One-velocity neutron diffusion calculations based on a two-group reactor model

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Many processes in reactor physics are described by the energy dependent neutron diffusion equations which for many practical purposes can often be reduced to one-dimensional two-group equations. Though such two-group models are satisfactory from the standpoint of accuracy, they require rather extensive computations which are usually iterative and involve the use of digital computers. In many applications, however, and particularly in dynamic analyses [1], where the studies are performed on analogue computers, it is preferable to avoid iterative calculations. The usual practice in such situations is to resort to one group models, which allow the solution to be expressed analytically. However, the loss in accuracy is rather great particularly when several media of different properties are involved.

This paper describes a procedure by which the solution of the two-group neutron diffusion equations can be expressed analytically in the form which, from the computational standpoint, is as simple as the one-group model, but retains the accuracy of the two-group treatment. In describing the procedure, the case of a multi-region nuclear reactor of cylindrical geometry is treated, but the method applied and the results obtained are of more general application.

Another approach in approximate solution of diffusion equations, suggested by Galanin [2] is applicable only in special ideal cases.

STATEMENT OF THE PROBLEM

The one-dimensional, two-group approximation of the radial flux distribution in a multi-region nuclear reactor [3] results in the following set of diffusion equations in vector notation:

$$\begin{Vmatrix} \nabla^2 - a_{11}^i & a_{12}^i \\ a_{21}^i & \nabla^2 - a_{22}^i \end{Vmatrix} \cdot \begin{Vmatrix} F^i(r) \\ N^i(r) \end{Vmatrix} = \|A^i\| \overrightarrow{Q^i(r)} = 0 \qquad (1)$$

which describe the fast and thermal flux $F^i(r)$, $N^i(r)$ in the ith region, defined by the interval:

$$R_{i-1} \leqslant r < R_i \quad i = 1,2, \ldots I \qquad (2)$$

I being the number of reactor regions.

Coefficients a_{nm}^i of the matrix $\|A^i\|$ are determined by the material and geometrical constants of the ith reactor region.**

The solution $\overrightarrow{Q^i(r)}$ of Eq. (1) is subject to the following conditions:

(a) at the centre of the reactor $r = R_0 = 0$

$$\frac{d}{dr} \overrightarrow{Q^1(0)} = \frac{d}{dr} \overrightarrow{Q_0^1} = 0 \qquad (3)$$

(b) at the extrapolated boundary of the reactor; $r = R_I$

$$\overrightarrow{Q^I(R_I)} = \overrightarrow{Q_I^I} = 0$$

(c) at the interface between any two regions; $r = R_i$

$$\begin{Vmatrix} D^i \end{Vmatrix} \frac{d}{dr} \vec{Q}_i^i = \begin{Vmatrix} D^{i+1} \end{Vmatrix} \frac{d}{dr} \vec{Q}_i^{i+1}$$

$$\vec{Q}_i^i = \vec{Q}_i^{i+1}$$

where $\vec{Q}_i^{i+1} = \vec{Q}^{i+1}(R_i)$, and

$$\begin{Vmatrix} D^i \end{Vmatrix} = \begin{Vmatrix} D_f^i & 0 \\ 0 & D_n^i \end{Vmatrix}$$

D_f^i and D_n^i being diffusion coefficients for the fast and thermal group in the region i.

The problem thus formulated requires iterative adjustment of two parameters to satisfy the boundary conditions. In most cases these two parameters are:

(a) ratio of the fast and thermal flux at the centre of the reactor; $q_0 = F_0^1/N_0^1$, and

(b) the thickness of one of the regions; i.e. $r = R_k$, between regions k and $k+1$.

The usual procedure in solving such a problem consists of representing the solution of the two mutually coupled Eqs. (1) as a linear combination of solutions of homogeneous equations in the form:

$$\overrightarrow{Q^i(r)} = \|B^i\| \overrightarrow{W^i(r)} = \begin{Vmatrix} b_{11}^i & b_{12}^i \\ b_{21}^i & b_{22}^i \end{Vmatrix} \cdot \begin{Vmatrix} X^i(r) \\ Y^i(r) \end{Vmatrix} \qquad (4)$$

where for cylindrical geometry $X^i(r)$ and $Y^i(r)$, coordinates of vector $\overrightarrow{W^i(r)}$, are solutions of [3]:

$$(a) \quad \frac{d^2}{dr^2} X^i(r) + \frac{1}{r} \cdot \frac{dX^i(r)}{dr} + \mu_i^2 i X^i(r) = 0$$

$$(b) \quad \frac{d^2}{dr^2} Y^i(r) + \frac{1}{r} \cdot \frac{dY^i(r)}{dr} - \nu_i^2 Y^i(r) = 0 \qquad (5)$$

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** See Appendix.

while the initial and boundary conditions (3) take the form:

$$(a)\ \text{at } r=R_0:\ \frac{\mathrm{d}}{\mathrm{d}r}\overrightarrow{W_0{}^1}=0$$

$$(b)\ \text{at } r=R_I:\ \overrightarrow{W_I{}^I}=0$$

$$(c)\ \text{at } r=R_i:\ \overrightarrow{W_1{}^{i+1}}=\|G^i\|\cdot\overrightarrow{W_i{}^i}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\overrightarrow{W_i{}^{i+1}}=\|H^i\|\cdot\frac{\mathrm{d}}{\mathrm{d}r}\overrightarrow{W_i{}^i} \tag{6}$$

where $\|G^i\|=\|B^{i+1}\|^{-1}\cdot\|Bi\|$, and

$$\|H^i\|=[\|D^{i+1}\|\cdot\|B^{i+1}\|]^{-1}\cdot\|D^i\|\cdot\|B^i\|$$

The formulae for calculating elements $b^i{}_{nm}$ of the matrix $\|B^i\|$ are given in the Appendix.

Appropriate values of initial conditions for Eq. (5) are then sought by iterative adjustment of the two variable parameters using a digital computer [4].

TRANSFORMATION INTO RICCATI EQUATIONS

To avoid iterative computation it is necessary to formulate the problem in such a way that the two variable parameters can be determined separately. This suggests the use of first order equations with appropriate conditions that can be satisfied by adjusting one parameter. Since in most practical cases the nature of the diffusion process is such that there is always one point in the reactor where either the flux or its derivative is zero, we may try the substitution:

$$U^i(r)=\frac{\mathrm{d}X^i(r)}{\mathrm{d}r}\cdot\frac{1}{X^i(r)}$$

$$V^i(r)=\frac{\mathrm{d}Y^i(r)}{\mathrm{d}r}\cdot\frac{1}{Y^i(r)} \tag{7}$$

which will eliminate the initial ratio of the fast and thermal flux as a variable parameter.

Introducing substitution (7) into Eqs. (5) one obtains [5]:

$$(a)\quad \frac{\mathrm{d}U^i(r)}{\mathrm{d}r}+[U^i(r)]^2+\frac{U^i(r)}{r}+\mu_i{}^2=0$$

$$(b)\quad \frac{\mathrm{d}V^i(r)}{\mathrm{d}r}+[V^i(r)]^2+\frac{V^i(r)}{r}-\nu_i{}^2=0 \tag{8}$$

Using substitution (7) and designating $Y_i{}^i/X_i{}^i$ by $Z_i{}^i$, boundary and initial conditions (6) become:

$$(a)\ \text{at } r=R_0=0:\ U^1(0)=U_0{}^1=0;\ V_0{}^1=0$$

$$(b)\ \text{at } r=R_I:\ [U_I{}^I]^{-1}=0;\ [V_I{}^I]^{-1}=0$$

$$(c)\ \text{at } r=R_i:\ U_i{}^{i+1}=\frac{h_{11}{}^iU_i{}^i+h_{12}{}^iV_i{}^iZ_i{}^i}{g_{11}{}^i+g_{12}{}^iZ_i{}^i}$$

$$V_i{}^{i+1}=\frac{h_{21}{}^iU_i{}^i+h_{22}{}^iV_i{}^iZ_i{}^i}{g_{21}{}^i+g_{22}{}^iZ_i{}^i} \tag{9}$$

The original Bessel Eqs. (5) are thus transformed into Riccati Eqs. (8) which are of the first order.

EXHIBITED PROPERTIES OF RICCATI EQUATIONS

As can be seen from Eqs. (9) the initial values of $U^i(r)$ and $V^i(r)$ at the centre of the reactor do not depend on the initial values $F_0{}^1$ and $N_0{}^1$. As a result, $U^i(r)$ and $V^i(r)$ are independent of the parameter q_0, and the original problem of iterative adjustment of two parameters has now been reduced to finding only the critical radius R_k. As a further consequence of Eqs. (9) the values of $U^i(r)$ and $V^i(r)$ are completely specified both at the centre of the reactor and at the extrapolated boundary.

Another valuable property [5] exhibited by Eq. (8*b*) is that its solutions can be represented by the following asymptotic expression:

$$V_j{}^i=\nu_i\,[-(2R_j\nu_i)^{-1}+(-1)^{(i+j)}\times\sqrt{1+(2R_j\nu_i)^{-2}}] \tag{10}$$

which follows from the fact that the value of $V^i(r)$ at one end of the reactor region is independent of the initial value at the other end if the thickness of the region is sufficiently large. In other words, values $V_j{}^i$ are dependent only on the coefficient ν_i and corresponding radius R_j.

As will be seen later, it is now possible by solving Eqs. (8), starting both from the centre ($i=1$) and from the extrapolated boundary ($i=\mathrm{I}$) and using conditions in Eq. (9*c*), to reduce the total number of regions to only two, and find the critical radius R_k which will satisfy conditions in Eqs. (9).

DERIVATION OF THE ANALYTICAL EXPRESSION FOR THE FLUX EQUATION

Owing to the previous transformations it is possible to determine the critical radius and the coefficients

$$Z_j{}^i,\ V_j{}^i \text{ and } U_j{}^i \tag{11}$$

for all regions and radii.

Taking into account that the values of function $Y^i(r)$ and its derivatives $\mathrm{d}Y^i(r)/\mathrm{d}r$ at both boundaries $r=R_j$ of each region can be expressed as

$$Y_j{}^i=X_j{}^iZ_j{}^i$$

$$\frac{\mathrm{d}}{\mathrm{d}r}Y_j{}^i=X_j{}^iV_j{}^iZ_j{}^i \tag{12}$$

where $j=i-1$ and i for the left and right boundary, respectively, and that the values $X_j{}^i=X^i(R_j)$ may be obtained by solving the one-group diffusion equation:

$$\frac{\mathrm{d}^2X^i(r)}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}X^i(r)}{\mathrm{d}r}+X^i(r)\cdot\mu_i{}^2=0 \tag{13}$$

with known ratio of initial conditions:

$$\frac{\mathrm{d}X_j{}^i}{\mathrm{d}r}\cdot\frac{1}{X_j{}^i}=U_j{}^i, \tag{14}$$

it remains only to determine the function $Y^i(r)$ within the boundaries of each region.

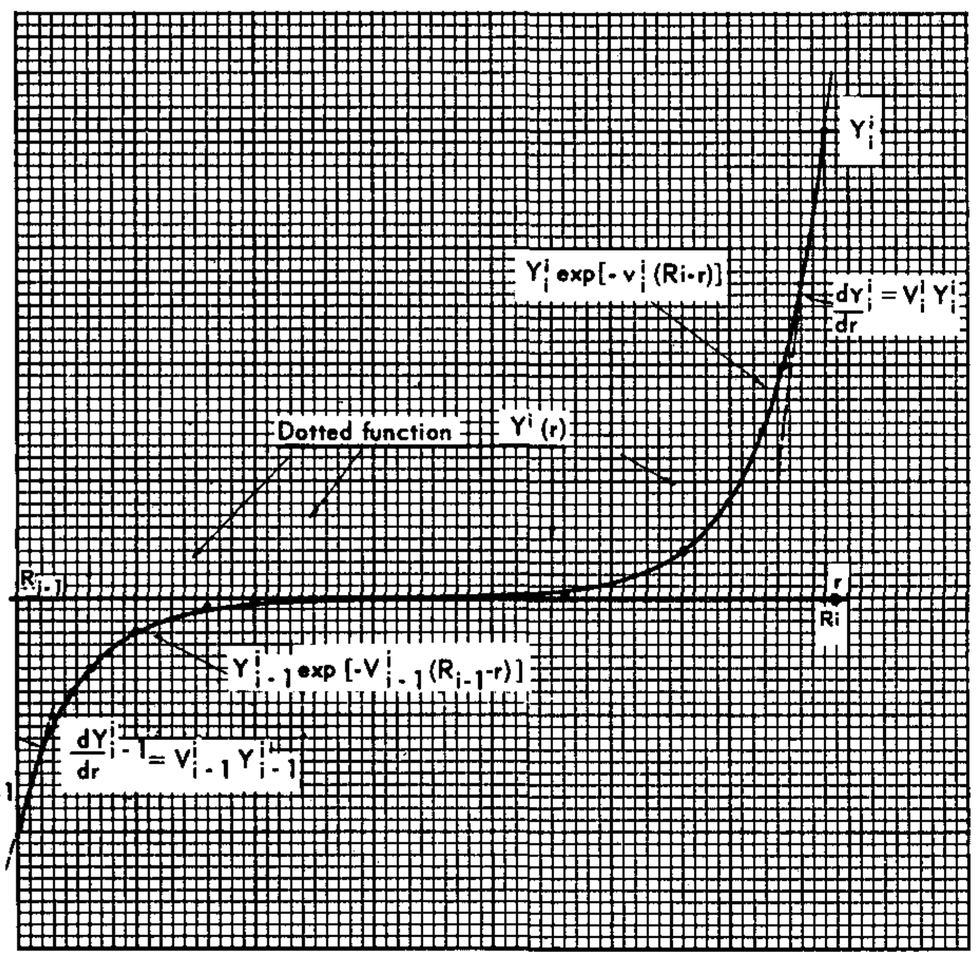

Figure 1. Typical form of function $Y^i(r)$ with two corresponding exponential functions

From Eq. (4) the thermal flux is given as:

$$N^i(r) = b_{21}{}^i X^i(r) + b_{22}{}^i Y^i(r) \tag{15}$$

A typical form of the function $Y^i(r)$ obtained by solving Eq. (8*b*) on an analogue computer is shown in Fig. 1. Taking into account Fig. 1 and using the exact values $Y_j{}^i$ and $\mathrm{d}Y_j{}^i/\mathrm{d}r$, the functions $Y^i(r)$ may be approximated by two exponential functions in the following way:

$$Y^i(r) \approx X^i{}_{i-1} Z^i{}_{i-1} \exp\left[-V^i{}_{i-1}(R_{i-1}-r)\right] + X_i{}^i Z_i{}^i \exp\left[-V_i{}^i(R_i - r)\right] \tag{16}$$

For comparison, these two exponential functions are also represented in Fig. 1.

Due to the fact that boundary values $Y_j{}^i$ are maximum and that these values are known exactly with their derivatives at these points, errors due to approximation are negligible. Thus, using Eqs. (12) and (15) the thermal flux distribution can be expressed as:

$$N^i(r) = b_{21}{}^i X^i(r) + b_{22}{}^i X_i{}^i Z_i{}^i \exp\left[-V_i{}^i(R_i - r)\right] + b_{22}{}^i X^i{}_{i-1} Z^i{}_{i-1} \exp\left[-V^i{}_{i-1}(R_{i-1} - r)\right] \tag{17}$$

NORMALIZATION OF RICCATI EQUATIONS

Instead of determining coefficients

$$Z_j{}^i,\ V_j{}^i \text{ and } U_j{}^i \tag{18}$$

by solving Riccati Eqs. (8) for each particular set of reactor parameters, it is possible to use a series of curves which represent the solutions of normalized Riccati equations, obtained once and for all [6].

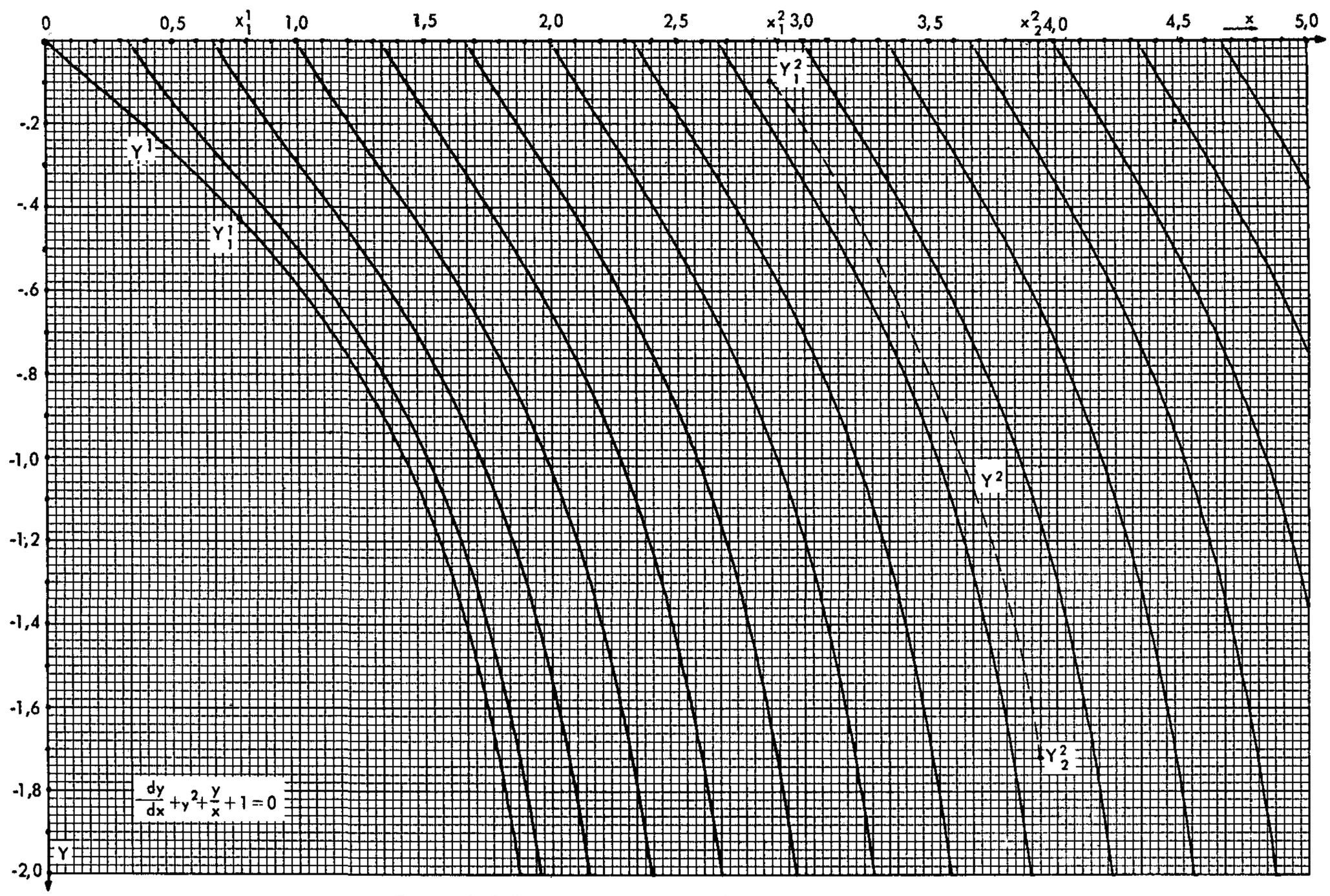

Figure 2. Solutions of the normalized Riccati equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y^2 + \frac{y}{x} + 1 = 0 \text{ for different initial conditions}$$

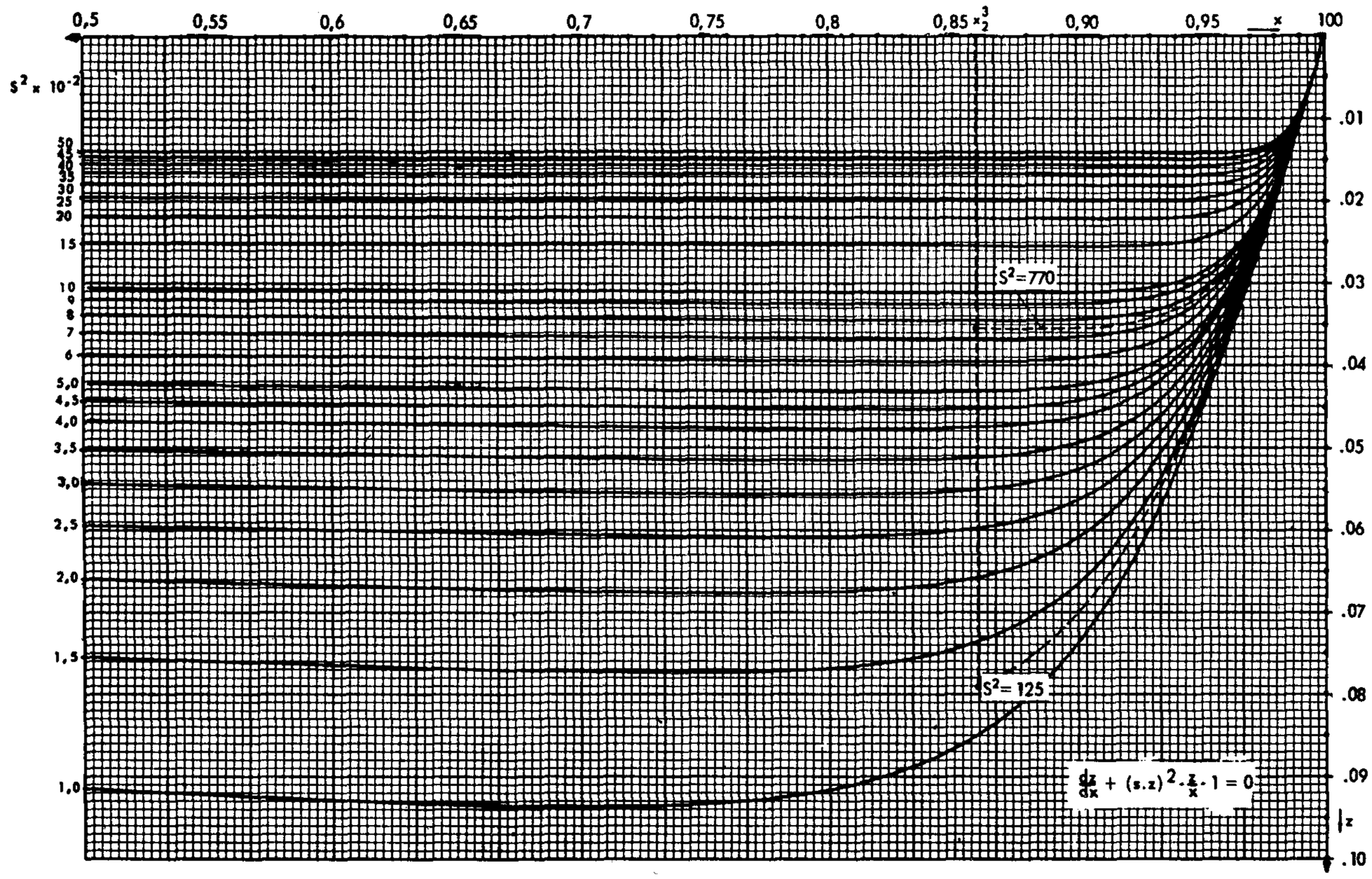

Figure 3. Solutions of the normalized Riccati equation

$$\frac{dz}{dx} + (s \cdot z)^2 - \frac{z}{x} - 1 = 0, \text{ for various } s. \text{ (Initial conditions: } x = 1,\ z = 0)$$

To normalize Eq. (8*a*) we introduce the substitutions:

$$U^i(r) = y^i(x) \cdot \mu_i \text{ and } r = \frac{x}{\mu_i} \tag{19}$$

and obtain:

$$\frac{dy^i(x)}{dx} + [y^i(x)]^2 + \frac{y^i(x)}{x} + 1 = 0 \tag{20}$$

The solutions $y^i(x)$ for different values of $y_j^i = y^i(x_j^i)$ and $x_j^i = R_j \cdot \mu_i$, obtained on an analogue computer are shown in Fig. 2. Figure 3 represents the solutions $z(x)$ of the normalized equation

$$\frac{dz(x)}{dx} + [s \cdot z(x)]^2 - z(x)/x - 1 = 0 \tag{21}$$

for various s, which are necessary for $i = I$ (last region).

In deriving Eq. (21) we used substitutions:

$$z(x) = \frac{1}{R_I U^I(r)},\ z(x) = \frac{1}{R_I V^I(r)},\ x = \frac{r}{R_I} \tag{22}$$

The procedure of determining coefficients (18) using Figs. 2 and 3 is explained in the given example.

CONCLUSION

As has been shown, all coefficients appearing in Eq. (17) may be easily determined from material and geometrical constants of the given reactor using simple formulae and the family of curves shown in Figs. 2 and 3. Since these formulae and curves are derived from a two-group reactor model, the accuracy attainable with Eq. (17) approaches that of the two-group treatment.

On the other hand, since, in addition to the two exponential functions, Eq. (17) contains only the function $X^i(r)$ which is a solution of the one-group reactor model, it can be concluded that the simplicity of analytical treatment of Eq. (17) is compatible with one-group approximation.

EXAMPLE

To illustrate the practical application of this procedure consider a gas-cooled power reactor with two active zones and one reflector. Values of the material constants for all regions are given in Table 1:*

Table 1

	$i=1$	$i=2$	$i=3$
$\Sigma_f^i \cdot 10^3$	2.5058	2.9158	2.9112
$\Sigma_n^i \cdot 10^3$	5.3793	3.6946	0.3561
D_f^i	1.7042	1.5253	1.15596
D_n^i	1.3651	1.2123	0.9119
k_∞^i	1.0282	1.0815	0.0000

* Symbols used are the same as in Ref. [3].

The geometrical constants are:

axial buckling $\alpha_z^2 = 0.2486 \cdot 10^{-4}$

The radius between the first and second region $r = R_1$ is unknown.

Other radii are: $R_2 = 472$ cm

$R_3 = 550$ cm

Matrices $\|a_{nm}{}^i\|$ from Eq. (1) may be found using given material and geometrical constants of the reactor. These matrices are:

Table 2

	$i=1$		$i=2$		$i=3$	
$10^{+3}\|a_{nm}{}^i\|$	−1.495	3.241	−1.912	2.618	−2.543	0.000
	1.841	−3.932	2.403	−3.096	3.193	−0.415

Matrices $\|B^i\|$ which transform diffusion equations into the Bessel equations, Eq. (4), are:

Table 3

	$i=1$		$i=2$		$i=3$	
$\|B^i\|$	1.000	1.000	1.000	1.000	0.000	1.000
	0.462	1.223	0.765	1.201	1.000	1.523

Arguments of Bessel functions (3) become:

Table 4

	$i=1$	$i=2$	$i=3$
$\mu_i^2 \,.\, 10^{+3}$	0.00495	0.06915	−2.54238
$\nu_i^2 \,.\, 10^{+3}$	5.46615	5.08442	0.41537

Matrices $\|G^i\|$ and $\|H^i\|$ which define conditions at the interface between any two regions may be determined using Eqs. (4) and (6). These matrices are:

Table 5

	$i=1$		$i=2$	
$\|G^i\|$	0.8457	−0.0113	1.0000	1.0000
	0.1536	1.1132	2.2659	0.2993
$\|H^i\|$	0.9475	−0.0193	1.3193	1.3193
	0.1698	1.1367	2.9952	0.3854

From Fig. 3, which gives the values of $U^I{}_{I-1}$ and $V^I{}_{I-1}$, we may for $x_2{}^3 = R_2/R_3 = 0.861$, determine:

(a) $s^2 = R_3{}^2\mu_3{}^2 = 125$

$$z = (U_2{}^3 \cdot R_3)^{-1} = 3.5 \cdot 10^{-2}$$

(b) $s^2 = R_3{}^2\nu_3{}^2 = 770$

$$z = (V_2{}^3 \cdot R_3)^{-1} = 7.91 \cdot 10^{-2} \quad (23)$$

From Eq. (10): $V_2{}^2 = 0.07024$

Knowing $U_2{}^3$; $V_2{}^3$ and $V_2{}^2$ using Eq. (9c), which represent a system of two algebraic equations with two unknowns, we find:

$$Z_2{}^2 = -0.2273$$
$$U_2{}^2 = \quad 0.01423 \quad (24)$$

According to Eq. (10) for 300 cm $\leqq r = R_1 <$ 350 cm we have:

$$V_1{}^1 = \quad 0.07191 \pm 1.2 \cdot 10^{-3}$$
$$V_1{}^2 = -0.07152 \pm 1.2 \cdot 10^{-3} \quad (25)$$

Value $y_2{}^2 = U_2{}^2\mu_2 = 1.71$ defines one end point of the solution $y^i(x^i)$, which corresponds to $x_2{}^2 = 3.92$ ($i = 2$ and $r = R_2$). Also, $U_0{}^1 = 0$ and $r = R_0$, Eq. (9a), defines one end point ($y_0{}^1; x_0{}^1 = 0; 0$) for the first region ($i = 1$).

Other end points ($y_1{}^1, x_1{}^1$) and ($y_1{}^2, x_1{}^2$) of these functions have to satisfy the following conditions:

(a) $R_1 = x_1{}^1/\mu_1 = x_1{}^2/\mu_2$, and

(b) $$U_1{}^1 = \frac{g_{11}{}^1}{h_{11}{}^1} U_1{}^2 + \frac{g_{21}{}^1 h_{12}{}^1}{g_{22}{}^1 h_{11}{}^1} \cdot V_1{}^1 \left[1 - \frac{h_{22}{}^1 V_1{}^1}{g_{22}{}^1 V_1{}^2}\right]^{-1} \quad (26)$$

Condition (26b) is derived eliminating $Z_1{}^1$ from Eq. (9c) and taking into account the relations:

$$\frac{h_{12}{}^1 h_{21}{}^1}{h_{11}{}^1 h_{22}{}^1} \ll 1 \quad \frac{g_{12}{}^1 g_{21}{}^1}{g_{11}{}^1 g_{22}{}^1} \ll 1$$

Using calculated $V_1{}^1$ and $V_1{}^2$ and curves in Fig. 2 from conditions (26) we get

$$U_1{}^1 = y_1{}^1\mu_1 = -0.915 \cdot 10^{-3}$$
$$U_1{}^2 = y_1{}^2\mu_2 = -0.847 \cdot 10^{-3} \quad (27)$$

and consequently the critical radius is

$$r = R_1 = \frac{x_1{}^1}{\mu_1} = \frac{x_1{}^2}{\mu_2} = 343 \text{ cm.}$$

From Eq. (9c) we may now determine

$$Z_1{}^1 = -0.113525$$

Using Eq. (6c) it may be shown that the ratio $Z_1{}^2 = Y_1{}^2/X_1{}^2$ is as follows:

$$Z_1{}^2 = \frac{g_{21}{}^1 + g_{22}{}^1 Z_1{}^1}{g_{11}{}^1 + g_{12}{}^1 Z_1{}^1} = 0.0468 \quad (28)$$

Finally, according to Eq. (17) and curves 2 and 3, the thermal flux distribution through active regions may be approximated with the expression:

$$N^i(r) = \begin{cases} N^1(r) & \text{for } R_0 \leqslant r < R_1 \\ N^2(r) & \text{for } R_1 \leqslant r < R_2 \end{cases} \quad (29)$$

where $N^i(r)$ is given with Eq. (15).

It should be pointed out that the radial distribution of fast flux $F^i(r)$ may also be similarly obtained, if required. Figure 4 represents thermal and fast flux distributions, obtained by the proposed method.

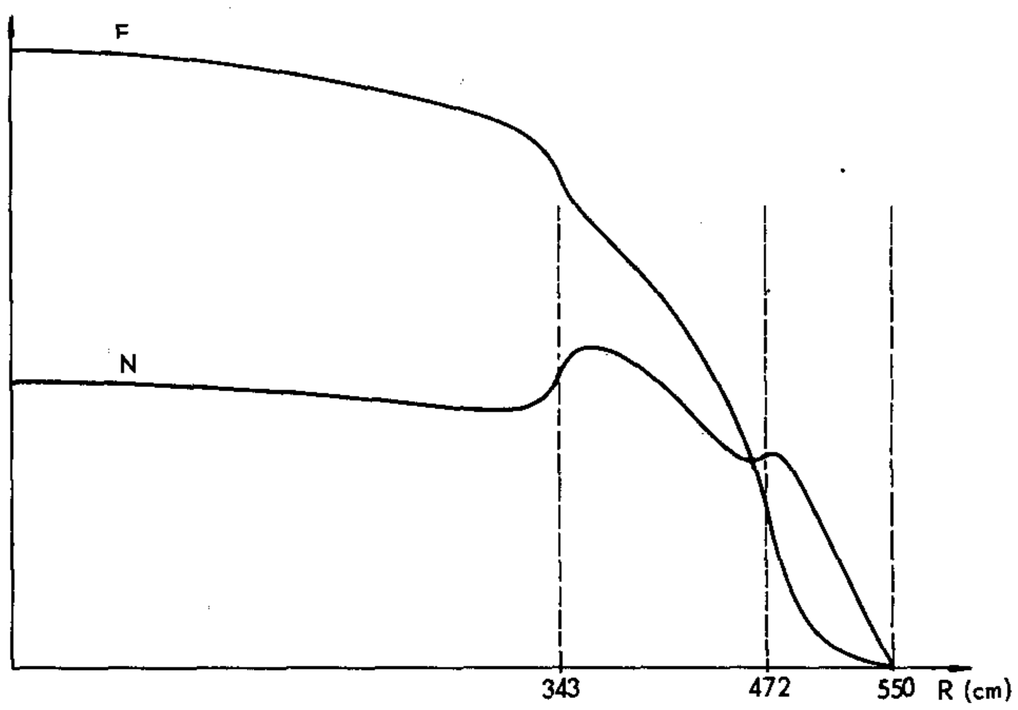

Figure 4. Thermal and fast flux distribution obtained by the proposed method

As has been explained earlier all constants

$Z_j^i,\ U_j^i,\ V_j^i,\ b_{nm}^i \qquad i=1, 2, \ldots I.$

$j=1-1, i.$

$n, m=1, 2.$

may be easily determined using the obtained family of curves and the set of formulae derived in the paper and appendix.

APPENDIX

Review of the formulae which are used in the determination of the coefficients a_{nm}^i, b_{nm}^i, μ_i, ν_i, g_{nm}^i and h_{nm}^i.

$$\frac{b_{21}^i}{b_{11}^i}=\frac{1}{2a_{12}^i}\left[a_{11}^i-a_{22}^i+\sqrt{[(a_{11}^i-a_{22}^i)^2+4a_{12}^i\,a_{21}^i]}\,\right]$$

$$\frac{b_{22}^i}{b_{12}^i}=\frac{1}{2a_{12}^i}\left[a_{11}^i-a_{22}^i-\sqrt{[(a_{11}^i-a_{22}^i)^2+4a_{12}^i\,a_{21}^i]}\,\right]$$

Two of four b_{nm}^i may be chosen arbitrarily.

$$\mu_i^2=a_{11}^i-a_{12}^i\frac{b_{21}^i}{b_{11}^i}=a_{22}^i-a_{21}^i\frac{b_{11}^i}{b_{21}^i}$$

$$\nu_i^2=a_{11}^i-a_{12}^i\frac{b_{22}^i}{b_{12}^i}=a_{22}^i-a_{21}^i\frac{b_{12}^i}{b_{22}^i}$$

$$\|G^i\|=\|g_{nm}^i\|$$

$$=\frac{1}{|B^{i+1}|}\begin{Vmatrix} b_{11}^ib_{22}^{i+1}-b_{21}^ib_{12}^{i+1} & b_{12}^ib_{22}^{i+1}-b_{22}^ib_{12}^{i+1} \\ b_{21}^ib_{11}^{i+1}-b_{11}^ib_{21}^{i+1} & b_{22}^ib_{11}^{i+1}-b_{12}^ib_{21}^{i+1}\end{Vmatrix}$$

$$\|H^i\|=\|h_{nm}^i\|=\frac{1}{|B^{i+1}|}\begin{Vmatrix} p_f^ib_{11}^ib_{22}^{i+1}-p_n^ib_{21}^ib_{12}^{i+1} & p_f^ib_{12}^ib_{22}^{i+1}-p_n^ib_{22}^ib_{12}^{i+1} \\ p_n^ib_{21}^ib_{11}^{i+1}-p_f^ib_{11}^ib_{21}^{i+1} & p_n^ib_{22}^ib_{11}^{i+1}-p_f^ib_{12}^ib_{21}^{i+1}\end{Vmatrix}$$

where $p_f^i=D_f^i/D_f^{i+1}$, $p_n^i=D_n^i/D_n^{i+1}$, and

$$|B^i|=\det\|B^i\|=b_{22}^ib_{11}^i-b_{12}^ib_{21}^i.$$

$$a_{11}^i=\frac{\Sigma_f^i}{D_f^i}+\alpha z^2 \qquad a_{12}^i=\frac{\Sigma_n^i}{D_f^i}\,.\,k_\infty^i$$

$$a_{21}^i=\frac{\Sigma_f^i}{D_n^i} \qquad a_{22}^i=\frac{\Sigma_n^i}{D_n^i}+\alpha z^2$$

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ABSTRACT—RÉSUMÉ—АННОТАЦИЯ—RESUMEN

A/706 Yougoslavie

Calculs de la diffusion de neutrons à une seule vitesse à partir d'un modèle à deux groupes

par S. Bingulac et *al.*

Quand on évalue les dimensions critiques et les distributions de flux dans des réacteurs nucléaires à plusieurs régions, il est avantageux de disposer d'une méthode qui conserve la simplicité de calcul du traitement à un groupe tout en s'approchant de la précision que l'on peut obtenir avec un modèle à deux groupes. De telles représentations analytiques simples, suffisamment précises, sont très importantes dans les études de dynamique pour lesquelles il faut obtenir simultanément la solution de l'équation de flux et de la fonction d'importance sur un calculateur analogique.

Le mémoire décrit une procédure permettant de représenter la distribution de flux par une expression de la forme:

$$\phi^i(r) = \phi^i{}_1(r) + Z^i{}_i \phi^i{}_1(R_i) \exp\left[-V^i{}_i(R_i - r)\right] + Z^i{}_{i-1}\phi^i{}_1(R_{i-1}) \exp\left[-V^i{}_{i-1}(R_{i-1} - r)\right]$$

où $\phi^i{}_1(r)$ est la solution de l'équation de diffusion à un groupe dans la *i*ème région du réacteur, définie par l'intervalle: $R_{i-1} \leqslant r < R_i$, tandis que $Z^i{}_i$, $Z^i{}_{i-1}$, $V^i{}_{i-1}$ et $V^i{}_i$ sont des constantes.

A chaque interface entre deux régions, la fonction $\phi_1{}^i(r)$ doit satisfaire à une condition limite définie par:

$$\lim_{r \to R_i} \frac{d\phi^i{}_1(r)}{dr} \cdot \frac{1}{\phi^i{}_1(r)} = U^i{}_i$$

Pour cette expression un utilise un modèle unidimensionnel à deux groupes, adapté à l'utilisation d'un calculateur analogique. Les équations de diffusion à deux groupes sont transformées en équations de Riccati avec les conditions initiales et les conditions aux limites correspondantes. En utilisant les propriétés de la solution des équations de Riccati, on réduit à deux régions seulement le réacteur à plusieurs régions, avec un ensemble simple d'équations pour déterminer les dimensions critiques et les coefficients:

$$Z^i{}_i,\ Z^i{}_{i-1},\ V^i{}_i,\ V^i{}_{i-1},\ U_i{}^i$$

pour toutes les régions.

On peut résoudre une fois pour toutes l'ensemble normalisé de ces équations, ce qui donne une famille de courbes dont on peut déduire les dimensions critiques et les coefficients pour différents paramètres de réacteurs.

On peut utiliser cette méthode pour des géométries sphériques et cylindriques, ainsi que pour toute disposition des zones actives et des réflecteurs. On discute à l'aide d'exemples concrets l'application pratique et la précision de ce procédé.

A/706 Югославия

Односкоростное вычисление диффузии нейтронов, основанное на двухгрупповой модели реактора

С. Бингулаи *et al.*

При определении критических размеров и распределения потока у многозонных ядерных реакторов полезно располагать методом, который в отношении точности приближается к двухгрупповой трактовке, а с точки зрения вычисления сохраняет свойства одногруппового метода. Такой достаточно точный, но аналитически несложный метод удобен, в частности, для динамических анализов на моделирующих устройствах, которые часто требуют одновременного определения распределения нейтронного потока и его сопряженной функции.

В настоящей работе предлагается метод, с помощью которого распределение нейтронного потока можно представить в форме

$$\varphi^i(r) = \varphi_1^i(r) + Z_i^i \varphi_1^i(R_i) \exp\left[-V_i^i(R_i - r)\right] + Z_{i-1}^i \varphi_1^i(R_{i-1}) \exp\left]-V_{i-1}^i(R_{i-1} - r)\right],$$

где $\varphi_1^i(r)$ является решением одногруппового диффузионного уравнения в зоне i, определенной интервалом $R_{i-1} \leqslant r < R_i$; Z_i^i, Z_{i-1}^i и V_{i-1} — постоянные.

На границе между двумя соседними зонами функции $\varphi_1^i(r)$ должны удовлетворять следующим граничным условиям:

$$\lim_{\to R_i} \frac{d\varphi_1^i(r)}{dr} \frac{1}{\varphi_1^i(r)} = U_i^i.$$

При определении выражения для потока использована одномерная двухгрупповая модель реактора, приспособленная для моделирующего устройства. Двухгрупповые диффузионные уравнения преобразовались в дифференциальные уравнения Риккати с соответствующими начальными и граничными условиями. Благодаря обнаруженным свойствам уравнения Риккати многозонный реактор сводится к реактору лишь с двумя эквивалентными зонами и несложной системе уравнений, решение которой определяет критические размеры и коэффициенты

$$Z_i^i,\ Z_{i-1}^i,\ V_i^i,\ V_{i-1}^i,\ U_i^i$$

для всех зон реактора.

Путем перенормировки этой системы уравнений получается семейство кривых, из которых легко можно определить коэффициенты и критические размеры для различных значений реакторных параметров.

Изложенный метод можно применить как

к сферической, так и к цилиндрической геометрии. Порядок активных зон и отражателей может быть произвольным. Практическое применение и точность метода продемонстрированы на специфическом примере.

A/706 Yugoslavia

Cálculos de difusión neutrónica a una velocidad, basados en un modelo de reactor de dos grupos

por S. Bingulac et *al.*

Al evaluar las dimensiones críticas y distribuciones de flujo en reactores de varias regiones, es ventajoso el empleo de métodos que poseen las ventajas de la simplicidad de cálculo del tratamiento en un grupo, y cuya precisión se aproxime a la lograda con el modelo en dos grupos. Estas ecuaciones de flujo, analíticamente simples pero suficientemente precisas, son particularmente importantes en estados dinámicos en los que se requiere la obtención simultánea de la solución de la ecuación de flujo y su adjunta, por medio de una calculadora analógica.

El artículo describe un procedimiento que permite representar la distribución de flujo, mediante una expresión de la forma,

$$\phi^i(r) = \phi^i{}_1(r) + Z^i{}_i \phi^i{}_1(R_i) \exp\left[-V^i{}_i(R_i - r)\right] + Z^i{}_{i-1}\phi^i{}_1(R_{i-1}) \exp\left[-V^i{}_{i-1}(R_{i-1} - r)\right]$$

donde $\phi^i{}_1(r)$ es la solución de la ecuación de difusión de un grupo en la i-ésima región del reactor, definida por el intervalo: $R_{i-1} \leqslant r < R_i$ mientras que $Z^i{}_i$, $Z^i{}_{i-1}$, $V^i{}_i$ y $V^i{}_{i-1}$ son constantes.

En cada interfase entre dos regiones, la función $\Phi^i{}_1(r)$ debe satisfacer una condición de contorno definida por,

$$\lim_{r \to R_i} \frac{d\phi^i{}_1(r)}{dr} \cdot \frac{1}{\phi^i{}_1(r)} = U^i{}_i$$

Para la deducción de esta expresión, se utiliza un modelo de reactor monodimensional en dos grupos, adaptado a la resolución con una calculadora analógica. Las ecuaciones de difusión de dos grupos se transforman en ecuaciones de Riccati, con las condiciones iniciales y de contorno asociadas. Las propiedades de las ecuaciones de Riccati permiten la reducción del reactor en varias regiones a otro de solo dos regiones, lo que únicamente precisa un sistema de ecuaciones simple, que determinan las condiciones críticas y los coeficientes,

$$Z^i{}_i,\ Z^i{}_{i-1},\ V^i{}_i,\ V^i{}_{i-1},\ U^i{}_i$$

para todas las regiones.

Este sistema de ecuaciones normalizado puede resolverse una vez por todas, con lo que se obtiene una familia de curvas que permiten la determinación fácil de las dimensiones críticas y de los coeficientes, para varios parámetros del reactor.

Este procedimiento es aplicable a geometrías esférica y cilíndrica, y a cualquier disposición de regiones activas y reflectores. Se discute en ejemplos específicos la aplicación práctica y la precisión del método.