Transient Temperature Distribution in a Reactor Core with Cylindrical Fuel Rods and Compressible Coolant

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TRANSIENT TEMPERATURE DISTRIBUTION IN A REACTOR CORE WITH CYLINDRICAL FUEL RODS AND COMPRESSIBLE COOLANT

by

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ABSTRACT

Applying linearisation and Laplace transformation the transient temperature distribution and weighted temperatures in fuel, canning and coolant are calculated analytically in two-dimensional cylindrical geometry for constant material properties in fuel and canning. The model to be presented includes previous models as special cases and has the following novel features: compressibility of the coolant is accounted for. The material properties of the coolant are variable. All quantities determining the temperature field are taken into account. It is shown that the solution for fuel and canning temperature may be given by the aid of 4 basic transfer functions depending on only two variables. These functions are calculated for all relevant rod geometries and material constants. The integrals involved in transfer functions determining coolant temperatures are solved for the most part generally by application of coordinate and Laplace transformation.

The model was originally developed for use in steam cooled fast reactor analysis where the coolant temperature rise and compressibility are considerable. It may be applied to other fast or thermal systems after suitable simplifications.

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1. INTRODUCTION

The two approaches to the determination of the transient temperature distribution in a reactor core differ in the solution of the relevant partial differential equations. An analytical solution can only be obtained after certain physical simplifications have been accepted. These are discussed below. In non-analytical models the approximations refer mainly to the mathematical treatment with less restrictions on the physics. The solution is achieved by the difference technique which essentially implies a division of the core into a number of subregions. The use of the latter models is advantangeous when dealing with large transients where nonlinear effects become appreciable. Large computer capacities and the uncertainty as to the number of subregions are two of the main disadvantages of non-analytical models.

Analytical models seem to be a powerful tool for investigating moderate transients and, in addition, may be used to determine the number of subregions for the non-analytical models as will be outlined. It will be shown that only very few physical simplifications are needed for arriving at an analytical solution.

This solution will be given in a general and for the most part integral-free form.

The transient temperature anywhere in the core is completely determined by the following four independent variables:

- nuclear power
- inlet coolant temperature
- coolant mass flow
- coolant pressure

All previous analytical models are incomplete in that

- they do not take into account all four variables

- they simplify the geometrical system

- they neglect time and space variance of material properties.

The assumption that power is separable in space and time is generally used. With the exception of Buffoni and Galati [1] most authors assume a constant radial power distribution in the fuel rod. Coolant temperature is assumed either time and space invariant [1], [2], or space invariant only [3] Storrer [4] considers both space and time dependence. A variation of coolant velocity affects the coolant transit through the core which is neglected in most models (exception Caldarola et al. [5] who investigate the step response), and, in addition, the heat transfer coefficient can-coolant which has not been treated.

For compressible media the effect of pressure arises which has not been investigated.

As to geometrical assumptions the canning has often been neglected [1, 4, 5] or treated as heat resistance [3] Kirchenmayer [2] seems to be the only one who treated the influence of the canning in cylindrical geometry.

In all quoted references the material properties in fuel and coolant are considered to be constant in space and time.

The present model avoids several of the above-mentioned shortcomings as may be seen from Table I.

Comparison of assumptions applied in previous models and the present model on the transient core temperature distribution

Author	Radial power distribution	Coolant temperature	Coolant velocity	Pressure	Material properties in coolant	Canning	Gap
Buffoni, Galati [1]	variable	constant	constant	constant	constant	none	none
Kirchenmayer [2]	constant	"	"	"	"	included	"
Iriarte [3]	"	spatially constant	"	"	"	considered as heat resistance	considered as heat resistance
Storrer [4]	"	variable	"	"	"	none	none
Caldarola et al. [5]	"	"	step	"	"	none	none
Present model	"	"	variable	variable	variable	included	considered as heat resistance

Table I

2. TEMPERATURE IN THE FUEL ROD

2.1 Basic equations

The transient temperature distribution in a fuel rod is determined by 3 partial differential equations of the form

$$\rho_i c_i \frac{\partial T_i(r,z,t)}{\partial t} = \lambda_i \frac{\partial^2 T_i(r,z,t)}{\partial r^2} + \frac{\lambda_i}{r} \frac{\partial T_i(r,z,t)}{\partial r} + N_i(r,z,t)$$

$$i = 1,2,3 \qquad (1)$$

ρ, c, λ are density, specific heat and heat conductivity, respectively and $N(r,z,t)$ is the specific power (per unit volume). The subscripts 1, 2, 3 refer to fuel, gap and canning, respectively, see Fig. 1.

The boundary conditions are

$$\frac{\partial T_1(o,z,t)}{\partial r} = 0 \qquad (2)$$

and

$$\lambda_i \frac{\partial T_i(r,z,t)}{\partial r} = \lambda_{i+1} \frac{\partial T_{i+1}(r,z,t)}{\partial r} \qquad \text{at boundaries.} \quad (3)$$

These equations have to be solved for both local temperatures of interest, e.g. hottest temperature in fuel and canning, and average temperatures as needed for reactivity calculation.

In the further equations we shall employ the heat flux q_i, and thermal diffusivity $\varkappa$ defined as

$$q_i(z,t) = -\lambda_i \frac{\partial T(r_i,z,t)}{\partial r} \qquad (4)$$

$$\varkappa_i \equiv \frac{\lambda_i}{\rho_i c_i} \qquad\qquad (5)$$

It is assumed that nuclear power is released in the fuel only, and separability may be applied, thus

$$N(z,t) \equiv N(z)[1 + \frac{\delta N(t)}{N}] \qquad\qquad 1$$

$$N_i(r,z,t) = 0 \qquad\qquad \text{for } i = 2 \qquad (6)$$

$$0 \qquad\qquad\qquad\qquad 3$$

2.2 Solution

Application of Laplace transformation to eqs. (1) to (4) and (6) and use of eq. (5) yields the following result for the local fuel temperature (written as deviation from its initial value)

$$\delta T_1(r,z,s) = \frac{1}{\rho_1 c_1 s} \delta N(z,s) - \frac{I_0(r\sqrt{\frac{s}{\varkappa_1}})}{\lambda_1 \sqrt{\frac{s}{\varkappa_1}} I_1(r_1\sqrt{\frac{s}{\varkappa_1}})} \cdot \delta q_1(z,s)$$

$$(7)$$

The I's are Bessel functions of imaginary arguments since the Laplace variables are imaginary. For convenience we define

$$\omega_i \equiv \sqrt{\frac{s}{\varkappa_i}} \qquad\qquad (8)$$

Solution of eqs. (1) and (3) for the gap and canning (i = = 2,3) reveals that all temperatures may be expressed as a function of $\delta N(z,s)$ and $\delta q_1(z,s)$ with the use of four basic functions:

$$y_{oo}(x, a) \equiv I_o(x) K_o(ax) - K_o(x) I_o(ax) \qquad (9)$$

$$y_{11}(x, a) \equiv I_1(x) K_1(ax) - K_1(x) I_1(ax) \qquad (10)$$

$$y_{1o}(x, a) \equiv ax [I_o(x) K_1(ax) + K_o(x) I_1(ax)] \qquad (11)$$

$$y_{o1}(x, a) \equiv \frac{x}{2} \frac{I_o(ax)}{I_1(x)} \qquad (12)$$

The K's are Bessel functions, too, x is an imaginary argument, a is real. Omitting details of the calculation (which is straightforward) we arrive at

$$\delta T_1(r, z, s) = \frac{1}{\lambda_1 \omega_1^2} [\delta N(z, s) - \frac{2}{r_1} y_{o1}(r_1 \omega_1, \frac{r}{r_1}) \cdot \delta q_1(z, s)]$$

$$(13)$$

$$\delta T_3(r, z, s) = y_{1o}(r \omega_3, \frac{r_2}{r}) \delta T_3(r_2, z, s) -$$

$$- \frac{r_1}{\lambda_3} y_{oo}(r \omega_3, \frac{r_2}{r}) \delta q_1(z, s) \qquad (14)$$

If the heat capacity of the gap may be neglected the temperature drop is obtained from

$$\delta T_3(r_2, z, s) = \delta T_1(r_1, z, s) - \frac{1}{\alpha_{13}} \delta q_1(z, s) \qquad (15)$$

where α_{13} is the proper heat transfer coefficient.

$$\alpha_{13} = \frac{\lambda_2}{r_1 \ln \frac{r_2}{r_1}} \approx \frac{\lambda_2}{\Delta r_2} \tag{16}$$

The equation for the heat flux from canning to coolant, obtained from eqs. (4) and (14), reads:

$$\delta q_3(z, s) = \frac{r_1}{r_3} y_{1o}\left(\omega_3 r_2, \frac{r_3}{r_2}\right) \delta q_1(z, s) +$$

$$+ \lambda_3 r_2 \omega_3^2 y_{11}\left(\omega_3 r_2, \frac{r_3}{r_2}\right) \delta T_3(r_2, z, s) \tag{24}$$

2.3 Basic transfer functions

The problem of determining the local temperature anywhere in the fuel rod has thus been reduced to determining 4 basic functions (y_{ik}), $\delta N(z, s)$ and $\delta q_1(z, s)$. The y_{ik}'s are calculated in Figs. 2 to 5, and may be used for all relevant rod geometries and materials. Geometry essentially determines the parameter a where $0 \leq a \leq \frac{r_3}{r_2}$. Given a specific geometry different material properties result in a parallel displacement of the curves along the frequency axis.

In many applications the exact transfer functions may be approximated by simpler terms. When x is small i. e. either for low frequencies, small radii or large thermal diffusivity an expansion of the Bessel functions in Eqs. (9) through (12) yields:

$$y_{oo}(x, a) \approx \frac{1}{2}(1-a)(3-a) \qquad , a \approx 1 \tag{9 a}$$

$$y_{11}(x, a) \approx \frac{1}{2} (1-a)(3-a) \qquad , \; a \approx 1 \tag{10 a}$$

$$y_{10}(x, a) \approx \frac{1}{2}(1-a)^2 x^2 + 1 \quad , \; a \approx 1 \tag{11 a}$$

$$y_{01}(x, a) \approx \frac{1 + \frac{1}{4}a^2 x^2}{1 + \frac{1}{8}x^2} \tag{12 a}$$

2.4 Average temperatures

The average fuel and canning temperatures are obtained by radial and axial integration of eqs. (13) and (14). Defining the total heat flows δq_i and heat capacities C_i as

$$\delta q_i(s) \equiv \int_0^{z_4} 2\pi r_i \, \delta q_i(z, s) \, dz \tag{18}$$

$$C_1 \equiv z_4 \pi r_1^2 \rho_1 c_1 \tag{19}$$

$$C_3 \equiv z_4 \pi (r_3^2 - r_2^2) \rho_3 c_3 \tag{20}$$

the average temperatures become

$$\delta T_1(s) = \frac{1}{C_1 s} [\delta N(s) - \delta q_1(s)] \tag{21}$$

$$\delta T_3(s) = \frac{1}{C_3 s} [\delta q_1(s) - \delta q_3(s)] \tag{22}$$

This result is expected from physical reasons.

3. COOLANT TEMPERATURE

3.1 Basic equations

The calculation of coolant temperature requires the solution of 4 equations relating to the heat transfer from canning to coolant, the conservation laws for mass and energy and the equation of state of the coolant.

The heat flux may be expressed as

$$q_3(z,t) = a_3 \left(\frac{W_4(t)}{W_4} \right)^{\nu} [T_3(r_3, z, t) - T_4(z, t)]^n \tag{23}$$

where n, ν are given exponents, e.g. $\nu = 0.8$, $n = 1$, and a_3 is a constant. Radial mixing of the coolant is assumed so that T_4 is only a function of axial position and time.

Linearisation and application of Laplace transformation yields

$$\delta q_3(z, s) = \alpha_3^* [\delta T_3(r_3, z, s) - \delta T_4(z, s)] +$$

$$+ \nu q_3(z, o) \frac{\delta W_4(s)}{W_4} \tag{24}$$

where

$$\alpha_3^* \equiv \frac{n q_3(z, o)}{T_3(r_3, z, o) - T_4(z, o)} \tag{25}$$

α_3^* is referred to as the dynamic heat transfer coefficient and equals the static coefficient for $n = 1$. When eq. (24) is substituted into

eq. (17) and the fuel rod temperatures are eliminated, the heat fluxes δq_1 and δq_3 may be expressed as:

$$\delta q_1(z, s) = \frac{1}{Y_2 + Y_3} \{ \frac{r_1}{2} Y_1 \, \delta N(z, s) - \alpha_3^* \, \delta T_4(z, s) +$$

$$+ \nu q_3(z, o) \frac{\delta W_4(s)}{W_4} \} \tag{26}$$

$$\delta q_3(z, s) = \frac{1}{Y_2 + Y_3} \frac{r_1}{r_3} \{ Y_5 \frac{r_1}{2} \, \delta N(z, s) + Y_4(-\alpha_3^* \, \delta T_4(z, s) +$$

$$+ \nu q_3(z, o) \frac{\delta W_4(s)}{W_4}) \} \tag{27}$$

The following transfer functions have been introduced:

$$Y_1 \equiv \frac{2}{r_1 \rho_1 c_1 s} [r_2 \rho_3 c_3 s \, y_{11} (\omega_3 r_3, \frac{r_2}{r_3}) +$$

$$+ \alpha_3^* \, y_{10} (\omega_3 r_3, \frac{r_2}{r_3})] \tag{28}$$

$$Y_2 \equiv \frac{r_1}{r_3} y_{10} (\omega_3 r_2, \frac{r_3}{r_2}) + \alpha_3^* \frac{r_1}{\lambda_3} y_{00} (\omega_3 r_3, \frac{r_2}{r_3}) \tag{29}$$

$$Y_3 \equiv \lceil y_{o1}(r_1 \omega_1, 1) + \frac{r_1 \rho_1 c_1 s}{2} \frac{1}{\alpha_{13}} \rceil Y_1 \tag{30}$$

$$Y_4 \equiv y_{10}(\omega_3 r_2, \frac{r_3}{r_2}) + 2 \frac{r_2}{r_1} \frac{r_3 \rho_3 c_3}{r_1 \rho_1 c_1} y_{11} (\omega_3 r_3, \frac{r_2}{r_3}) [y_{o1}(r_1 \omega_1, 1) +$$

$$+ \frac{r_1 \rho_1 c_1 s}{2 \alpha_{13}}] \tag{31}$$

$$Y_5 \equiv \frac{2\alpha_3^*}{r_1 \rho_1 c_1} \frac{1}{s} [y_{10}(\omega_3 r_3, \frac{r_2}{r_3}) \, y_{10}(\omega_3 r_2, \frac{r_3}{r_2}) -$$

$$- r_2 r_3 \omega_3^2 y_{11}(\omega_3 r_3, \frac{r_2}{r_3}) \, y_{00}(\omega_3 r_3, \frac{r_2}{r_3})] \qquad (32)$$

The balance equations read (when omitting the arguments)

$$\frac{\partial}{\partial t} A_4 \rho_4 = - \frac{\partial W_4}{\partial z} \qquad (33)$$

$$\frac{\partial}{\partial t} [A_4 \rho_4 (h_4 - p_4 v_4)] = 2\pi r_3 q_3 - \frac{\partial}{\partial z} W_4 h_4 \qquad (34)$$

A_4 is the cross section of the coolant channel, h and v are the specific enthalpy and volume, respectively. Enthalpy is pressure and temperature dependent.

$$dh = C \, dT + v \left(1 - \frac{T}{v} \left(\frac{\partial v}{\partial T} \right)_p \right) dp \qquad (35)$$

where c is the specific heat. The dimensionless constant

$\frac{T}{v} \left(\frac{\partial v}{\partial T} \right)_p$ will be denoted by ν_v

$$\nu_v \equiv \frac{T}{v} \left(\frac{\partial v}{\partial T} \right)_p \qquad (36)$$

and may be calculated from the equation of state. For an ideal gas $\nu_v = 1$, for H_2O-steam ν_v is presented in Fig. 6.

As the pressure drop in the coolant channel is normally small (several bars) we may neglect the pressure's space-dependence. Combining eqs. (33) to (36) yields an equation for the local transient temperature $T_4(z, t)$ which may be linearised, and Laplace transformed. When substituting δq_3 from eq. (27) into this result we arrive at

$$\frac{d}{dz} \delta T_4(z, s) + \delta T_4(z, s) \left[\frac{1}{c_4(z, o)} \frac{dc_4(z, o)}{dz} + \frac{A_4 \rho_4(z, o)}{W_4} s + \right.$$

$$\left. + \frac{2\pi r_3 \alpha_3^*}{c_4(z, o) W_4} \frac{r_1}{r_3} \frac{Y_4}{Y_2 + Y_3} \right] = \frac{f_1(z) N}{c_{4i} W_4} \left[\frac{Y_5}{Y_2 + Y_3} \frac{\delta N(s)}{N} - \right.$$

$$\left. - (1 - \frac{r_1}{r_3} \frac{\nu Y_4}{Y_2 + Y_3}) \frac{\delta W_4(z, s)}{W_4} \right] + [f_2(z) + f_3(z) s] \delta p_4(s)$$

$$\tag{37}$$

The boundary condition for Eq. (37) is

$$\delta T_4(o, s) = \delta T_{4i}(s) \tag{38}$$

Eq. (37) contains the following space dependent factors

$$f_1(z) \equiv \frac{2\pi r_3 q_3(z, o)}{N} \frac{c_{4i}}{c_4(z, o)} \tag{39}$$

$$f_2(z) \equiv \frac{1}{c_4(z, o)} \frac{d}{dz} \left[v_4(z, o) \left(\nu_{v4}(z, o) - 1 \right) \right] \tag{40}$$

$$f_3(z) \equiv \frac{A_4 \nu_{v4}(z, o)}{c_4(z, o) W_4} \tag{41}$$

3.2 Solution

Eq. (37) is a linear first order differential equation which may be solved readily. Defining the square bracket on the left hand side of eq. (37) as σ,

$$\sigma(z, s) \equiv \frac{1}{c_4(z, o)} \frac{dc_4(z, o)}{dz} + \frac{A_4 \rho_4(z, o)}{W_4} s +$$

$$+ \frac{2\pi r_3 \alpha_3^*}{W_4 c_4(z, o)} \frac{r_1}{r_3} \frac{Y_4}{Y_2 + Y_3} \tag{42}$$

then the solution of eqs. (37) and (38) reads

$$\delta T(z,s) = \exp\left(- \int_0^z \sigma(z,s)\,dz\right)\left[\delta T_{4i}(s) + \int_0^z \left\{ \frac{f_1'(z)\,N}{c_{4i}\,W_4}\left[\frac{Y_5}{Y_2+Y_3}\,\frac{\delta N(s)}{N} \right.\right.\right.$$

$$\left. - \left(1 - \frac{r_1}{r_3}\frac{\nu Y_4}{Y_2+Y_3}\right)\frac{\delta W_4(z,s)}{W_4} \right] + [f_2(z) + sf_3(z)]\delta p_4(s)\right\} \times$$

$$\left. \times \exp\left(\int_0^z \sigma(z,s)\,dz\right) dz \right] \tag{43}$$

Eq. (43) determines the temperature field quite generally. The only assumptions involved refer to the constant material properties in the fuel rod and the spatial invariance of pressure. Basically 4 different transfer functions have to be calculated which determine the influence of inlet coolant temperature, power, flow and pressure on the local temperature.

3.3 Basic transfer functions

The integrations involved in the above-mentioned transfer functions can not be performed in a closed form for all geometries and operating conditions that may apply to a coolant channel.

Integral-free transfer functions are obtained, however, if the space dependence of α_3^* may be neglected. We shall further assume that

$$\sigma(z,s) \equiv \sigma_1(z) + \sigma_2(z)\,\sigma_3(s) \tag{44}$$

which does not imply a severe loss of generality since the last two terms in eq. (42) are both separable. Then the transfer functions

involved in eq. (43) have the following form

$$Y = \exp\left(- \int_0^z [\sigma_1(z) + \sigma_2(z) \cdot \sigma_3(s)] dz\right) \int_0^z f(z) \exp\left(\int_0^z [\sigma_1(z) + \right.$$

$$\left. + \sigma_2(z) \sigma_3(s) dz\right) dz \tag{45}$$

We now define

$$Z \equiv \int_0^z \sigma_1(z) dz \tag{46a}$$

$$\zeta \equiv \int_0^{z'} \sigma_2(z) dz \tag{46b}$$

transfer the coordinate system by eq. (46 b) and denote the

function $\dfrac{f(z) e^{Z(z)}}{\sigma_2(z)}$ in the new system as $F(\zeta)$

$$F(\zeta) \equiv \frac{f[z(\zeta)] \exp\left(Z[z(\zeta)]\right)}{\sigma_2[z(\zeta)]} \tag{47}$$

Then eq. (40) reads

$$Y = e^{-Z}\left[e^{-\zeta \sigma_3} \int_0^\zeta F(\zeta') e^{\zeta' \sigma_3} d\zeta' \right] \tag{48}$$

Applying the same technique as the author used previously [6] the bracket on the right hand side of eq. (48) may be expressed in terms of Laplace transforms of the function F.

Consider ζ' and σ_3 as variables in "time" and Laplace domain, respectively, and denote the corresponding Laplace transformation by $L\{\ \}$ then Y may be shown to read:

$$Y = e^{-Z}\left[L\{F(\zeta-\zeta')\} - L\{F(-\zeta')\}e^{-\sigma_3\zeta} \right] \qquad (49)$$

It is stressed that Laplace transformation has been applied for two different purposes. The t, s transformation rendered possible the solution of partial differential equations wheras the ζ', σ_3 transformation facilitated the solution of transfer functions (already given in the s domain). By the latter the integrations involved in eqs. (43), (45) are carried out for all sorts of spatial distributions of nuclear power, specific heat and density. The main assumption that lead to this general solution was the spatial invariance of the heat transfer coefficient.

Applying the result developed above for the various f_i's in eq. (43) yields

$$\delta T(z,s) = \frac{N}{c_{4i}W_4} Y_6(z,s)\left[\frac{Y_5}{Y_2+Y_3} \frac{\delta N(s)}{N} - \left(1- \frac{r_1}{r_3} \frac{\nu Y_4}{Y_2+Y_3}\right) \frac{\delta W_4(s)}{W_4} + \right.$$

$$\left. + \left[Y_7(z,s) + s Y_8(z,s)\right]\delta p_4(s) + Y_9(z,s)\,\delta T_{4i}(s) \right. \qquad (50)$$

where

$$Y_{5+i}(z,s) = e^{-Z}\left[L\{F_i(\zeta-\zeta')\} - L\{F_i(-\zeta')\}e^{-\sigma_3\zeta} \right]$$

$$i = 1,2,3 \qquad (51)$$

$$Y_9(z,s) = e^{-Z-\zeta\sigma_3} \qquad (52)$$

Note that the transfer functions in eqs. (51) and (52) depend on z and s through Z, ζ, and σ_3, respectively.

If the flow perturbations are slow with respect to the coolant transit time through the core the space dependence of $\delta W(z, s)$ may be neglected, which is assumed in eq. (50).

3.4 Weighted coolant temperature

Apart from local temperatures (where the core exit temperature is of major interest) weighted temperatures are needed to determine the reactor behaviour. These are calculated from

$$\delta \tilde{T}(s) = \frac{1}{z_4} \int_0^{z_4} g(z)\, \delta T(z, s)\, dz \tag{53}$$

As a special case, $g(z) = 1$ yields the average temperature. The most general solution is obtained by inserting eq. (43) into eq. (53). If the assumptions employed in the derivation of eq. (50) apply, the solution to eq. (53) is

$$\delta \tilde{T}_4(s) = \frac{N}{c_{4i} W_4} \tilde{Y}_6(s) \left[\frac{Y_5}{Y_2 + Y_3} \frac{\delta N(s)}{N} - \left(1 - \frac{r_1}{r_3} \frac{\nu Y_4}{Y_2 + Y_3} \right) \frac{\delta \dot{W}_4(s)}{W_4} \right] +$$

$$+ \left[\tilde{Y}_7(s) + s\, \tilde{Y}_8(s) \right] \delta p_4(s) + \tilde{Y}_9(s)\, \delta T_{4i}(s) \tag{54}$$

where the transfer functions $\tilde{Y}$ are defined as

$$\tilde{Y}_{5+i}(s) = \frac{1}{z_4} \int_0^{z_4} g(z) \cdot Y_{5+i}(z, s)\, dz, \quad i = 1, \ldots, 4 \tag{55}$$

Here, too, the coordinate transformation to ζ is suitable, see eq. (46 b), and in analogy to the function F, eq. (47) we define

$$G(\zeta) = \frac{g[z(\zeta)]\exp\left(-Z[z(\zeta)]\right)}{\sigma_2[z(\zeta)]} \tag{56}$$

With the use of eqs. (51) and (52), eq. (55) reads

$$\tilde{Y}_{5+i}(s) = \frac{1}{z_4}\int\limits_0^{\zeta_4} G(\zeta)\left[L\{F_i(\zeta-\zeta')\} - L\{F_i(-\zeta')\}\right. \times$$

$$\times\left. e^{-\sigma_3\zeta}\right]d\zeta, \quad i = 1,2,3 \tag{57}$$

$$\tilde{Y}_9(s) = \frac{1}{z_4}\int\limits_0^{\zeta_4} G(\zeta)e^{-\sigma_3\zeta}\,d\zeta \tag{58}$$

ζ_4 is the upper limit of integration corresponding to core exit.

These transfer functions are of the same form as those derived previously [6] for the weighted temperature of an incompressible coolant where the temperature dependence of the material properties was neglected. The previous solution thus applies directly to eqs. (57) and (58). As result it is found that $\tilde{Y}_9(s)$ may be given in an integral-free form which reads:

$$\tilde{Y}_9(s) = \frac{1}{z_4}\left[L\{G(\zeta')\} - L\{G(\zeta_4+\zeta')\}e^{-\sigma_3\zeta}\right] \tag{59}$$

Since $L\{F_i(-\zeta')\}$ in eq. (57) does not depend on ζ eq. (57) may be written as

$$\tilde{Y}_{5+i}(s) = \frac{1}{z_4}\int\limits_0^{\zeta_4} G(\zeta)\,L\{F_i(\zeta-\zeta')\}\,d\zeta - \tilde{Y}_9(s)\,L\{F_i(-\zeta')\} \tag{60}$$

The remaining integral in eq. (60) cannot be solved in general without some knowledge on the functions G or F_i. Its further treatment is dealt with elsewhere [6].

4. APPLICATION

The results for local and weighted core temperatures given

above were included in a more extensive model for a steam cooled

fast reactor plant [7]. To save computer capacity without exceed-

ing the desired accuracy the accurate transfer functions Y_i, i = 1, .., 9

were calculated and approximated by convenient expressions of the form

$$Y_i \approx A_i \frac{(1+\tau_1 s)(1+\tau_3 s) \ldots}{(1+\tau_2 s)(1+\tau_4 s) \ldots} \tag{61}$$

Having specified the frequency range of interest (up to

20 c/sec), and accuracy (2 db in the gains) the time constants may

easily be determined graphically or by the aid of a specially developed

computer programme. A maximum of 2 time constants in the denominator

and one in the numerator proved to be sufficient to achieve the quoted ac-

curacy in this special case. Moreover, apart from differences in the A_i's,

a common function applied to all Y_i, i = 6,7,8. This is an essential

capacity saving and renders possible the calculation of the dynamic

behaviour of steam cooled fast reactor plant on an analogue computer

(utilising the accuracy of the digital computer).

For low frequencies (s → 0) the approximations given in

eq. (61) include essentially only one time constant τ_2. In this case

it may be shown that the distributed model yields results which are

of the same form as those from the lumped model. The validity or

accuracy of a lumped model may thus be directly assessed by comparison

of the accurate and lumped model.

In non-analytical models a lumped model is applied to each

subregion. The number of those regions is essentially equal to the

number of time constants in eq. (61) if the non-analytical model is

to give the same accuracy as the analytical model in the desired frequency range. This fact provides a means of estimating the number of subregions and determining the proper time constants.

ACKNOWLEDGEMENTS

NOMENCLATURE

A   = area

a   = constant

C   = heat capacity

c   = specific heat

F   = space dependent function, see eq. (47)

f   = space dependent function, see eqs. (39) - (41)

G   = space dependent function, see eq. (56)

g   = weighting function

h   = specific enthalpy

I   = Bessel function

K   = Bessel function

N   = power

n   = exponent, see eq. (23)

p   = pressure

q   = heat flux

r   = radius

s   = Laplace variable

T   = temperature

t   = time

v   = specific volume

W   = mass flow

Y, y = transfer function

z   = height

α = heat transfer coefficient

α^* = dynamic heat transfer coefficient

δ = denotes deviation from steady state

Z, ζ = defined by eq. (46)

$\varkappa$ = thermal diffusivity

λ = heat conductivity

ν = exponent, see eq. (23)

ν_v = defined by eq. (36)

ρ = density

σ = defined by eq. (42)

τ = time constant

ω = defined by eq. (8)

Subscripts 1, 2, 3, 4 refer to fuel, gap, canning and coolant, respectively, see Fig. 1.

The specification of local, average, time or Laplace transformed variables is achieved by the proper use of arguments:

Example:

$T_1(r, z, t)$ = local fuel temperature

$T_1(z, s)$ = Laplace transform of radial average fuel temperature

T_1 = steady state average fuel temperature

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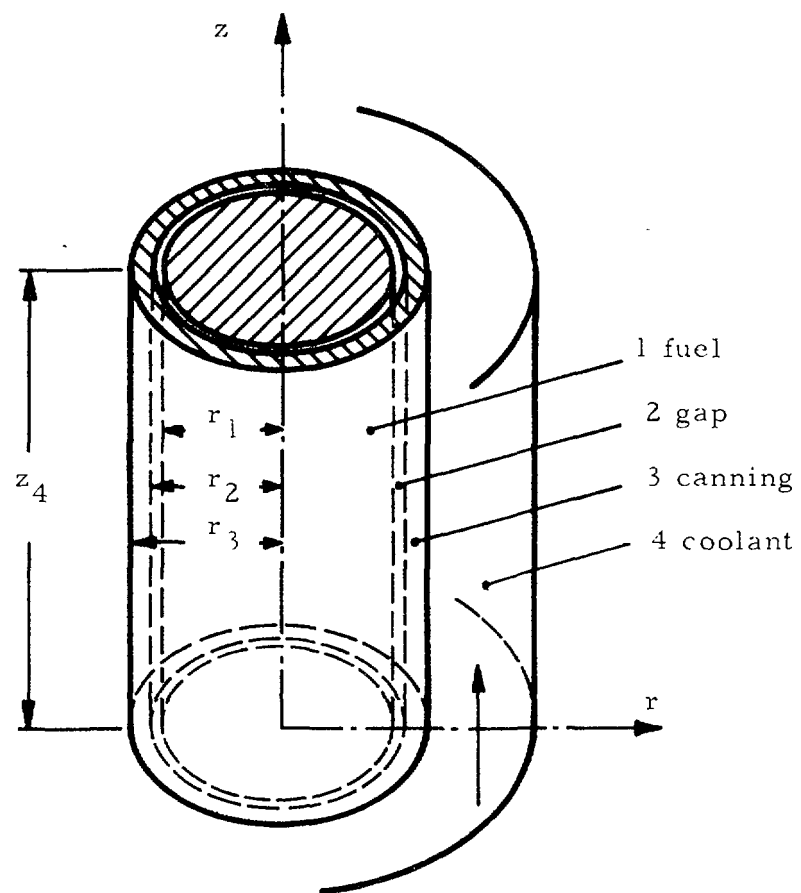
    (AE-316.)
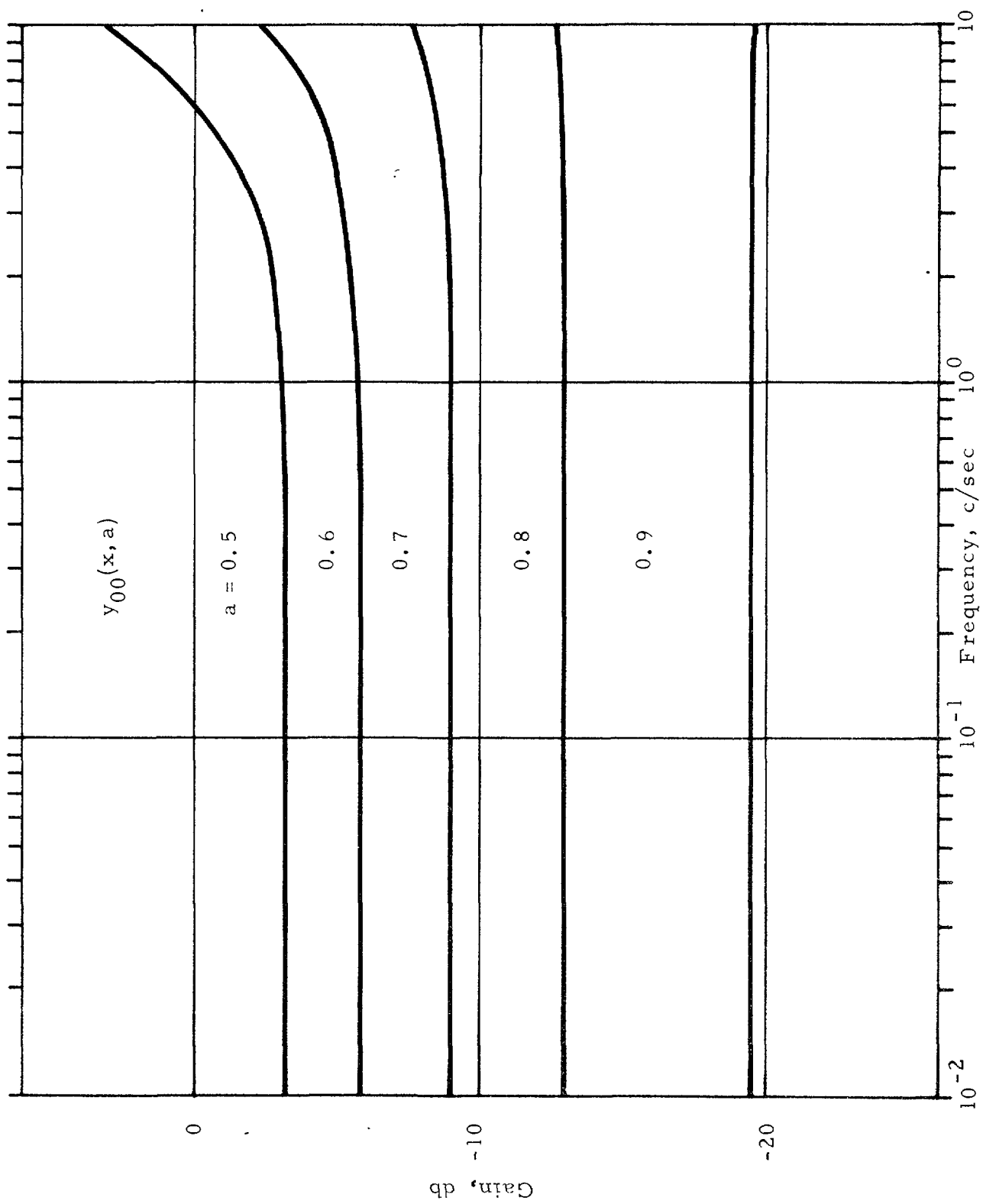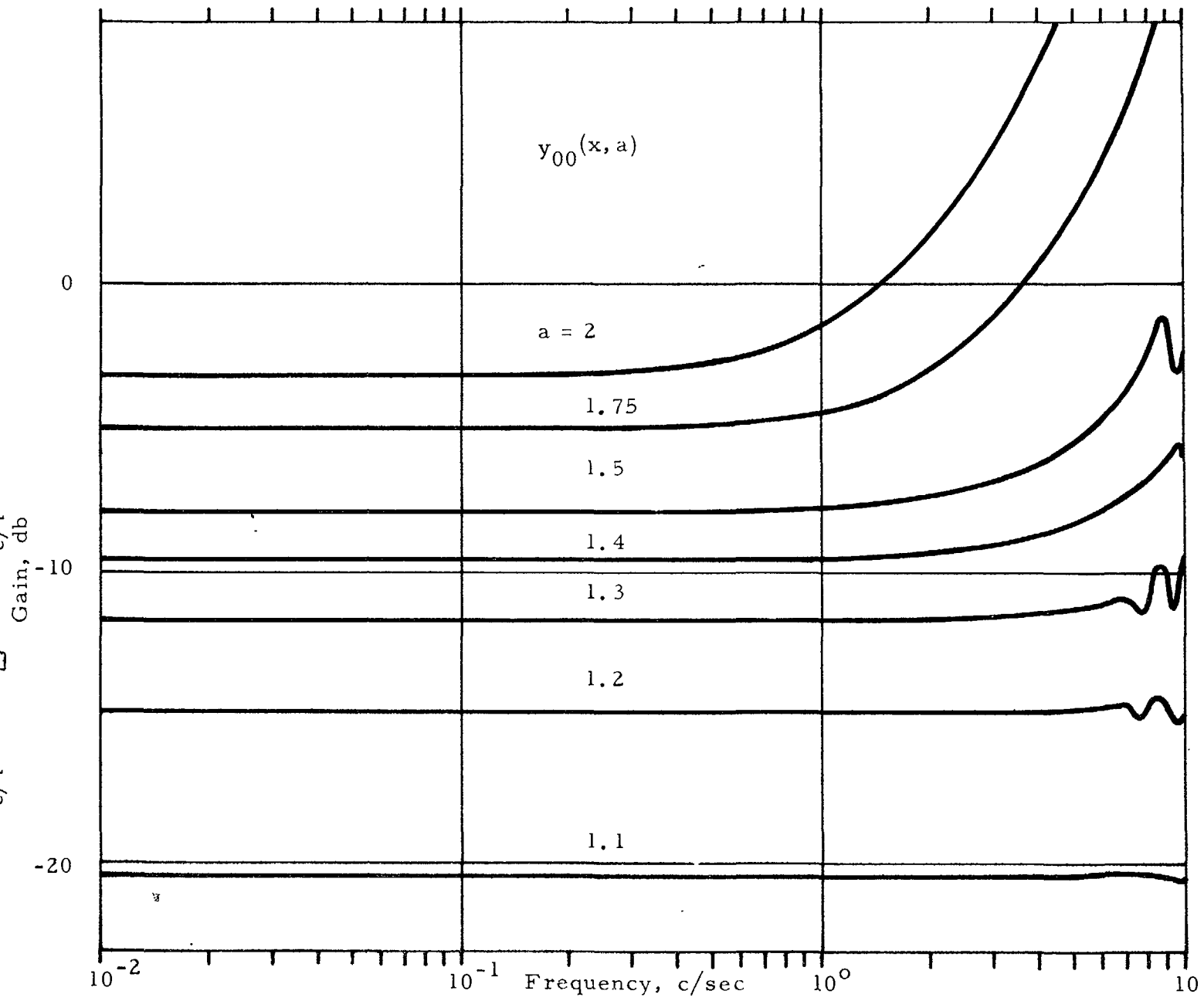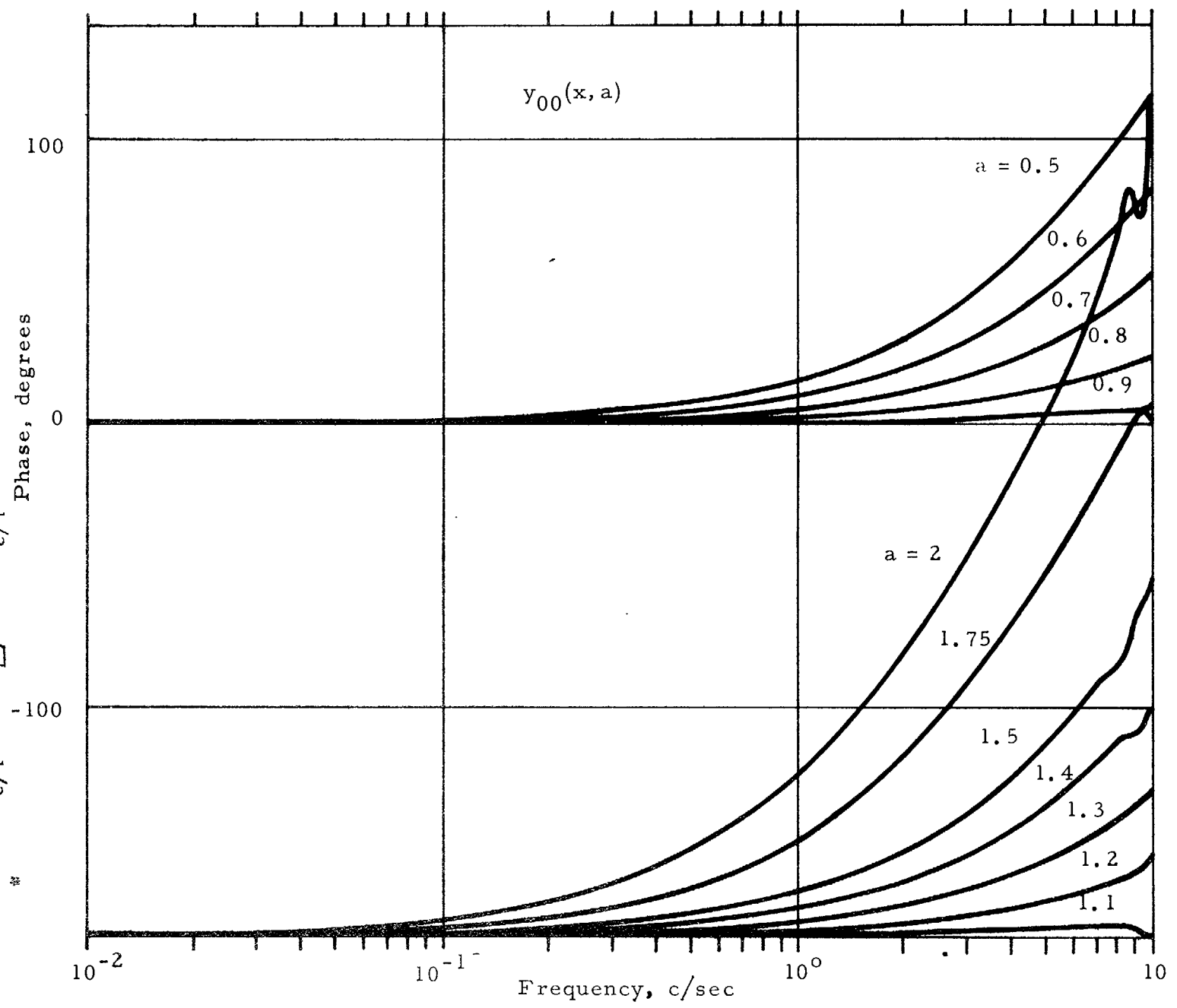
Fig 1   Sketch of the system

Fig 2a $y_{00}(x, a)$ where $x = r(2\pi j \nu/\varkappa)^{1/2}$ and $r/\sqrt{\varkappa} = 1 \sec^{1/2}$

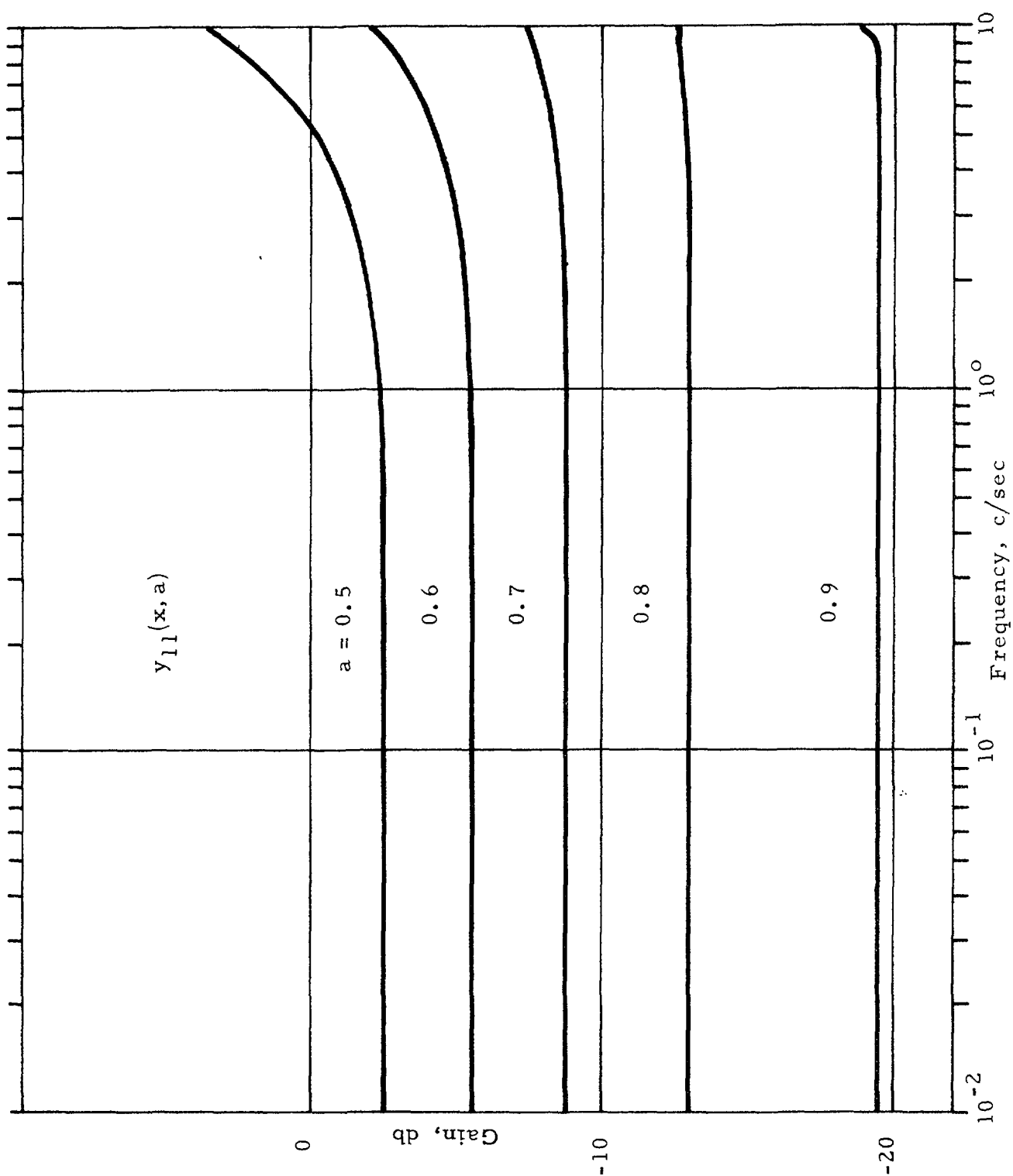Fig 2 b $y_{00}(x, a)$ where $x = r(2\pi j \nu / \varkappa)^{1/2}$ and $r/\sqrt{\varkappa} = 1$ sec$^{1/2}$

Fig 2 c $y_{00}(x, a)$ where $x = r(2\pi j \nu / \varkappa)^{1/2}$ and $r/\sqrt{\varkappa} = 1\ \mathrm{sec}^{1/2}$

Fig 3a $y_{11}(x, a)$ where $x = r(2\pi j\nu/\varkappa)^{1/2}$ and $r/\sqrt{\varkappa} = 1 \ \sec^{1/2}$

$y_{11}(x, a)$

a = 0.5

0.6

0.7

0.8

0.9

Frequency, c/sec

Gain, db

0

-10

-20

10^{-2} 10^{-1} 10^{0} 10

Fig 3 b $y_{11}(x, a)$ where $x = r(2\pi j\nu/\kappa)^{1/2}$ and $r/\sqrt{\kappa} = 1 \sec^{1/2}$

Fig 3 c $y_{11}(x, a)$ where $x = r(2\pi j\nu/\varkappa)^{1/2}$ and $r/\sqrt{\varkappa} = 1 \ sec^{1/2}$

Fig 4a $y_{10}(x, a)$ where $x = r(2\pi jv/\kappa)^{1/2}$ and $r/\sqrt{N}\kappa = 1 \text{ sec}^{1/2}$

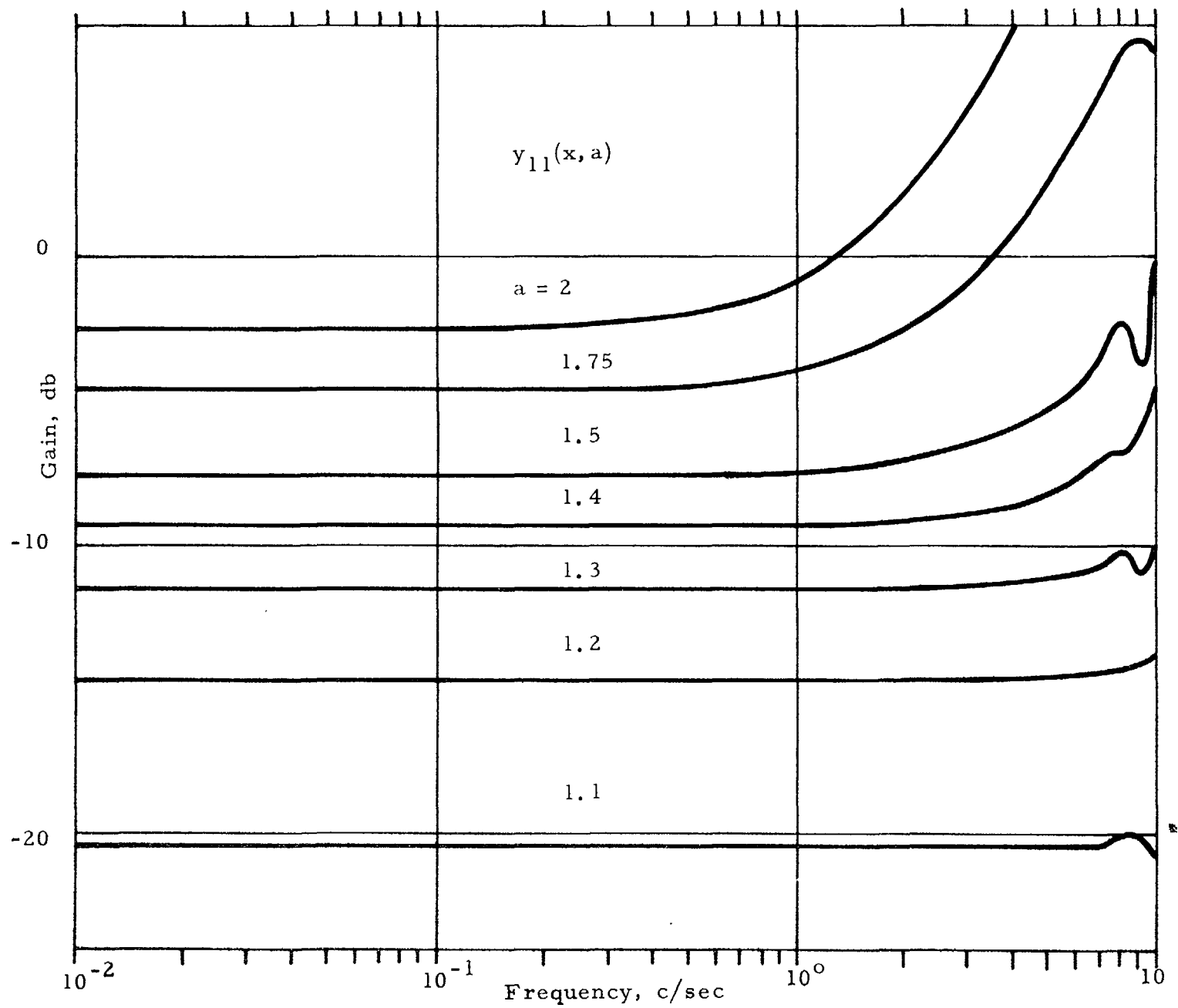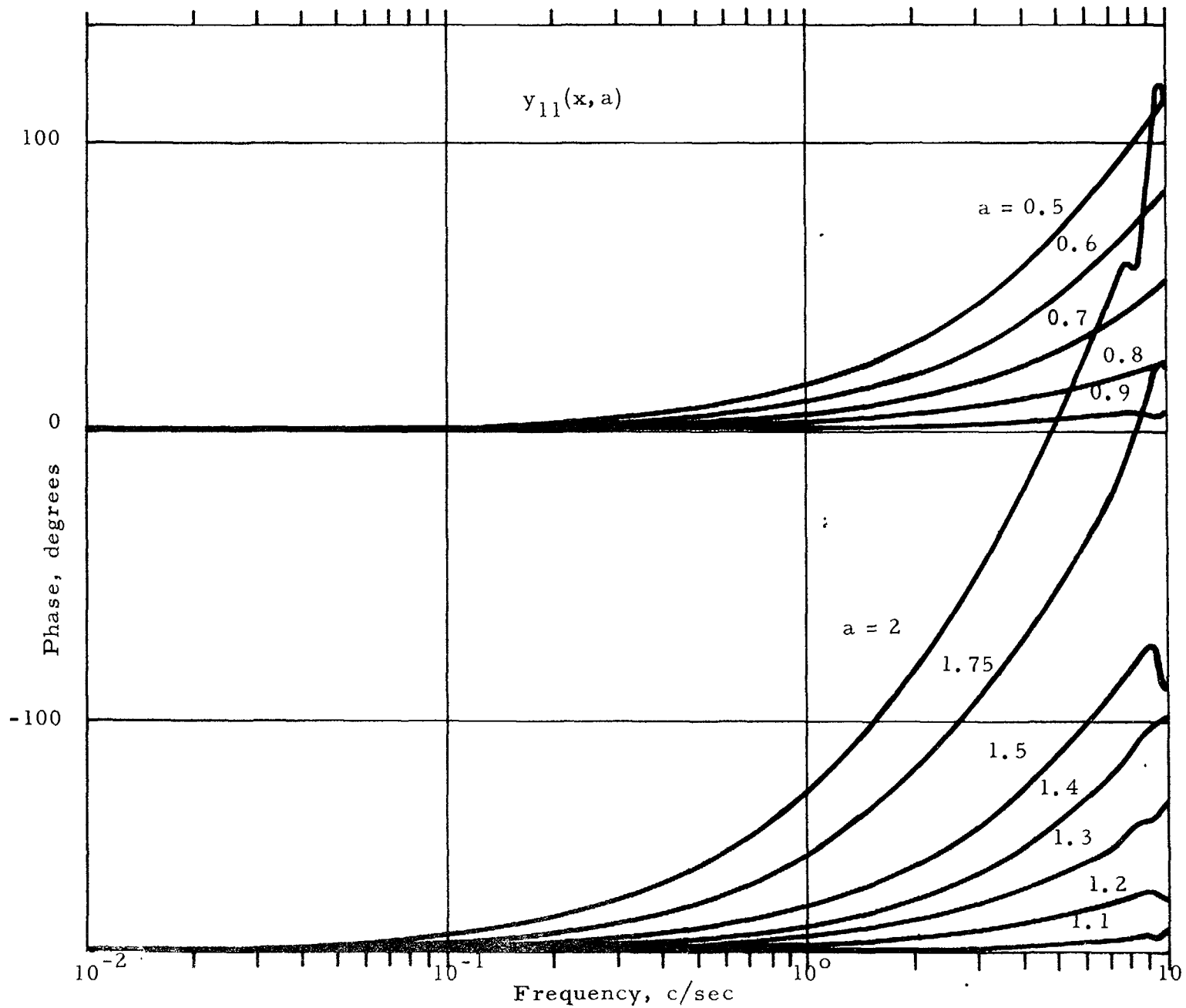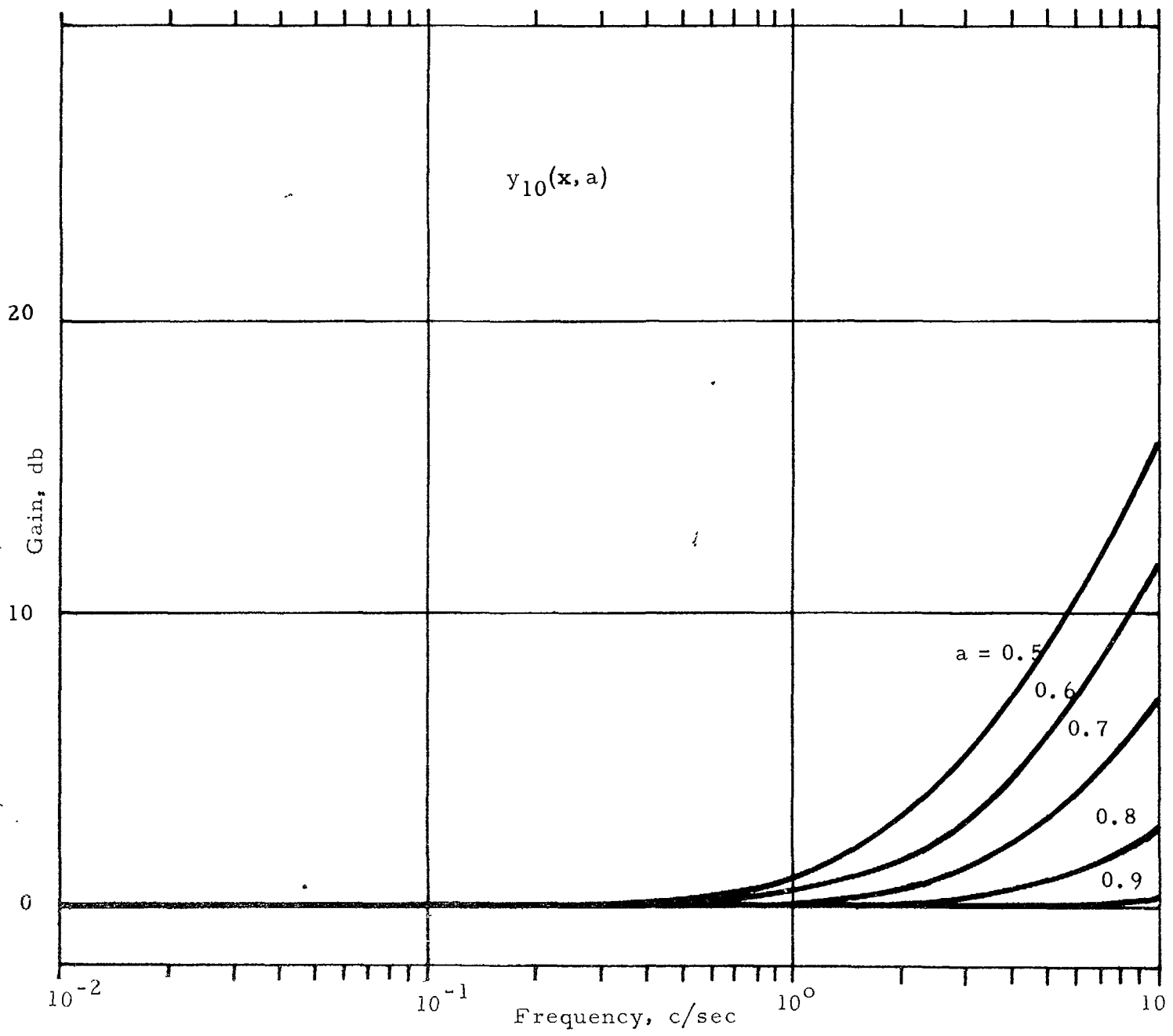Fig 4 b $y_{10}(x, a)$ where $x = r(2\pi j\nu/\kappa)^{1/2}$ and $r/\sqrt{\kappa} = 1 \sec^{1/2}$

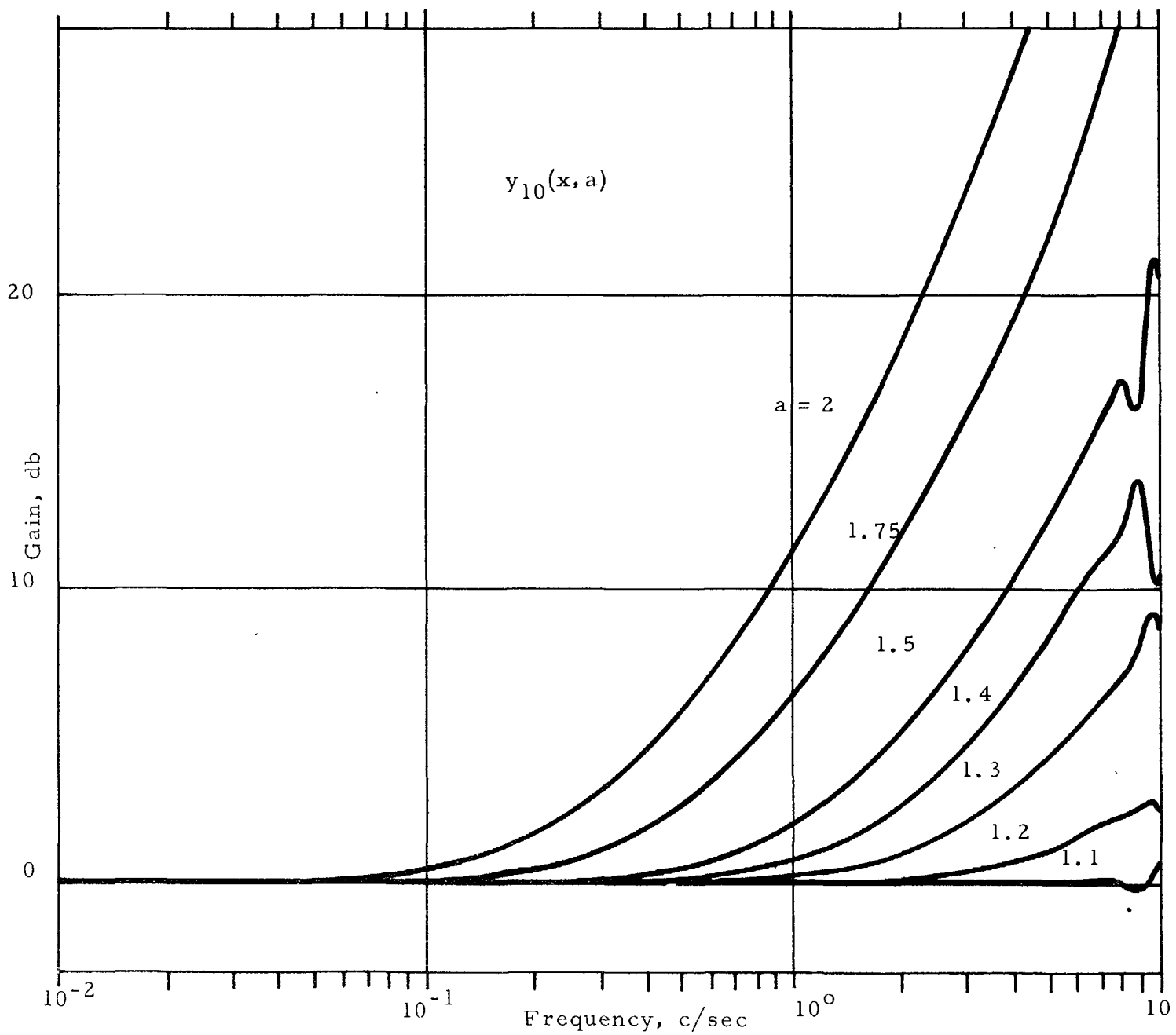$y_{10}(x, a)$

Fig 4 c $y_{10}(x, a)$ where $x = r(2\pi j\nu/\kappa)^{1/2}$ and $r/\sqrt{\kappa} = 1 \sec^{1/2}$

Phase, degrees

100

$a = 0.5$

0.6

0.7

0.8

0.9

0

-100

10^{-2} 10^{-1} 10^{0} 10

Frequency, c/sec

Fig 4d $y_{10}(x, a)$ where $x = r(2\pi j\nu/\varkappa)^{1/2}$ and $r/\sqrt{\varkappa} = 1\ \sec^{1/2}$

$y_{10}(x, a)$

a = 2

1.75

1.5

1.4

1.3

1.2

1.1

Phase, degrees

100

0

-100

10^{-2} 10^{-1} 10^{0} 10

Frequency, c/sec
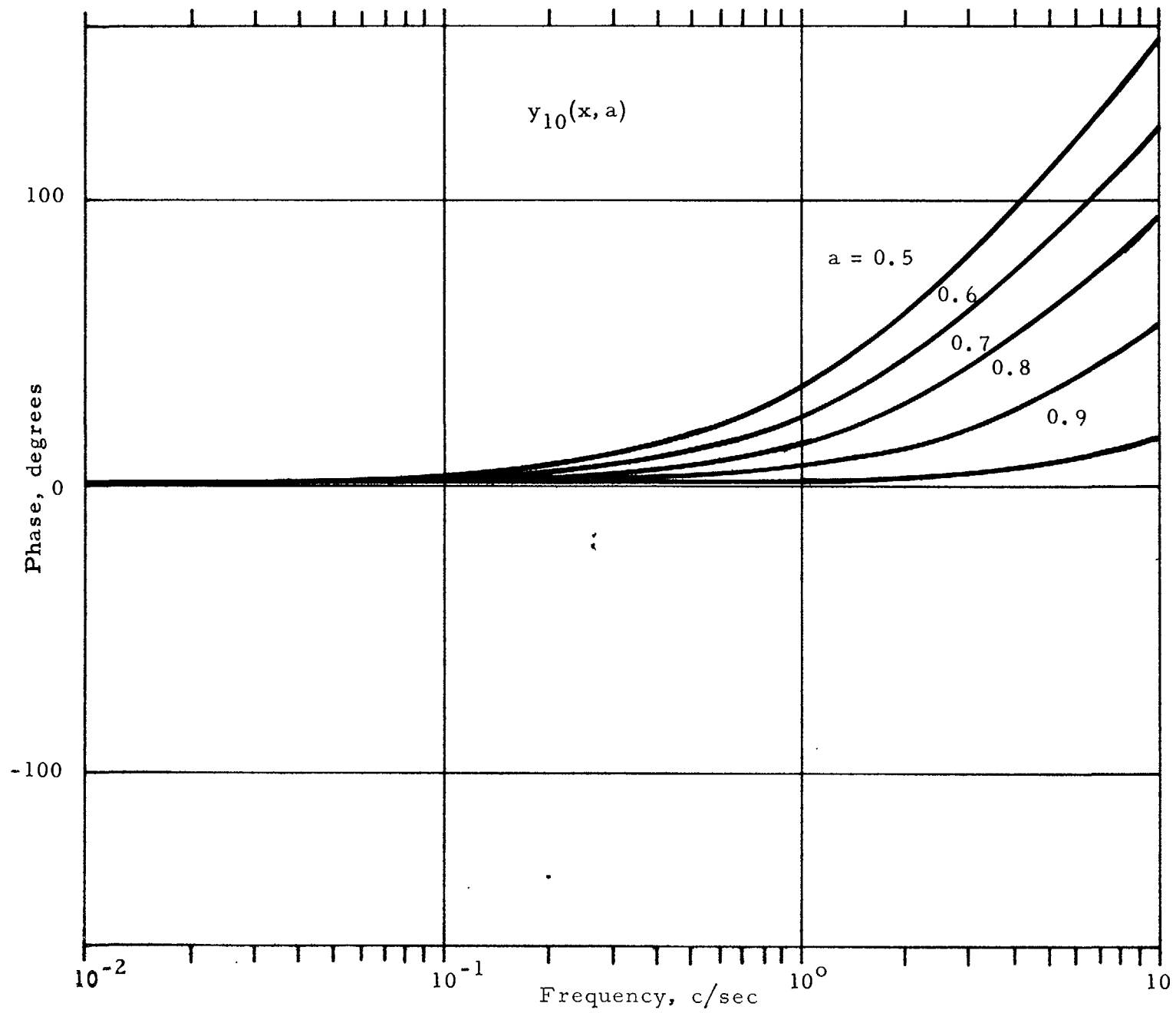
Fig 5 $y_{01}(x, a)$ where $x = r(2\pi j\nu/\varkappa)^{1/2}$ and $r/\sqrt{N_\varkappa} = 1 \text{ sec}^{1/2}$

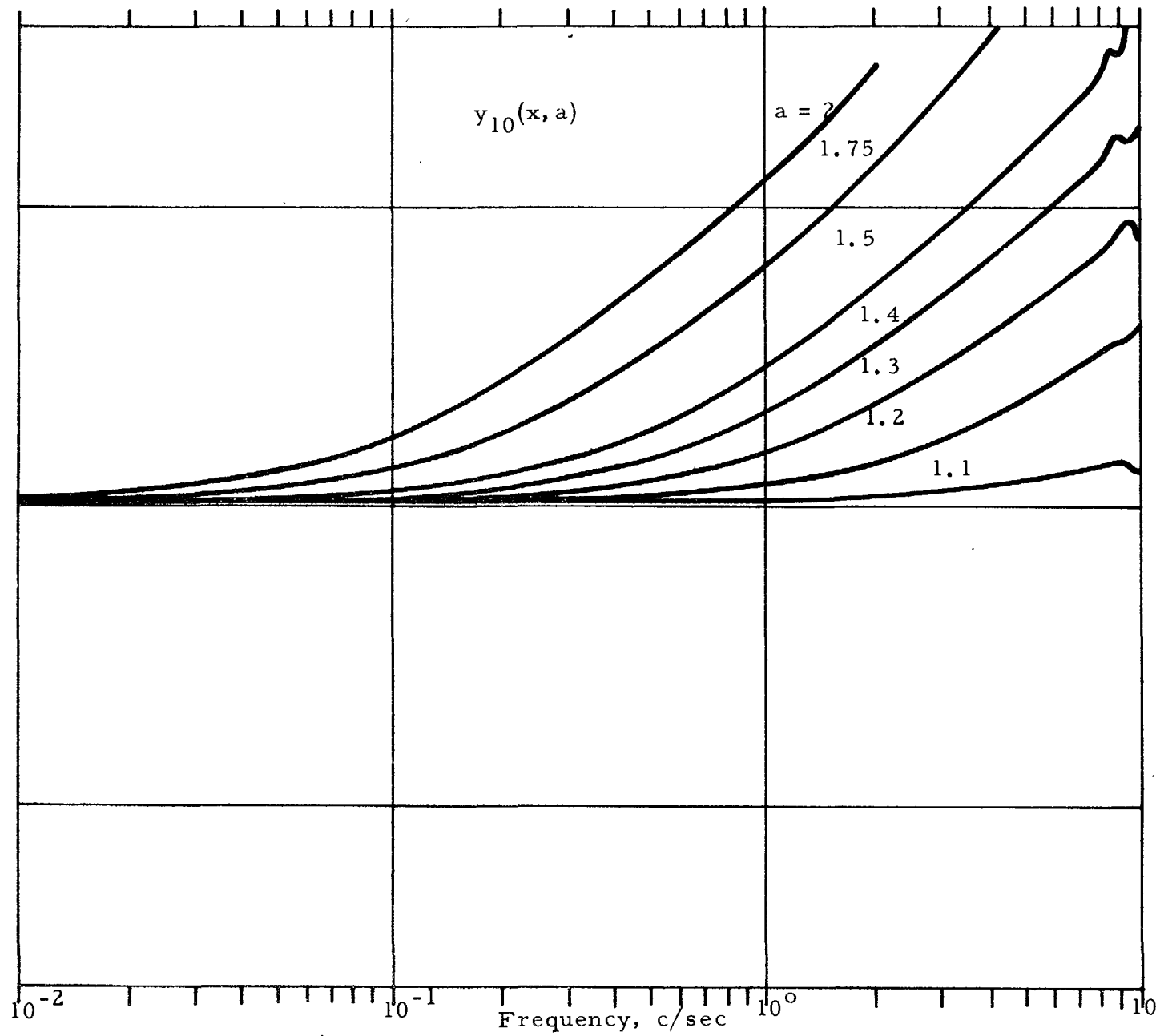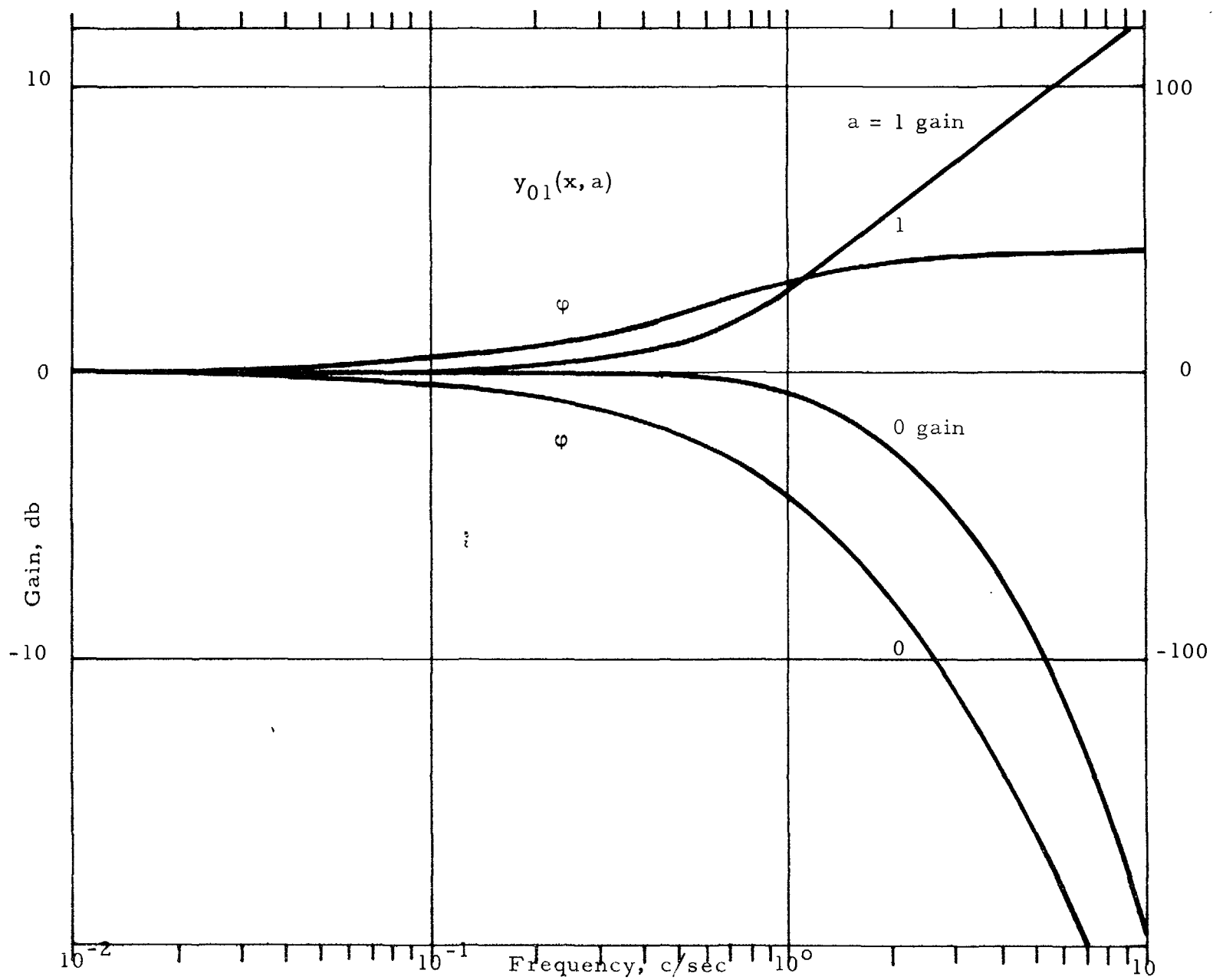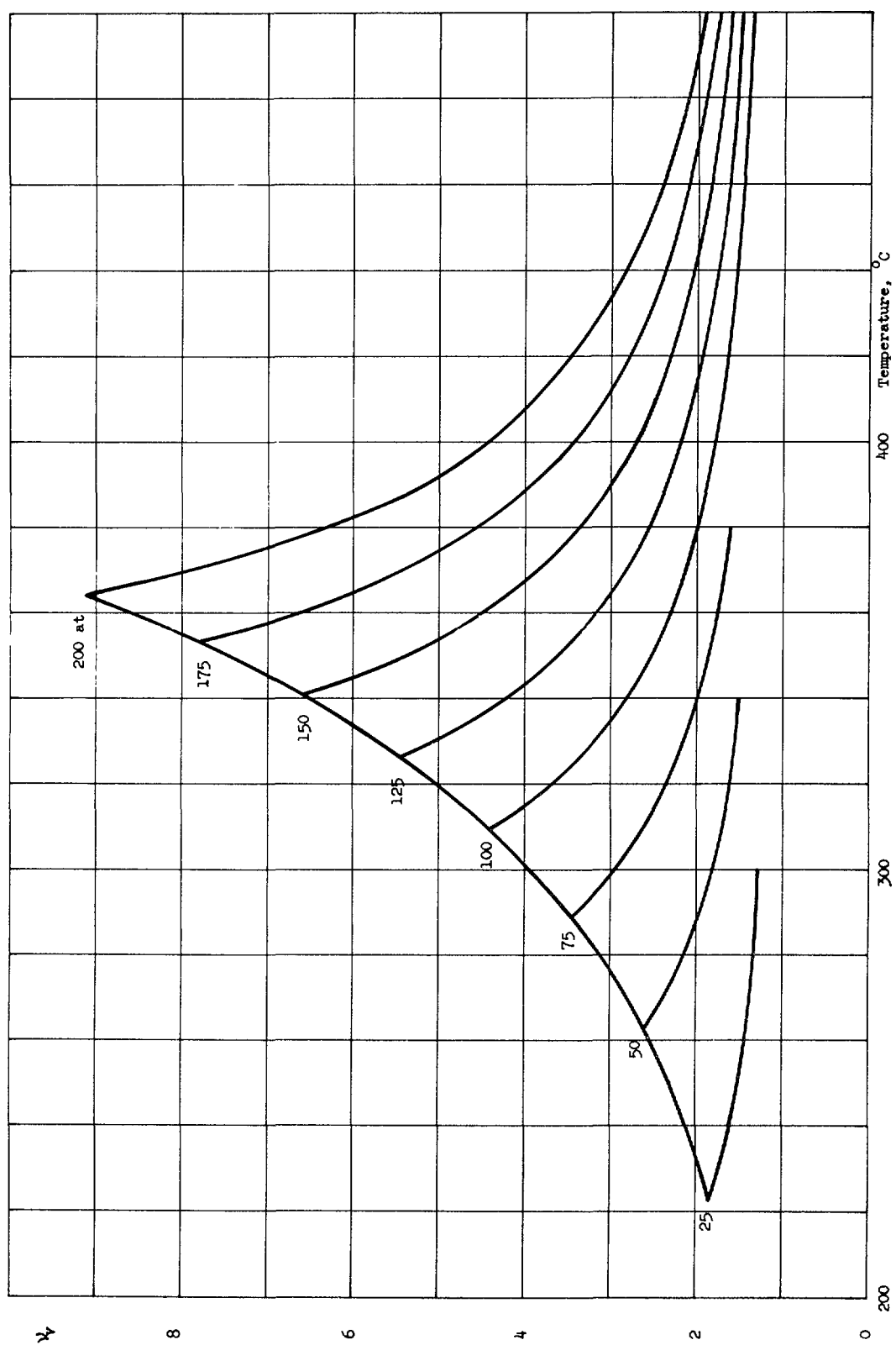Fig. 6   The normalised partial derivative $\nu_v = \frac{T}{v} \left(\frac{\partial v}{\partial T} \right)_p$ for H_2O

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