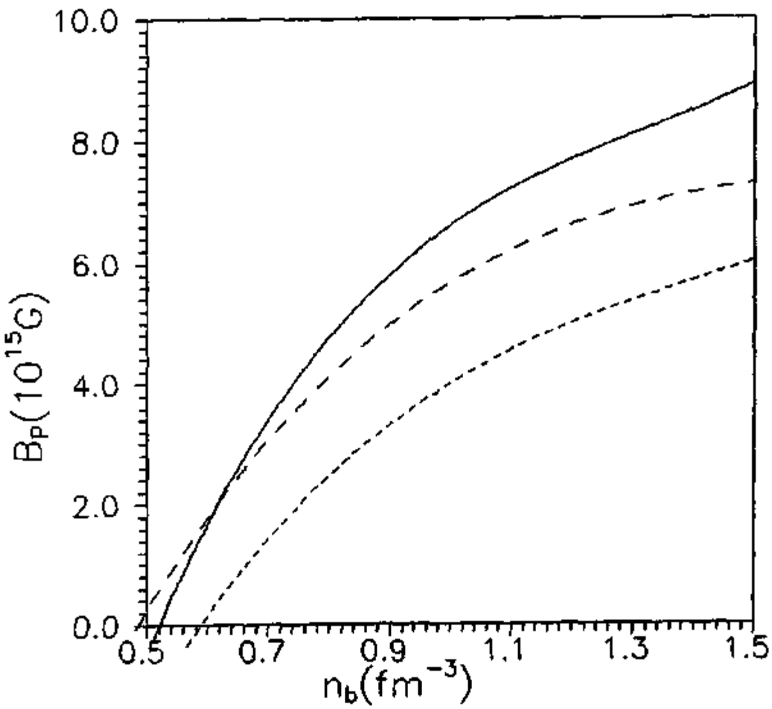

$B_P(10^{15}G)$
0.0
2.0
4.0
6.0
8.0
10.0
0.5
0.7
0.9
1.1
1.3
1.5
$n_b(fm^{-3})$

Fig. 10