BILATERAL ELETRIC ENERGY CONTRACTS: RETURN AND RISK

Laura K. Gunn and Elisa B. Silva and Paulo B. Correia

***Abstract*—In Brazil electricity is traded through three segments: the Spot market that balances offer and demand, with prices calculated by a cost-based computational model; the Regulated Market , where prices are settled in public auctions, and the Free Market for bilateral contracts.**

As Spot and Regulated Market prices are public information, a seller is able to calculate his opportunity price to trade a bilateral contract in the Free Market by using the non-arbitrage principle. Thus, the seller searches the price of a bilateral contract in the Free Market that balances his/her revenues with the value expected in case it were negotiated in the Regulated and the Spot Market.

Besides the expected revenue, the seller may also consider the CVaR to measure the risk of her/his bilateral contract in the Free Market. So this paper develops a binomial lattice approach to price bilateral contracts in the Free Market, considering the seller's opportunity of negotiations in both Regulated and Spot Markets, and measuring the contract risk directly.

***Index Terms*—Energy Market, Bilateral contracts, Risk And Return**

I. INTRODUCTION

ECONOMICAL decisions are rarely taken under conditions of absolute certainty. In general investments are evaluated according to their return, and if it is not possible to know in advance what the return will be, there is an uncertainty or risk situation.

The total elimination of risks is economically unviable or even impossible. And risk situations many times may offer great profit possibilities. Economical decisions are taken in a return-risk contexts, that is, decisions which demand a greater risk are only acceptable if they provide greater returns. The investment decisions always include two components: the expected return analysis and the assumed risk analysis.

This article approaches the pricing of bilateral contract (forward contract) of electric energy in restructured markets. A forward contract is one of the simplest derivatives. It is a purchase or sale agreement of an asset on a certain future date for specified price. [1] The forward contract is a price guarantee tool for both the buyer and the seller. However, the electrical energy sale and purchase contract present great pricing difficulty, mainly due to the great price volatility electricity and for a great number of dealing flexibilities that these contracts allow. One of the main difficulties for the use of contracts results from the difficulty of formulating models which has expected prices that are similar to those verified later. Too big energy price variations increase the risks involved in their negotiations making it difficult to find a non-inhibitory price that enables the agreement of sellers and buyers.

To assess a return contract the Binominal model is used as a tool and to evaluate the risk the Value-at-risk (VAR) and the Conditional Value-at-risk (CVAR) tools are used.

II. BINOMIAL LATTICE

In this section, a model based on the one developed by [2], will be presented. It is called Binomial lattice Model nowadays. Originally developed to price assets, this model is discussed in this work only with the purpose of pricing electrical energy forward contract and measure its risks.

As it is a very useful and popular technique due to its simplicity and easy implementation, it involves the construction of a binominal lattice to represent the random parameters of a contract [1]. Differently from other markets, the spot price of electric energy in Brazil is not determined only by the market and its offer and demand conditions. It is formed by mathematical models and includes other variables, such as the hydroelectric plants reservoir storage levels, the predicted evolution for the demand of energy and the future and present availability of power plants and transmission lines.

Initially the binomial lattice is used to estimate the spot price expected value, which is represented by the S variable. The binomial lattice analyses the S dynamics considering that at each contract time interval, the price has an p probability of rising with u rate and the $(1-p)$ probability of dropping with the d rate, with the additional imposition that $u > 1$ and $d = 1/u$.

The binomial lattice is expanded from $t = 0 \rightarrow T$, and one of its properties is that the number of levels of each stage is given by $n = t$. The analysis with binomial lattices includes two phases:

- With a known current S price, the lattice is expanded using u and d to produce all the possible and unfolding S_T^n price at the T horizon;
- With the possible S_T^n prices, the lattice is contracted using p and $(1-p)$ to result in S_0^0, the expected price $\bar{S}$, discounted with the risk-free return $(1+f)$.

Figure 1 lattice, for example, could indicate the evolution of spot price along four weeks. The spot price known in the period of beginning of the analysis has a p probability of going up with a u rate and a probability $(1-p)$ of decreasing with a d rate. Thus, if a spot price is worth S in the beginning of the period it will be worth Su or Sd in the next period and so on until the horizon of the contract.

In the expansion phases, prices are calculated using:

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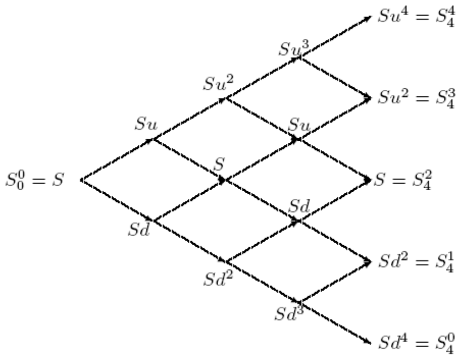

Fig. 1. Binominal laticce: *forward*

$$S_{t+1}^{n+1} = S_t^n u \tag{1}$$
$$S_{t+1}^{n} = S_t^n d \tag{2}$$

Figure 2 shows the lattice contraction leaving from the knots corresponding to the contract horizon until it results in the expected price, this discounted of the F rate.

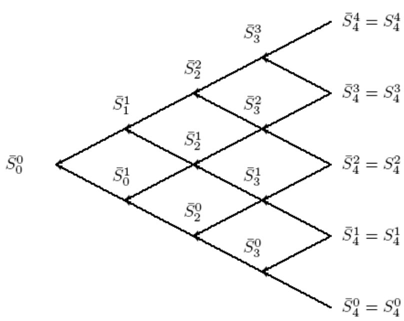

Fig. 2. Binominal lattice: *backward*

In the contraction phase the expected value of the prices discounted with the F rate are calculated as:

$$\bar{S}_t^n = \frac{1}{F}\left[p\bar{S}_{t+1}^{n+1} + (1-p)\bar{S}_{t+1}^{n}\right] \tag{3}$$

The expected value of spot price is $\bar{S} = \bar{S}_0^0$. The coupling of the 2 analyses phases is simple by doing $S_T^n = \bar{S}_T^n$, para $n = 0, \ldots, T$.

The construction of the binomial lattices requires the parameter calculation u, d and p, which must be estimated. Actually the binomial lattice represents an exploration of the future from the past history as Figure 3 shows.

The binomial lattice uses a price multiplication model which allows to calculate the growth rate and the monthly volatility of spot price. In the following way:

$$\nu = E[\ln(S_T/S_0)] \tag{4}$$
$$\sigma = \sqrt{\mathrm{var}[\ln(S_T/S_0)]} \tag{5}$$

Now with the discrimination interval Δt known, it is possible to calculated:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{6}$$
$$d = e^{-\sigma\sqrt{\Delta t}} \tag{7}$$
$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{\nu}{\sigma}\right)\sqrt{\Delta t} \tag{8}$$

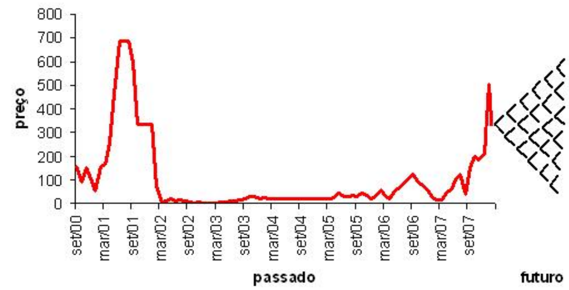

Fig. 3. Price History and Future laticce

Usually the interests rate and the volatility of an asset are calculated from there historical series. However, in the case of the spot price in Brazil, due to fact that its history is very recent, it is more convenient to use the synthetic series of CMO produced by NEWAVE or DECOMP models, using a history of the past affluences. It is also interesting to calculate the monthly volatility of spot price because the CMO is usually seasonal and has a great volatility, with greater dispersion in wet periods (November to April).

In case studies that show the use of this analyses technique, we chose to use 2000 synthetic series of CMO produced by Newave for the year 2010 in the Southeast Submarket. This series are used by the ONS to do the mid term planning of the operation. The minimum and the maximum SPOT prices were limited at levels of R$ 15.57 and R$ 569.59 per MWh, respectively. As the CMO is calculated according to the charge (cargas) levels (light , mediun and heavy) average was calculated considering monthly average charge (carga ponderada) of these levels. With this series it was possible to calculate the average prices for each month of the year 2010, as shown in I.

The lattice may be built with daily or week prices variation depending on the Δt used, affecting consequently values of u, d and p. The ν and σ calculation also modify according to the time adopted (daily or weekly). We opted to use lattices with weekly variations, were $\Delta t = 1 \div 48 = 0,0208$, $\nu_d = \nu \div 4$ e $\sigma_d = \sigma \div \sqrt{4}$. The discounted rate used was of 10% a year. The other parameters for the year 2010 are as show in Table II.

Given the inicial spot price of January 2010 presented in Table I, there is a possibility of price rise or drop. The spot price variations are weekly obtaining a total of four variations within a month. The different routes followed by spot price in the month of 2010 are presented in Figure 4.

The spot price variations of the following months of 2010 are obtained the same way as in Figure 4.

TABLE II
PARÂMETERS OF SPOT PRICE LATTICES - 2010

Month	ν	σ	u	d	p	$1-p$
1	1.456E-05	0.8422	1.1293	0.8855	0.5000	0.5000
2	1.495E-04	0.8848	1.1362	0.8801	0.5000	0.5000
3	2.354E-04	0.8598	1.1321	0.8833	0.5000	0.5000
4	2.257E-04	0.8395	1.1288	0.8859	0.5000	0.5000
5	2.104E-04	0.8072	1.1236	0.8900	0.5000	0.5000
6	2.223E-04	0.7600	1.1159	0.8961	0.5000	0.5000
7	2.327E-04	0.7545	1.1150	0.8968	0.5000	0.5000
8	1.636E-04	0.7689	1.1174	0.8950	0.5000	0.5000
9	1.463E-04	0.7628	1.1164	0.8957	0.5000	0.5000
10	1.484E-04	0.7871	1.1203	0.8926	0.5000	0.5000
11	1.209E-04	0.8094	1.1239	0.8897	0.5000	0.5000
12	1.633E-04	0.8163	1.1251	0.8888	0.5000	0.5000

TABLE I
AVERAGE SPOT PRICE 2010

Month	R$/MWh
1	129,24
2	130,32
3	135,52
4	159,55
5	150,59
6	158,23
7	162,64
8	157,64
9	165,50
10	160,61
11	146,93
12	150,92

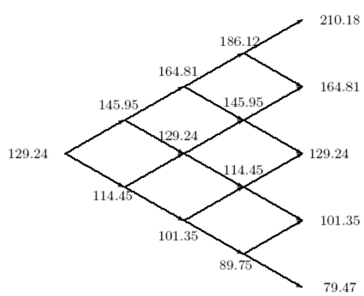

Fig. 4. Spot price route

III. SPOT PRICE EXPECTED REVENUE

The spot price expected revenue of annual forward contract where the charge remains constant for all the year months may be calculated with the expected value of the spot price as in the lattice of Figure 2 using the equation 3.

A Figure 5 shows the spot price expected value in January 2010 which is equivalent to 132.01 R$/MWh. The expected values of the other months of 2010 are found in a similar way to what was demonstrated for January 2010.

The spot price expected values for the months of 2010 are shown in Table III.

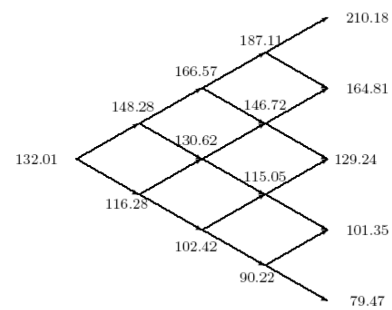

Fig. 5. Expected value for the spot price in January 2010

TABLE III
EXPECTED VALUE FOR THE SPOT PRICE - 2010

Month	R$/MWh
1	132,01
2	133.52
3	138.59
4	162.93
5	153.45
6	160.74
7	165.16
8	160.23
9	168.15
10	163.43
11	149.74
12	153.88

IV. VAR AND CVAR

The risk value, known as Value-at-risk (VAR), was development by JPMorgan Bank to measure, in an efficient and practical way, the loss that an institution could suffer. The VAR is the maximum loss in a determined time horizon and given an occurrence probability (confidence level) that the investor is subject to when investing in a determined financial asset. For example, if an investment portfolio has an R$ 10,000 VaR in a certain month or period with the confidence interval of 95%, this means that there is a 5% probability for the portfolio to lose less than R$10,000. Its simplicity in briefing the risk evaluation of an institution by using only one number would make it a market standard in the late 90s. The confidence level usually adopted is between 94% and 99% and it is possible to take into account the degree of risk aversion each investor is exposed to and the cost of a loss that exceeds the VAR.

According to [3], the VAR must be seen as a necessary procedure but not enough for a total risk control. VAR must not be used as an independent risk manager, but be controlled and limited. For [4] VAR is a viable measure for risk analysis that takes into account many synthetic risk factors and provides one sole number to evaluate the risk effect. For [5], VAR is instable and difficult to work with. A great deficiency of VAR is that it does not provide an indication of the extension of the losses which the investor is exposed to, that is, it does not distinguish between situations in which the losses could be slightly worse or completely crushing.

With the evolution and a better understanding of events related to the distribution of the asset return, a risk measure that uses in its structure information about events that occur in the distribution probability tail appears. This measure, named Conditional- value -at risk (Cvar) has stood out in recent literature about risks and leads to linear models of great dimensions when used to compose portfolios.

For continuous distribution, Cvar maybe described as the average losses resident in the $\propto$ portion of the distribution tail. That is, Cvar at a $\propto$ reliability level maybe described as the expected conditional value of the losses of a portfolio, considering that the losses to be accounted are bigger or the same as VAR. For example, for $\propto = 95\%$, Cvar is obtained by the average of 5% of the greater losses.

Adopting Cvar as a risks metrics of a portfolio is characterized as a more conservative risk management strategy than VAR . This is because a portfolio Cvar of a reliability level $\propto$ % will never be smaller than its respective Var.

V. Risk to Spot

In addition of permitting estimating the expected value of the Spot price, the binomial lattice model could also be used to estimate the VaR and CVaR [6] which may be associated with the future behavior of the PLD. These concepts are usually associated to income or benefits, but as we are dealing with constant loads in these contracts, they may be translated in terms of prices.

In the context of this article the VaR reflects, in the form of loss, the difference between the expected price and the one that may occur with a certain probability of risk. For example, a contract could have a expected price $\bar{S}$ but there might be a 5% probability for the price S_T to be lower than the risk price - RaR [7]. While the $CVaR$ is understood as the expected value of the price conditioned to it lower or equal to the RaR.

For example, the admitted level of risk is 5%, then you must have:

$$\Pr[S_T \le RaR] = 0,05 \quad (9)$$

$$VaR = \bar{S} - RaR \quad (10)$$

$$CVaR = E[S_T | S_T \le RaR] \quad (11)$$

These measures can be identified directly on the binomial lattice, therefore it is enough to calculate the cumulative probabilities of achieving the price associated to the last stage levels of the lattice. Figure 6 shows the probabilities of occurrence for a price lattice of four stages.

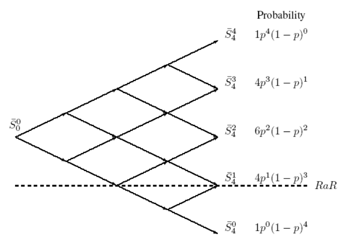

Fig. 6. VaR and $CVaR$ in binomial lattice

It could be noted that the number of alternative paths, from the initial knot to each knot of the last stage, is given by the binomial coefficient $\binom{4}{n}$, for $n = 0, \ldots, 4$.

Generalizing to a lattice with T stages, the probability of the final price S_T be equal to is given by

$$\Pr[S_T = S_T^n] = \binom{T}{n} p^n (1-p)^{T-n}, \quad \text{para } n = 0, \ldots, T \quad (12)$$

Where

$$\binom{T}{n} = \frac{T!}{n!(T-n)!} \quad (13)$$

Thus, the cumulative probability of the final price S_T to be less than or equal to S_T^n is given by:

$$\Pr[S_T \le S_T^n] = \sum_{j=1}^{n} \left[\binom{T}{j} p^j (1-p)^{T-j} \right] \quad (14)$$

The procedure for calculating the risk parameters can be summarized with the following steps. For the terminal knot of the lattice $n = 0, \ldots, T$:

- Calculate the terminal price S_T^n;
- Calculate the cumulative of probabilities $\Pr[S_T \le S_T^n]$;
- Find out the $RaR = S_T^n | \Pr[S_T \le S_T^n] \le 0,005$ (5%);
- Calculate the $VaR = \bar{S} - RaR/R^T$;
- Calculate the $CVaR = E[S_T^n | S_T^n \le RaR]$

Figure 7 shows this procedure of risk analysis applied to the first month, January 2008, with a confidence interval of 94%. The expected value of PLD for this month would be R$ 132.01, but there is a 6% chance of being worth less than R$ 79.47 (RAR). The present value of RAR discounted for one month is R$ 78.81. Therefore, the VaR in this month would be R$ 53.19.

VI. Forward contract pricing

The basic model of the binomial lattice presented previously now will be used in a forward contract pricing using the non-arbitrage principle [6]. In spite of the difficulty for the use of arbitrage in the electrical sector due to impossibility of stocking electrical power in great scales we must not consider

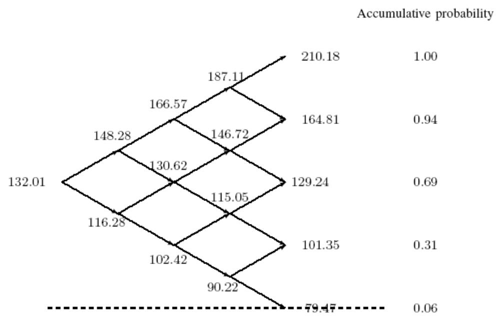

Fig. 7. Risk in January 2010

the impossibility of the use of this principle to price contracts [8].

A generator of the electrical energy may sell it in three markets: the spot market, the regulated market and the free market as shown in Figure 8. Consider that it intends to price a forward contract at the Free Market and for that considers the energy E_R which was placed in the regulated Market at average price R, and the energy E_S which was exposed at spot price. The problem is to estimate the price L of the negotiated energy at a forward contract of the free market.

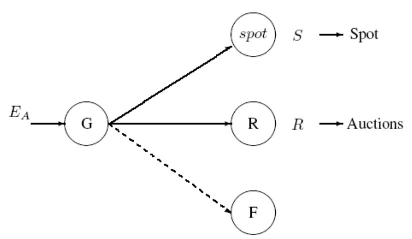

Fig. 8. Energy sale by generator

A. Equivalency: Free Market × (spot + Regulated Market)

One alternative is for the generator to look at the forward contract for a benefit equivalent to its position at the spot plus regulated market. Let us consider that the generator splits its negotiated energy E in two parts, the E_S amount negotiated in spot market and the E_R amount negotiated at the regulated market. That is:

$$E = E_S + E_R \tag{15}$$

Admitting that the spot price S, calculated as demonstrated previously, S may increase by factor u or decrease by factor d. As for price R negotiated in the regulated market, it will remain constant, and also being known by the generator. This situation is shown in Figure 9.

Now the generator intends to sell part of its negotiated energy E at the Free Market, for price L, which is constant, according to Figure 9. But, which price should be negotiated in this market?

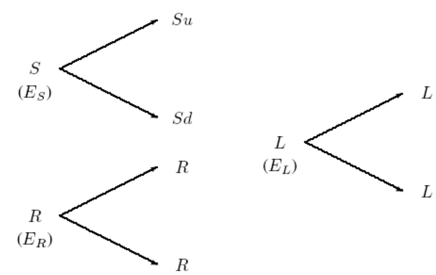

Fig. 9. Equivalency: *spot* + Regulated market →forward

To find the price L of a forward contract negotiated at the Free Market its is possible to use the same principle to obtain a benefit equivalent to that reached with the R prices practiced in the Regulated Market and the expected price of the Spot $\bar{S}$. The binomial lattice of the previous section can also be use to estimate $\bar{S}$.

$$\bar{S}E_S + RE_R = (E_S + E_R)L \tag{16}$$

So, to price this contract we start by building a lattice with the price routes of spot price according to Figure 1. Later we build the new lattice according to Figure 10.

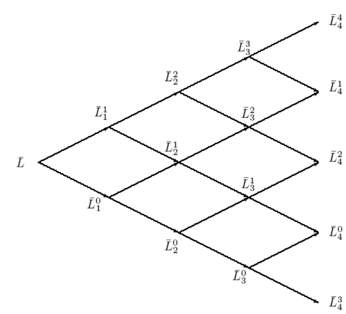

Fig. 10. Inflexible forward contract pricing

The lattice contraction phase is started with the calculation of

$$\bar{L}_T^n = \frac{\bar{S}_T^n E_S + RE_R}{E_S + E_R} \tag{17}$$

The expected benefit of the forward contract for each lattice knot is then calculated with:

$$\bar{L}_t^n = \frac{1}{F}\left[p\bar{L}_{t+1}^{n+1} + (1-p)\bar{L}_{t+1}^n\right] \tag{18}$$

The substitution of equation 17 in 18 results in

$$\bar{L}_t^n = \frac{1}{F}\left[p\left(\frac{\bar{S}_{t+1}^{n+1}E_S + RE_R}{E_S + E_R}\right) + (1-p)\left(\frac{\bar{S}_{t+1}^{n}E_S + RE_R}{E_S + E_R}\right)\right] \tag{19}$$

$$\bar{L}_t^n = \frac{1}{F}\left[\frac{p\bar{S}_{t+1}^{n+1} + (1-p)\bar{S}_{t+1}^{n} + RE_R}{E_S + E_R}\right] \tag{20}$$

$$\bar{L}_t^n = \frac{1}{F}\left[\frac{\bar{S}_t^n E_S + RE_R}{E_S + E_R}\right] \tag{21}$$

The expected price for the forward contract is calculated by

$$\bar{L} = \frac{\bar{S}E_S + RE_R}{E_S + E_R} \tag{22}$$

In the calculation of the expected price $\bar{L}$ of the forward contract, the spot price volatility is decreased by the regulated market price. The expected benefit of the forward contract is given by

$$\bar{B} = K - \bar{L} \tag{23}$$

VII. Case Studie

In this case study, only for the purpose of exemplifying, let us initially suppose that part of the energy (up to 30%) of the UHE Santo Antonio on the Madeira river were allocated in the Free market as a Forward contract. We have to consider that 70% of its energy was negotiated at the regulated market at 79.00 per R\$/MWh, the winning call of the auction. The initial analyses considers 12-month contracts for the year 2010.

According to what was previously discussed if we used the equivalency principle for the prices in the spot market and the regulated markets which would be the sale opportunities available, we could find a reference unit value (R\$/MHh) for the Forward contract dealt at the Free Market.

We start by building a lattice with the spot price routs for the month of 2010 according to Figure 4. By using the formulas presented in section VI-A, it is possible to build a new lattice for the forward contract pricing for January 2010 according to Figure11.

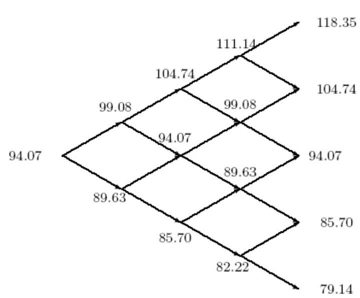

Fig. 11. Forward contract value in January 2010

The Forward contract is this case would be worth 94.07 R\$/MWh in January 2010. This means that if the sale opportunities were given the generator would have hope in probabilistic terms to reach some benefit if it sold a forward contract in the Free market for a higher value than this. The same procedure is repeated for all months of 2010 and their expected value as shown in Table IV.

If the forward contract were signed for the whole of 2010, its price references in the contract should correspond to the average of values found for each month which is 100.49 R\$/MWh in this case.

When pricing this contract taking into consideration the Regulated Market Prices, its risk was already minimized if compared to the contract priced only predicting the spot price. However, this contract still has a certain risk level, once its calculation also considered as spot prediction for the year 2010. But this risk may be measured by using the risk analyses procedures presented in section V. If we determine the reliability level of these contracts in 94%, we will have the results presented in Table V for the year 2010.

TABLE IV
Forward contract value - 2010

Month	R\$/MWh
1	94.07
2	94.40
3	95.96
4	103.17
5	100.48
6	102.77
7	104.09
8	102.59
9	104.95
10	103.48
11	99.38
12	100.58
Average	100.49

TABLE V
Risk in 2010

Month	VAR R\$/MWh	CVAR R\$/MWh
1	15,26	78,81
2	16,85	77,55
3	14,15	81,81
4	5,71	97,45
5	6,77	93,71
6	1,58	101,19
7	-0,24	104,34
8	2,30	100,29
9	-0,71	105,66
10	2,37	101,11
11	8,07	91,31
12	7,15	93,42

VIII. Conclusions

The methodology for pricing of contracts used in this article, the binomial lattices, is reasonably used in the financial market for pricing option contracts. This technique has proved to be an appropriate tool of analysis for pricing electric energy, besides also enabling the risk analysis of contracts, through the VaR and $CVaR$ measures. These analyses could be characterized within the lattice, without the need of importing data into a curve of distribution of probabilities. Curiously, this type of analysis has never been found in any work using lattices before. The use of lattices has also allowed an easy computational in modeling, in addition to not require complex calculations.

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