# Linear Dynamics Model for Steam Cooled Fast Power Reactors

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## LINEAR DYNAMICS MODEL FOR STEAM COOLED FAST POWER REACTORS

by

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#### ABSTRACT

A linear analytical dynamic model is developed for steam cooled fast power reactors. All main components of such a plant are investigated on a general though relatively simple basis. The model is distributed in those parts concerning the core but lumped as to the external plant components. Coolant is considered as compressible and treated by the actual steam law.

Combined use of analogue and digital computer seems most attractive.

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## LIST OF CONTENTS

		Page
Abstract		1
1.	Introduction	3
2.	Scope of the model	5
2.1	Brief description	5
2.2	General assumptions	5
3.	Core dynamics	6
3.1	Reactivity and nuclear power	6
3.2	Core temperatures	8
3.3	Coolant density	10
4.	Plant dynamics	12
4.1	Steam dome	12
4.2	Steam generator	13
4.3	Compressor	17
4.4	Turbine	21
4.5	Heat exchanger	23
4.6	Pipes and valves	24
5.	Application	25
Acknowlegements		26
List of references		27
List of figures		30
Nomenclature		31
Appendix A		33
Figu	res	

#### 1. INTRODUCTION

As power density in modern reactor systems increases and their designs become less conservative, the need for reliable dynamic investigations increases accordingly. These become inevitable when studying new systems such as steam cooled fast breeders. There are four features typical of the latter systems which have bearings on the dynamic behaviour and which prevent a direct application of previous models.

- High core power density
- Short neutron life time
- Fast and strong coupling between core and external plant in case of direct cycle systems
- Equation of state for saturated and superheated steam

The first two items represent a general difference between fast and thermal reactors and cause the faster response of fast reactors to perturbations. (Compact core design, thin fuel rods.) Another consequence of high specific power is the large coolant temperature rise along the core so that assumptions of constant material properties frequently employed in linear models for thermal reactors do not apply.

The third item reflects the main difference between a steam and sodium cooled fast breeder. The external plant - defined in this context by all those components outside the core which affect the dynamic behaviour of the entire plant to an appreciable extent - of a sodium cooled breeder consists of a heat exchanger. Slow and damped responses result from its large heat capacity.

The fourth item finally characterises differences between steam and helium cooled fast systems. The simpler equation of state and no

- 3 -

risk for two-phase operation imply simplifications for the helium cooled breeder.

An extensive list of references on fast reactor kinetics is given by McCarthy and Okrent [1]. It is obvious that sodium cooled systems have received most interest [2-6]. The weak coupling between core and external plant, the relatively simple external plant and the adequacy of considering sodium as incompressible constitute the main reasons why the above mentioned models cannot be applied to steam cooled breeders. Helium cooling is treated by Fortescue et al. [7] The model employed is non-analytical; more precisely, the partial differential equations describing the core dynamics are solved by the difference technique. This approach is chosen in other references [2,5] too.

An early paper on steam cooled fast reactors was published by Schmid [8]. His model's main shortcomings are the oversimplified description of coolant temperature and density and the heat removal system. Results of dynamic calculations performed at Karlsruhe [9-12] are based on a non-analytical model which is to be published.

The model to be presented here is an attempt to avoid the insufficiencies of previous models. It is analytical to take advantage of savings in computational time capacity.

- 4 -

#### 2. SCOPE OF THE MODEL

#### 2.1 Brief description

The dynamic behaviour of a steam cooled fast reactor plant depends on the plant design and operational conditions. Input data to a dynamic model has consequently to refer to these two directly. As there are several possible arrangements of the main plant components, one of them is shown in Fig. 1, it is convenient to treat these components separately.

The external plant may be conceived as a system which may be influenced by core coolant mass flow and enthalpy and which responds by pressure and temperature. Thus the problem of determining the entire plant behaviour consists of:

- Calculation of fuel and canning temperatures. These are affected by nuclear power, coolant temperature and mass flow
- Calculation of coolant temperature and density as a function of heat flux, inlet temperature and pressure
- Calculation of net reactivity to which core feedback effects and external perturbations contribute.

A schematic block diagram is included in Fig. 2.

#### 2.2 General assumptions

The model is linear and distributed as far as the core is concerned but lumped as to the external plant. The core is represented by an average channel consisting of fuel, gap, canning and coolant referred to by subscripts 1 to 4 respectively. Material properties

- 5 -

are assumed constant only in the fuel rod. The compressibility of the coolant is accounted for. In all those expressions containing kinetic and inner energy terms the former are neglected. For the neutron flux separability in space and time is postulated.

#### 3. CORE DYNAMICS

## 3.1 Reactivity and nuclear power

In most dynamic calculations for both thermal and fast reactors the kinetic equations are employed to determine power

$$\frac{\mathrm{dN}}{\mathrm{dt}} = \left[k(1-\beta)-1\right] \frac{N}{\ell} + \Sigma \lambda_{j}^{*}C_{j}^{*} + S^{*}$$
(1)

$$\frac{dC_{j}^{*}}{dt} = -\lambda_{j}^{*}C_{j}^{*} + \beta_{j} k \frac{N}{\ell}$$
(2)

The symbols have their usual meaning and the asterisk is applied because  $\lambda$ , C and S are reserved for heat conductivity, heat capacity and surface, respectively. Values for  $\lambda^*$ ,  $\beta$ ,  $\ell$  may be found in the work of Paxton and Keepin [13]. The effective multiplication factor varies due to external perturbations  $\delta k_{ext}$ , (e.g. by control rods) and due to temperature and density changes in the core (reactivity feedback effects).

Several reactivity effects occurring in a fast reactor have been discussed [1]. Their relative importance in a given reactor depends on its design. It appears that Doppler reactivity,  $\delta k_1$  and reactivity due to coolant density  $\delta k_4$  and possibly thermal expansion of the cladding  $\delta k_3$  are of major interest in a steam cooled fast reactor.

Thus,

$$k = 1 + \delta k_1 + \delta k_3 + \delta k_4 + \delta k_{ext}$$
(3)

The various feedback reactivities are space integrals of the form

$$\delta \mathbf{k} = \frac{\partial \mathbf{k}}{\partial \mathbf{x}} \frac{1}{\mathbf{V}} \int_{\mathbf{V}} \mathbf{g}(\mathbf{r}, \mathbf{z}) \ \delta \mathbf{x}(\mathbf{r}, \mathbf{z}, \mathbf{t}) \ d\mathbf{V}$$
(4)

where x represents either temperature T or density  $\rho$ , and the local reactivity coefficient is written as the product of the total coefficient and the normalised distribution.

Nordheim [14] gives an excellent account of the Doppler coeffi cient which is inversely proportional to  $T_1^n$ , where n is close to 1 for soft spectra. Greebler and Hutchin [15] showed that the actual Doppler coefficient may be approximately 20 % larger than the isothermal coefficient (g(r, z) = 1). With regard to uncertainties in the determination of the isothermal Doppler coefficient we simplify eq. (4) for the reactivity due to fuel temperature changes to read

$$\delta \mathbf{k}_{1}(t) = \frac{\partial \mathbf{k}}{\partial T_{1}} \quad \delta T_{1}(t)$$
(5)

where  $\delta T_1$  is the average fuel temperature deviation from steady state and  $\frac{\partial k}{\partial T_1}$  includes the properly weighted Doppler effect and fuel expansion. Similarly we describe the reactivity effect of canning thermal expansion by

$$\delta k_3(t) = \frac{\partial k}{\partial T_3} \quad \delta T_3(t) \tag{6}$$

The influence of coolant density changes on reactivity is calculated from

$$\delta k_{4}(t) = \frac{\partial k}{\partial \rho_{4}} \frac{1}{z_{4}} \int_{0}^{z_{4}} g(z) \, \delta \rho(z, t) \, dz \equiv \frac{\partial k}{\partial \rho_{4}} \, \delta \widetilde{\rho}(t)$$
(7)

The integration extends from core inlet to its outlet  $(z_4)$  of the average channel.  $\tilde{\rho}$  is referred to as weighted density.

## 3.2 Core temperatures

An analytical solution for the fuel, canning and coolant temperature has been derived by the author elsewhere [16]. The results to be given here are obtained for material properties constant in the fuel rod only, spatially invariant heat transfer coefficient (canning-coolant) and read

$$\delta T_{j}(s) = \frac{1}{C_{j}s} \lfloor \delta N(s) - \delta q_{j}(s) \rfloor$$
(8)

$$\delta T_{3}(s) = \frac{1}{C_{3}s} \left[ \delta q_{1}(s) - \delta q_{3}(s) \right]$$
(9)

$$\delta T_{4}(s) = \frac{N}{c_{4i}W_{4}} Y_{6} \left[ \frac{Y_{5}}{Y_{2}+Y_{3}} \frac{\delta N(s)}{N} - (1 - \frac{r_{1}}{r_{3}} \frac{\nu Y_{4}}{Y_{2}+Y_{3}}) \frac{\delta W_{4}(s)}{W_{4}} \right]$$

+ 
$$[Y_7 + sY_8] \delta p_4(s) + Y_9 \delta T_{4i}(s)$$
 (10)

The total heat flows from fuel to canning  $q_1$  and canning to coolant  $q_3$  entering the equations for the average temperatures are calculated from

.

$$\delta q_{1}(s) = \frac{N}{Y_{2}+Y_{3}} \left[ Y_{1} \frac{\delta N(s)}{N} + \frac{r_{1}}{r_{3}} \left( v \frac{\delta W_{4}(s)}{W_{4}} - \frac{S_{3}a_{3}^{*}}{N} \delta T_{4}(s) \right) \right] \quad (11)$$

$$\delta q_{3}(s) = \frac{N}{Y_{2}+Y_{3}} \left[ Y_{5} \frac{\delta N(s)}{N} + Y_{4} \frac{r_{1}}{r_{3}} \left( v \frac{\delta W_{4}(s)}{W_{4}} - \frac{S_{3}a_{3}}{N} \delta T_{4}(s) \right) \right] (12)$$

The  $Y_i$ 's in eqs. (8) till (12) are (space independent) transfer functions summarised in Appendix A. c, r, S denote specific heat, radius and surface, respectively where the subscripts 1, 2, 3, 4 refer to fuel, gap, canning and coolant as already mentioned.  $a_3^{\#}$  is a dynamic heat transfer coefficient which is  $\mu$  times the static coefficient. Finally  $\mu$  and  $\vee$  are exponents describing the relationship between heat flux, mass flow and temperature drop

$$q_{3}(z,t) \equiv \text{const} \left[\frac{W_{4}(t)}{W_{4}}\right]^{\nu} \left[\Delta T_{34}(z,t)\right]^{\mu}$$
(13)

Apart from the average temperatures, the exit coolant temperature is needed for its influence on the external plant. The proper equation is similar to the equation for the average temperature see eq. (10)

$$\delta T_{40}(s) = \frac{N}{a_{4i}W_4} Y_{60} \left[ \frac{Y_5}{Y_2 + Y_3} \frac{\delta N(s)}{N} - (1 - \frac{r_1}{r_3} \frac{\nu Y_4}{Y_2 + Y_3}) \frac{\delta W_4(s)}{W_4} \right]$$
  
+  $\left[ Y_{70} + s Y_{80} \right] \delta p_4(s) + Y_{90} \delta T_{4i}(s)$  (14)

The transfer functions  $Y_{io}$ , i = 6,..,9 are related to  $Y_i$  as is shown in Appendix A.

#### 3.3 Coolant density

Local coolant density or specific volume is a function of temperature and pressure and may be calculated from the equation of state p = p(T, v). For small variations

$$\frac{\delta\rho}{\rho} = \frac{1}{\nu_{\rho}} \left( \frac{\delta p}{p} - \nu_{T} \frac{\delta T}{T} \right)$$
(15)

where the dimensionless constants

$$\nu_{\rho} \equiv -\frac{v}{p} \left(\frac{\partial p}{\partial v}\right)_{T}$$
(16)

$$v_{\rm T} \equiv \frac{\rm T}{\rm p} \left(\frac{\rm \partial p}{\rm \partial T}\right)_{\rm V} \tag{17}$$

have been introduced. These become unity for an ideal gas and are displayed in Figs. 3 and 4 for  $H_2O$  steam. The equation of state has been taken from VDI tables [17].  $v_p$  and  $v_T$  are related to  $v_v$  introduced previously [16] by

$$v_{\rm T} = v_{\rm \rho} v_{\rm v} \tag{18}$$

Since appreciable deviations of both  $\nu_{\rho}$  and  $\nu_{T}$  from unity occur at high pressure and close to saturation temperature the necessity of using the proper steam law instead of the gas law becomes obvious.

The weighted density change as needed for feedback reactivity is obtained by substituting eq. (15) into eq. (7) and becomes

$$\delta \widetilde{\rho}_{4}(t) = \frac{\partial \widetilde{\rho}_{4}}{\partial p} \delta p_{4}(t) - \frac{1}{z_{4}} \int_{0}^{z_{4}} \frac{\partial \widetilde{\rho}_{4}}{\partial T}(z) \delta T(z, t) dz$$
(19)

where

$$\frac{\partial \widetilde{\rho}_4}{\partial p} = \frac{1}{z_4} \int_0^{z_4} \frac{g(z)\rho(z,0)}{v_{\rho}(z,0)p(z,0)} dz$$
(20)

$$\frac{\partial \widetilde{\rho}_4}{\partial T}(z) = \frac{1}{T(z,0)} g(z)\rho(z,0)v_v(z,0)$$
(21)

The integral in eq. (19) is seen to be a weighted temperature with  $\frac{\partial \tilde{\rho}_4}{\partial T}$  as weighting function. Its calculation is performed elsewhere [16] and results in an expression similar to that given for the average temperature i. e.

$$\delta \widetilde{T}_{4}(s) = \frac{N}{c_{4i}W_{4}} \widetilde{Y}_{6} \left[ \frac{Y_{5}}{Y_{2}+Y_{3}} \frac{\delta N(s)}{N} - \left(1 - \frac{r_{1}}{r_{3}} \frac{\nu Y_{4}}{Y_{2}+Y_{3}}\right) \frac{\delta W_{4}(s)}{W_{4}} \right]$$
  
+  $\left(\widetilde{Y}_{7} + s\widetilde{Y}_{8}\right) \delta p_{4}(s) + \widetilde{Y}_{9} \delta T_{4i}(s)$  (22)

The transfer functions  $\widetilde{Y}_i$ , i = 6,...,9 are dealt with in Appendix A.

Eq. (19) may thus be written as

$$\delta \widetilde{\rho}_4(\mathbf{s}) = \frac{\partial \widetilde{\rho}_4}{\partial \mathbf{p}} \delta \mathbf{p}_4(\mathbf{s}) - \delta \widetilde{\mathbf{T}}_4(\mathbf{s})$$
 (23)

#### 4. PLANT DYNAMICS

In this chapter the most important components of a steam cooled fast reactor plant are described without combining them in a specific flow diagram.

## 4.1 Steam dome

Both above and below the core large volumes of steam may exist that primarily determine the transient reactor pressure. Given the flow and its enthalpy at the inlet, the pressure, temperature and enthalpy of the steam dome are calculated from the volume, mass, energy balance and the equation of state. Omitting arguments they read:

$$dV = d(mv) = o \tag{24}$$

$$\frac{\mathrm{dm}}{\mathrm{dt}} = W_{i} - W_{o}$$
(25)

$$\frac{d}{dt}[m(h-pv)] = W_i h_i - W_o h_o$$
(26)

The equation of state is given by eq. (15). Linearising and combining eqs. (24) to (26) and applying Laplace transformation yields

$$\delta h(s) = \frac{1}{1 + \frac{m}{W_i} s} \left[ \delta h_i(s) + \frac{V}{W_i} s \delta p(s) \right]$$
(27)

Specific enthalpy is related to temperature and pressure as follows

$$\delta h = c \,\delta T + v(1 - v_{y}) \,\delta p \tag{28}$$

where

$$v_{\rm v} = \frac{T}{v} \left(\frac{\partial v}{\partial T}\right)_{\rm p} \tag{29}$$

 $v_v$  may be expressed in terms of  $v_p$  and  $v_T$  as shown in eq. (18). Utilising eq. (28), the temperature is obtained from

$$\delta \mathbf{T} = \frac{1}{c} \left[ \delta \mathbf{h} + \mathbf{v} (\upsilon_{v} - 1) \delta \mathbf{p} \right]$$
(30)

Finally pressure is calculated from eq. (15) with the aid of eqs. (24) and (25)

$$\frac{\delta p}{p} = \frac{\nu_{\rho} W_{i}}{ms} \left( \frac{\delta W_{i}}{W_{i}} - \frac{\delta W_{o}}{W_{i}} \right) + \nu_{T} \frac{\delta T}{T}$$
(31)

The outlet flow from the steam dome to the following component is determined by the pressure there so that eqs. (27) to (31) completely determine the dynamic behaviour of the steam dome.

### 4.2 Steam generator

The dynamics of the steam generator are more involved and depend on its actual design and operation. The basic problem consists of transferring the energy of the superheated steam to the feedwater so as to produce slightly superheated steam. The two technical methods of achieving this are either to spray feedwater into hot steam (spray type steam generator) or to blow hot steam through feedwater (Löffler boiler). An attempt is made to find a theoretical description that includes both alternatives. For this purpose consider the sketch of the steam generator as shown in Fig. 5. The water (denoted by an asterisk) and steam in the steam generator are schematically divided into two different volumes and it is assumed that only a part ( $W_i^* - w_o^*$ ) of the feedwater flow  $W_i^*$  exchanges its energy with the superheated incoming steam,  $W_i$ . The rest  $w_o^*$  is added to the water. Further energy exchange is accomplished by condensation of less superheated steam,  $w_o$ , and evaporation of water,  $w_i$ . The spray type steam generator is thus simulated by  $w_o^* = 0$  and the Löffler boiler by  $w_o^* = W_i^*$ . No restriction is made on the state of the steam leaving the generator which may be saturated or supersaturated.

As in section 4.1 we make use of the volume, mass and energy balances for both steam and water and obtain:

$$\frac{d}{dt}(mv + m^*v^*) = 0$$
 (32)

$$\frac{dm}{dt} = W_{i} + (W_{i}^{*} - w_{o}^{*}) - w_{o} + w_{i} - W_{o}$$
(33)

$$\frac{dm}{dt} = w_0^* + w_0 - w_1$$
(34)

$$\frac{d}{dt} [m(h - pv)] = W_i h_i + (W_i^* - w_o^*) h_i^* - w_o h + w_i h^* - W_o h$$
(35)

$$\frac{d}{dt}(m^{*}h^{*}) = w_{o}^{*}h_{i}^{*} + w_{o}h - w_{i}h^{*}$$
(36)

Further, the specific enthalpy depends on pressure and temperature as given by eq. (28) which may be simplified for the water phase

$$\delta h^{*} = c^{*} \delta T^{*}$$
(37)

Finally, the equations of state read

$$\frac{\delta p}{p} = v_{\rm T} \frac{\delta T}{T} - v_{\rho} \frac{\delta v}{v}$$
(38)

$$\delta v^* = \frac{\partial v^*}{\partial T} \delta T^*$$
(39)

It is pointed out that these equations are not sufficient to determine the transient behaviour of the steam generator. Further equations describing the heat exchange between steam and water phase are necessary but these are subject to the detailed design. The following relations are more intuitive and have to be verified or revised by proper experiments.

That volume of the feedwater which does not come in contact with the hot steam is (for a certain design) likely to be dependent upon pressure, steam and feedflow (velocity effect) and temperatures. For small variations we may thus postulate

$$\delta w_{o}^{*} = \frac{\partial w_{o}^{*}}{\partial W_{i}^{*}} \delta W_{i}^{*} + \frac{\partial w_{o}^{*}}{\partial W_{i}} \delta W_{i} + \frac{\partial w_{o}^{*}}{\partial p} \delta p + \frac{\partial w_{o}^{*}}{\partial T_{i}} \delta T_{i} + \frac{\partial w_{o}^{*}}{\partial T_{i}^{*}} \delta T_{i}^{*}$$
(40)

Similarly, a relation of the same type might hold for the condensing steam where pressure and the temperature difference between steam and water may be of major importance.

$$\delta \mathbf{w}_{o} = \frac{\partial \mathbf{w}_{o}}{\partial \mathbf{T}} \delta \mathbf{T} + \frac{\partial \mathbf{w}_{o}}{\partial \mathbf{T}^{*}} \delta \mathbf{T}^{*} + \frac{\partial \mathbf{w}_{o}}{\partial p} \delta \mathbf{p}$$
(41)

Finally, the evaporating water may be considered as depending on the temperature difference between its actual temperature and the saturation temperature.

$$\delta w_{i} = \frac{\partial w_{i}}{\partial T} \left( \delta T^{*} - \delta T_{sat} \right)$$
(42)

where

$$\delta T_{sat} = \frac{dT_{sat}}{dp} \delta p$$
(43)

Given the flows and enthalpies at the inlet of the steam generator, the pressure and enthalpy at its outlet may be determined. The only variable that is still unknown is the outlet flow since this depends upon the pressure in the following component.

Linearising eqs. (32) to (36) and eliminating v, v, m, m<sup>\*</sup> and  $h^*$  yields:

$$\delta h = \frac{1}{1 + \frac{m}{W_{0} + w_{0}} s} \left\{ \frac{W_{i}}{W_{0} + w_{0}} \delta h_{i} + \frac{h_{i} - h}{W_{0} + w_{0}} \delta W_{i} - \frac{h - h_{i}^{*} - pv}{W_{0} + w_{0}} \delta W_{i} + \frac{mvs}{W_{0} + w_{0}} \delta p + \frac{h - h_{i}^{*} - pv}{W_{0} + w_{0}} \delta w_{0}^{*} - \frac{h - h_{i}^{*} - pv}{W_{0} + w_{0}} \delta W_{i} + \frac{mvs}{W_{0} + w_{0}} \delta p + \frac{h - h_{i}^{*} - pv}{W_{0} + w_{0}} \delta w_{0}^{*} - \frac{h - h_{i}^{*} - pv}{W_{0} + w_{0}} \delta w_{0}^{*} + \frac{h - h_{i}^{*} - pv}{W_{0} + w_{0}} \delta w_{0}^{*} + \frac{h - h_{i}^{*} - h}{W_{0} + w_{0}} \delta r^{*} \right\}$$

$$\delta T^{*} = \frac{1}{1 + \frac{m^{*}}{w_{i}} s} \left\{ \frac{w_{0}^{*}}{c^{*}w_{i}} \delta h_{i}^{*} + \frac{w_{0}}{c^{*}w_{i}} \delta h - \frac{h^{*} - h_{i}^{*}}{c^{*}w_{i}} \delta w_{0}^{*} + \frac{h - h^{*}}{c^{*}w_{i}} \delta w_{0} \right\}$$

Substituting eqs. (33), (34) and (38) into eq. (32) yields

$$\frac{\delta p}{p} = \frac{v_{\rho}(W_{o}^{+} w_{o})}{m s \left[1 - \frac{v_{T}(v_{v}^{-1})pv}{c_{p}T}\right]} \left\{\frac{m v_{v}}{c_{p}T(W_{o}^{+} w_{o})} s \delta h + \frac{m^{*}}{(W_{o}^{+} w_{o})v} \frac{\partial v^{*}}{\partial T} s \delta T^{*} + \frac{\delta W_{i}^{+} \delta W_{i}^{*} - \delta W_{o}}{W_{o}^{+} w_{o}} + \frac{v_{v}^{-}v^{*}}{v} \frac{\delta w_{i}^{-} \delta w_{o}^{-} \delta w_{o}^{*}}{W_{o}^{+} w_{o}}\right\}$$
(46)

The eqs. (28) and (40) to (46) form a complete set of equations for the variables h, p, T, T<sup>\*</sup>, T<sub>sat</sub>,  $w_o^*$ ,  $w_i$  and  $w_o$ . No further elimination is made partly because the variables are needed in the further calculation and partly due to the preliminary character of eqs. (40) and (42).

Finally it is pointed out that in a system where both the water and steam phases are assumed to be at saturation during any transient, the partial derivatives  $\frac{\partial w}{\partial T}$  and  $\frac{\partial w}{\partial p}$  in eqs. (41) and (42) become uniquely defined due to the fact that both enthalpy and temperature are only pressure dependent. In this case these coefficients need not be determined from experiments.

#### 4.3 Compressor

As will be shown below the problem of determining the dynamic behaviour of the compressor may be reduced to the solution of two characteristic equations and the momentum balance equation.

The first characteristic equation relates the outlet pressure of the compressor to inlet pressure, temperature, flow and speed and depends obviously on the special design. Traupel [18] shows that the pressure ratio

$$\Pi = \frac{P_0}{P_1}$$
(47)

most generally is a function of only two normalised variables  $\boldsymbol{\xi}$  and  $\boldsymbol{\zeta}$  defined as

$$\xi(t) = \frac{W(t)}{W} \frac{P_i}{P_i(t)} \sqrt{\frac{j_i(t)}{j_i}}$$
(48)

$$\zeta(t) \equiv \frac{n(t)}{n} \sqrt{\frac{j_i}{j_i(t)}}$$
(49)

where

$$j_{i}(t) = \frac{\kappa}{\kappa - 1} p_{i}(t) v_{i}(t)$$
(50)

Subscript i refers to inlet, o to outlet and n, W,  $\varkappa$  denote speed, mass flow and the isentropic exponent, respectively.

Consequently, the first characteristic equation may be written as

$$\Pi = \Pi(\xi, \zeta)$$
(51)

Similarly, it is found that the internal efficiency  $\eta$  may be expressed by the aid of  $\Pi$  and  $\zeta$  so that

$$\eta = \eta(\Pi, \zeta) \tag{52}$$

which forms the second characteristic equation.

The internal efficiency is defined as the ratio of the "total" enthalpies for isentropic and polytropic compression. Since the "total" enthalpy is composed of the specific enthalpy, h and kinetic energy,  $\frac{u^2}{2}$  where the latter term is small compared to the former, no serious error will be introduced when neglecting  $\frac{u^2}{2}$  in the internal efficiency which thus becomes

$$\eta = \frac{h_0^* - h_i}{h_0 - h_i}$$
(53)

The isentropic enthalpy increase may be written as

$$h_{o}^{*} - h_{i} = c T_{i}(\Pi^{\frac{N-1}{N}} - 1)$$
 (54)

Eqs. (51) to (54) will be used to calculate the pressure and specific enthalpy at the compressor outlet. After linearisation and substitution of eqs. (47) to (50) and eq. (38) into eqs. (51) to (54) we obtain

$$\frac{\delta \mathbf{p}_{o}(\mathbf{t})}{\mathbf{p}_{o}} = \frac{\delta \mathbf{p}_{i}(\mathbf{t})}{\mathbf{p}_{i}} \left[1 - \frac{\xi}{\Pi} \frac{\partial \Pi}{\partial \xi} + \frac{1}{2} \left(\frac{\xi}{\Pi} \frac{\partial \Pi}{\partial \xi} - \frac{\zeta}{\Pi} \frac{\partial \Pi}{\partial \zeta}\right) \left(1 - \frac{1}{\nu_{\rho i}}\right) \right] + \frac{\delta \mathbf{T}_{i}(\mathbf{t})}{\mathbf{T}_{i}} \left[\frac{1}{2} \nu_{v i} \left(\frac{\xi}{\Pi} \frac{\partial \Pi}{\partial \xi} - \frac{\zeta}{\Pi} \frac{\partial \Pi}{\partial \zeta}\right)\right] + \frac{\delta \mathbf{W}(\mathbf{t})}{\mathbf{W}} \frac{\xi}{\Pi} \frac{\partial \Pi}{\partial \xi} + \frac{\delta \mathbf{n}(\mathbf{t})}{\mathbf{n}} \frac{\zeta}{\Pi} \frac{\partial \Pi}{\partial \zeta}$$
(55)

$$\frac{\delta h_{o}(t) - \delta h_{i}(t)}{h_{o} - h_{i}} = \frac{\delta p_{o}(t)}{p_{o}} \left[ \frac{\kappa - 1}{\kappa} - \frac{1}{\kappa} - \frac{1}{\eta} \frac{\partial \eta}{\partial \Pi} \right] + \frac{\delta p_{i}(t)}{1 - \Pi^{\kappa}} + \frac{\delta p_{i}(t)}{\eta} \left[ -\frac{\kappa - 1}{\kappa} - \frac{1}{\kappa} - \frac{1}{\eta} \frac{\partial \eta}{\partial \Pi} - \frac{1}{2} \left( \frac{1}{\nu_{pi}} - 1 \right) \frac{\zeta}{\eta} \frac{\partial \eta}{\partial \zeta} \right] + \frac{\delta T_{i}(t)}{1 - \Pi^{\kappa}} \left[ 1 + \frac{1}{2} \nu_{vi} \frac{\zeta}{\eta} \frac{\partial \eta}{\partial \zeta} \right] - \frac{\zeta}{\eta} \frac{\partial \eta}{\partial \zeta} \frac{\delta n(t)}{n}$$
(56)

It is seen that in this linearised form only 4 constants of the characteristic curves have to be specified. These are the slopes of pressure ratio and efficiency with respect to the variables  $\xi$ ,  $\zeta$  and  $\Pi$ .

By the aid of eq. (30) the outlet temperature may be calculated since  $\delta h_0$  and  $\delta p_0$  are given above.

The compressor speed has to be determined from the momentum equation which reads for the turbo compressor

$$(J_t + J_c) \frac{dn}{dt} = \delta M_t(t) - \delta M_c(t) - \delta M_\ell(t)$$
(57)

the subscript t refers to the drive turbine and  $\ell$  indicates loss. The loss torque is often expressed as

$$M_{\ell} = a_{\ell 1} + a_{\ell 2} n \tag{58}$$

where  $a_{\ell 2} = \frac{0}{\text{finite}} \text{ for } \frac{n(t)}{n} \stackrel{<}{\geq} 0.7$  (59)

Both the compressor and turbine torque are related to power and speed by

$$M(t) = \frac{N(t)}{n(t)}$$
(60)

which reduces to

$$\frac{\delta M(t)}{M} = \frac{\delta N(t)}{N} - \frac{\delta n(t)}{n}$$
(61)

for small deviations from the corresponding steady state value.

As the specific kinetic energy was assumed to be small compared to the specific enthalpy, the power depends only upon flow and the enthalpy so that

$$\frac{\delta N}{N} = \frac{\delta W}{W} + \frac{\delta h_o - \delta h_i}{h_o - h_i}$$
(62)

This completes the set of equations determining the compressor.

In summary it is seen that eqs. (55) to (57) determine outlet pressure, enthalpy and the speed if the state variables at the inlet and the flow is specified. The torques in eq. (57) are given by eqs. (58), (61) and (62).

#### 4.4 Turbine

The procedure for determining the dynamic behaviour of the turbine is very similar to that used for the compressor and applies both for a back pressure and condenser turbine.

Both types of turbine occur in a steam cooled fast reactor power plant where the drive turbine will be of the former and the main turbine of the latter type.

Following practice in turbine techniques the pressure ratio is defined as

$$\Pi_{t} \equiv \frac{P_{i}}{P_{O}}$$
(63)

and the characteristic equation is written in the following form, [18]

$$\xi = \xi(\Pi_{+}, \zeta) \tag{64}$$

 $\xi$  and  $\zeta$  have the same meaning as for the compressor, see eqs. (48) and (49).

The internal efficiency for the turbine is defined as

$$\eta_{t} = \frac{h_{i} - h_{o}}{h_{i} - h_{o}}$$
(65)

and the characteristic equation for  $\boldsymbol{\eta}_t$  reads

$$\Pi_{t} = \Pi_{t}(\Pi_{t}, \zeta)$$
(66)

Following the same procedure as in section 4.3 eqs. (64) and (66) are solved to give:

$$\frac{\delta W(t)}{W} = -\frac{\Pi_{t}}{\xi} \frac{\partial \xi}{\partial \Pi_{t}} \frac{\delta p_{o}(t)}{p_{o}} + \ln + \frac{\Pi_{t}}{\xi} \frac{\partial \xi}{\partial \Pi_{t}} + \frac{1}{2}(1 + \frac{\zeta}{\xi} \frac{\partial \xi}{\partial \zeta})(\frac{1}{\nu_{pi}} - 1) \int \frac{\delta p_{i}(t)}{p_{i}} + \frac{1}{2}(1 + \frac{\zeta}{\xi} \frac{\partial \xi}{\partial \zeta})(\frac{1}{\nu_{pi}} - 1) \int \frac{\delta p_{i}(t)}{p_{i}} + \frac{1}{2}(1 + \frac{\zeta}{\xi} \frac{\partial \xi}{\partial \zeta})(\frac{1}{\nu_{pi}} - 1) \int \frac{\delta p_{i}(t)}{p_{i}} + \frac{1}{2}(1 + \frac{\zeta}{\xi} \frac{\partial \xi}{\partial \zeta})(\frac{1}{\nu_{pi}} - 1) \int \frac{\delta p_{i}(t)}{p_{i}} + \frac{1}{2}(1 + \frac{\zeta}{\xi} \frac{\partial \xi}{\partial \zeta})(\frac{1}{\nu_{pi}} - 1) \int \frac{\delta p_{i}(t)}{p_{i}} + \frac{\delta p_{i}(t)}{p_{i}} + \frac{\delta p_{i}(t)}{p_{i}} \int \frac{\Pi_{t}}{p_{i}} \frac{\partial \eta_{t}}{\partial \Pi_{t}} + \frac{\delta p_{i}(t)}{p_{i}} \int \frac{\Pi_{t}}{\eta_{t}} \frac{\partial \eta_{t}}{\partial \Pi_{t}} + \frac{1}{2}(1 + \frac{1}{2} \frac{\zeta}{\eta_{t}} \frac{\partial \eta_{t}}{\partial \zeta} (\frac{1}{\nu_{pi}} - 1) + \frac{\delta p_{i}(t)}{\eta_{t}} \int \frac{\Pi_{t}}{\eta_{t}} \frac{\partial \eta_{t}}{\partial \Pi_{t}} + \frac{1}{2}(1 + \frac{1}{2} \frac{\zeta}{\eta_{t}} \frac{\partial \eta_{t}}{\partial \zeta} (\frac{1}{\nu_{pi}} - 1) + \frac{\delta p_{i}(t)}{\eta_{t}} \int \frac{1}{\eta_{t}} \frac{\delta \eta_{t}}{\partial \zeta} + \frac{\delta p_{i}(t)}{\eta_{t}} \int \frac{1}{\eta_{t}} \frac{\delta q_{t}}{\partial \zeta} (\frac{1}{\nu_{pi}} - 1) + \frac{\delta q_{t}}{\eta_{t}} \int \frac{\delta q_{t}}{\partial \zeta} \int \frac{\delta q_{t}}{\eta_{t}} \int \frac{\delta q_{t}}{\eta_{t}} \frac{\delta q_{t}}{\delta \zeta} + \frac{\delta q_{t}}{\eta_{t}} \int \frac{\delta q_{t}}{\eta_{t}} \int \frac{\delta q_{t}}{\delta \zeta} \int \frac{\delta q_{t}}{\eta_{t}} \int \frac{\delta q_{t}}$$

The power and torque delivered to the compressor or generator are found in eqs. (62) and (61), respectively. They are valid for constant mechanical efficiency.

For the main turbine it may be adequate to assume constant speed and thus a momentum equation of the form of eq. (57) becomes trivial.

We have thus established a set of equations that completely determines the turbine. As for the compressor only four slopes of characteristic curves need be known in order to calculate the dynamic behaviour of the turbine for small perturbations.

#### 4.5 Heat exchanger

The heat exchanger in a steam cooled fast reactor plant does not play the same prominent rôle as it does in a sodium cooled system. It is mainly used as reheater and may present some complications as far as its dynamic description is concerned as will be explained. In contrast to heat exchangers used in thermal reactor systems a large temperature drop may occur on the primary side of the reheater and the compressibility of the steam and space dependence of density, specific heat and velocity must be accounted for. Similar conditions may also apply to the secondary side of the heat exchanger.

Gyftopoulos and Smets [19], Eurola [20] treated the dynamics of heat exchangers mainly in view of an application to thermal reactors and the above mentioned specific features are not included. Direct use of these calculations may be made by dividing the heat exchanger into a few subregions and applying the models to them. Also to account for the steam compressibility the previous models have to be revised.

- 23 -

#### 4.6 Pipes and valves

The main dynamic effect of the pipes interconnecting the different components of a large steam cooled fast reactor power plant is of smoothing and delaying nature.

The delay is simply a matter of tube length and area and steam velocity. Smoothing occurs due to compressibility, heat losses and flow regime of the steam. We assume the heat losses to be small and the radial velocity to be constant.

Large pipes may be treated by the methods developed for the steam dome, see section 4.1, where the compressibility of the steam is properly accounted for. Otherwise, a sufficiently accurate model should be

$$\delta h_{o}(t) = \delta h_{i}(t - \theta)$$
(69)

$$\delta \mathbf{p}_{o}(t) = \delta \mathbf{p}_{i}(t) \tag{70}$$

$$\delta W_{o}(t) = \delta W_{i}(t) \tag{71}$$

where  $\theta$  is the steam transit time through the pipe. Outlet temperature and density are then determined by the aid of eqs. (30) and (15), respectively. For short transit times a further mathematical simplification is justifiable:

$$\delta h_{o}(s) = e^{-\theta s} \delta h_{i}(s) \approx \frac{1}{1+\theta s} \delta h_{i}(s)$$
 (72)

It is noted that eq. (70) does not necessarily imply no frictional pressure drop along the pipe. However, we shall make this assumption and represent the pressure drop at adequate places. Thus it is reasonable to lump the pipe's frictional pressure drop into a value at the outlet of the pipe.

The pressure drop through a valve may be calculated from

$$W = A(\frac{p_{o}}{p_{i}})^{\frac{1}{n}} \sqrt{\frac{n}{n-1} \left[1 - (\frac{p_{o}}{p_{i}})^{\frac{n-1}{n}}\right]} \sqrt{2\frac{p_{i}}{v_{i}}}$$
(73)

A is the area of the valve.

For small perturbations and for  $\frac{p_i - p_o}{p_i} \ll 1$ , we obtain

$$\frac{\delta W(t)}{W} = \frac{\delta A(t)}{A} + \frac{1}{2} \left(1 - \frac{2}{\pi} \frac{p_i - p_o}{p_i}\right) \frac{\delta p_i(t) - \delta p_o(t)}{p_i - p_o} + \frac{1}{2} \frac{\delta \rho_i}{\rho_i}$$
(74)

Since we did not consider heat losses

$$\delta h_{o}(t) = \delta h_{i}(t)$$
(75)

Eqs. (74) and (75) determine the value completely.

#### 5. APPLICATION

So far, the dynamic behaviour of the most important plant components has been calculated based on the assumptions that the state at the inlet is known. When investigating a specific plant design the various components are interconnected essentially by stating that a variable at the inlet of a component is identical to the variable at the exit of the preceding. The set of equations derived in sections 3 and 4 supplemented by the identity statements may be solved with either the "accurate" transfer functions or approximations of the form

$$Y = A \frac{(1 + \tau_1 s)(1 + \tau_3 s)...}{(1 + \tau_2 s)(1 + \tau_4 s)...}$$
(76)

The latter is convenient for analogue computers and may be obtained by the aid of a digital computer programme when the accuracy and the frequency range is specified. By this procedure the speed and flexibility of an analogue computer is combined with the accuracy of a digital computer.

The model has been applied to different plant designs for various operational conditions. For further details reference is made to a separate report [21].

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#### LIST OF FIGURES

Fig. 1Flow diagram of a steam cooled fast reactor power plant.Fig. 2Simplified block diagram of a dynamics model for steam<br/>cooled fast power reactors.Fig. 3The normalized partial derivative  $v_p = -\frac{v}{p} \left(\frac{\partial p}{\partial v}\right)_T$  for  $H_2O$ .

Fig. 4 The normalized partial derivative  $v_T = \frac{T}{p} \left(\frac{\partial p}{\partial T}\right)_v$  for  $H_2O$ .

Fig. 5 Sketch of the steam generator.

### NOMENCLATURE

А area С heat capacity c \* concentration of delayed emitters specific heat с weighting function g specific enthalpy h J inertia effective multiplication factor k ł neutron life time M · torque m mass Ν power speed n pressure р heat flux q radius r surface S s\* source Laplace variable s temperature (absolute) Т time t V volume specific volume v W,w mass flow Y transfer function axial coordinate  $\mathbf{z}$ 

a	heat transfer coefficient	
β	delayed neutron fraction	
δ	denotes deviation from steady state	
ς	variable defined by eq. (49)	
η	internal efficiency	
θ	transit time	
н	isentropic exponent	
* λ	decay constant	
μ	exponent in eq. (13)	
ν	exponent in eq. (13)	
$v_{v}, v_{\rho}, v_{T}$ normalised slopes of state variables as defined by eqs. (29), (16), (17) respectively		
ξ	variable defined by eq. (48)	
П	pressure ratio	
ρ	density	
τ	time constant	

## Subscripts:

- 1 refers to fuel
- 2 " " gap
- 3 <sup>11</sup> <sup>11</sup> canning
- 4 " " coolant
- i " " inlet
- o " " outlet

Note: In order to achieve a clear and simple denotation we define the use of arguments as follows.

- f(t) time dependent function
- f(s) Laplace transform of f(t)
- f = f(t = 0)

### APPENDIX A

#### Summary of transfer functions for local and weighted temperature

Of the transfer functions  $Y_i$ , i = 1, ..., 9, determining both local and weighted temperature one group, namely  $Y_i$ , i = 1, ..., 5, does not depend on coolant properties and is thus common to both local and weighted temperature in contrast to the remaining functions. For spatially constant heat transfer coefficient  $a_3^*$  the functions were shown to read [16]

$$Y_{1} = \frac{2}{r_{1}\rho_{1}c_{1}s} \left[ r_{2}\rho_{3}c_{3}s y_{11}(\omega_{3}r_{3}, \frac{r_{2}}{r_{3}}) + a_{3}^{*}y_{10}(\omega_{3}r_{3}, \frac{r_{2}}{r_{3}}) \right] \quad (1)$$

$$Y_{2} \equiv \frac{r_{1}}{r_{3}} y_{10}(\omega_{3}r_{2}, \frac{r_{3}}{r_{2}}) + \alpha_{3}^{*} \frac{r_{1}}{\lambda_{3}} y_{00}(\omega_{3}r_{3}, \frac{r_{2}}{r_{3}})$$
(2)

$$Y_{3} \equiv [y_{01}(r_{1}\omega_{1},1) + \frac{r_{1}\rho_{1}c_{1}s}{2} \frac{1}{a_{13}}]Y_{1}$$
(3)

$$Y_{4} = y_{10}(\omega_{3}r_{2}, \frac{r_{3}}{r_{2}}) + 2 \frac{r_{2}}{r_{1}} \frac{r_{3}^{\rho} \beta_{3}c_{3}}{r_{1}^{\rho} \beta_{1}c_{1}} y_{11}(\omega_{3}r_{3}, \frac{r_{2}}{r_{3}}) [y_{01}(r_{1}\omega_{1}, 1) + \frac{r_{1}^{\rho} \beta_{1}c_{1}s}{2a_{13}}]$$
(4)

$$Y_{5} = \frac{2\alpha_{3}^{*}}{r_{1}\rho_{1}c_{1}} \frac{1}{s} \left[ y_{1o}(\omega_{3}r_{3}, \frac{r_{2}}{r_{3}}) y_{1o}(\omega_{3}r_{2}, \frac{r_{3}}{r_{2}}) - r_{2}r_{3}\omega_{3}^{2} y_{11}(\omega_{3}r_{3}, \frac{r_{2}}{r_{3}}) y_{oo}(\omega_{3}r_{3}, \frac{r_{2}}{r_{3}}) \right]$$
(5)

where

$$\omega_{i} = \sqrt{\frac{s\rho_{i}c_{i}}{\lambda_{i}}}$$
(6)

The  $y_{ik}$ 's are combinations of Bessel functions of imaginary argument, defined as

$$y_{oo}(x, a) \equiv I_{o}(x) K_{o}(ax) - K_{o}(x) I_{o}(ax)$$
(7)

$$y_{11}(x, a) \equiv I_1(x) K_1(ax) - K_1(x) I_1(ax)$$
 (8)

$$y_{0}(x, a) \equiv ax [I_{0}(x) K_{1}(ax) + K_{0}(x) I_{1}(ax)]$$
 (9)

$$\mathbf{y}_{01}(\mathbf{x},\mathbf{a}) \equiv \frac{\mathbf{x}}{2} \frac{\mathbf{I}_{0}(\mathbf{a}\mathbf{x})}{\mathbf{I}_{1}(\mathbf{x})}$$
(10)

The transfer functions  $Y_i$ , i = 6,...9, read for the local temperature:

$$Y_{5+i}(z,s) = e^{-Z} [L\{F_i(\zeta - \zeta')\} - L\{F_i(-\zeta')\}e^{-\sigma_3\zeta}]$$
(11)

$$i = 1, 2, 3$$
  
 $Y_{9}(z, s) = e^{-Z - \zeta \sigma_{3}}$ 
(12)

L{ } denotes Laplace transformation with respect to the variable  $\zeta'$ , the Laplace variable is  $\sigma_3$ . Both space functions Z and F<sub>i</sub> are expressed in terms of  $\zeta$  where

$$\zeta \equiv \int_{0}^{z} \sigma_{Z}(z) dz$$
 (13)

$$Z(\zeta) = \int_{0}^{z} \sigma_{l}(z) dz \qquad z = z(\zeta) \qquad (14)$$

$$F_{i}(\zeta) = \frac{f_{i}(z) \exp Z(z)}{\sigma_{2}(z)} \qquad z = z(\zeta) \qquad (15)$$

Finally  $\mathbf{f}_i$  and  $\boldsymbol{\sigma}_i$  are defined as

$$f_{1}(z) = \frac{2\pi r_{3}q_{3}(z,0)}{q_{3}} \frac{c_{4i}}{c_{4}(z,0)}$$
(16)

.

$$f_{2}(z) \equiv \frac{1}{c_{4}(z,o)} \frac{d}{dz} [v_{4}(z,o) (v_{v}(z,o) - 1)]$$
 (17)

$$f_{3}(z) = \frac{A_{4} v_{v4}(z, o)}{c_{4}(z, o)W_{4}}$$
(18)

$$\sigma(z, s) \equiv \sigma_1(z) + \sigma_2(z) \sigma_3(s)$$
(19)

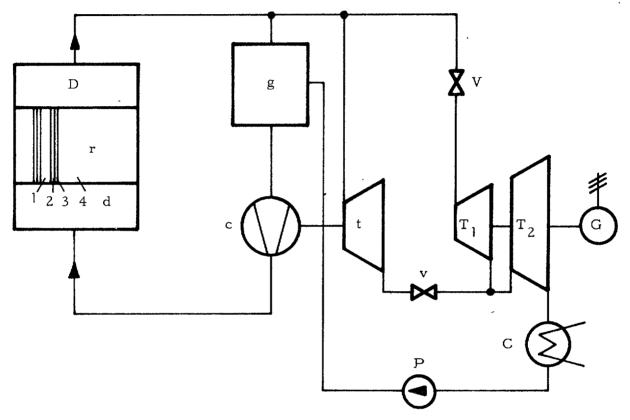
where the general expression for  $\sigma(z,s)$  is

$$\sigma_{(z,s)} \equiv \frac{1}{c_4(z,o)} \frac{dc_4(z,o)}{dz} + \frac{A_4\rho_4(z,o)}{W_4}s + \frac{2\pi r_3 a_3^{**}}{W_4 c_4(z,o)} \frac{r_1}{r_3} \frac{Y_4}{Y_2 + Y_3}$$
(20)

The transfer functions for weighted temperatures may be written as

$$\widetilde{Y}_{5+i}(s) = \frac{1}{z_4} \int_{0}^{z_4} g(z) Y_{5+i}(z,s) dz$$
,  $i = 1, ..., 4$  (21)

The integration involved in eq. (21) is treated elsewhere [16].



## Legend:

- C condenser
- c compressor
- D,d plenum at core outlet, inlet
- G generator
- g steam generator
- P pump
- r reactor core; 1,2,3,4, refer to fuel, gap, canning, coolant, respectively
- T main turbine
- t compressor drive turbine
- V, v valves

Fig. 1 Flow diagram of a steam cooled fast reactor power plant

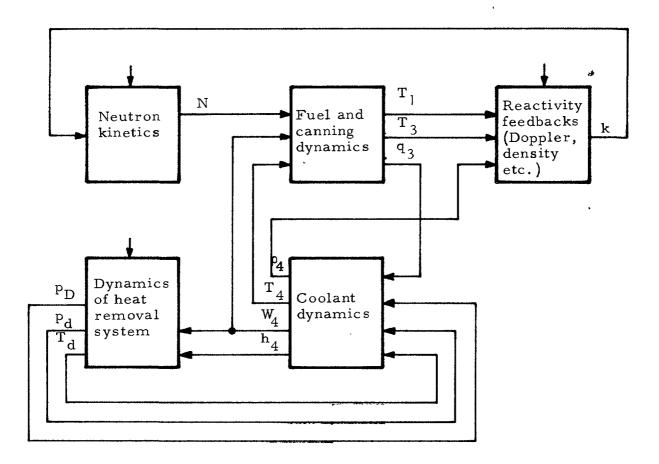


Fig. 2 Simplified block diagram of a dynamic model for steam cooled fast power reactors

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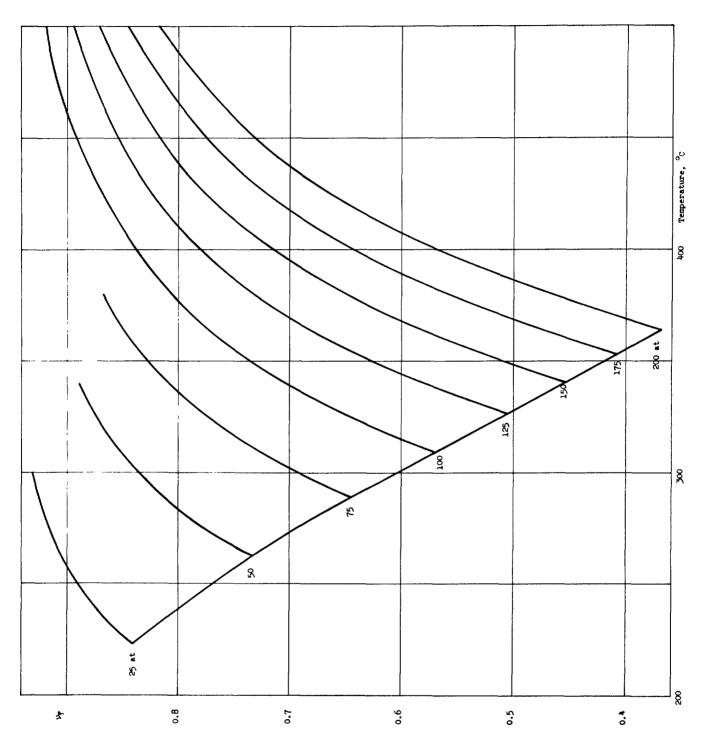
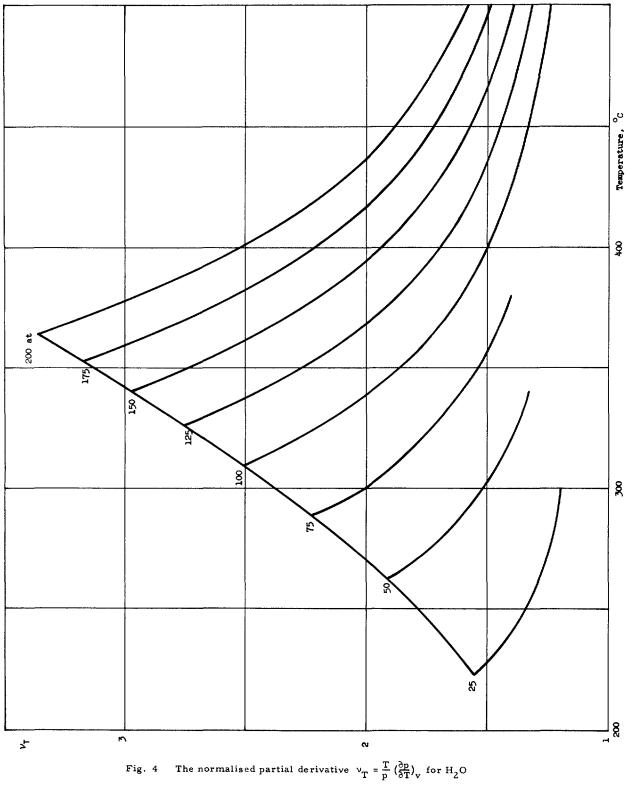
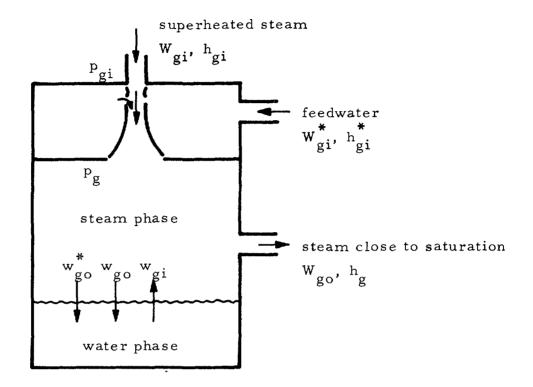


Fig. 3 The normalised partial derivative  $v_{\rho} = -\frac{v}{p} \left(\frac{\partial p}{\partial v}\right)_{T}$  for  $H_{2O}$ 





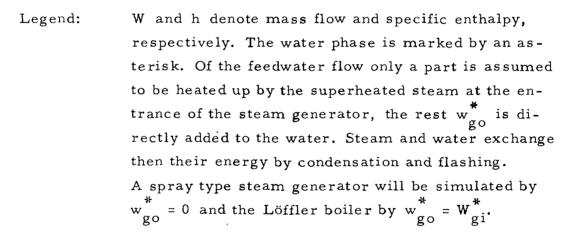


Fig. 5 . Sketch of the steam generator

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