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# Collision Probabilities for Finite Cylinders and Cuboids

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COLLISION PROBABILITIES FOR FINITE CYLINDERS AND CUBOIDS

by

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## SUMMARY

Analytical formulae have been derived for the collision probabilities of homogeneous finite cylinders and cuboids. The formula for the finite cylinder contains double integrals, and the formula for the cuboid only single integrals. Collision probabilities have been calculated by means of the formulae and compared with values obtained by other authors. It was found that the calculations using the analytical formulae are much quicker and give higher accuracy than Monte Carlo calculations.

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## 1. INTRODUCTION

The collision probability of a homogeneous body is the probability that particles from an isotropic constant source inside the body collide in the body in their first flight. Collision probabilities have been used to a great extent for the calculation of the fast effect and of the resonance absorption in nuclear reactors, but they are also important in other applications, for example neutron flux measurement by means of activation of small samples, calculation of the reactivity effect of small samples in a fast reactor, and the attenuation of gamma rays in a gamma source.

Analytical expressions have been given by Case et al. (1) for infinite slabs, infinite cylinders, spheres, hemispheres, oblate spheroids and oblate hemispheroids. In practice a sample is often given the form of a cuboid or a finite cylinder. These shapes are more complicated and collision probabilities for them cannot be expressed in simple functions only.

A general approach is to use the Monte Carlo method, and two such codes have been written. The CEP code by Foell, Berner and Tong (2) calculates the collision probability of a finite cylinder or annulus, which may be empty or not, and the code by Gubbins (3), which is also called CEP, calculates the collision probability of a finite cylinder or a cuboid.

McLeod (4) has derived an analytical formula for the collision probability of a finite cylinder. The formula contains only double integrals. All three papers contain numerical results.

We have derived a formula for the collision probability of a cuboid. For this geometry it is possible to express the collision probability in single integrals only. We were not aware of the work of McLeod, so we also derived an expression for the collision probability of a finite cylinder.

McLeod's approach and ours are so different that we hope that our derivation may still be of interest. McLeod utilizes formulae for chord distributions, whereas we use a more elementary method and calculate the currents out of the cylinder for a constant source distri-

bution. Our final expression looks simpler than McLeod's, but this may be due to the fact that his formula is written to allow a general absorption law. McLeod's expression covers explicitly only cases with height to diameter larger than one, but the generalization should be simple. It can no doubt be proved that the two formulas are equivalent.

In the next two paragraphs the formulae are derived. The last paragraph shows sample calculations and comparisons with results of the other authors.

A typical computing time for a collision probability of a cylinder with an accuracy of  $10^{-4}$  is something like one second on IBM 7044. The computing time for a cuboid is shorter. This is very fast when compared to Monte Carlo calculations. On the other hand a Monte Carlo method is more general and can easily be extended to still more complicated geometries or to inclusion of non-flat source distributions.

## 2. COLLISION PROBABILITY FOR A FINITE CYLINDER

Consider a cylinder of radius  $a$  (diameter  $d = 2a$ ) and height  $h$ , measured in mean free paths. Let the collision probability be  $P_c$ , and set

$$P_c = 1 - P_e - P_m \quad (1)$$

where  $P_e$  is the probability of escape through the two end surfaces and  $P_m$  is the probability of escape through the curved surface.

$P_e$  is calculated first. With notations from figure 1

$$P_e = \frac{2}{\pi a^2 h} \int_0^a \rho d\rho \int_0^{2\pi} d\varphi \int_0^h d\zeta \int_0^a r dr \int_0^{2\pi} d\psi \frac{e^{-R}}{4\pi R^2} \frac{\zeta}{R} \quad (2)$$

$\frac{\zeta}{R}$  is the cosine of the angle between the inward normal at the escape point and the neutron path.

Integration over one angle can be performed directly.

With  $\varphi - \psi = \alpha$

$$P_e = \frac{1}{\pi a^2 h} \int_0^a \rho d\rho \int_0^{2\pi} d\alpha \int_0^a r dr \int_0^h \zeta d\zeta \frac{e^{-R}}{R^3} \quad (3)$$

where

$$R^2 = t^2 + \zeta^2; \quad t^2 = r^2 + \rho^2 - 2r\rho \cos \alpha \quad (4)$$

Integrate over R instead of  $\zeta$ ;  $\zeta d\zeta = R dR$

$$\int_0^h \zeta d\zeta \frac{e^{-R}}{R^3} = \int_t^{\sqrt{t^2+h^2}} dR \frac{e^{-R}}{R^2} = \frac{E_2(t)}{t} - \frac{E_2(\sqrt{t^2+h^2})}{\sqrt{t^2+h^2}} \quad (5)$$

where  $E_2(z)$  is an exponential integral

$$E_n(z) = \int_1^{\infty} \frac{e^{-zu}}{u^n} du \quad (6)$$

Set

$$P_e = \frac{1}{\pi a^2 h} [I(0) - I(h)] \quad (7)$$

$$I(z) = \int_0^a \rho d\rho \int_0^a r dr \int_0^{2\pi} d\alpha \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} \quad (8)$$

The last two integrals over  $r$  and  $\alpha$  represent an integration over the surface of a circle. Integrate instead over  $t$  and  $\beta$  of figure 2.

$$\int_0^a \rho d\rho \int_0^a r dr \int_0^{2\pi} d\alpha \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} = \int_0^a \rho d\rho \int_0^{a+\rho} t dt \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} \int_{-\beta_1}^{+\beta_1} d\beta \quad (9)$$

$$\begin{aligned} \text{For } 0 \leq t \leq a - \rho & \quad \beta_1 = \pi \\ a - \rho \leq t \leq a + \rho & \quad \beta_1 = \arccos \left[ \frac{\rho^2 + t^2 - a^2}{2 \rho t} \right] \end{aligned} \quad (10)$$

$$I(z) = \int_0^a \rho d\rho \left\{ 2\pi \int_0^{a-\rho} t dt \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} + \int_{a-\rho}^{a+\rho} t dt \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} \cdot 2 \arccos \left[ \frac{\rho^2 + t^2 - a^2}{2 \rho t} \right] \right\} \quad (11)$$

After some algebra one obtains

$$I(z) = \pi a^2 E_3(z) - 2 \int_0^a \rho d\rho \int_{a-\rho}^{a+\rho} \frac{dt}{t} E_3(\sqrt{t^2+z^2}) \frac{t^2 + a^2 - \rho^2}{\sqrt{4\rho^2 t^2 - (\rho^2 + t^2 - a^2)^2}} \quad (12)$$

Exchange the order of integration. According to figure 3 and observing that the integrand is a function of  $s = \rho^2$ , one obtains

$$\int_0^a 2\rho d\rho \int_{a-\rho}^{a+\rho} f(t, \rho^2) = \int_0^a dt \int_{a-t}^a 2\rho d\rho f + \int_a^d dt \int_{t-a}^a 2\rho d\rho f = \quad (13)$$

$$= \int_0^a dt \int_{(a-t)^2}^{a^2} ds f + \int_a^d dt \int_{(a-t)^2}^{a^2} ds f = \int_0^d dt \int_{(a-t)^2}^{a^2} ds f$$

$$f = \frac{1}{t} E_3(\sqrt{t^2+z^2}) \frac{t^2 + a^2 - s}{\sqrt{4t^2 s - (s + t^2 - a^2)^2}} \quad (14)$$

Set

$$\left. \begin{aligned} t^2 + a^2 - s &= 2atu, \quad ds = -2at du \\ f ds &= -\frac{1}{t} E_3(\sqrt{t^2+z^2}) \frac{2at u du}{\sqrt{1-u^2}} \end{aligned} \right\} \quad (15)$$



$$I(z) = \pi a^2 E_3(z) - \int_0^d dt E_3(\sqrt{t^2 + z^2}) \int_{\frac{t}{d}}^1 \frac{2 au du}{\sqrt{1-u^2}} \quad (16)$$

$$I(z) = \pi a^2 E_3(z) - \int_0^d dt E_3(\sqrt{t^2 + z^2}) \sqrt{d^2 - t^2} \quad (17)$$

$$P_e = \frac{1}{h} [E_3(0) - E_3(h)] - \frac{1}{\pi a^2 h} \int_0^d dt [E_3(t) - E_3(\sqrt{t^2 + h^2})] \sqrt{d^2 - t^2} \quad (18)$$

For  $P_m$ , the probability of leakage through the cylindrical surface, one obtains with notations from figure 4

$$P_m = \frac{1}{\pi a^2 h} \int_0^h 2\pi a dz \int_0^a \rho d\rho \int_0^{2\pi} d\varphi \int_0^h d\zeta \frac{e^{-R}}{4\pi R^2} \cos \alpha' \quad (19)$$

Change the integration over  $\rho$  and  $\varphi$  to  $t$  and  $\alpha$  as indicated in figure 5:

$$\left. \begin{aligned} \rho d\rho d\varphi &= t dt d\alpha \\ R^2 &= t^2 + (z - \zeta)^2 \\ \cos \alpha' &= \frac{t \cos \alpha}{R} \end{aligned} \right\} \quad (20)$$

$$\begin{aligned} P_m &= \frac{1}{2\pi a h} \int_0^h dz \int_0^h d\zeta \int_0^d t dt \int_0^{\arccos \frac{t}{d}} d\alpha \frac{e^{-R}}{R^2} \frac{t \cos \alpha}{R} = \\ &= \frac{1}{\pi a h} \int_0^h dz \int_0^h d\zeta \int_0^d t^2 dt \frac{e^{-R}}{R^3} \sqrt{1 - \frac{t^2}{d^2}} \end{aligned} \quad (21)$$

$$\text{Set } u = z - \zeta \quad (22)$$

The integrand depends on  $z$  and  $\zeta$  only as a function of  $u^2$ .  
It is easily found that

$$\int_0^h dz \int_0^h d\zeta f(u^2) = 2 \int_0^h (h-u) f(u^2) du \quad (23)$$

$$P_m = \frac{2}{\pi a h} \int_0^h (h-u) du \int_0^d t^2 dt \frac{e^{-R}}{R^3} \sqrt{1 - \frac{t^2}{d^2}} \quad (24)$$

$$R^2 = t^2 + u^2$$

The complete formula is then

$$\left| P_c^{\text{cylinder}} = 1 - \frac{1}{h} [E_3(0) - E_3(h)] + \frac{4}{\pi d^2 h} \int_0^d dt [E_3(t) - E_3(\sqrt{t^2 - h^2})] \sqrt{d^2 - t^2} - \frac{4}{\pi d^2 h} \int_0^h (h-u) du \int_0^d t^2 dt \frac{e^{-\sqrt{u^2 + t^2}}}{(t^2 + u^2)^{3/2}} \sqrt{d^2 - t^2} \quad (25) \right.$$

It is possible to elaborate this expression further, although we do not believe that it gives any improvement. However, it will be pointed out how the last term can be expressed formally in single integrals.

Consider the integral

$$I_1 = I_2 - I_3$$

$$I_2 = h \int_0^h du \int_0^d t^2 dt \frac{e^{-R}}{R^3} \sqrt{d^2 - t^2} \quad (26)$$

$$I_3 = \int_0^h u du \int_0^d t^2 dt \frac{e^{-R}}{R^3} \sqrt{d^2 - t^2}$$

One integration can be performed directly in  $I_3$

$$I_3 = \int_0^d t^2 \sqrt{d^2 - t^2} \int_0^h \frac{e^{-R}}{R^3} u \, du \quad (27)$$

$$R^2 = u^2 + t^2 \quad ; \quad u \, du = R \, dR \quad (28)$$

$$I_3 = \int_0^d t^2 \sqrt{d^2 - t^2} \left[ \frac{E_2(t)}{t} - \frac{E_2(\sqrt{h^2 + t^2})}{\sqrt{h^2 + t^2}} \right] \quad (29)$$

In  $I_2$  set

$$u = R \sin \varphi, \quad t = R \cos \varphi, \quad du \, dt = R \, dR \, d\varphi \quad (30)$$

The integration is split up in integrations over three areas; I, II, and III, according to figure 6:

$$I_2 = I_I + I_{II} - I_{III} \quad (31)$$

$$I_I = \int_0^d d R e^{-R} \int_0^{\frac{\pi}{2}} \cos^2 \varphi \sqrt{d^2 - R^2 \cos^2 \varphi} \, d\varphi \quad (32)$$

$$I_{II} = \int_d^{\sqrt{d^2+h^2}} d R e^{-R} \int_{\arccos \frac{d}{R}}^{\frac{\pi}{2}} \cos^2 \varphi \sqrt{d^2 - R^2 \cos^2 \varphi} \, d\varphi \quad (33)$$

$$I_{III} = \int_h^{\sqrt{d^2+h^2}} d R e^{-R} \int_{\arcsin \frac{h}{R}}^{\frac{\pi}{2}} \cos^2 \varphi \sqrt{d^2 - R^2 \cos^2 \varphi} \, d\varphi \quad (34)$$

The integrals over  $\varphi$  are elliptic. In the case of  $I_I$  and  $I_{II}$  they can be expressed in terms of complete elliptic integrals, but the elliptic integral of  $I_{III}$  is incomplete.

A computer routine COLCYL was written based on equation (25). In order to remove the singularity at  $u = t = 0$ , the last integral was written

$$\begin{aligned} & \int_0^h (h-u) du \int_0^d t^2 dt \frac{e^{-R}}{R^3} \sqrt{d^2 - t^2} = \\ & = \int_0^h (h-u) du \int_0^d \frac{t^2 dt}{R^3} \left[ e^{-R \sqrt{d^2 - t^2}} - d \right] + \\ & + \frac{h}{2} \left[ \left( \frac{d}{h} \right)^2 + \frac{d}{h} \sqrt{1 + \left( \frac{d}{h} \right)^2} + \ln \left( \frac{d}{h} + \sqrt{1 + \left( \frac{d}{h} \right)^2} \right) \right] \end{aligned} \quad (35)$$

Further, in the numerical integration  $\sqrt{d-t}$  was used as a variable to remove the infinite derivative of the integrand at  $t = d$ .

### 3. COLLISION PROBABILITY FOR A CUBOID

As in the case of the cylinder, the mean free path is used as the unit of length. Let the cuboid have sides  $a$ ,  $b$ , and  $c$  (figure 7), and assume an isotropic, constant source density inside the cuboid. If  $P_{xy}$  denotes the escape probability through a surface with sides  $x$  and  $y$ , the collision probability  $P_c$  is

$$P_c = 1 - 2P_{ab} - 2P_{bc} - 2P_{ca} \quad (36)$$

With notations from figure 1, one gets for  $P_{ab}$  that is the probability for escape through the bottom surface,

$$P_{ab} = \frac{1}{abc} \int_0^a d\xi \int_0^b d\eta \int_0^c d\zeta \int_0^a \int_0^b \frac{dx dy}{4\pi R^2} \frac{\zeta}{R} e^{-R} \quad (37)$$

Integrate first over  $\zeta$

$$\left. \begin{aligned} R^2 &= (x - \xi)^2 + (y - \eta)^2 + \zeta^2 \\ R \, dR &= \zeta \, d\zeta \end{aligned} \right\} \quad (38)$$

$$\int_{\underline{0}}^c d\zeta \frac{\zeta}{R^3} e^{-R} = \int_{R_1}^{R_2} \frac{e^{-R} dR}{R^2} = \frac{E_2(R_1)}{R_1} - \frac{E_2(R_2)}{R_2} \quad (39)$$

where

$$\begin{aligned} R_1 &= \sqrt{(x - \xi)^2 + (y - \eta)^2} \\ R_2 &= \sqrt{(x - \xi)^2 + (y - \eta)^2 + c^2} \end{aligned} \quad (40)$$

As in paragraph 2

$$E_n(z) = \int_1^{\infty} \frac{e^{-zu}}{u^n} du \quad (41)$$

Define

$$I_{ab}(z) = \int_0^a dx \int_0^b dy \int_0^a d\xi \int_0^b d\eta \frac{E_2(\sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2})}{\sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2}} \quad (42)$$

Then

$$P_{ab} = \frac{1}{4\pi abc} [I_{ab}(0) - I_{ab}(c)] \quad (43)$$

Consider the integration over  $y$  and  $\eta$  and set

$$y - \eta = u, \quad dy = du \quad (44)$$

The integrand is a function of  $u^2$  only. One obtains (compare equation 23)

$$\int_0^b dy \int_0^b d\eta f(u^2) = 2 \int_0^b (b-u) f(u^2) du \quad (45)$$

The integrations over  $x$  and  $\xi$  can be handled in the same way and the result is

$$I_{ab}(z) = 4 \int_0^a (a-v) dv \int_0^b (b-u) du \frac{E_2(\sqrt{u^2+v^2+z^2})}{\sqrt{u^2+v^2+z^2}} \quad (46)$$

Now  $(a-v)(b-u) = ab - au - bv + uv$  and the corresponding four terms in  $I_{ab}(z)$  are

$$\begin{aligned} I_1 &= 4ab \int_0^a dv \int_0^b du \frac{E_2(\sqrt{u^2+v^2+z^2})}{\sqrt{u^2+v^2+z^2}} \\ I_2 &= 4a \int_0^a dv \left[ E_3(\sqrt{v^2+z^2}) - E_3(\sqrt{v^2+b^2+z^2}) \right] \\ I_3 &= 4b \int_0^b du \left[ E_3(\sqrt{u^2+z^2}) - E_3(\sqrt{u^2+a^2+z^2}) \right] \\ I_4 &= 4 \cdot \frac{1}{2} \left\{ \int_0^a v dv \left[ E_3(\sqrt{v^2+z^2}) - E_3(\sqrt{v^2+b^2+z^2}) \right] + \right. \\ &\quad \left. + \int_0^b u du \left[ E_3(\sqrt{u^2+z^2}) - E_3(\sqrt{u^2+a^2+z^2}) \right] \right\} \end{aligned} \quad (47)$$

In  $I_1$  set

$$u = r \sin \phi, \quad v = r \cos \phi, \quad du dv = r dr d\phi \quad (48)$$

According to figure 8 and with

$$g(r) = \frac{E_2(\sqrt{r^2+z^2})}{\sqrt{r^2+z^2}}$$

one gets

$$\int_0^a \int_0^b \int_0^{\sqrt{a^2+b^2}} \frac{\pi}{2} \int_0^{\sqrt{a^2+b^2}} \arccos \frac{a}{r} \int_0^{\sqrt{a^2+b^2}} \frac{\pi}{2} \int_0^{\arcsin \frac{b}{r}} r dr g(r) \int_0^{\frac{\pi}{2}} d\phi - \int_a^{\sqrt{a^2+b^2}} r dr g(r) \int_0^{\frac{\pi}{2}} d\phi - \int_b^{\sqrt{a^2+b^2}} r dr g(r) \int_{\arcsin \frac{b}{r}}^{\frac{\pi}{2}} d\phi \quad (49)$$

and after some algebra

$$I_1 = 4ab \left\{ \frac{\pi}{2} E_3(z) - a \int_a^{\sqrt{a^2+b^2}} E_3(\sqrt{r^2+z^2}) \frac{dr}{r\sqrt{r^2-a^2}} - b \int_b^{\sqrt{a^2+b^2}} E_3(\sqrt{r^2+z^2}) \frac{dr}{r\sqrt{r^2-b^2}} \right\} \quad (50)$$

Set in the first integral

$$r^2 - a^2 = u^2 ; \quad \frac{dr}{r\sqrt{r^2-a^2}} = \frac{du}{a^2+u^2} \quad (51)$$

and analogously in the second integral

$$I_1 = 4ab \left\{ \frac{\pi}{2} E_3(z) - a \int_0^b E_3(\sqrt{a^2+u^2+z^2}) \frac{du}{a^2+u^2} - b \int_0^a E_3(\sqrt{v^2+b^2+z^2}) \frac{dv}{b^2+v^2} \right\} \quad (52)$$

Combining all terms one gets

$$I_{ab}(z) = 2\pi ab E_3(z) - 2 \int_0^a \left[ (2a-v) E_3(\sqrt{v^2+z^2}) + \frac{v^2-2av+b^2}{b^2+v^2} v E_3(\sqrt{v^2+b^2+z^2}) \right] dv - 2 \int_0^b \left[ (2b-u) E_3(\sqrt{u^2+z^2}) + \frac{u^2-2bu+a^2}{a^2+u^2} u E_3(\sqrt{a^2+u^2+z^2}) \right] du \quad (53)$$

and the collision probability  $P$  is expressed in single integrals as

$$P_c^{\text{cuboid}} = 1 - \frac{1}{2\pi abc} \left[ I_{ab}(0) - I_{ab}(c) + I_{bc}(0) - I_{bc}(a) + I_{ca}(0) - I_{ca}(b) \right] \quad (54)$$

It is in fact possible to do some more analytical integrations in equation 53 before treating it numerically. However, the gain in computing time, if any, would be rather small, and it is also possible that some accuracy would be lost. In particular, it is preferable to work with exponential integrals of one single order as in equation 53.

#### 4. NUMERICAL CALCULATIONS

Two simple FORTRAN routines were written for the calculation of collision probabilities, COLCYL for the cylinder and COLCUB for the cuboid. Gaussian quadrature was used for the integrations. COLCYL is based on equation 25 with the remarks following the equation, and COLCUB is based on equation 54. The number of points in the integrations can be varied, and the calculations were performed with increasing number of points in order to study the convergence.

The approximations for the exponential integrals were taken from Handbook of Mathematical Functions (5), formulae 5.1.53 and 5.1.56.

In tables 1 to 4 we compare collision probabilities calculated by means of COLCUB and COLCYL with results given by the other authors for selected values of the parameters.

Table 1 gives values for cubes. They are compared with Monte Carlo results by Gubbins (3). In table 2 values for cylinders with height to diameter equal to one are compared with the corresponding Monte Carlo values by Gubbins, and in table 3 values for cylinders with height to diameter equal to two are compared with Monte Carlo values of Foell et al. (2).

The agreement with Monte Carlo values is as good as can be expected. Foell et al. do not quote explicitly the accuracy of their



results. The numbers include five decimal places, but the number of significant decimals is probably three to four.

Table 4 finally shows the comparison with McLeod's results for cylinders. The agreement is good except for the very largest cylinders. We have used Gaussian 16-, 20-, and 24-point formulae over both integration ranges 0 to  $\Sigma d$  and 0 to  $\Sigma h$ ,  $d$  being the diameter and  $h$  the height of the cylinder. One can see from the convergence in the tables that the 24-point formula does not give more than three correct decimal digits in cases when both the diameter and the height are more than 20 mean free paths. If COLCYL - or COLCUB - were to be used for such large bodies, it would be advisable to divide the integration ranges in subregions, so that more points could be used.

On the other hand, for cylinders of diameter less than five mean free paths, all results agree within 0.00002 except in one case where the difference is 0.00006.

## 5. CONCLUSIONS

Collision probabilities for cuboids and finite cylinders have been calculated by means of the FORTRAN codes COLCUB and COLCYL, which are based on analytical expressions. An absolute accuracy of about  $5 \times 10^{-5}$  can be obtained in one or two seconds on IBM 7044, if the linear dimensions of the bodies are not too large measured in mean free paths. The corresponding computing time with a Monte Carlo code would be at least several minutes, so one should resort to Monte Carlo methods only for more complicated geometries.

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TABLE 1

Collision probabilities for cubes  
 Comparison with Monte Carlo results of Gubbins

$\Sigma a$	COLCUB			CEP Gubbins
	8-point formula	12	16	
0.1	0.04337	0.04337	0.04337	0.0434
0.4	0.15770	0.15770	0.15770	0.157
1.1	0.35426	0.35427	0.35427	0.356
3.0	0.62585	0.62585	0.62585	0.626
8.0	0.83145	0.83147	0.83147	0.833
14.0	0.89915	0.89918	0.89918	0.8993
20.0	0.92809	0.92812	0.92812	0.9280

TABLE 2

Collision probabilities for cylinders  
with height equal to diameter.

Comparison with Monte Carlo results of Gubbins

$\Sigma d = \Sigma h$	COLCYL				CEP (Gubbins)
	8-point formula	16	24	$P_c$	Standard deviation
0.1	0.04069	0.04066	0.04065	0.0406	0.00084
0.4	0.14901	0.14891	0.14890	0.149	0.0030
1.1	0.33948	0.33922	0.33920	0.340	0.0061
3.0	0.61338	0.61265	0.61260	0.612	0.0086
8.0	0.82927	0.82719	0.82707	0.826	0.0070
17.0	0.92024	0.91517	0.91491	0.915	0.0057

TABLE 3

Collision probabilities for cylinders  
with height/diameter equal to 2.

Comparison with Monte Carlo results of Foell et al.

$\Sigma d$	$\Sigma h$	COLCYL			CEP (Foell, Berner, Tong)
		8-point formula	16	24	
0.8	1.6	0.30366	0.30369	0.30369	0.30646
2.0	4.0	0.54499	0.54503	0.54503	0.54682
4.0	8.0	0.72522	0.72524	0.72524	0.72649
6.0	12.0	0.80684	0.80682	0.80682	0.80691
8.0	16.0	0.85191	0.85179	0.85179	0.85220
12.0	24.0	0.89964	0.89920	0.89920	0.89932

TABLE 4

Collision probabilities for cylinders.  
Comparison with results of McLeod.

$\Sigma d$	$\Sigma h$	COLCYL			McLeod
		16-point formula	20	24	
20.0	20.0	0.927596	0.927369	0.927287	0.92656
"	60.0	0.941347	0.941975	0.942219	0.94187
"	200.0	0.945675	0.947362	0.947696	0.94723
5.0	5.0	0.738986	0.738932	0.738912	0.73889
"	15.0	0.783825	0.783994	0.784058	0.78412
"	50.0	0.799964	0.799989	0.799986	0.79997
1.0	1.0	0.316346	0.316335	0.316331	0.31633
"	3.0	0.372257	0.372292	0.372304	0.37232
"	10.0	0.394533	0.394512	0.394505	0.39450
0.2	0.2	0.078916	0.078914	0.078913	0.078912
"	0.6	0.099236	0.099243	0.099245	0.099249
"	2.0	0.109865	0.109851	0.109847	0.10985
0.05	0.05	0.020638	0.020637	0.020637	0.020637
"	0.15	0.026408	0.026409	0.026410	0.026411
"	0.50	0.029801	0.029788	0.029785	0.029784
0.01	0.01	0.004178	0.004178	0.004178	0.0041781
"	0.03	0.005373	0.005373	0.005373	0.0053740
"	0.10	0.006115	0.006103	0.006100	0.0060939
0.001	0.001	0.0 <sup>3</sup> 420	0.0 <sup>3</sup> 420	0.0 <sup>3</sup> 420	0.0 <sup>3</sup> 41896
"	0.003	0.0 <sup>3</sup> 540	0.0 <sup>3</sup> 540	0.0 <sup>3</sup> 540	0.0 <sup>3</sup> 53950
"	0.010	0.0 <sup>3</sup> 629	0.0 <sup>3</sup> 617	0.0 <sup>3</sup> 614	0.0 <sup>3</sup> 61322

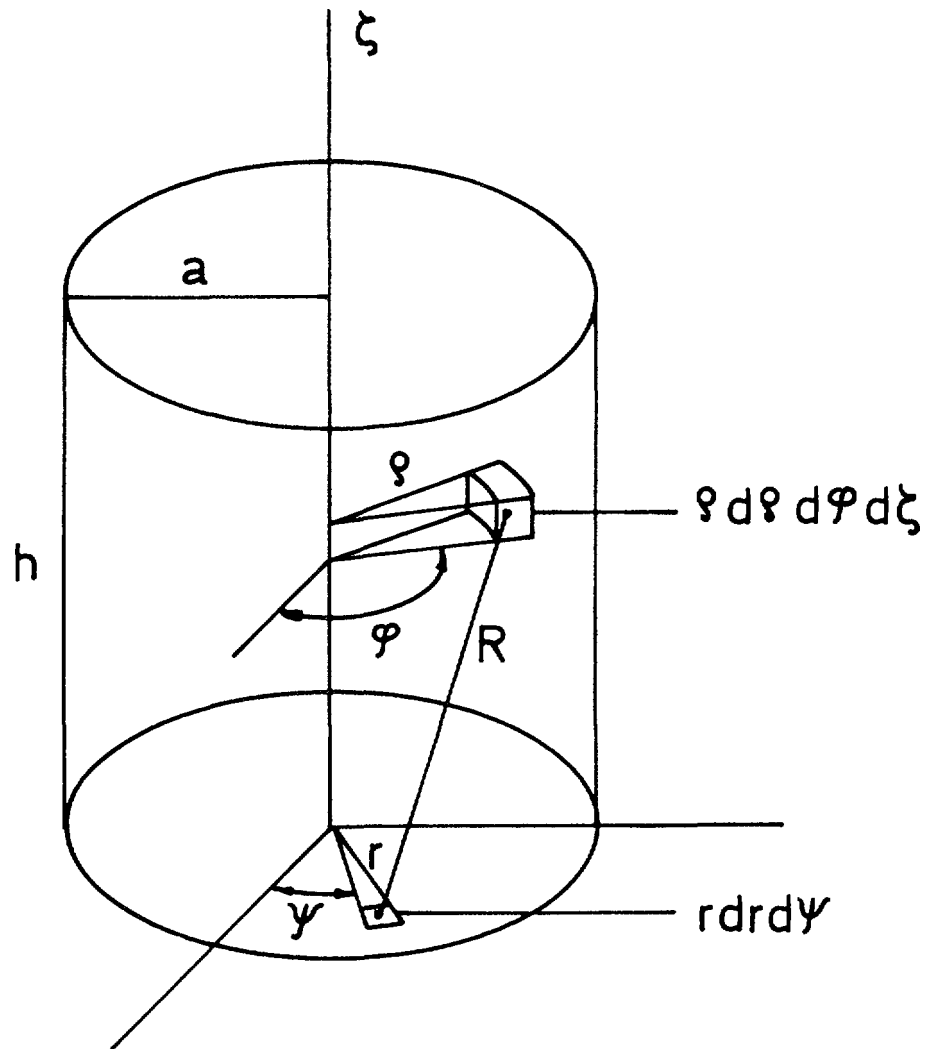


Figure 1

Notations for the calculation of escape through the bottom surface of a cylinder

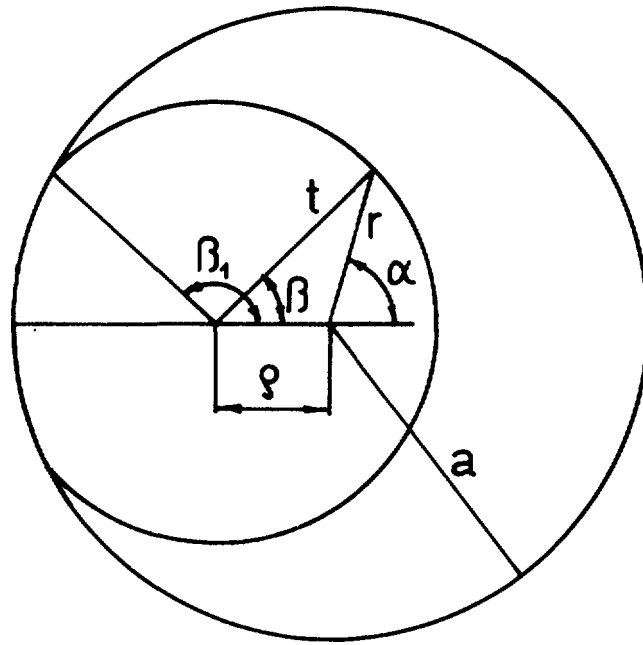


Figure 2

Change of variables (equation 9)



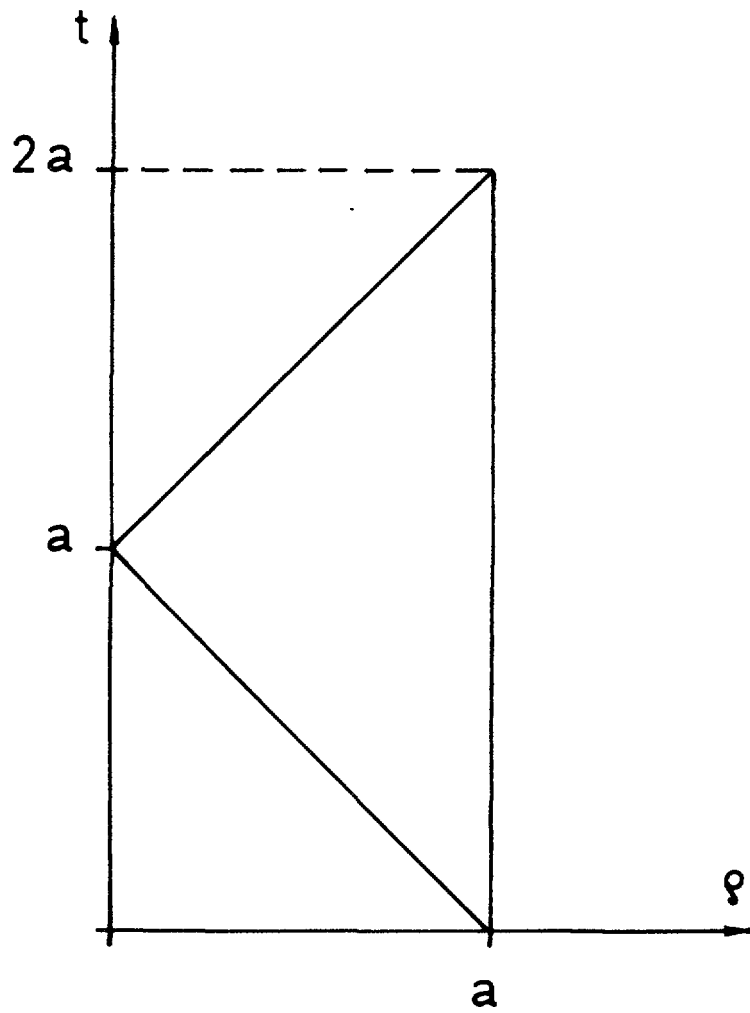


Figure 3

Change of variables (equation 13)

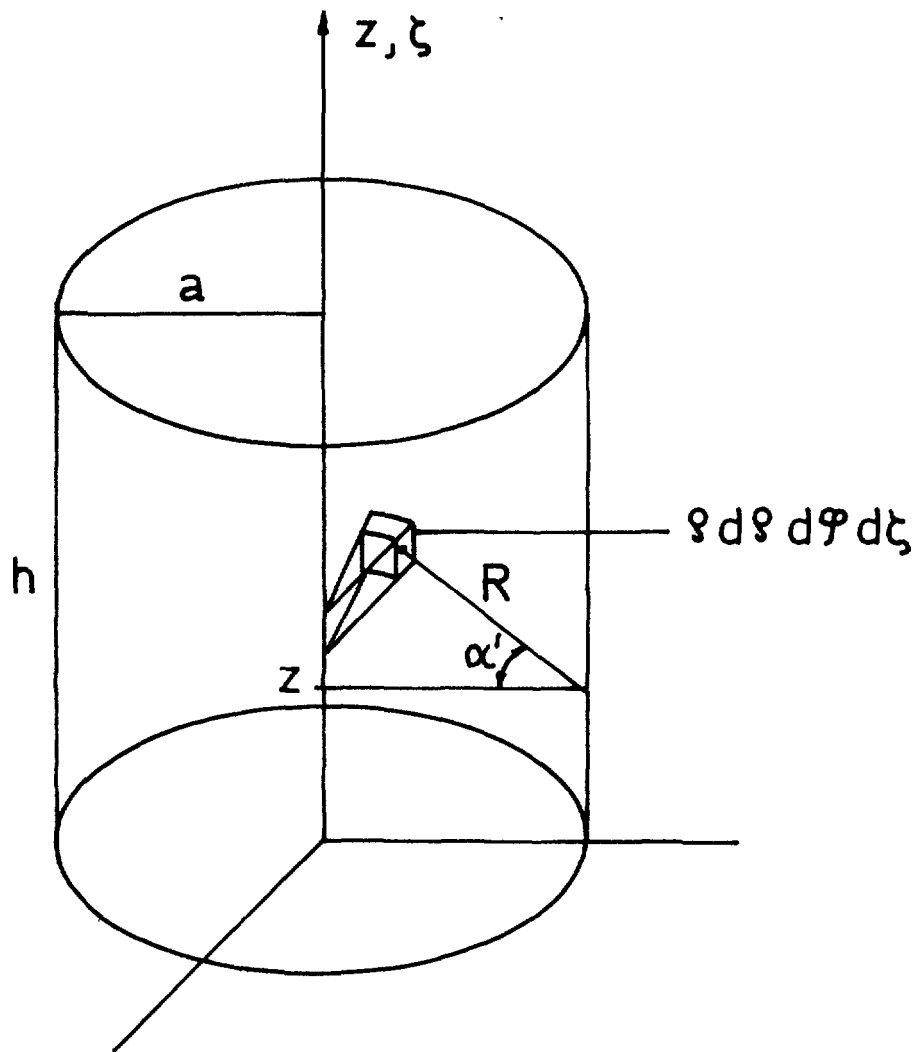


Figure 4

Notations for the calculation of escape through the curved surface of a cylinder

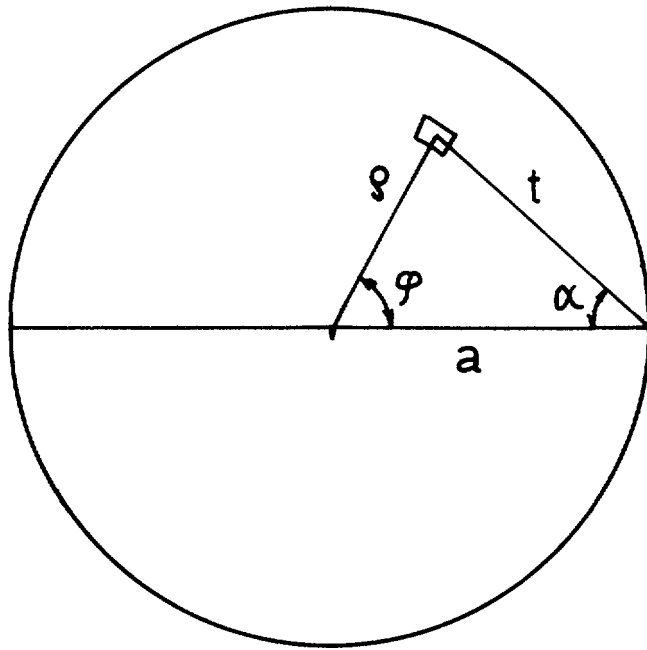


Figure 5

Change of variables (equation 20)

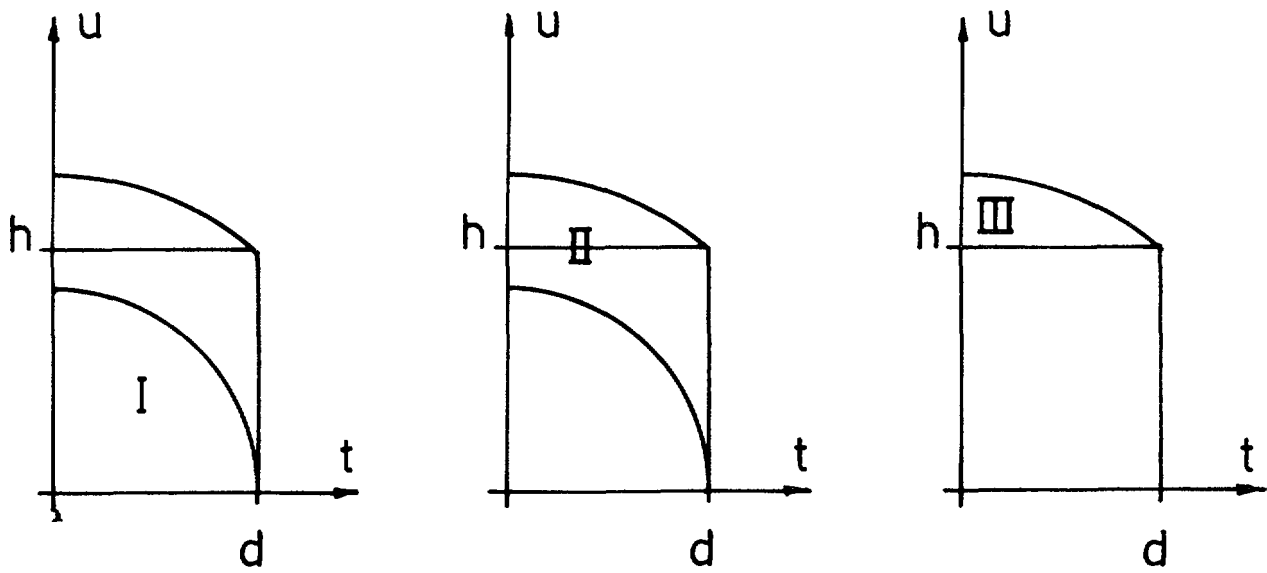


Figure 6

Regions of integration (equation 31)

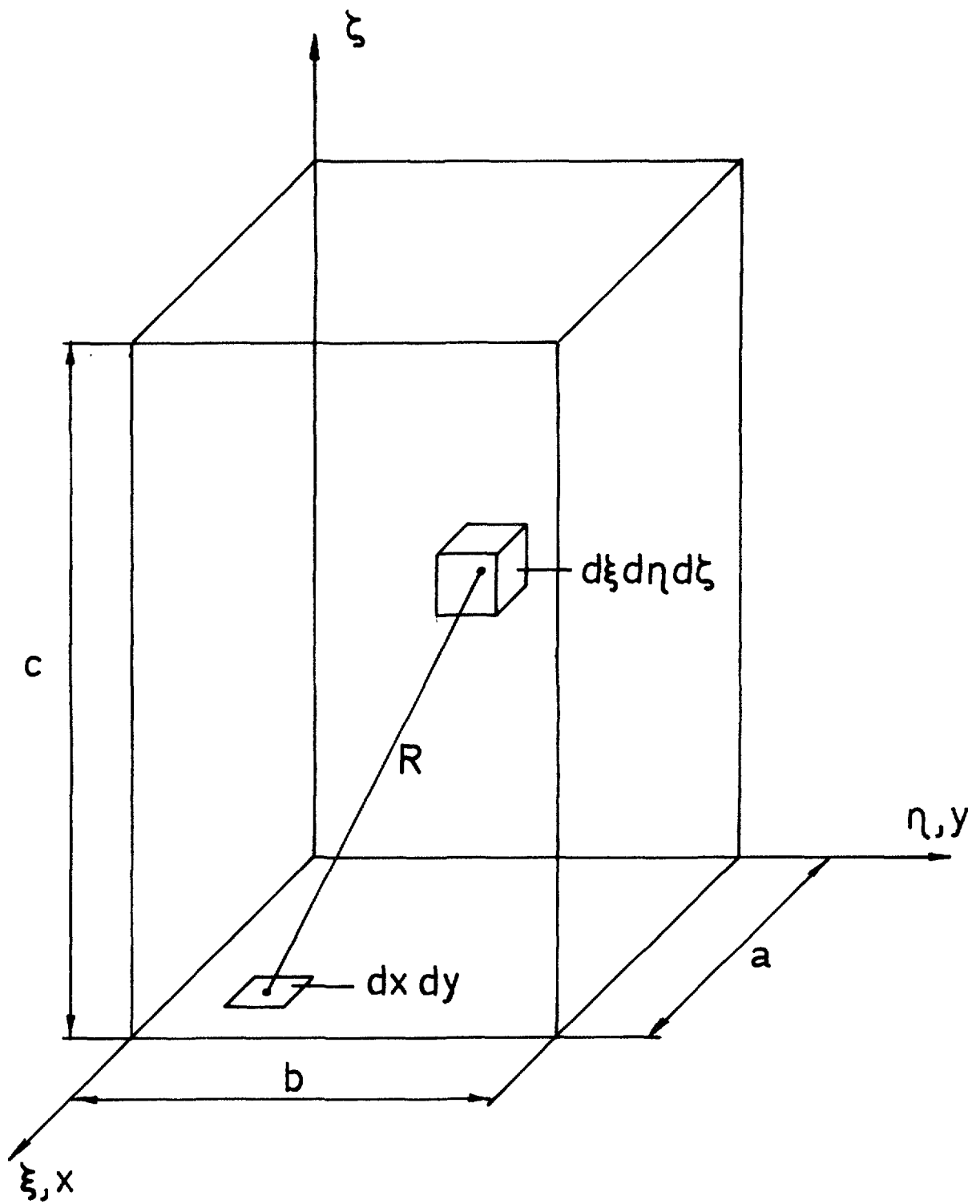


Figure 7

Notations for the calculation of escape through the bottom surface of a cuboid

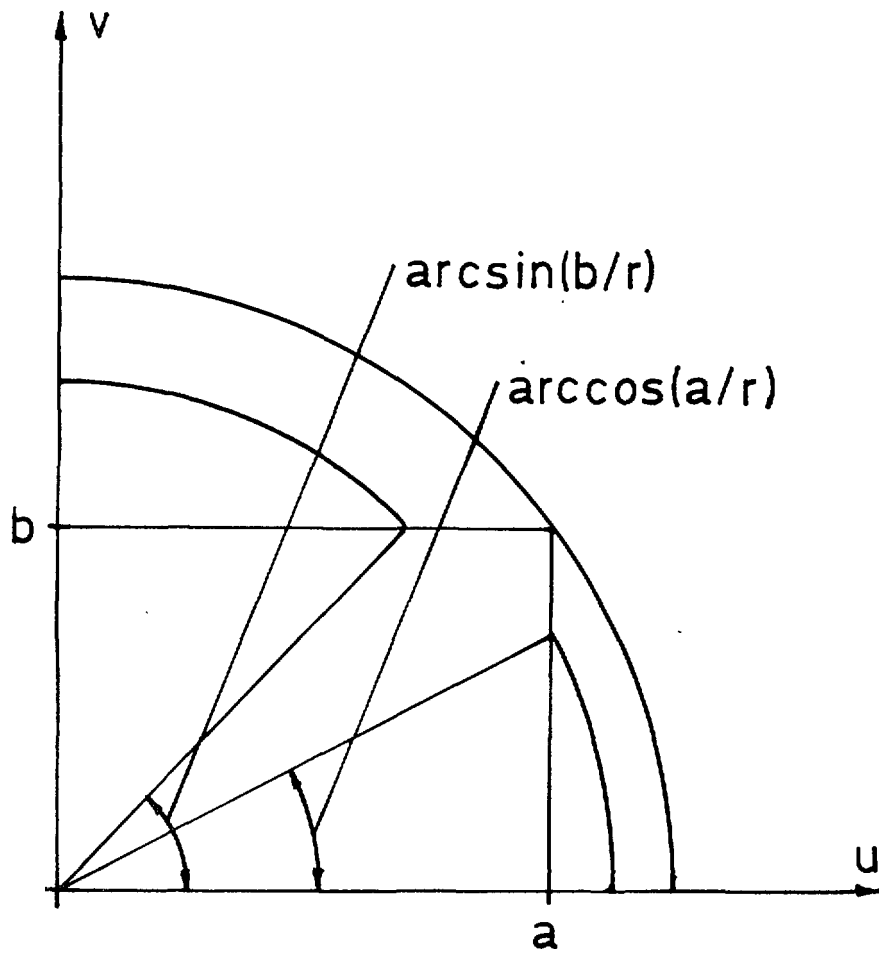


Figure 8

Change to polar coordinates (equation 49)



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