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STATIC AND QUASI-STATIC SOLUTIONS OF THE REAL PROCA FIELD

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ABSTRACT

We consider the theory of the massive real vector field with spin 1, (the real Proca field), and its solutions. First the field equations with dual symmetry [1] are written, and the 4-pseudo vector is chosen to be zero. The constants of motion for the real Proca field, constant "electric" real Proca field, the uniform motion of a point charge in the real Proca field, uniform motions in the "Coulomb" field, dipole and multi-pole free momentum, constant "magnetic" field, and the field of a point charge in motion, are presented.

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1 Introduction

The conventional Maxwell equations are based on the hypothesis that the photon has zero mass.

Some experiments are based on the idea, whether a photon has a mass, or not [2]. But new experiments are done by quasi-real photon-proton collisions, the photon remnant produced in resolved photon interaction has been isolated. The selected events contain two high-p_T jets with $p_T > 6GeV$, and $\eta < 1.6$, and $130 < W_{\gamma_p} < 270GeV$. The mean value of p_T for photon remnant, 2.1±0.2GeV, is substantially larger than the Monte Carlo expectation [3]. It is important to mention that physicists are interested in self interaction, preacceleration, runaway solutions, and finite-size effects. Some calculations are presented for nucleus-nucleus collisions at $p_{lab} = 200GeV/c$. The experimental data would then signal the onset of new phenomena such as a quark-gluon plasma [4].

It is known that since 1930, when the conventional Maxwell Lagrangian was modified by a mass term, Proca Lagrangian was obtained in 1936 [5]. Luis de Broglie [6] had a large influence on a younger generation like Proca and Petiau, and he tried to explain that the photon has a mass.

Yukawa first introduced mesons in 1934, when he postulated the existence of a "heavy particle" which would mediate the transition by a proton state to neutron state [7]. The new field of force had a potential function satisfying the Klein-Gordon equation and an interaction energy between two particles given by what is now known as the Yukawa potential. In 1936 Proca [5] elaborated on the theory by generalizing the Maxwell equations to fields with nonzero mass (the Proca equations).

Today, the Proca field may be interesting for physics. Proca's work was discussed very rigorously. Yukawa and Sakata [8] constructed the theory of the charged scalar field.

Historical, dual symmetry first appeared in classical electrodynamics, where Maxwell equations for the free electromagnetic field are invariant under the transformations

$$\vec{E} \longrightarrow \pm\vec{B}, \quad \vec{B} \longrightarrow \mp\vec{E}$$

Heaviside observed this peculiarity, but its meaning, as a symmetry, appeared much later (Larmor, 1928).

In the general case, Maxwell equations can include the magnetic current [9]. In that case we have to discuss the magnetic monopole. Dirac was the first to propose the monopole to quantize the electric charge [10].

In 1974, 't Hooft and Polyakov found the solution of the magnetic monopole making spontaneous symmetry breaking. In this case the monopole will come out as a topologically non-trivial finite-energy solution.

Today, physicists are interested in mono-pole solutions in supersymmetric theories [12]. We would like to consider the Proca field on the dual-symmetry basis as a mathematical artifact, in which we include dual fields, and the dual tensor [1]. It should be a general Lagrangian. Also we would like to find some static and quasi-static solutions of that field.

The paper is organized as follows. The Lagrange equations, the canonical Proca field equations with the source, density and a current energy, are discussed in Sect.2. The constant "electric" real Proca field, "electric" energy of a point charge, the field of a point charge in the "Coulomb" field, dipole and multi-pole orbital momentum, and the system of charges in an exterior field will be discussed in Sec.3. In Sec. 4. the constant "magnetic" field, "magnetic" momentum, the relationship of the "magnetic" and the mechanical momentum will be discussed. Our results will be found in Sect.5.

2 Lagrangian equations

The Lagrangian of the real Proca field generated by point sources in the dual symmetry formulation is:

$$\mathcal{L} = \mathcal{L}_P + \mathcal{L}_{intr} + \mathcal{L}_p, \tag{1}$$

$$\begin{aligned} \mathcal{L} &= \frac{1}{8\pi}\left[-\frac{1}{2}F_{\alpha\beta}F^{\alpha\beta} + F^2 - G^2 + \kappa^2\left(A_\alpha A^\alpha - b_\alpha b^\alpha\right)\right] - \\ & \quad j_{q^\alpha}A^\alpha - j_{g^\alpha}b^\alpha - m_q\sqrt{1-v_{q^2}} - m_g\sqrt{1-v_{g^2}}, \end{aligned} \tag{2}$$

where

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - \frac{1}{2}\varepsilon_{\alpha\beta\xi\varsigma}\left(\partial^\xi b^\varsigma - \partial^\varsigma b^\xi\right), \tag{3}$$

$$G = \partial_\alpha A^\alpha, \tag{4}$$

$$F = \partial_\alpha b^\alpha, \tag{5}$$

Here A^α is a 4-vector field, and b^α is a 4-pseudo-vector field, with κ being a scalar constant. Indices "q" and "g" denote the charge for the current, and charge for the pseudo-current respectively. The Lorentz metric is $\eta_{\mu\nu} = diag(1,-1,-1,-1)$, and also $\hbar = c = 1$. The field equations are:

$$\left(\partial_\eta\partial^\eta + \kappa^2\right)A^\mu = 4\pi j^{\mu_q}, \tag{6}$$

$$\left(\partial_\eta\partial^\eta + \kappa^2\right)b^\mu = 4\pi j_g^\mu \tag{7}$$

These are clearly invariant under the dual transformations [1]

$$A'^\alpha = A^\alpha cos\lambda + b^\alpha sin\lambda, \tag{8}$$

$$b'^\alpha = b^\alpha cos\lambda - A^\alpha sin\alpha, \tag{9}$$

where λ is a free constant. According to (3) $F^{\alpha\beta}$ and $\tilde{F}^{\alpha\beta}$ transform as

$$F^{\alpha\beta} \longrightarrow F^{\alpha\beta}, \quad \tilde{F}^{\alpha\beta} \longrightarrow -F^{\alpha\beta}$$

or

$$F'_{\alpha\beta} = F_{\alpha\beta} cos\lambda + \tilde{F}_{\alpha\beta} sin\lambda, \tag{10}$$

$$\tilde{F'}_{\alpha\beta} = \tilde{F}_{\alpha\beta} cos\lambda - F_{\alpha\beta} sin\lambda, \tag{11}$$

where

$$\tilde{F}^{\alpha\beta} = \partial^\alpha b^\beta - \partial^\beta b^\alpha + \frac{1}{2}\varepsilon^{\alpha\beta\xi\zeta}\left(\partial^\xi A^\zeta - \partial^\zeta A^\xi\right),$$

The currents also transform as:

$$j_q'^\alpha = j_q^\alpha cos\lambda + j_g^\alpha sin\lambda, j_q'^\alpha = j_q^\alpha cos\lambda + j_g^\alpha sin\lambda, \tag{12}$$

$$j_g'^\alpha = j_g^\alpha cos\lambda - j_q^\alpha sin\lambda,$$

2.1 The canonical equations for the real Proca field with sources

The Lagrangian density is

$$\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{8\pi}\left[\left(-\partial_0\vec{A} - \nabla A^0\right)^2 - (rot\vec{A})^2 - \left(-\partial_0\vec{b} - \nabla b^0\right)^2 + (rot\vec{b})^2\right] \\
& -\frac{1}{4\pi}\left[\left(-\partial_0\vec{A} - \nabla\vec{b}\right)^2 - \left(\partial_0 A^0 + \nabla\vec{A}\right)^2\right] \\
& +\frac{1}{8\pi}\kappa^2\left[\left(A^{0^2} - \vec{A}^2\right) - \left(b^{0^2} - \vec{b}\right)\right] \\
& -\rho_q A^0 - \vec{j}_q - \rho_g b^0 - \vec{j}_g\vec{b} + \mathcal{L}_p.
\end{aligned} \tag{13}$$

where

$$A^\alpha = (A^0, \vec{A}), \quad b^\alpha = (b^0, \vec{b}).$$

Now we will write the canonical field equations:

$$\partial_\alpha(F^{\alpha\beta} + \eta^{\alpha\beta}) + \kappa^2 A^\beta = 0, \tag{14}$$

$$\partial_\alpha(\tilde{F}^{\alpha\beta} + \eta^{\alpha\beta}F) + \kappa^2 b^\beta = 0'$$

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \frac{1}{2}\varepsilon^{\alpha\beta\xi\zeta}(\partial^{xi} b^\zeta - \partial^{zeta} b^\xi),$$

$$\tilde{F}^{\alpha\beta} = \partial^\alpha b^\beta - \frac{1}{2}\varepsilon^{\alpha\beta\xi\zeta}(\partial^\xi A^\zeta - \partial^\zeta b^\xi),$$

$$G = \partial_\alpha A^\alpha, \quad F = \partial_\alpha b^\alpha,$$

and it is a simple exercise to show

$$rot\vec{B} = \partial_0\vec{E} + gradG - \kappa^2\vec{A} + 4\pi\vec{j}_q, \tag{15}$$

$$rot\vec{E} = -\partial_0\vec{B} - gradF + \kappa^2\vec{b} - 4\pi\vec{j}_g,$$

$$div\vec{E} = -\partial_0 G - \kappa^2 A^0 + 4\pi\rho_q,$$

$$
\begin{aligned}
div\vec{B} &= -\partial_0 F - \kappa^2 b^0 + 4\pi\rho_g, \\
\vec{E} &= -\partial_0\vec{A} - gradA^0 - rot\vec{b}, \\
\vec{B} &= -\partial_0\vec{b} - gradb^0 + rot\vec{A}, \\
G &= \partial_0 A^0 + div\vec{A}, \\
F &= \partial_0 b^0 + div\vec{b}.
\end{aligned} \tag{16}
$$

Also, it is simple to see that the canonical equations, (15) and (16) have a proper dual symmetry. Equations (16) define the vector of "electric" and "magnetic" field and the Lorentz condition.

2.2 Density and the current of energy

Dual symmetry combined with the 4-vector field and 4-pseudo vector field (eqs. (14), (15), (16)), will be separated in two solutions:

$$
A^\alpha \neq 0, \quad b^\alpha = 0, \tag{17}
$$

$$
b^\alpha \neq 0, \quad A^\alpha = 0.
$$

In this paper we would like to consider the case when 4-pseudo vector is chosen to be zero. Then equations (15), (16) become:

$$
rot\vec{B} = \partial_0\vec{E} - \kappa^2\vec{A} + 4\pi\vec{j}_q, \tag{18}
$$

$$
\begin{aligned}
rot\vec{E} &= -\partial_0\vec{B}, \\
div\vec{E} &= -\kappa^2 A^0 + 4\pi\rho_q, \\
div\vec{B} &= 0, \\
\vec{E} &= -\partial_0\vec{A} - gradA^0, \\
\vec{B} &= rot\vec{A}, \\
\partial_0 A^0 &+ div\vec{A} = 0,
\end{aligned} \tag{19}
$$

and, after a short calculation, and after the integration in the whole space, and neglecting the element in which the field in the infinite goes to zero, we obtain our first result, the energy density and the Pointing vector of the real Proca field, respectively:

$$
\mathcal{E} = \frac{1}{8\pi}\left[\vec{E}^2 + \vec{B}^2 + \kappa^2\left(A^0 + \vec{A}^2\right)\right]. \tag{20}
$$

$$
\vec{S} = \frac{1}{4\pi}\left(\vec{E}\times\vec{B} + \kappa^2 A^0\vec{A}\right). \tag{21}
$$

Using the Noether theorem, we have the energy-momentum 4-vector:

$$P^0 = \frac{1}{8\pi}\int[\vec{E}_{\vec{A}}^2 + \vec{B}_{\vec{A}}^2 + G^2 + \kappa^2(\vec{A}^2 + A^{0^2}) - 2\partial^0 A^0 G + 2A^0\partial^0 G]d^3x, \tag{22}$$

$$P^i = \frac{1}{4\pi}\int[\vec{E}_{\vec{A}} \times \vec{B}_{\vec{A}} + \kappa^2\vec{A}A^0 + G\nabla A^0 + \vec{A}\partial^0 G]d^3x. \tag{23}$$

Having a good choice of the scalar constant G, it is easy to see that (22), and (23) agree with (20), and (21).

3 Constant "electric" real Proca field (the equations of the constant "electric" real Proca field, the "Coulomb" law)

Equations (18) and (19) simplify to

$$div\vec{E} = -\kappa^2 A^0 + 4\pi\rho_q, \tag{24}$$

$$rot\vec{E} = 0,$$

$$\vec{E} = -gradA^0,$$

where the "electrostatic" potential A^0 satisfies

$$\left(\Delta - \kappa^2\right) A^0 = -4\pi\rho_q. \tag{25}$$

Solving this equation, we get, [4, 13]

$$A^0\left(\vec{r}\right) = \int A(\vec{k})e^{i\vec{k}\bullet\vec{r}}\frac{d\vec{k}}{(2\pi)^3}, \tag{26}$$

$$\begin{aligned} A^0(\vec{r}) &= \int \frac{4\pi q}{k^2+\kappa^2}e^{i\vec{k}(\vec{r}-\vec{r}_q)}\frac{d\vec{k}}{(2\pi)^3} = \frac{2}{\pi}\frac{q}{|\vec{r}-\vec{r}_q|}\int_0^{+\infty}\frac{k signk|\vec{r}-\vec{r}_q|}{k^2+\kappa^2}qdk \\ &= \frac{q}{|\vec{r}-\vec{r}_q|}e^{-\kappa|\vec{r}-\vec{r}_q|}. \end{aligned} \tag{27}$$

and

$$\vec{E} = q\left(\frac{1}{|\vec{r}-\vec{r}_q|^3} + \frac{\kappa}{|\vec{r}-\vec{r}_q|^2}\right)e^{-\kappa|\vec{r}-\vec{r}_q|}q\left(\vec{r}-\vec{r}_q\right). \tag{28}$$

The "electrostatic" potential propagates to the distance fixed by the constant κ, and it is the "Coulomb" potential of the real Proca field. Now we can discuss eq. (28):

a. $\frac{1}{|\vec{r}-\vec{r}_q|} \ll \kappa$, then we have that the first element in (28) dominates.

b. $\frac{1}{|\vec{r}-\vec{r}_q|} \gg \kappa$, then we have that the second element dominates.

c. $\kappa \longrightarrow \infty$, then we can neglect at once the first element in eq. (28), but after some calculation we can neglect, also the second element.

One region exists, which is the total difference of electromagnetic field, and we can say that effective action of the Proca field increases with $1/r^2$. Probably, we can expect an essential difference with the electromagnetic field on a distance $\Delta r = 10^{-9}m$.

3.1 The "electric" energy of a point charge

According to 4-vector energy momentum, the "electric" energy is determinated by

$$P^0 = \frac{1}{8\pi}\int \left(\vec{E}^2 + \kappa^2 {A^0}^2\right) d^3x. \tag{29}$$

Knowing that $\vec{E} = \nabla A^0$, and $div\vec{E} = -\kappa^2 A^0 + 4\pi\rho$, eq. (29) becomes

$$P^0 = \frac{1}{2}\int \rho A^0 d^3x. \tag{30}$$

Now, it is interesting to write the "electric" energy for two point charges

$$P^0 = \frac{1}{2}\rho_1 A_1^0 + \frac{1}{2}\rho_2 A_2^0 + \frac{1}{2}\rho_1 A_2^0 + \frac{1}{2}\rho_2 A_1^0. \tag{31}$$

The first two elements present the self "electrostatic" energy of charges, and the second two elements present their interaction. It is easy to see that the self-energy of a point charged particle is infinite as in the electromagnetic case. This shows that the interaction of the field and a point charge is generally not well defined. With the radiative correction, this interaction can be solved as in the electrodynamics field [4, 13, 14], but this problem is still open. The energy of the interaction charges can be written as

$$\frac{1}{2}\rho_1 A_2^0 + \frac{1}{2}\rho_2 A_1^0 = \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|} e^{-\kappa|\vec{r}_1 - \vec{r}_2|}, \tag{32}$$

In general, for more charges

$$\frac{1}{2}\sum_{i,j} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|} e^{-\kappa|\vec{r}_i - \vec{r}_j|}. \tag{33}$$

3.2 The field of a point charge which has a uniform motion

At this point we can calculate the field of a uniformly moving charge

$$A^0 = \frac{q}{r*} e^{-r*/\sqrt{1-(v/c)^2}}, \tag{34}$$

$$\vec{A} = \frac{q\vec{v}/c}{r*} e^{-r*/\sqrt{1-(v/c)^2}},$$

where

$$r*^2 = (x - vt)^2 - \left(1 - (v/c)^2\right)\left(y^2 + z^2\right) \tag{35}$$

and

$$\vec{E} = q\frac{\vec{R}}{R^3}\frac{1-(v/c)^2}{\left(1-(v/c)^2 sin^2\theta\right)^{3/2}}\left(1 + \kappa R\sqrt{\frac{1-(v/c)^2 sin^2\theta}{1-(v/c)^2}}\right) e^{-\kappa R}\sqrt{\frac{1-(v/c)^2 sin^2\theta}{1-(v/c)^2}}. \tag{36}$$

Here θ denotes the angle between the direction of motion and radius vector, $\vec{R}$. For $\theta = 0$

$$\vec{E}_{\|} = q\frac{\vec{R}}{R^3}\left(1-(v/c)^2\right)\left(1 + \kappa R\frac{1}{\sqrt{1-(v/c)^2}}\right) e^{-\kappa R\frac{1}{\sqrt{1-(v/c)^2}}}, \tag{37}$$

and for $\theta = \pi/2$ we get

$$\vec{E}_{\perp} = q\frac{\vec{R}}{R^3}\frac{1}{\sqrt{1-(v/c)^2}}(1+\kappa R)\,e^{-\kappa R}. \tag{38}$$

3.3 The motion of a point charge in the "Coulomb" field

Another interesting exercise is the motion of a point charge in the "Coulomb" field. The Lagrangian of the system is

$$\mathcal{L}_p = mc^2\sqrt{1-(v/c)^2} + \frac{q}{c}\vec{A}\bullet\vec{v} - qA^0. \tag{39}$$

which is formally equal to that in the Maxwell theory. So the equations of motion are analogous to those in electrodynamics,

$$\frac{d\vec{p}}{dt} = q\vec{E} + \frac{q}{c}\vec{v}\times\vec{B}, \tag{40}$$

which leads to the following equation in polar coordinates

$$\frac{d^2u}{d\phi^2} + u = \frac{m_q}{\ell^2}\left(1+\frac{\kappa}{u}\right)e^{-\kappa/u}. \tag{41}$$

3.4 Dipole and multi-pole orbital momentum

At distant points of a point charge system (26) takes the usual multi-pole expansion form

$$A^0(\vec{r}) = \sum_i q_i\frac{e^{-\kappa r}}{r} - \sum_i q_i\vec{r}_i grad\frac{e^{-\kappa r}}{r} + 1/6 D_{nm}\frac{\partial^2}{\partial x_n \partial x_m}, \tag{42}$$

where

$$D_{nm} = \sum_{i,n,m} q_i\left(3x_{in}x_{im} - r^2\delta_{nm}\right), \tag{43}$$

which is the quadripole momentum system of charges [15]. Equation (42) gives

$$\vec{E} = \frac{1}{r^5}\left[\left(3\vec{d}\bullet\vec{r}\right)\vec{r}\,(1+\kappa r)\,e^{-\kappa r} - r^2\vec{d}\left(1+\kappa r - r\kappa^2\right)e^{-\kappa r}\right]. \tag{44}$$

for the dipole "electric" field. This is one of our results. One useful result is the potential energy of two dipoles

$$P^0 = \frac{1}{r^5}\left\{r^2\left(\vec{d}_1\vec{d}_2\right)\left(1+\kappa R - \vec{r}r\kappa^2\right)e^{-\kappa R} - 3\left(\vec{r}\bullet\vec{d}_1\right)\left(\vec{r}\bullet\vec{d}_2\right)(1+\kappa R)\,e^{-\kappa R}\right\}. \tag{45}$$

Up to now we have worked on the electric solutions of our equations. Let us now turn our attention to the magnetic solutions.

4 "Magnetic" field

Like in electrodynamics, the constant "magnetic" field does not exist in the Proca field. That constant "magnetic" field should appear as a consequence of the motion of a point charge in the finite part of space-time. This motion is stationary and it is interesting to consider the time averaging effect of this motion. Equation (18) denote the "magnetic" field, and after time averaging, and knowing that elements which have time derivation for the finite motion, are equal to 0, it becomes

$$rot\bar{\vec{B}} = -\kappa^2\bar{\vec{A}} + 4\pi\bar{\vec{j}}, \tag{46}$$

$$div\bar{\vec{B}} = 0,$$

$$\bar{\vec{B}} = rot\bar{\vec{A}},$$

$$div\bar{\vec{A}} = 0.$$

The particular solution of $\Delta\bar{\vec{A}} = -4\pi\bar{\vec{j}}_q$, according to (43) and (45) (for the point charge) is

$$\bar{\vec{A}} = q < \frac{\vec{v_q}|\vec{r} - \vec{r_q}|}{e}^{-\kappa|\vec{r}-\vec{r_q}|} >_t \tag{47}$$

and for the system of the point charges the vector potential is

$$\bar{\vec{A}} = \sum_i q_i < \frac{\vec{v_i}}{|\vec{r} - \vec{r_i}|} e^{-\kappa|\vec{r}-\vec{r_i}|} >_t . \tag{48}$$

Knowing that $\bar{\vec{B}} = rot\bar{\vec{A}}$, and after some calculation, we get a new result, the "magnetic" vector for a point charge

$$\bar{\vec{B}} = rot\bar{\vec{A}} = q < \frac{\vec{v} \times (\vec{r} - \vec{r_q})}{|\vec{r} - \vec{r_q}|} (1 + \kappa\,|\vec{r} - \vec{r_q}|)\, e^{-\kappa|\vec{r}-\vec{r_q}|} >_t . \tag{49}$$

This corresponds to the Biot-Savart law in electrodynamics. In the limit of $\kappa \longrightarrow 0$, these expressions agree with electrodynamics. For a large κ, and $\kappa\,|\vec{r} - \vec{r}'| >> 1$, the activity of the field is small, and the second element in the small brackets is much larger than the first one. In other cases the activity of the field is limited according to the system of charges.

4.1 "Magnetic" momentum

By analogy with electrodynamics, we consider the time averaged "magnetic" field which is far from the system of charges. Equation (48) can be expanded in $1/r$, and keeping the first element of this expansion, we get

$$\bar{\vec{A}} = \frac{1 + \kappa r}{2r^3} e^{-\kappa r} \sum_i q_i < (\vec{r_i} \times \vec{v_i}) \times \vec{r} >_t, \tag{50}$$

where $\sum_i q_i \vec{v}_i = d/dt \sum_i q_i \vec{r}_i = 0$. Equation (50) can be written in the form:

$$\vec{A} = \frac{1+\kappa r}{r^3} e^{-\kappa r (\vec{m} \times \vec{r})} = \left(\nabla e^{-\kappa r}/r\right) \times \vec{m}, \tag{51}$$

where

$$\vec{m} = 1/2 \sum_i q_i \vec{r}_i \times \vec{v}_i. \tag{52}$$

The corresponding vector of "magnetic" field is our result

$$\vec{B} = \left(e^{-\kappa r}/r^5\right) \left\{ \left(\vec{r}\vec{r}\vec{m}\right) \left[3\,(r+\kappa r) + \kappa^2 r^2\right] - \vec{m}\,(1+\kappa r)\, r^2 \right\}. \tag{53}$$

4.2 "Larmor' theorem

By analogy of electrodynamics we consider the system of points charges in the "magnetic" field. According to (40) the force which acts on the system is

$$\vec{F} = \sum_i q_i \vec{v}_i \times \vec{H}. \tag{54}$$

The time averaging of this force equals zero, but the time averaged momentum of the force is not equal to zero:

$$\vec{K} = 1/2 \sum_i q_i < (\vec{r}_i \times \vec{v}_i) \times \vec{H} >_t . \tag{55}$$

The Lagrangian function is

$$\mathcal{L} = \sum_i m_i/2 \left(\vec{v}_i + \vec{\Omega} \times \vec{r}_i\right)^2 - U, \tag{56}$$

where

$$\vec{v'} = \vec{v} + \vec{\Omega} \times \vec{r}, \tag{57}$$

and $\vec{\Omega} = e/2m\vec{B}$ is the rotating angular velocity of the moving system, and U is the potential energy for the charge in the "electric" field. Also, $\vec{v}$ is the velocity in the new coordinate system, and $\vec{v'}$ is the velocity in the old coordinate system. On the other hand, the Lagrangian function for the close system of charges in a homogeneous "magnetic" field, is

$$\mathcal{L}_H = \sum_i q_i \vec{A}_i \bullet \vec{v}_i = \sum_i q_i/2\,(\vec{r}_i \times \vec{v}_i)\,\vec{B}. \tag{58}$$

Comparing (56) and (57), it is easy to see that equation (56) represents the Lagrangian for the system of charges in a constant homogeneous "magnetic" field in the static system. Using the two relations for the Lagrangian functions, one can see that in the non-relativistic limit, the behavior of the system of points of charges with finite motions in the central-symmetric "electric" field, and also in the "magnetic" weak field, is equivalent to the motion of the same system of points of charges in the same "electric" field in the coordinate system with uniform motion, and angular velocity. This is the theorem which corresponds to the Larmor theorem in electrodynamics.

5 Conclusion

The real Proca field contains one vector field (A_α) and one pseudo-vector field (b_α).We have considered the case when the 4-pseudo vector is chosen to be zero.

The static and quasi-static real Proca field is considered in the dual-symmetry field theory. The real Proca field is considered as a physical object equal to the electromagnetic field, but this theory is not a gauge theory.

Interesting results in this article are: The constant of motion, in a general way, including one vector field, and one pseudo-vector, the "Pointing" vector (21), energy density (20),"electrostatic" field (28), the parallel and the orthogonal vector of the "electric" field (37), (38), "electric" field for the dipole and multi-pole orbital momentum (44), the potential energy of the points charges (45), the "magnetic" vector of a point charge (49), the vector of the "magnetic" field (53), the"Larmor" theorem.

The main results of electrostatics and magnetostatics [15] are very similar to those in the real Proca field, but the essential difference appears in the exponential factor which suppresses this field.

The motion of one particle in the static real Proca field is very interesting. When κ (mass term) goes to 0, then one can obtain the Biot-Savart law, but when κ goes to infinity, then one has a different result for $\vec{\bar{H}}$.

The fundamental problem in electrodynamics is the self field, and now, we see that it persists here too.

One of the open questions is the solution of the "magnetic" mono-pole, and in that case it is important to include also 4-pseudo vector field.

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