## The Calculation of Resonance $C$ tire in Cr-nular Fuels

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## ABS ${ }^{\prime 2} R^{\prime} C T$

The methods used in the UK for the calculation of resonance capture in granular fin fuels follow the lonzestablished path of determining a "geometric equivalencel which equates the resonance shieldirs to that in a homogeneous mixture of the resorance absorber in hydrogen.

Simple collision probability arguments, usually for the black limit, were used for $\delta G R$ and SGFi, systems. For granular fuel a 'grey' equivalence, convenient for numerical use, has been adopted, and the geometric solution performed in two ways: by a synthetic collisinn probability model which is rapid arc appropriate for design work and by a Monte Carlo model which allows details of the grain lattice structure to bs studied.

The results are in good agreement, and are shown to give good results compared kith measured relative conversion ratios in the NASTOR stack experiments.

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## Introduction

The basic model for equivel, ding heterogeneous, th hydrogenous resonance integrals is based upor the use of the national approximation to approximate the fuel self collie: n probaisility

$$
\begin{equation*}
P(今 \rightarrow f) \quad \approx \frac{\Sigma \bar{I}}{1+\Sigma} \bar{I} \tag{1}
\end{equation*}
$$

This leads to a form fir the absorptions in an asymptotic source which is identical in form to tare "drcesnous equation.

$$
\begin{align*}
\text { R.I. }=A b: \text { orptions } & =\int \Sigma \sum_{a} \not \partial d u  \tag{2}\\
& =\int \frac{\Sigma^{\frac{a}{} \Sigma_{p}}}{\Sigma_{p}+\Sigma_{t}} \cdot d u \tag{3}
\end{align*}
$$

where $\Sigma_{0}$ is the sum of the true potential scatter inside the fuel and a 'surface' term $\Sigma_{x}$. It will be seen from equation 1 that

$$
\begin{equation*}
\Sigma_{x} \simeq 1 / \overline{1} \tag{4}
\end{equation*}
$$

for a non-re-entrant body. A factor, $a$, is normally introduced to correct for the approximation inherent in equation 1 (the Bell factor), and a further term, $\gamma$, for interaction between adjacent rods in a lattice (the Dancoff factor). Thus

$$
\begin{equation*}
\Sigma_{x}=a \gamma / \bar{I} \tag{5}
\end{equation*}
$$

Now equation 1 is exact at the black limit $\Sigma \rightarrow \infty$. As a first step to a numerical calculation for more complicated geometry, it was proposed in Reference 1 to replace this procedure by a direct calculation of the fuel self collision probability for a 'grey' fuel, whose cross-section was chosen to give a suitable approximation to the average resonance integral. Now

$$
\begin{align*}
\overline{f-P(f \rightarrow f)} & =\int \Sigma_{a}(1-P(f \rightarrow f)) d u / \int \Sigma_{a} \partial u  \tag{6}\\
& =I\left(\Sigma_{p}\right) / I(\infty)  \tag{7}\\
& =\frac{\Sigma_{p}}{\Sigma_{p}+\Sigma}  \tag{8}\\
\text { e we use } \bar{\Sigma} & =\Sigma_{p}\left(\frac{I(\infty)}{I\left(\Sigma_{p}\right)}-1\right) \tag{9}
\end{align*}
$$

In fact the proposals of Reference 1 are slightly more advanced than this, in that they propose to use a flux solution rather than the collision probebilities to determine the equivalence: this permits us to deal approximately with scattering materials of intermediate or large atomic weight outside the fuel. Although one of the codes described here uses this method, the differences are not important for typical HTR fuels.

Before turning to the calculation of the collision probabilities for Eramlar fuel, it is co:ver er, $\hat{i}$, for corpleceness, to review briefly the energy treatment used in the dI'S $(2,3)$ code. Tnis is basically a few group treatrint in which partjal reconance intecrals for oach grour are converted into eroup constanis in a maner consistent with the heteroce. . ty calculation to follon. For a sumple two-region cell uis results in a form

$$
\begin{equation*}
\Sigma_{A}(\text { fuel })=\frac{I / \Delta U}{1-I_{/ \Sigma_{p}} \Delta U} \tag{10}
\end{equation*}
$$

Other corrections for the effect of absorption on the group removal crosssections are also made for consistency. The resultant solution then allows for both spatial and energy depletion of the slowing down density, the former usually neslected in simple expasitions.

Given these manipulations the quentity $I \Delta U$ tabulated for each group is closely the classicel resonance integral. It is cəouced from exact solutions for homogeneous hyarogen/fuel mixtures by the inverse of the wist model.

Extersive comparijons between this model and detailed Nonte Carlo studies have been published (4) for rod, plate and cluster geometries, where the lionte Carlo calculation has used identical cross-sections for the resonances.

In Reference 4 a slight adjustment to the calculated resonance integrals (a deduction of 0.9 barns) was recommended. We currently use 0.6 barns in place of this. The resultant resonance integral at $300^{\circ} \mathrm{K}$ is shown in Fig 2 plotted asainst $\sigma p$. Note that for $U$ metal the fuel contributes 2 b to $\sigma p$, for $\mathrm{VO}_{2}$ the corresponding figure is 9 b . These effects are typically neglicible for $H M R$ lattices where $\sigma_{p} \simeq 300, I \simeq 30$ barns.

The PPOCOL Collision Probability Method
Using an initial guess of the appropriate "effective scattering" crosssection of the fuel refion, a value for the resonance integral of the fuel is calculated. From this an eifective cross section for the grain is calculated and hence collision probabilities are synthesised for all regions of the cell, including the kernel and coating of the average grain. The neutron balance equation for the cell is then solved giving the flux in each region. From the flux distribution a new value of the effective cross section is conputed. Iterations around this cycle continue until the difference between the last two consecutive values of the effective scattering cross section is less than a specified value. Finally the "Bell factor" is calculated."

An accurate value of the fuel transport cross section is not required as this quentity is small in comparison to the effective absorption cross section; twenty per cent errors in magnitude of the scattering cross section resulting in errors of less than one per cent in the effective cross section of the fuel.
*This relates to a feature of the WIMS code rather than to any physical effect. WIUS smears all "fuel" materials together into a rod of appropriate volume. This will include, in the present fuel design, an inner can, which thus contributes to the in fuel (volume) scattering rather than logically to the surface term in the resonance equivalence. The "Bell factor" modifies the outer surface contribution to give the required overe? effective scatter within this peculiar model: it could be negative in extreme cases. This form is dictated by existing input facilities of WIMS: in principle $\Sigma_{p}$ would be input directly with a Dancoff factor of unity.

The iterative method used in PROCOL reguires an initial guess at the effective cross section of the fuel ( $\Sigma_{0}$ ), $w=$ is used in a modafied form of the $\mathfrak{l l a}$ astone resonance inteeral auproination.

$$
I_{p}=3.0\left(\frac{\Sigma_{p}}{N_{8}}+9\right)^{0.415}
$$

where $i_{8}$ is the nuber $d s$ sity of the fuel material
$\Sigma_{\mathrm{p}}$ is the effective cros section of the fuel
$I_{p}$ is the $r$ oncuce integral
This apmovinates to the one-, roup resonence intecra for $U-\overline{2} S$ in WIAS to a sufficicni accuacy for deter ining $\Sigma_{0}$ in the ranje $\Sigma_{0} \leqslant 500$. The nethod described here could be used in few-sroups with the WIIS . Jibulation for each Eroup beinz assesser iirectly.

The total cross section $\Sigma_{T}$ is then evaluated for the fuel from

$$
\Sigma_{T}=\Sigma_{a}+\Sigma_{p}\left(\frac{I_{\infty}}{I_{p}}-1.0\right)
$$

where $\quad \Sigma_{a}$ is the cross section of the fuel ignoring the absorption due to the resonant nuclide $\mathrm{U}-238$.
$I_{\infty}$ is the resonance integral for an infinitely dilute mixture of $U-233$ : aproximately 280.0 barns

Because the grains are not isolated, but set in a matrix of material which has its own scatterine properties, the matrix material is smeared into an additional spherical shell, around the fuel grain, whose volume is given by

$$
V_{B l 1}=v_{o}\left(\frac{1-f}{f}\right)
$$

where $V_{\text {Bli }}$ is the volume of the smeared matrix around the grain
$V_{0}$ is the volume of the grain
$f$ j.s the 'packing fraction' or volume fraction of fuel grains in the matrix

Having obtained the geometry of the fuel grain and associated matrix the spherical collision probabilities $P(i, j)$ for the various layers of the grain are calculated using the collision probability routine PRCB(4). The probability of a neutron born in layer $i$ of the grain escaping from the grain is

$$
P(i, s)=1.0-\sum_{i=1}^{J} P(i, j)
$$

where $J$ is the total nurber of layers comprising the grain. Hence the probability of a neutron born in any layer of the grain escaping to the surface is

$$
P(0, s)=\sum_{i=1} P(i, s)
$$

The probability of a neutron from outside the Erain enterins any layer i of the grain is civen by $P(s, i)$ and values of $P(s, i)$ are obtained from values of $P(i, s)$ by reciprocity, thus assumino an isotropic invard filux distribution.

$$
P(s, i)=\frac{4 \Sigma_{i} y_{i} P(i, s)}{A_{s}}
$$

where $A_{s}$ is the surface area of the grain
$V_{i}$ is the volume of the ith layer

Therefore the probability of a neutron fren outside the crain suffering its next interaction in the grain is

$$
\sum_{i=1}^{J} P(s, i)=\sum_{i=1}^{J}\left(\frac{4.0 \sum_{i} V_{i} P(i, s)}{A_{S}}\right)
$$

J
$\sum_{i=1}^{\sum(s, j)}$ is also are $d$ to equal the probabilit: of that same neutron uniering and interactine in an e uivalent homoe e ised grain. Fqus

$$
\sum_{i-1}^{J}\left(\frac{4 . C \Sigma_{i} V_{i} P(i, s)}{A_{s}}\right)=4.0\left(\frac{V_{0}}{A_{s}}\right)^{\Sigma_{t}} P\left(\Sigma_{t}^{0}, s\right)
$$

Where $\stackrel{\circ}{\Sigma}_{t}$ is the effective cruss section of the homogerised grain.
$F\left(\bar{\Sigma}_{t}, s\right)$ is the probability of a neutron born in the equivalent homogenised grain escaping to the surface.

$$
0 \quad 0
$$

An estinate of $\Sigma_{t}$ is made and $\mathrm{P}\left(\Sigma_{t}, s\right)$ čiculated usins the $P R O B$ routine, the above icentity then beina calculated; if tue balance is not within a set tolerance a new value of $\mathcal{E}_{t}$ is calculated from

$$
\stackrel{o}{t}_{t}=\sum_{i=1}^{J} \frac{\Sigma_{i} v_{i} P(i, s)}{P\left(\Sigma_{t}, s\right) V_{o}}
$$

- 

The new values of $\Sigma_{t}$ are used to rapeat the iteration $u$ til the required accuracy is obtained.
:
As the effective cross section of the grain is idertical to that of the layer, the anrular collision probabilities, $P(I, J)$ can be calculated using the routine ᄃIESUUS(5).

In order to obtain a one-dimensional flux solution, the spherical layers of the fuel grains must be represented by series of concentric annuli. By summing the volume of each type of material in a layer for all grains, the volume of the equivalent annular layers is obtained. The equivalent annular layers are ordered in an outward direction in the same order as in the fuel grain, Figure 1 refers. Collision probabilities for the equivalent annular regions of the cell are computed from the annular and spherical collision prababilities obtained for the cell and for the grains using the following equations, in which annular layers of the physical cell are represented by suffices $I$ and $J$ and the surface of the cell by $S$. Layers of individual grains are represented by $m$ and $n$ and the surface of the grain by s. Arinular layers where the cell has been reduced to a set of equivalent annular rings are represented by suffixes $K$ and $L$.
i Correction of Annular Cross Sections for Infinite Array

$$
P^{*}(I, J)=P(I, J)+\frac{P(I, S) P(S, J)}{(1.0-P(S, S))}
$$

where $P(I, J)$ is the probability of a neutron travelling from $I^{\text {th }}$ annular layer to $J^{\text {th }}$ arnular layer of a cell before suffering interaction.

P* $(I, J)$ is the probability of a neutron travelling from the Ith - annular layer to the Jth annular layer of an infinite array of cells before suffering an interaction.

```
P(I, S) is the probability of a leutron soing from the I th annular
                    layer of a cell to its surface
P(S,J) is th probability of a neutron cntering a cell suffering
    an in eraction in the Jth annular layes.
P(S, S) is the probability of a neutron leaving the surface of one
cell and crte ng the surface of another cell.
```

Correction for Both Annular I, ers Unfuelled
$P^{*}(K, L)=P(I, J)$
$P^{*}(K, L)$ is the probability of a neutron going from the $K^{\text {th }}$ layer to the $L^{\text {th }}$ layer of an equivalent cell before suffering its next interaction.

Correction for Annular Laver I Unfuelled, L: jer J Fuelled

$$
P *(K, I)=\frac{V_{L}\left(\Sigma_{t}\right)_{L} P(L, S) P(I, J)}{V_{0} \Sigma_{t}^{0}(1.0-P(0,0))}
$$

where
$V_{L} \quad$ is the volume of the $L^{\text {th }}$ equivalent layer

$V_{0} \quad$ is the volume of the grain
$\left(\Sigma_{t}\right)_{L} \quad$ is the cross section of the $L^{\text {th }}$ equivalent layer
$\Sigma_{t} \quad$ is the cross section of the smeared Efrain
$P(0,0)$ is the self collision probability of the homogenised grain
$P(L, S)$ is the probability of a neutron from region $L$ of the grain in Layer $u$ escaping from the grain

Correction for Layer I Fuelled, Layer J Unfuelled

$$
P *(K, I)=\frac{P(L, S) P(I, J)}{P(O, S)}
$$

where $P(O, S)$ is the probability of a neutron leaving the smeared grain in Layer I suffering its next collision in another grain.

Correction for Layers I and J Fuelled

$$
P *(K, L)=P(m, n)+\frac{\left(S_{A}\right)_{I}\left(\Sigma_{t_{n}}\right)_{J}\left(V_{n}\right)_{J} P(K, S)_{I} P(L, S)_{J} P(I, J)}{\left(S_{A}\right)_{J}\left(\Sigma_{t}\right)_{J}\left(V_{0}\right)_{J} P(O, S)_{I} P(O, S)_{J}}
$$

where
$P(m, n)$ is the probability of a neutron in spherical layer $m$ sustaining its next interaction in spherical layer $n$ of a fuel grain in annular region $I$.
$\Sigma_{t_{n}} \quad$ is the macroscopic cross section of spherical layer $n$
$V_{n} \quad$ is the volume of spherical layer $n$
$P(K, S)_{I}$ is the probability of a neutron born in spherical layer $K$ of grain in region I escaping from the surface of the grain
$P(I, S)_{J}$ is the probability of a neutron born in spherical layer $L$ of grain in region $J$ escaping from the surface of the grain

Having obtained the cor lete set of probsbilities, $\mathrm{P}^{*}$, the radial flux equations

$$
\Sigma_{t_{K}} V_{K} \phi_{K}=\left[\sum_{I=1}^{N}\left(\lambda_{L} \Sigma_{S L} \bar{\varnothing}+\left(1-\lambda_{L}\right) \Sigma_{S L} \phi_{L}\right) V_{L} P^{*}(L, K)\right]_{K}
$$

where $\bar{\varnothing}$ is the mesn flux or $r$ the cell
$\varnothing_{\mathrm{L}}$ is the flux in region L
N is the total number of regions
$\lambda$ is the hirogeneous factor for nuclide in region $L$ from Reference 7.
are solved, hence enabing a new value of $\Sigma_{p}$ to be svaluated for the fuel region using

$$
\Sigma_{p}^{\prime}=\Sigma_{T}\left(\frac{\phi}{1-\phi}\right)
$$

where $\begin{array}{ll}\Sigma_{p}^{\prime} & \text { is the now value of } \Sigma_{p} \\ \varnothing^{\prime} & \text { is the flux in the fuel region }\end{array}$
If the value of $\Sigma_{p}^{\prime}$ is now within pre-specified limits of $\Sigma_{p}$, it is taken as a new guess for $\sum_{p}$ and the cycle repeated. On final convergence all annular layers with fuel type spectrum are smeared on a volumetric basis and the Bell factor calculated from:-

$$
\text { BELL }=4.0\left(\frac{\text { Volume }}{\text { Surface area }}\right)_{\substack{\text { smeared } \\ \text { region }}} . \quad\binom{\text { Volume of fuel }}{\text { in cluster }}-\left(\lambda \Sigma_{S}\right)_{\text {smear }}
$$

where $\lambda \Sigma_{S}$ is the volume weighted, mean hydrogen equivalent scattering cross section for non resonance materials in the smear.

4 The ARJi:OC : :onte Carlo Apnroach

### 4.1 Geometrical model for spherical Erains in an annulus

In order to adequately simulate the passage of neutrons through a HTR fuel element which contains fuel grains an accurate geometrical model describing grains packed into an annular container is required. Such a model will have to satisfy the following conditions.

1 The packing density must agree with measured values,
2 The observed fuel elements have a high degree of ordering, hence we require a model that includes this feature.

3 In order that the method has validity over a wide range of experimental and design conditions the packing fraction and, to some extent, the structure of the grains must be adjustable.

The packing fraction for experimental fuel elements used in NESTOR was of the order of 0.66 . For an amorphous structure the packing fraction is less than 0.63 whereas for regular lattice structures it can vary from 0.5 to 0.75 . Hence conditions 1 and 2 are complimentary in that the large packing fractions observed in experimental fuel elements entail some regularity in the grain structure.

In the light of these considerations the geometrical model used to describe the grains in a container was what might be described as a distorted lattice model. A lattice structure with approximately the correct packing fraction is chosen and then the vectors joining the centres of grains
distorted so that the structure conforms to the contours of the container. As an illustration ve consider a cubic lattice in the form of a slab. The disto ted lattice suructure would then be formed by rolling this slab up, in a sirilar fashion to rolling up a carpet, and honce form an annulus.

In general a recular lattice is unicuely des mibed by three basic vectors. These vectors in cartecian ceonetry are given by the equation

$$
\begin{aligned}
& \underset{\sim}{i}=\left[a_{1} \hat{\sim}+b_{1} \hat{y}+c_{1} \hat{\sim}\right] d . \\
& \dot{j}=\left[a_{2} \hat{X}+b_{2} \hat{X}+c_{2} \hat{\sim}\right] d . \\
& \underset{\sim}{k}=\left[a_{3} \hat{\sim}+b_{3} \hat{y}+c_{3} \hat{\sim}\right] d .
\end{aligned}
$$

and $\underset{\sim}{\underset{\sim}{x}}, \hat{\sim}, \underset{\sim}{\underset{z}{x}}$ are unit vectors in the respective directions.
For example, in a cubic lattice all the values of $a_{i}, b_{i} c_{i}$, are zero except $a_{1}, b_{2}$ and $c_{3}$. Any vector joining eqrivalent points in the lattice will then have the form

$$
1 i \underset{\sim}{i}+m j n \underset{\sim}{k} \quad l, m, n \text { intesers }
$$

On distorting the lattice the constants $a_{i}, b_{i}, c_{i}$ will becone functions of ( $x, y, z$ ) in cartesian ceometry, ${ }^{i}$ the ${ }^{i}$ form of these functions derending on the distortion. The usefulness of this technique comes when these functions are very simple, or only a few of the constants become functions of a variable.

To facilitate the use of this method when treating annular fuei elements we return to our illustration of rolling a slab into an annulus. This operation can be performed by the transformation

$$
\begin{aligned}
& x \rightarrow r w \\
& y \longrightarrow R_{\text {out }}-r \quad \text { where } R_{\text {out }} \text { is the outer radius of the annulus } \\
& z \longrightarrow z
\end{aligned}
$$

where $r, w, z$ are cylindrical co-ordinates.
The lattice vectors then transform as

$$
\begin{aligned}
& \hat{x} \longrightarrow \underset{\sim}{\hat{W}} / r \\
& \hat{z} \longrightarrow-\hat{\sim} \\
& \hat{\sim} \\
& \hat{\sim} \longrightarrow z
\end{aligned}
$$

Under this transformation the layers in the $x, z$ plane and hence at constant radius will retain their regularity but they will tend to slide one over the other.

If we choose our lattice so that $b_{1}=b_{3}=0$ then in the distorted lattice

$$
\left.\begin{array}{l}
\underset{\sim}{i}=\left[a_{1} \hat{\sim} / r+c_{1} \underset{\sim}{\hat{z}}\right] d . \\
\underset{\sim}{k}=\left[a_{3} \hat{\sim} / r+c_{3} \hat{\sim}\right.
\end{array}\right] d .
$$

In the cuse of an annular fuel the la:crs are taken to be concentric and hence $r$ is the mean radius of a given layer. Hence we can rowrite eq 6 as

$$
\begin{aligned}
& {\underset{\sim}{i}}^{n}=\left[a_{1}^{n} \underset{\sim}{\underset{w}{w}}+c_{1} \underset{\sim}{\underset{\sim}{z}}\right] d . \\
& {\underset{\sim}{k}}^{n}=\left[a_{3}^{n} \underset{\sim}{\underset{w}{w}}+c_{3} \underset{\sim}{\hat{z}}\right] d .
\end{aligned}
$$

where $\dot{i}^{n}$ refers to the $n^{\text {th }}$ layer counting from the outside and

$$
a_{i}^{n}=a_{i} / \bar{r}_{n} \quad \vec{r}_{n}=\text { mean radius of } n^{t h} \text { layer }
$$

The distortion of one laver with respect to another will be described by vector $\underset{\sim}{j}$. Putting

$$
\underset{\sim}{j}=\left[\left(a_{2}^{n}+\alpha^{n}\right) \underset{\sim}{\hat{w}}-\left(b_{2}+\beta^{n}\right) \underset{\sim}{\bar{r}}+\left(c_{2}+\delta^{n}\right) \underset{\sim}{\underset{z}{z}}\right] d .
$$

then $\alpha^{n}, \beta^{n}, \delta^{n}$ are functions of $r, w, z$ which describe how one layer moves over the other. All these functions will be periodic.

We choose a simple value for $a^{n}$, where

$$
\begin{equation*}
\alpha^{n}=\operatorname{Modular}\left(\Delta_{n}\left(w-w_{n}\right) / a_{2}\right), \Delta_{n}=\frac{\bar{r}_{n-1}-\bar{r}_{n}}{\bar{r}_{n-1}} \tag{9}
\end{equation*}
$$

and $v_{n}$ is some constant.
The values of $\beta^{n}$ and $\delta^{n}$ are dependent on $\alpha^{n}$ and now the layers are constrained to move one over the other. Hence in the computer programme there are facilities for feeding in various functions for these quantities.

However there is one constraint. If the outer layer conforms to the shape of the wall of the container then in order that the inner layer conforms to the other wall of the container we require
$\Sigma_{n} \beta^{n}=$ constant independent of $w$.
The nodel thus satisfies two of our conditions. First there will be a high degree of ordering as the structure of the layers is similar to a pure lattice. Secondly, the model is general. Different lattices can be chosen and distorted in many vays.

The vectors in eq 1 all have length $d$ so that every grain is touching. However the model can be easily adapted so that grain to grain distances in any direction can be adjusted. This is accomplished by altering the values of the constants $a_{i}, b_{i}, c_{i}$ so that equation 2 is no longer obeyed. In this way the packing densiもy can be adjusted or the structure expanded or contracted in any direction.

Using the geometrical model presented here we can satisfy all the conditions stipulated in this section. The generality of the model lends it to studying details of the grain lattice structure.

### 4.2 Yonte Carlo Calculation

The geometrical model defines the positions of the grains in the fuel region for a given lattice structure by means of the functions $\alpha^{n}, \beta^{n}$ and $\delta n$. This model was incorporated in a simple one group Nonte Carlo calculation of the fuel to fuel probability $P_{f f}$ using as the fuel cross section the average cross section given by eq. 9 section 1 .
 boundary. $F$ nce we adopted a tectrique called :oojcock tracking (5). In this case the path length of a neutron is $\dot{c}$ icruined by a cross section $\Sigma$ tot wich is constant for all moteriols. $\sum$ tot is cefined by ' $\dot{v}$., , , the equation

$$
\Sigma_{\text {tot }}=\Sigma_{\text {true }}+\Sigma_{\text {dummy }}
$$


where $\Sigma$ true is the physical aross section for a given material. At every collision the there will be $\Sigma_{\text {true }} / \Sigma_{\text {iot }}$ piysical event and $\Sigma_{\text {dummy }} / \Sigma_{\text {tot }}$ dummy events. If an ev-nt is a durmy ミient the ieutron continues unchanged. The effect of using $\Sigma_{\text {dummy }}$ in this way is to remove the need to calculate the positions of grain ana kernel boundaries, altrough more collisions (real plus dumny) are recorded. This greatly reduced the complexity of the code and saved time.

In the calculations reported here $\Sigma$ tot was taken to equal the fuel cross section so that $\Sigma_{\text {dummy }}$ in the fuel wos zero.

### 4.3 Effects of Different Grain Structure

In addition to the results reported in sections 5 and 6 calculations have been carried out for 1 INSOR fuel element $9 / \mathrm{B}-\mathrm{B}$ using a variety of packing structures.

## 1 Results of chancing the lattice structure

Due to the geometry of the experimental fuel elemerts the desired packing fraction can only be achieved if interlocking hexagcnal lavers of particles are used in the model. Hence only small chanjes in the lattice structure are possible. The orientation of the basic lattice structure can be rotated through ancles from $0-30^{\circ}$ with respect to a horizontal plane perperdicular to the axis of the annulus. Results of calculations with varying rotations are shown in Table 1 and indicate that $P_{f f}$ is not sensitive to this parameter.

TABJ.E 1

| Angle of rotation | $0^{\circ}$ | $20.7^{\circ}$ | 300 |
| :--- | :---: | :---: | :---: |
| $P_{\text {ff }}$ | $0.836 \pm .0045$ | $0.833 \pm .0045$ | $0.833 \pm .0045$ |

## 2 Results of varying the density of packing with radius

It is possible that in realistic compacts used in IK III reactors the density of the layers of erains adjacent to the walls of the cen will be less than elsewhere in the compact. To estimate the effect of such density variations we also carried out a calculation of $P_{f f}$ with the layers adjacent to the walls of the can reduced by $9 \%$ with respect to the other layers but keeping the total fuel content constant. The results are shown in Table 2. Again $P_{f f}$ was not sensitive to this variation in packing.

| Reciuction in iensity <br> of outer jayers |  |  |
| :--- | :---: | :---: |
| $P_{f f}$ | -0 | $0 \%$ |

## Comparisons: $3+\cdots e e n$ Theories

Table 3 below shows the comp rison between the two aypoaches.
TABLE 3

| Core Number | $\Sigma_{p}$ |  |  | $r$ | PROCOL lionte |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | PROCOL | ronte $\pm 1 \%$ | Corrccted ionte |  |  |
| AGN 3/C-1 | 6.573 | $6.709 \pm 0.057$ | $6.441 \pm 0.064$ | 0.959 | $0.732 \pm 0.067$ |
| AG: $4 / C-1 / \sqrt{2}$ | 6.742 | $6.700 \pm 0.067$ | $6.679 \pm 0.065$ | 0.9954 | $0.063 \pm 0.067$ |
| AGN $6 / 8-3 / \sqrt{2}$ | 4.059 | $4.089 \pm 0.041$ | $4.070 \pm 0.047$ | 0.9951 | -0.011 $\pm 0.047$ |
| AGi: $7 / P-B / \sqrt{2}$ | 4.765 | $4.772 \pm 0.048$ | $4.750 \pm 0.048$ | 0.9953 | $0.015 \pm 0.048$ |
| AGI: $8 / \mathrm{P}-\mathrm{B}$ | 4.635 | $4.772 \pm 0.048$ | $4.575 \pm 0.046$ | 0.9571 | $0.060 \pm 0.046$ |
| AGN $9 / B-B$ | 3.949 | $4.089 \pm 0.041$ | $3.912 \pm 0.039$ | 0.9549 | $0.037 \pm 0.039$ |
| AGI 10/3-B/Cu | 3.793 | $4.089 \pm 0.041$ | $3.912 \pm 0.039$ | 0.9549 | -0.119 $\pm 0.039$ |
| LGN $11 / \mathrm{B}-\mathrm{B} / \mathrm{C}$ | 4.269 | $4.500 \pm 0.045$ | $4.345 \pm 0.043$ | 0.9643 | $-0.076 \pm 0.01+3$ |
| AGN 12/G-1 | 7.230 | $7.690 \pm 0.077$ | $7.390 \pm$ ¢. 074 | 0.9691 | $-0.160 \pm 0.074$ |
| AGN 13/A-3 | 2.969 | $3.013 \pm 0.03$ | $2.881 \pm 0.029$ | 0.9530 | $0.088 \pm 0.029$ |

It will be noted that Dancoff factor has to be allowed for explicitly in AIIJ,OC which considers an isolated fuel element: the correction is of modest size. We conclude that, within the statistics of the Monte Carlo method, the two approaches agree.

## 6 Comparisons with Exleriment

Table 4 shows relative conversion ratios calculated using ANJHOC equivalences for the $N E S T O R$ experiments as presented to the 9 th $\operatorname{DCPM}$ in llay 1970. The experimental data have been corrected for the small environmental mismatch where appropriate.

| Core <br> identification | $\begin{gathered} \text { Lattice } \\ \text { Ditch } \\ \left(m_{r}\right) \\ \hline \end{gathered}$ | Fuel |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\text { Central } 5 \times 5$ arrey | Outer zone | $\begin{gathered} \text { Reasured } \\ \text { R.C.R. } \end{gathered}$ | $\begin{gathered} \text { Discrepancy } \\ \left(\%^{\prime}\right) \end{gathered}$ |
| AGN 2/C-2 | 83.8 | C | $2 \mathrm{pin} \mathrm{jO}_{2}$ | 4.59 | $-1.3$ |
| AGN 3/C-1 | 11 | C | $1 \mathrm{pin} \mathrm{UO}_{2}$ | 3.97 | +1.2 |
| AGN $4 / 0-1 / \sqrt{2}$ | 118.4 | C | 1 pin $\mathrm{UO}_{2}$ | 2.49 | $1+0.3{ }_{1}$ |
| AGN $6 / 3-B / \sqrt{2}$ | " | B | B | 2.90 | $+0.5$ |
| AGR 9/B-B | 83.8 | B | 11 | 4.88 | + 1.4 |
| AGN 10/B-3/Cu* | 11 | B | 11 | 5.24 | + +1.0 |
| AGN $11 / \mathrm{B}-3 / \mathrm{C}^{+}$ | 11 | B | 1 | 4.86 | + 1.4 |
| AGN 12/G-1 | 11 | G | $1 \mathrm{pin} \mathrm{UO}_{2}$ | 3.85 | $+0.1$ |
| AGN $13 / \mathrm{A}-3$ | " | A | $3 \operatorname{pin} \mathrm{UO}_{2}$ | 6.12 | -0.4 |
| AGN 14/G-1(S) | 11 | G | $\begin{aligned} & 1 \text { pin } \mathrm{VO}_{2}+ \\ & \text { graphite } \\ & \text { sleeve } \end{aligned}$ | 3.68 | $\begin{aligned} & +1.8 \\ & \therefore \end{aligned}$ |

* Copper poison tubes in central voids of each fuel pin.
tGraphite dowels " " " " " " ".
Agreement is seer to be good.
:


## 7 Conclusions

Methods of geometric equivalence which have been used for AGR and SGYH systems have been extended to deal with granular HTR fuels. Two approaches to the calculation of the equivalence have been tried, and shown to give similar results. The Honte Carlo studies show that these are insensitive to modest changes in the grain packing assumed, which is a necessary condition for the simple theory.

No full space/energy Monte Carlo has been carried out on this problem, and the efficacy of this model is thus not fully demonstrated. On the other hand, comparisors with experiment for a range of grain sizes show good agreement. Together with the experience previously gained in simpler geometries, this permits a modest confidence that the model is, in fact, of adequate accuracy.

## References

1 J R Askew. A resonance equivalence procedure convenient for numerical use. AEDW Mi. 892

2 J R Askew et al. A general description of the lattice code WIMS. Jour. Brit. Nucl. Energy Soc. p. 564 October 1966.

3 J R Askew. The calculation of resonance captures in a few-sroup approximation. AEEV R. 4891966.

4 F J Fayers et al. Uranium and Plutonium fuelled lattices with graphite and water moderation - a comparison of experiment and theory. The physics problems in thermal reactor design.
p. 15 BNES London 1967.
5. E R Woodcock et al. ANL 7050 p. 557 August 1955.


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