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## INTRODUCTION

During a very large reactor accident, the reactor core could be vaporized into a uranium gas bubble with internal pressures of the order of 100 kilobars. The time required to produce the energy, vaporize the core, and produce the high gas pressure is so short (a few microseconds), that a shock wave is established. The shock wave is transmitted through the materials surrounding the core. If these materials are compressible, relief of high gas bubble pressure is obtained by expansion of the gas bubble by the same volume that the surrounding material is compressed by the shock wave. This mechanism of pressure relief occurs very quickly after the formation of the shock wave. After the shock wave has passed completely through the surrounding material, further pressure relief is obtained by outward movement of the material without compression. The problem of shock wave transmission and attenuation has been solved analytically for a very limited number of cases, and then only when simple geometric and material conditions are used. None of the cases have been found to be applicable to the geometry and materials usually associated with reactors. Consequently, it has become common practice in reactor accident analysis to neglect the shock wave with consequent material compression, and assume that all pressure relief is obtained by material movement with no compression.

## METHOD

Recently, it was pointed out by Prof. H. A. Bethe, that the shock wave equations could be written for the geometry and material configuration which is typical of many reactors. The particular configuration is that of a spherical gas bubble surrounded by one or more concentric regions of compressible material. In order to make it possible to obtain a solution for the problem being considered, three basic assumptions were made. The first assumption is that the gas bubble is an ideal gas expanding adiabatically. The restriction that this expansion is adiabatic can be shown to be justifiable. Non-adiabatic work, e.g., radiation, can be shown to be negligible compared to adiabatic work done during the expansion. The restriction that the gas be an ideal gas was made because it is approximately true with respect to the available experimental data and it simplifies the problem. It can be relaxed into any gas law that one might wish to write. The second and third deal with the crushing that occurs. The second assumption is that when the shield material is crushed, the extent of the crushing is independent of the shock pressure. The third assumption is that once the material is compressed, the material remains compressed without further compression or expansion. These two assumptions appear to be physically reasonable within the pressure levels (kilobars) and time scales (microseconds) which are of interest. These assumptions were used to write down for a particular problem certain analytic expressions which can be treated numerically. The specific approach used will be demonstrated for the case of a gas bubble surrounded by one crushable region and the method will be outlined for the case of a gas bubble surrounded by two concentric crushable regions. The basic hydrodynamic equation in the crushed material is the force balance equation

$$-\frac{1}{\rho_1}\frac{\partial P}{\partial r} = \dot{u} + u\frac{\partial u}{\partial r}$$

where

$\rho_1$ = density of crushed material behind shock wave, assumed to be independent of pressure

P = pressure at radial point r and time t

u = material velocity at radial point r and time t

The continuity equation, which is a statement that the material is incompressible after crushed by the shock wave, can be written as

$$u = \frac{F(t)}{r^2}$$

The third equation which must be considered is the Hugoniot relation

$$u_{sh}^2 = P_{sh}\,\Delta V = \frac{f P_{sh}}{\rho_o}$$

where

$$f = \frac{\Delta V}{V_o} = \frac{\rho_1 - \rho_o}{\rho_1}$$

$\rho_o$ = uncrushed density of compressible material

$P_{sh}$ = shock front pressure

$u_{sh}$ = material velocity at shock front

Finally, in order to relate the crushing behavior to the gas bubble behavior, a particular gas law must be introduced. For this study, the simplest form, the ideal gas, expanding adiabatically, was chosen. This can be expressed as

$$P_i = P_o \left(\frac{R_i}{R_o}\right)^{-3\gamma}$$

where

$P_o$ = initial gas bubble pressure

$R_o$ = initial gas bubble radius

$P_i$ = gas pressure when bubble radius is $R_i$

$\gamma$ = expansion constant

The method of solving these equations is the following. The continuity equation can be differentiated to obtain

$$\frac{\partial u}{\partial t} = \frac{\dot F}{r^2} \qquad\qquad u\frac{\partial u}{\partial r} = -\frac{2F^2}{r^5}$$

The force balance equation can then be written as

$$-\frac{1}{\rho_1}\frac{\partial P}{\partial r} = \frac{\dot F}{r^2} - 2\frac{F^2}{r^5}$$

This can be integrated to obtain

$$\frac{P}{\rho_1} = \frac{\dot F}{r} - \frac{F^2}{2r^4} + G(t)$$

where G(t) is an integration constant. Evaluated at the shock front, at radius $R_s$ it becomes

$$\frac{P_{sh}}{\rho_1} = \frac{\dot F}{R} - \frac{F^2}{2R^4} + G(t)$$

By use of the Hugoniot relation, the integration constant can be shown to be

$$G = \left(\frac{1}{f} - \frac{1}{2}\right)\frac{F^2}{R^4} - \frac{\dot F}{R}$$

Thus, the integrated force balance equation can be written as

$$\frac{P}{\rho_1} = F^2\left(\frac{1}{2r^4} + \frac{1}{fR^4} - \frac{1}{2R^4}\right) + \dot F\left(\frac{1}{r} - \frac{1}{R}\right)$$

If this is evaluated at the outer radius of the gas bubble, where the pressure is $P_i$ and the radius is $R_i$, it becomes

$$\frac{P_i}{\rho_1} = \dot{F}\left(\frac{1}{R_i} - \frac{1}{R}\right) + F^2\left[-\frac{1}{2R_i^4} + \left(\frac{1}{F} - \frac{1}{2}\right)\frac{1}{R^4}\right]$$

Now the gas law must be introduced to obtain

$$\frac{P_o}{\rho_1}\left(\frac{R_i}{R_o}\right)^{-3\gamma} = \dot{F}\left(\frac{1}{R_i} - \frac{1}{R}\right) + F^2\left[-\frac{1}{2R_i^4} + \left(\frac{1}{F} - \frac{1}{2}\right)\frac{1}{R^4}\right]$$

The above expresses all four basic equations, with the three basic assumptions in one differential equation. However, a form more amenable to solution can be obtained by changing the independent variable from time to $R_i$. This is done by using the relation

$$\dot{F} = \frac{dF}{dt} = \frac{dF}{dR_i}\cdot\frac{dR_i}{dt}$$

$$= \frac{dF}{dR_i}\cdot\dot{R}_i = \frac{dF}{dR_i}\cdot\frac{F}{R_i^2} = \frac{1}{2R_i^2}\cdot\frac{d(F^2)}{dR_i}$$

which uses the continuity equation. The differential equation can then be written as

$$\frac{P_o}{\rho_1}\left(\frac{R_i}{R_o}\right)^{-3\gamma} = \frac{1}{2R_i^2}\left(\frac{1}{R_i} - \frac{1}{R}\right)\frac{d(F^2)}{dR_i} + F^2\left[-\frac{1}{2R_i^4} + \left(\frac{1}{F} - \frac{1}{2}\right)\frac{1}{R^4}\right]$$

Changing variables to

$$y = F^2 \qquad x = \frac{R_i^3}{R_o^3(1-f)}$$

and using the gas volume relation that

$$V_{gas} = \frac{4\pi}{3} R_i^3 = \frac{4\pi}{3}\left[R_o^3 + f\left(R^3 - R_o^3\right)\right]$$

the differential equation can be written as

$$3\left[1 - f^{\frac{1}{3}}(x-1)^{-\frac{1}{3}}\right] x \frac{dy}{dx} + \left[-1 + (2-f) f^{\frac{1}{3}}\left(\frac{x}{x-1}\right)^{\frac{4}{3}}\right] y = \frac{2 P_o R_o^4 (1-f)^{\frac{7}{3}-\gamma}}{\rho_o} x^{\frac{4}{3}-\gamma}$$

This equation cannot be solved analytically, but it is amenable to solution by finite difference methods on a digital computer.

Using the above notation, where $x$ is the independent variable and $y$ the dependent variable, the expressions for a number of interesting quantities can be written. They are

$$t = \frac{R_o^3(1-f)}{3} \int_{x_o}^{x} \frac{dx'}{\sqrt{y}}$$

$$R_i = R_o (1-f)^{\frac{1}{3}} x^{\frac{1}{3}}$$

$$P_{gas} = P_o (1-f)^{-\gamma} x^{-\gamma}$$

$$P_{sh} = \frac{\rho_o f^{\frac{1}{3}}}{(1-f)^{4/3} R_o^4} \cdot \frac{y}{(x-1)^{4/3}}$$

$$u_{sh} = \left(\frac{f}{1-f}\right)^{2/3} \cdot \frac{1}{R_o^2} \cdot \frac{\sqrt{Y}}{(x-1)^{2/3}}$$

$$R_{sh} = R_o \left[\frac{1-f}{f} \cdot (x-1)\right]^{\frac{1}{3}}$$

$$\text{KINETIC ENERGY} = \frac{2\pi\rho_o Y}{R_o(1-f)^{4/3}} \left[x^{-\frac{1}{3}} - \left(\frac{x-1}{f}\right)^{-\frac{1}{3}}\right]$$

$$\text{CRUSHING ENERGY} = \frac{2\pi\rho_o}{3R_o} \left(\frac{f}{1-f}\right)^{\frac{1}{3}} \int_{x_o}^{x} \frac{y\,dx'}{(x'-1)^{4/3}}$$

$$\text{TOTAL ENERGY} = \frac{4\pi P_o R_o^2 (1-f)^{1-\gamma}}{3} \int_{x_o}^{x} (x')^{-\gamma}\,dx'$$

$$\text{MOMENTUM} = \frac{4\pi\rho_o R_o \sqrt{Y}}{(1-f)^{2/3}} \left[\left(\frac{x-1}{f}\right)^{\frac{1}{3}} - x^{\frac{1}{3}}\right]$$

The same method can be used in the case in which the gas bubble is surrounded by two concentric crushable regions. The above differential equation is valid in the inner crushable region. In order to treat the outer crushable region, a similar equation for that region must be used. This equation must take into account the density and compressibility of the outer region and it must satisfy the boundary condition that the pressure must be continuous at the interface between the two regions. Such an equation was derived and solved by the same method that was used on the equation for the inner region.

The particular numerical method used was the Runge-Kutta-Gill method, with an initial value for the derivative being determined from a Taylor series expansion of the differential equation around the starting point. The method of solution was programmed for the IBM-7090 into a series of programs designated as the SWAP series. Typical running time for a complete problem is about one minute, which includes the plotting of the output in graphical form on a cathode ray tube.

COMPARISON ANALYSIS

Using this approach, a study was made in which the physical characteristics of the compressible shield regions and the expansion characteristics of a gas were assumed to be parameters, and a systematic parameter study was made. The purpose of this study was two-fold:

(1) to determine under what conditions alternative materials can be substituted for the reactor materials in model experiments, and

(2) to determine the validity of substituting a chemical, explosive for expanding uranium gas, in model experiments.

The geometry used was typical for model experiments. The initial radius of the gas bubble was fixed at 2.66 cm. The initial outer radius of the inner crushable region was fixed at 19.0 cm., and the outer radius of the outer crushable region was fixed at 26.6 cm., unless noted otherwise.

In the model, the inner crushable region has been assigned low compressibility values ($\Delta V/V_o$ = 0.01, .05), which one might expect for sodium and water. In the outer crushable region, the compressibility and density have been varied to simulate block graphite, granulated graphite, sand, and a fictitious material with the density of granulated graphite and the compressibility of block graphite. Also, some cases were done with uranium gas as the driving medium and others were done with the detonation products of TNT as the driving medium.

The results of seven cases are given in Table 1. In all cases the problem was considered to be terminated when the shock wave reached the outer radius of the second crushable region. Note that the outer radius of the second crushable region is large for Case No. 111 than it is for the other cases. The larger outer radius was used in Case No. 111 to give the second crushable region the same mass in both cases 106 and 111 for comparison of the density effect.

For a first analysis of the results, comparisons of two similar cases with only one important difference can be made. The possible comparisons are as follows:

| Comparison | Cases | Variable | Values |
|---|---|---|---|
| I | 107, 106 | Gas | U, TNT |
| II | 106, 108, 109 | Material in second crushable region | Block graphite, granulated graphite, sand |
| III | (106, 109) | $f_2$ | .25, .5 |
| IV | 106, 110 | $\rho_2$ | 1.6, 1.0 gm/cm$^3$ |
| V | 106, 111 | Outer radius of second region | 26.6 cm, 29.6 cm |
| VI | 106, 102 | $f_1$ | .01, .05 |

Listed below are some comments and conclusions about each comparison.

Comparison I

For this comparison, the kinetic energies differ by about 10 per cent and the momenta by about 5 per cent. However, the total crushing energy differs by a large a mount (a factor of two). This indicates that except for the amount of work done by crushing (which is only a small part of the total work done) there is no large difference in the results when TNT detonation products are used to simulate uranium vapor.

Comparison II

From this comparison, it appears that the kinetic energy and momentum given to each region is strongly dependent upon the material in the outer crushable region. In general, the granulated graphite gives a better simulation of the block graphite, with respect to kinetic energy but the sand gives better simulation of the block graphite, with respect to momentum. For the purpose of designing conservative experiments, sand probably is a better simulant of block graphite than is granulated graphite. The momentum and kinetic energy imparted to the outer region are greater for sand than they are for block graphite, while they are less for granulated graphite than they are for block graphite.

Comparison III

Comparison of case 106 against case 109 gives comparison of results when only the compressibility of the outer region is varied. From the results there is only about a 10 per cent difference in the kinetic energies and momenta for similar cases. This indicates that the compressibility of the outer region is only loosely coupled to kinetic energy and momentum. Consequently, it appears that one can make material substitutions in model experiments with little regard for the compressibilities of the materials being consider~~able~~ed for the outer region.

Comparison IV

Comparison of the results for cases 106 and 110 indicates that the kinetic energies and momenta are strongly dependent upon the bulk material density in the outer region. The differences in the kinetic energy and momenta in the two cases run as high as 33 per cent. This indicates that it is quite important to preserve the prototype bulk material density of the shield when designing model experiments with substitute materials, when all dimensions are being scaled.

Comparison V

Because the previous comparison showed that the density considerations are important, for a fixed size system, one then should investigate the comparison for systems with a fixed total mass. The density in the outer region was specified as 1.6 $gm/cm^3$ in case 106 and 1.0 $gm/cm^3$ in case 111; all other material parameters were the same for the two cases. However, the outer radius of the second region was increased in case 111 so that both cases, 106, and 111, have the same mass in outer region. Examination of the results of the two cases shows that the momenta and kinetic energies differ by 10 per cent or less. This leads one to conclude that if one has the freedom to specify the outer radius of the design of a model for experiments suitable results can be obtained with substitute materials if the outer radius is chosen so that the outer region contains the same mass with the substitute shield material as it would with the prototype material.

Comparison VI

Finally, for the sake of completeness, two cases with different compressibilities in the inner crushable region are considered. In case 106, $f_1$ = .01 and in case 102, $f_1$ = .05. As might be expected, far more work was put into crushing in case 102. Consequently, the kinetic energies and momenta differ by large amounts. From these results it appears that one should not use substitute shield materials in the inner crushable regions in the design of explosion experiments if the compressibility of the substitute material differs appreciably from that of the prototype material. It should be remembered that the compressibility used in these calculations is the irreversible volume deformation caused by the application of a high pressure. Such deformation is very small for liquids such as water or sodium, even though the reversible compression of a liquid may be 5 to 20 per cent at 10 kilobars. In contrast, a material such as balsa wood has approximately the same density as water but has a very large value for irreversible compression. Consequently, such a material apparently would not be adequate as a substitute for water or sodium in the design of model experiments.

## Summary of Comparisons

From these comparisons, one could draw the following conclusions. One can design models for experiments which employ substitute materials from those used in the prototype. Only small and probably tolerable differences will be produced if TNT is used as an energy source rather than uranium. However, the design should be made along the lines that uses a material in the inner crushable region with very nearly the same compressibility as that in the prototype. In the outer crushable region the total mass of material is the important factor. This can be obtained by using the proper dimensions and the proper density or by adjusting both the dimensions and the density of the regions so as to obtain the proper total mass. The effect of the compressibility is only a secondary effect.

## ABSOLUTE ANALYSIS

Case 108 discussed above is a reasonable representation of the material configuration used in a model experiment performed by the Naval Ordnance Laboratory.[1]

In the model experiment the vessel surrounding the inner crushable region had a 20 per cent increase in the enclosed volume and the vessel surrounding the outer crushable region has a 50 per cent increase in the enclosed volume as a result of the experiment. Using a reasonable strain energy density curve it was found that the strain energy required to produce the observed strain in the two vessels would be 30 kcal and 8 kcal respectively. For the calculated case, if it is assumed that the kinetic energy which is in each region is converted into strain energy in the vessel immediately surrounding the region, the calculated strain energies would be 50 kcal and 10 kcal, respectively. Thus, the strain energies are reasonably close, especially when the differences of the geometries are considered. Consequently, the results of this method can be considered to be in rather good agreement with experiment and can be used in the design of reactors and in the planning of model experiments.

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[1] Wise, W. R. Jr., "Enrico Fermi Shield Plug Response to TNT Simulated Accidents," NOLTR-62-207, to be published.

TABLE I

Pertinent Results from Seven Shock Wave Problems

| Case<br>Gas<br>Outer Crushable Material | 107<br>U<br>Block Graphite | 106<br>TNT<br>Block Graphite | 108<br>TNT<br>Granulated Graphite | 109<br>TNT<br>Sand | 110<br>TNT<br>Fictitious Material | 111<br>TNT<br>Fictitious Material | 102<br>TNT<br>Block Graphite |
|---|---|---|---|---|---|---|---|
| $f_1$ - compressibility - Region I | .01 | .01 | .01 | .01 | .01 | .01 | .05 |
| $\rho_1$, gm/cm$^3$ - initial density - Region I | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| $f_2$ - compressibility - Region II | .25 | .25 | .33 | .50 | .25 | .25 | .25 |
| $\rho_2$, gm/cm$^3$ - initial density - Region II | 1.6 | 1.6 | 1.0 | 1.6 | 1.0 | 1.0 | 1.6 |
| $\gamma$ | 1.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 |
| Values at Problem Termination | | | | | | | |
| Total Energy, kcal | 87.5 | 90.3 | 90.3 | 90.3 | 90.3 | 90.3 | 90.3 |
| Total Kinetic Energy, kcal | 67.6 | 60.2 | 62.0 | 55.8 | 63.2 | 58.6 | 36.3 |
| Kinetic Energy, Region 1, kcal | 53.7 | 47.8 | 52.1 | 39.8 | 54.3 | 44.4 | 29.4 |
| Kinetic Energy, Region 2, kcal | 13.8 | 12.3 | 9.9 | 15.8 | 8.8 | 14.1 | 6.9 |
| Total Crushing Energy, kcal | 20.6 | 33.0 | 31.0 | 37.7 | 29.8 | 34.5 | 55.4 |
| Total Momentum, x $10^9$ gm/cm-sec | 1.027 | .969 | .914 | .961 | .911 | .972 | .737 |
| Momentum, Region 1, x $10^9$ gm/cm-sec | .703 | .663 | .696 | .612 | .707 | .647 | .520 |
| Momentum, Region 2, x $10^9$ gm/cm-sec | .324 | .306 | .218 | .349 | .204 | .324 | .217 |