AE-344

A Study of the Resonance Interaction Effect between ^{238}U and ^{239}Pu in the Lower Energy Region

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A STUDY OF THE RESONANCE INTERACTION EFFECT BETWEEN

^{238}U AND ^{239}Pu IN THE LOWER ENERGY REGION

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ABSTRACT

An investigation has been made of the statistical frequency function for the distances between the ^{238}U and the ^{239}Pu resonances in the region 4-244 eV. It is concluded that the frequency function is probably constant but that the distances diverge appreciably from a constant function in the actual case, and that the divergence is such that the interaction effect on the resonance integral is smaller than would be expected from statistical considerations. This is also confirmed by calculations on the interaction effect. These have been performed using three different methods, namely:

a)   considering the actual positions and widths of the resonances,

b)   assuming a constant frequency function for the resonance spacing and applying a theory developed by Rowlands and E A Fischer,

c)   applying a simplified, approximate method for calculations based on the statistical theory.

The calculations are made for two temperatures and two values of the plutonium enrichment. It is shown that the average cross sections are considerably larger than the statistical calculations indicate.

CONTENTS

1. INTRODUCTION

Effective cross sections in the resonance region can in principle be obtained with good accuracy if all resonances are resolved. Computer programmes have been developed which solve the slowing-down equation for a heterogeneous medium, e.g. SDR [1], or Rabble [2], or, for a homogeneous medium MC^2 [3]. But these programmes are generally not used for routine calculations of macroscopic cross sections, that is, for cases when cross sections for a large number of rather similar compositions are needed. In such cases one is satisfied with cruder approximations and a usual assumption in obtaining these is that the average interaction effect between two series of statistically independent resonances can be calculated assuming a constant frequency function for the spacing between such resonances, [4, 5, 6]. Effective cross sections can then be calculated separately for each isotope and the calculation of an approximate spectrum which gives the weight function for obtaining the macroscopic cross sections can be sufficiently fast for routine calculations. This method is used in the Dorix-Speng system [7].

In the statistical calculations we must consider several stochastic variables. The principal ones are the resonance energies, $E_r^{(i)}$, and the widths, $\Gamma_x^{(i)}$, where the index i stands for the isotope and x for a given reaction. Further we have to introduce a function $F_i(E, E_r^{(i)}, \Gamma_{x1}^{(i)}, \dots)$ which has as arguments the energy, E, and the resonance parameters. All variables are continuous. The expectation

value of the product $F_i F_j$ is obtained by using joint frequency functions for the resonance parameters. Because the resonance energies are independent of the other parameters, the expectation value is

$$E\{F_i F_j\} = \langle \int_{\Delta E} F_i \, dE_r^{(i)} \int_{\Delta E} F_j \, P(E_r^{(i)}, E_r^{(j)}) \, dE_{r,j} \rangle \qquad (1.1)$$

where the brackets indicate averaging over the resonance widths and $P(E_r^{(i)}, E_r^{(j)})$ is the joint frequency function for the resonance energies $E_r^{(i)}$ and $E_r^{(j)}$. The latter can also be written

$$P(E_r^{(i)}, E_r^{(j)}) = P_c(E_r^{(j)} \mid E_r^{(i)}) \, P(E_r^{(i)}) \qquad (1.2)$$

where $P_c(E_r^{(j)} \mid E_r^{(i)})$ is the conditional frequency function for $E_r^{(j)}$ when $E_r^{(i)}$ is known and $P(E_r^{(i)})$ is the marginal frequency function for $E_r^{(i)}$. Now, if P_c is constant, we can subtract $E_r^{(i)}$ without changing the value, that is,

$$P_c(E_r^{(j)} \mid E_r^{(i)}) = P_c(E_r^{(j)} - E_r^{(i)} \mid E_r^{(i)}) = \text{const.} \qquad (1.3)$$

The second expression in Eq. (1.3) is most suitable for statistical verification and we will return to it later. Now, making the additional assumption that the resonances are statistically independent, Eq. (1.1) assumes the form

$$E\{F_i F_j\} = \langle \frac{1}{\Delta E} \int_{\Delta E} F_i \, dE_r^{(i)} \rangle \langle \frac{1}{\Delta E} \int_{\Delta E} F_j \, dE_r^{(j)} \rangle \qquad (1.4)$$

Eq. (1.4) can be related to the calculation of the interaction effect. It has been shown by Rowlands, [4], and by Fischer, [5], that the interaction effect between two resonances under some assumptions can be described in terms of the following function and its derivatives with respect to β_k and β_ℓ:

$$F_0(\beta_k, \beta_\ell) = \frac{1}{\Gamma_k} \int_{E_a}^{E_b} \frac{\Psi_k \Psi_\ell dE}{(\Psi_k + \beta_k)(\Psi_\ell + \beta_\ell)} \tag{1.5}$$

Ψ_k and Ψ_ℓ are the symmetric resonance line shape functions for the resonances k and ℓ, respectively. Their arguments are $E - E_k$ and $E - E_\ell$. Γ_k is the total width of resonance k. β_k and β_ℓ are parameters which are related to the peak resonance cross sections and the so-called background cross section.

If the parameters belonging to the k and ℓ resonances are stochastic variables corresponding to the isotopes i and j respectively, the average cross section for isotope i is obtained for an energy interval which equals the average spacing, $\bar{D}_i$, between the corresponding resonances. The width of the integration interval, $E_b - E_a$, is related to $\bar{D}_i$, but it should be observed that tails from neighbouring resonances contribute to the cross section within this interval. This contribution is approximately obtained by extending the energy boundaries to zero and infinity for the integral defining the J-function. For the interaction effect we carry out a corresponding averaging of Eq. (1.5) by using Eq. (1.4). We introduce the following new variables:

$$E_1 = E - E_a \qquad (1.6)$$

$$E' = E - E_k \qquad (1.7)$$

$$E'' = E - E_\ell \qquad (1.8)$$

where E_k and E_ℓ are the resonance energies corresponding to ψ_k and ψ_ℓ. The lower boundary of the energy group is E_a and the group width is ΔE, so that $E_b = E_a + \Delta E$. The expectation value for $F_o(\beta_k, \beta_\ell)$ is then:

$$\overline{F}_o = \int_0^{\Delta E} \left\{ \left\langle \frac{1}{\Gamma_k} \int_{E_1 - \Delta E}^{E_1} \frac{\psi_k(E')}{\psi_k(E') + \beta_k} \left(\frac{dE'}{\Delta E}\right) \right\rangle \left\langle \int_{E_1 - \Delta E}^{E_1} \frac{\psi_\ell(E'')}{\psi_\ell(E'') + \beta_\ell} \right. \right.$$

$$\left. \left. \left(\frac{dE''}{\Delta E}\right) \right\rangle \right\} dE_1 \qquad (1.9)$$

Now, assume that ΔE is large compared to the resonance widths and consider a fixed value of E_1 between 0 and ΔE. Then the second and third integrals in Eq. (1.9) will be approximately independent of E_1 if this is not very close to zero or to ΔE. Only if E_1 or $\Delta E - E_1$ is of the order of or less than Γ_k or Γ_ℓ will the integrals depend on E_1. But this range is assumed to be small. Therefore, E_1 can be replaced by a constant energy, E_2, at the boundaries of the integrals, where

$$\left. \begin{array}{l} \dfrac{E_2}{\Gamma_n} \gg 1 \\[3em] \dfrac{\Delta E - E_2}{\Gamma_n} \gg 1 \end{array} \right\} \text{for } n = 1, 2 \qquad (1.10)$$

But the boundaries are then asymmetric relative to zero. A further

approximation is to replace them by $\pm\, \Delta E/2$. Because we have to

integrate over a constant function of the variable E_1, we finally ob-

tain

$$\overline{F}_o = \left\langle \frac{1}{\Gamma_k} \int_{-\frac{\Delta E}{2}}^{\frac{\Delta E}{2}} \frac{\psi_k(E)}{\psi_k(E)+\beta_k}\ dE \right\rangle \left\langle \int_{-\frac{\Delta E}{2}}^{\frac{\Delta E}{2}} \frac{\psi_\ell(E)}{\psi_\ell(E)+\beta_\ell}\ (\frac{dE}{\Delta E}) \right\rangle \qquad (1.11)$$

To obtain the total interaction effect, the average effect is

multiplied by the product $N_i N_j$, where N_i and N_j are the average

number of resonances belonging to the isotopes i and j respectively

in the interval ΔE. N_i and N_j should be large to obtain good accuracy

using the statistical theory.

In calculating the zero order group cross section for a single

isotope there is good reason to limit the number of resonances with-

in an energy group. This is because the actual composition is not

considered in a correct way and because the asymptotic collision

density is often assumed to be constant within a group, [8]. For

example, calculations of single-isotope cross sections in the re-

sonance region for the Swedish cross section library for fast re-

actors using the computer programme Dorix, [7], have been made

for ^{238}U using a group structure containing about 3 resonances per

group below 1 keV. It is then clear that the errors become large if

we calculate the interaction between ^{238}U and another resonance

absorber for such a group using Eq. (1.11). If, however, the average

spectrum varies only slowly over a large number of such groups,

the average interaction effect over such a range can be calculated with reasonable accuracy using Eq. (1.11). But in the low energy region where the variation of the spectrum over a large number of groups is usually large, Eq. (1.11) does not give even the average interaction effect with reasonable accuracy.

In the present work the interaction effect is investigated between ^{238}U and ^{239}Pu in the energy range between 4 and 244 eV. Because the resonances are resolved for both isotopes in this region, the assumption that the frequency function $P_c(E_r^{(j)} - E_r^{(i)} \mid E_r^{(i)})$ is constant can be checked, and calculations of the interaction can also be made without that assumption. The statistical investigation consists of a classification of the resonances and a chi-squared test of the hypothesis that P_c is constant. Because of the small statistical material, the significance of the result is of course not very large. But the ordered material will also be useful in the explanation of the result of the interaction calculation. The basic assumptions in our calculation of the interaction effect is that the resonances obey the Breit-Wigner formula and that the neutron flux per lethargy unit varies as an inverse total cross section. The first assumption is also used in the programmes Rabble and MC^2. The SDR programme uses cross sections calculated by the programme Genex, [1]. The basis for these cross sections is the multilevel formula, but this is approximated in such a way that single-level resonance parameters can be used. The most obvious effect of a rigorous multilevel formalism is to decrease

the effective cross sections for isotopes for which the total reso-
nance widths are not small relative to their spacings, [9]. Typi-
cal such isotopes are ^{235}U and, although not so pronounced, ^{239}Pu.
Comparison of theoretical and experimental values of the Doppler
effect in the fission activation of ^{235}U have given too large theo-
retical values both in using the single level formalism, [10], and
using the Genex-SDR method, [11]. On the other hand, the cal-
culated Doppler effect for ^{238}U is too large in the work described
in Ref. 10 and is calculated with good accuracy by the Genex-SDR
method. The influence of the single-level approximation upon the
interaction effect between a fissile and a fertile isotope remains
to be investigated.

For the neutron flux we have adopted in the Dorix-Speng
system the intermediate resonance approximation developed by
Goldstein [8]. The collision density is virtually constant,
but the resonance integral for zero temperature is equal to that
given by a second order approximation of the flux if the cross
section is assumed to be symmetric about the resonance energy.
In this approximation, the flux is non-symmetric about the re-
sonance energy but has the same asymptotic value below as above
a resonance, [12]. According to Goldstein, this approximation
may cause non-negligible errors in the Doppler effect, [8]. On
the other hand, a comparison between the narrow resonance ap-
proximation and a more accurate treatment of the flux is made
by Hwang, [13]. He concludes that the correction to the narrow
resonance approximation of the flux can be given as an additional

term and that this correction is small. Because the narrow reso-
nance approximation is a special case of the intermediate resonance
approximation we can assume that these conclusions also hold for
the Dorix-Speng method. But in this work the additional assumption
is made that the narrow resonance approximation holds. This as-
sumption will of course lead to very inaccurate resonance integrals
for the lowest resonances. But it is reasonable to assume that
this will not have a strong influence upon the interaction effect for
a given effective potential cross section.

2. THE DISTANCES BETWEEN THE ^{238}U AND THE ^{239}Pu RESONANCES

We have considered 26 resonances of ^{238}U and 94 reso-
nances of ^{239}Pu in the region 4 to 244 eV. The ^{238}U resonance
data are given in Table 1. They were obtained by M Wallin, [14],
by considering the evaluation by J J Schmidt, [15], and the experi-
mental data by Ashgar, [16]. The ^{239}Pu resonance data are taken
from the work by Derrien et al., [17].

We now consider a specific ^{238}U resonance with the energy
$E_r^{(28)}$. Starting from this energy we count the number of ^{239}Pu
resonances within small intervals, δ. The corresponding resonance
energies, $E_r^{(49)}$, are determined by the expression

$$E_r^{(28)} + (n-1)\ \delta \leq E_r^{(49)} < E_r^{(28)} + n\delta \qquad (2.1)$$

where $n = \pm 1, \pm 2$ etc.

The number of ^{239}Pu resonances defined by Eq. (2.1) is $N^{(49)}(\delta, n | E_r^{(28)})$. This quantity can be normalized in the following way:

$$P_c^{(49)}(\delta, n | E_r^{(28)}) = \frac{N^{(49)}(\delta, n | E_r^{(28)})}{\sum\limits_n N^{(49)}(\delta, n | E_r^{28})} \qquad (2.2)$$

The maximum number of n is arbitrary in Eq:s (2.1) and (2.2). We will later see what limits this number if δ is finite. But first we will give the connection between the given discontinuous quantities and continuous functions.

The quantity defined by Eq. (2.2) is obviously related to the conditional probability $P_c(E_r^{(j)} - E_r^{(i)} | E_r^{(i)})$ given in Eq. (1.3). But this quantity can only be obtained as a limit of results in a large number of trials. Instead of these trials we count $N^{(49)}(\delta, n | E_r^{(28)})$ for all values of $E_r^{(28)}$. The approximate value of $P_c(E_r^{(49)} - E_r^{(28)} | E_r^{(28)})$ is then

$$\overline{P}_c^{(49)}(\delta, n | E_r^{28}) = \frac{\overline{N^{(49)}}(\delta, n | E_r^{28})}{\sum\limits_{r, n} \overline{N^{(49)}}(\delta, n | E_r^{28})} \qquad (2.3)$$

where the index r runs over all resonances of ^{238}U and

$$\overline{N^{(49)}}(\delta, n | E_r^{(28)}) = \sum\limits_r N^{(49)}(\delta, n | E_r^{(28)}) \qquad (2.4)$$

The quantity defined in Eq. (2.3) is of course an approximation to $P_c(E_r^{(49)} - E_r^{(28)} | E_r^{(28)})$ only if the latter quantity is approximately constant over the energy range considered. But this is now our hypothesis. Thus when the quantities given in Eq. (2.4) are known,

we can make a test whether they are consistent with Eq. (1.3) or not, but we cannot show that $P_c(E_r^{(49)} - E_r^{(28)} | E_r^{(28)})$ is constant and that the resonances are independent. But these quantities also give us qualitative information about how the exactly calculated interaction effect may be expected to diverge from the effect obtained assuming a constant frequency function for the spacing.

For the statistical investigation of the distribution of the distances between the resonances we have made a χ^2 test, [18]. In this test, the intervals δ must be so large that they contain a sufficient number of resonances. In Ref. [18], Cramér states that the minimum frequency number for sufficient accuracy is 10. But on the other hand, the maximum value of $|n|$ cannot be too small. Then, for our small statistical material, $|n\delta|$ tends to be large. Now, assume that we count the number of ^{239}Pu resonances at a distance $(n-1)\delta$ to $n\delta$ from a given ^{238}U resonance, and that n is large and positive. The energy range for the ^{239}Pu resonances may then come close to the energy of a higher ^{238}U resonance and they will then be counted twice, namely, for a negative value of n as well. But in this case there will be a correlation between the frequency numbers in different energy intervals which is not taken into account. This type of error must be considered.

In the χ^2 test, we have used $\delta = 1.6$ eV and n has assumed the values -4 to 4. Then, the maximum distance between a uranium and a plutonium resonance which is counted is 6.4 eV, and the distances between the uranium resonances should not be smaller than

12.8 eV in order to avoid counting some plutonium resonances twice. This could only be achieved by eliminating some of the ^{238}U resonances which fortunately are weak. Most of these probably belong to the p-wave. The χ^2 test is performed with and without these resonances. The statistical material becomes too small in the latter case, but the investigation seems still to be worth while because the results do not differ too much in the both cases.

However, there is another drawback with the choice of such a large value of δ. The resonance widths are only in one case of the order of 1 eV, namely for the $J = 0$, $\ell = 0$ resonances in ^{239}Pu. The other resonances are on an average of the order of or less than 0.1 eV. The interaction effect between two resonances can be expected to be weak if the distance between them is much larger than 0.1 eV. The largest contribution to the average effect is given by those plutonium resonances which have a distance from a uranium resonance of the order of or less than the total width, that is, usually less than 0.1 eV. In order to be able to calculate this contribution with reasonable accuracy, the fractional error in the determination of the distance must be small. Thus we have no information about the magnitude of the interaction effect if the interval δ is not known with better accuracy than 0.1 eV. We have therefore tabulated the frequency number for resonances within an interval δ at a distance $(n-1)\delta$ to $n\delta$ for a number of δ-values between 0.05 and 1.6 eV. For all but the largest δ-value, n was given the values -8 to 8. Summing up these frequency numbers over all ^{238}U resonances in a given statistical assembly, the quantity given by Eq. (2.4) was

obtained. As mentioned, we have two assemblies, given by two groups of uranium resonances. In assembly 1, all resonances in Table 1 are considered. In assembly 2, the following resonance energies (given in eV) are neglected: 4.41, 10.25, 11.32, 19.60, 45.20, 63.60, 83.50, 93.20, 125.00, 153.00, 160.00, 173.00, 242.90. Thus only 13 resonances are retained. In this case we give only the frequency numbers for $\delta = 1.6$ eV. Table 2 shows the frequency numbers for assembly 1 when $\delta = 0.05$ to 0.8 eV. We see that for small δ the statistics is very poor. That means that very few plutonium resonances have a distance to some uranium resonance of the order of or less than 0.1 eV, or even 0.5 eV. This can also to some extent be seen from the frequency numbers for $\delta = 0.8$ eV. For $|n| \leq 1$, the average number is 6.0 but the average number for the total assembly is 8.0. We can therefore expect that the average interaction effect is smaller than that calculated by assuming a constant frequency function,

$$P_c(E_r^{(49)} - E_r^{(28)} | E_r^{(28)}).$$

The sum of the frequency numbers for $\delta = 0.8$ eV in Table 2 is 136. That means that some of the plutonium resonances are counted twice, because the total number of them is 94. This source of error is eliminated in assembly 2. Table 3 shows the results for both assemblies with $\delta = 1.6$ eV. For assembly 1, all numbers are now larger than 10 so that the statistical material is reasonably large in this respect. For assembly 2, the numbers are too small for some n-values. The result of a χ^2-test can

therefore not be expected to be very accurate. The hypothesis is that the frequency numbers are distributed in accordance with a constant frequency function. The expectation value is then for every n equal to the mean value, $\langle \overline{N^{49}} \rangle$, for all values of n. We then form the quantity:

$$X^2 = \frac{1}{\langle \overline{N^{49}} \rangle} \sum_n \left[\overline{N^{49}}(\delta, n \,|\, E_r^{28}) - \langle \overline{N^{49}} \rangle \right]^2 \tag{2.5}$$

where

$$\langle \overline{N^{49}} \rangle = \frac{1}{2N+1} \sum_{n=-N}^{N} \overline{N^{49}}(\delta, n \,|\, E_r^{28}) \tag{2.6}$$

The quantity X^2 can now be considered as a point on a statistical distribution, obtained by many equivalent trials, if one could make more than one within a given energy band. Now, if the hypothesis is right, the quantity X^2 approaches a statistical chi-squared distribution when the number of observations approaches infinity. If the maximum number of n-values is M, the number of degrees of freedom, ν, is M-1. In the cases considered in Table 3, we have $\nu = 8$. Now we accept a risk, p, equal to 5 % that we reject the hypothesis even though it is correct. Then,

$$X_\nu^2(p = 0.05) = 15.5$$

That means: If X^2, given by Eq. (2.5), obeys a distribution of type X_ν^2 with $\nu = 8$, it is equal to or larger than 15.5 in 5 % of all cases.

Now, inserting the values given in Table 3 into Eq:s (2.5) and (2.6)

we obtain the results given in Table 4. Thus X^2 is less than 15.5

by a large margin for both statistical assemblies, and the corre-

sponding frequency numbers may well correspond to a constant

frequency function. But we have still to expect a large inaccuracy

in the result from an interaction calculation using this statistical

assumption.

3. INTERACTION CALCULATIONS

3 different methods for calculating the interaction effect

between ^{238}U and ^{239}Pu are used. Their theoretical background

is given in Ref. [7] and [19], and we will here only give the out-

line of the method and some fundamental formulae. The nomen-

clature is the same as in Ref. [7].

In the narrow resonance approximation as well as in the inter-

mediate resonance approximation, the flux is assumed to vary

as the inverse of an effective total cross section. A fundamental

parameter is then the effective potential cross section, $\sigma_{p,i}^{eff}$. This

quantity differs in both approximations, but the flux is in both cases

written in the following form:

$$\Phi(E) = \frac{\sigma_{p,i}^{eff}}{\sigma_{res,i} + \sigma_{p,i}^{eff}} \left(\frac{1}{E}\right) \tag{3.1}$$

where σ_{res} is the total resonance cross section. All cross sections

are given per atom of a resonance absorber, i, for which we want

to calculate the average reaction cross section.

The total resonance cross section is composed of resonances which belong both to the isotope i and to the other isotopes which occur in the composition considered. It is, however, possible to distinguish between these resonances in such a way that the influence upon the average reaction cross section, $\bar{\sigma}_{x,g}^{(i)}$, (reaction x, group g), from other isotopes can be given as a correction to the corresponding single-isotope cross section. In the present case we therefore consider only the interaction between two isotopes, i and j, neglecting all other interaction effects. The most important additional approximation is then that the product of cross section and flux is expanded into another product which contains a geometric series. Furthermore, in calculating the interaction effect the interference between the potential and the resonance cross section is neglected.

Under the assumptions above, the most exact way to calculate the average cross section is to consider each resonance separately, without any statistical assumptions. The corresponding resonance integral can be written in second order approximation:

$$RJ_{x,g}^{(i)} = \frac{\sigma_{p,i}^{eff}}{E_g^*} \sum_k \Gamma_{x,k} \{J(\theta_k, \beta_k) + \sum_{\ell} [F_1(\beta_k, \beta_\ell) - F_2(\beta_k, \beta_\ell)]\} \qquad (3.2)$$

Here, E_g^* is an average group energy, $\Gamma_{x,k}$ is the resonance width for the reaction x in the resonance k, and $J(\theta_k, \beta_k)$ is the usual J-function. The index k runs over all resonances belonging to the isotope i. The functions $F_1(\beta_k, \beta_\ell)$ and $F_2(\beta_k, \beta_\ell)$ give the inter-

action effect, and the index ℓ runs over all resonances belonging to the isotope j. For the first of these functions we have:

$$F_1(\beta_k, \beta_\ell) = \beta_k \frac{\partial F_o(\beta_k, \beta_\ell)}{\partial \beta_k} \tag{3.3}$$

where $F_o(\beta_k, \beta_\ell)$ is given by Eq. (1.5). This function is always negative. The function $F_2(\beta_k, \beta_\ell)$ is always positive and small to second order. It will be neglected here.

The second order approximation of the average flux is given by the expression:

$$\overline{\Phi}_g = \frac{1}{E_g^*} \{ \Delta E - \sum_k \Gamma_k J(\theta_k, \beta_k) - \sum_\ell \Gamma_\ell J(\theta_\ell, \beta_\ell) -$$

$$- \sum_{k, \ell} [\Gamma_k F_1(\beta_k, \beta_\ell) + \Gamma_\ell F_1(\beta_\ell, \beta_k)] \} \tag{3.4}$$

where ΔE is the width of the energy group.

In Eq. (3.4) the second order terms are retained because they are already calculated in the resonance integrals. They will decrease the resulting cross section somewhat, but the consistent second order approximation which retains all terms in Eq. (3.2) as well would be still lower.

We define the total resonance integral as the sum of the resonance integrals for all reactions, x. The relative contribution to this quantity from the interaction effect is in the first order approximation:

$$B_{is}^{(i)} = \frac{\sum_k \Gamma_k \sum_\ell F_1(\beta_k, \beta_\ell)}{\sum_k \Gamma_k J(\theta_k, \beta_k)} \tag{3.5}$$

This expression is always negative, but the corresponding relative contribution to the total resonance cross section may be positive. Indeed, we will see that it is usually positive in the cases considered. This quantity can be written:

$$C_{is}^{(i)} = \frac{(1 + B_{is}^{(i)})(1 - h_i)}{1 - h_i - h_j + g_{ij}} - 1 \tag{3.6}$$

where

$$h_i = \frac{1}{\Delta E} \sum_k \Gamma_k J(\theta_k, \beta_k) \tag{3.7}$$

and a corresponding expression holds for h_j by changing the indices k to the indices ℓ. The quantity g_{ij} is the second order contribution to the average flux:

$$g_{i,j} = \sum_{k,\ell} [\Gamma_k F_1(\beta_k, \beta_\ell) + \Gamma_\ell F_1(\beta_\ell, \beta_k)] \tag{3.8}$$

For calculation of the functions $J(\theta_n, \beta_n)$, where n = k or ℓ, and $F_1(\beta_k, \beta_\ell)$ we need a method to calculate the ψ-function. It is shown in Ref. 20 that one can find a rational approximation to the ψ-function which gives sufficiently accurate results for calculation of the interaction effect. This rational approximation is here used throughout.

Instead of calculating the quantities $B_{is}^{(i)}$ and $C_{is}^{(i)}$, which must be done by considering each resonance separately, one can make the assumption about constant frequency functions. Eq:s (3.5) and (3.6) are then replaced by expressions which involve averages over

the resonances in each isotope. Following Fischer, [5], we define a function J^* by the following expression:

$$J^*(\theta_n, \beta_n) = J(\theta_n, \beta_n) + \beta_n \frac{\partial J(\theta_n, \beta_n)}{\partial \beta_n}$$

(3.9)

Further, we introduce the following average quantities:

$$h_i = \frac{\langle \Gamma J \rangle_i}{\overline{D}_i}$$

(3.10)

and

$$h_i^* = \frac{\langle \Gamma J^* \rangle_i}{\overline{D}_i}$$

(3.11)

where $\overline{D}_i$ is the mean spacing between the resonances in isotope i. When the resonances are resolved, h_i is identical with the h_i given by Eq. (3.7). Similarly,

$$h_i^* = \frac{1}{\Delta E} \sum_k \Gamma_k J^*(\theta_k, \beta_k)$$

(3.11a)

The relative contribution from the interaction effect to the total resonance integral is then

$$B_{stat}^{(i)} = - h_j (1 - \frac{h_i^*}{h_i})$$

(3.12)

The corresponding contribution to the total resonance cross section can be obtained from the following expression:

$$C_{stat}^{(i)} = \frac{1 + \dfrac{h_i^*}{h_i} h_j}{1 + h_i h_j - h_i h_j^* - h_i^* h_j} - 1 \tag{3.13}$$

Eq. (3.12) and the numerator of the r.h.s. of Eq. (3.13) are first order approximations. The second order terms are neglected here in order to be consistent with the corresponding expressions for isolated resonances, Eq:s (3.5) and (3.6).

In the Dorix-Speng system, the interaction between different isotopes is calculated from the average total resonance cross sections for different isotopes, $\langle \sigma_{r,g}^{(i)} \rangle$, corresponding to the effective potential cross sections, $\sigma_{p,i}^{eff}$. If the interaction between different resonances within isotope i is neglected, the following expression can easily be derived:

$$h_i = \frac{\langle \sigma_{r,g}^{(i)} \rangle}{\sigma_{p,i}^{eff} + \langle \sigma_{r,g}^{(i)} \rangle} \tag{3.14}$$

For h_i^*, however, we must make additional assumptions. These are:

a) the resonance parameters are all equal to their average values,

b) the interdependence between J and J^* is independent of the temperature.

A formula can then be derived which gives an approximate value of h_i^* as a function of average cross sections only, namely,

$$h^{*}_{i, app} = \frac{h_i}{2} \left[1 - \left(\frac{h_i}{h_{i, \infty}} \right)^2 \right] \tag{3.15}$$

where

$$h_{i, \infty} = \frac{\langle \sigma^{(i)}_{r, \infty, g} \rangle}{\sigma^{eff}_{p, i}} \tag{3.16}$$

and $\langle \sigma^{(i)}_{r, \infty, g} \rangle$ is the total resonance cross section for infinite dilution.

The third method for calculating the interaction was then obtained by inserting Eq. (3.15) into Eq:s (3.12) and (3.13). The relative contribution from the interaction effect to the total resonance integral is then $B^{(i)}_{app}$ and the contribution to the corresponding cross section is $C^{(i)}_{app}$ where

$$B^{(i)}_{app} = - \frac{h_j}{2} \left[1 + \left(\frac{h_i}{h_{i, \infty}} \right)^2 \right] \tag{3.17}$$

and

$$C^{(i)}_{app} = \frac{1 + \frac{h_j}{2} \left[1 - \left(\frac{h_i}{h_{i, \infty}} \right)^2 \right]}{1 + h_i h_j \left[\left(\frac{h_i}{h_{i, \infty}} \right)^2 + \left(\frac{h_j}{h_{j, \infty}} \right)^2 \right]} - 1 \tag{3.18}$$

Computations using the three methods have been made for the interaction effect between resonances in groups which have been 18 to 30 eV wide. In the following the indices i and j are replaced by the numbers 28 and 49, corresponding to ^{238}U and ^{239}Pu respectively. For the temperature 300 $^{\circ}$K we have assumed two different values for $\sigma_{p,\,49}^{\text{eff}}$, namely 210.3 b and 410.3 b. The former value is of the same order as those obtained for ZPR-III, core 48 and Zebra, core 6. The latter value is more typical for large fast power reactors. With $\sigma_{p,\,49}^{\text{eff}} = 410.3$ b, calculations have also been made for the temperature 1800 $^{\circ}$K. In all cases, $\sigma_{p,\,28}^{\text{eff}}$ is assumed to be 50.64 b. The relative number of plutonium atoms to uranium atoms is then 0.2 if $\sigma_{p,\,49}^{\text{eff}} = 210.3$ and 0.1 if $\sigma_{p,\,49}^{\text{eff}} = 410.3$.

One series of calculations is made to obtain the effect on the resonance integral and on the average cross sections of ^{239}Pu due to the interaction with ^{238}U. The symbol E_L in the tables means the lower energy boundary of each group. The highest energy boundary is 244 eV. N_{49} and N_{28} are the numbers of ^{239}Pu and ^{238}U resonances, respectively. The results are given in Tables 5-7 and in Figures 1-6. It will be observed that the statistical calculations generally give a much larger interaction effect than the more exact calculations. This means that the average resonance integral over the whole region is larger than that obtained by statistical calculations of the interaction effect. The cross section corresponding to statistical calculations is too small, but the correction is usually positive in all approximations. The increase of the cross section depends upon the effect of the plutonium resonances upon the average

flux and it is not at all negligible. We observe also that the approximation of the statistical calculations, leading to Eq:s (3.17) and (3.18), gives results which are rather close to the results from more exact statistical calculations, compared to the errors relative to the isolated-resonance calculations. The approximation is of course best for T = 300 ^{O}K. Further, although the interaction effect on the resonance integral of course increases considerably from 300 ^{O}K to 1800 ^{O}K, the relative change in cross section is so small that the error in the Doppler effect will not be large if the interaction effect is neglected. The interaction correction on the Doppler effect is largest for the energy group 196-220 eV, where the cross section change between 300 and 1800 ^{O}K is about 10 per cent without this correction. The change in $C_{is}^{(49)}$ is here 2.5 per cent. Finally, a change in the plutonium content will of course have only an insignificant effect upon the interaction effect for the same isotope.

Another series of calculations is made for the interaction effect on ^{238}U due to ^{239}Pu resonances. The results of them are given in Tables 8-10 and in Figures 7-12. The general conclusions we can draw are the same as from the earlier calculations on the reverse effect. But it is important to note that although the effect on the resonance integral of ^{238}U is somewhat smaller than the corresponding effect on ^{239}Pu, this is not the case concerning the average cross section change. For $\sigma_{p,49}^{eff}$ = 210.3 b, the changes in the uranium cross sections are remarkably high, achieving a value of 28.7 per cent in one energy group. The errors in the

approximate statistical method are for the uranium case somewhat higher than for the plutonium case, compared to the error between the method using an exact derivative of the J-function and the isolated-resonance method. This may be explained by the fact that the Doppler broadening of the uranium resonances is on an average larger than that for plutonium resonances. This can be seen from Tables 11 and 12, which give the quantities h_i and h_i^* for different isotopes and temperatures. The definitions of h_i and h_i^* are given in Eq:s (3. 7) and (3. 11 a), and the approximate value for h_i^* which we call $h_{i,app}^*$ is calculated from Eq. (3. 15). It may be observed that on an average

$$\left[\frac{h_{28,app}^*}{h_{28}^*} - 1 \right] > \left[\frac{h_{49,app}^*}{h_{49}^*} - 1 \right] \tag{3.19}$$

at least when we weight the numbers in the single groups with the resonance integrals in these groups. The resonance integrals are proportional to h_i, and comparing the results for T = 300 and T = 1800 we find:

$$\sum_{groups} \left[\frac{h_{49}(T = 1800)}{h_{49}(T = 300)} - 1 \right] = 0.133$$

$$\sum_{groups} \left[\frac{h_{28}(T = 1800)}{h_{28}(T = 300)} - 1 \right] = 0.304$$

From the assumption b) made in the derivation of Eq. (3.15) we then see that the inequality given by Eq. (3.19) is logical. Still, the highest relative errors, which are of the order of 40 per cent, occur for $h^{*}_{49, app}$, but they should be weighted with small cross sections in order to get the average effect. And we see from the results that if we accept the statistical approximation, we introduce an error which is on the average larger than the additional error we introduce by using the approximate value for h^{*}_{i}.

Assume now that we have calculated the interaction effect for some specific composition and temperature using the approximate statistical method. We then ask: Can we correct these results by using results from exact and approximate calculations for another composition and/or temperature? Our calculations indicate that the approximate values usually can be improved by a constant correction if we always use the same group division. For example, in the region 196-220 eV, an increase of 3.0-3.5 % in the correction of the ^{238}U cross section obtained by the approximate method should significantly improve the results in all three cases considered. But a definite answer to the question probably demands a larger amount of calculation.

4. CONCLUSIONS

The main results may be summarized as follows:

a.   The statistical frequency function for the distance between the ^{239}Pu and the ^{238}U resonances can be assumed to be constant. But in the region below 244 eV there are so few

resonances that the divergence from the constant function is large, especially in intervals which contribute significantly to the interaction effect.

b. The interaction effect in the region 4-244 eV is much smaller than that obtained by assuming a constant frequency function for the resonance spacing. As a consequence of this, the cross sections are usually higher than if the interaction is neglected, but the correction varies between -5 and +29 per cent. The correction is not very strongly dependent upon the temperature.

c. If the statistical approximation is accepted for practical reasons, an additional approximation can be accepted which makes the interaction effect very easy to calculate. This approximation is probably most valuable in the higher energy region where the statistical approximation is better.

d. The results from approximate statistical calculations for not too strongly varying compositions could probably be improved by using constant corrections for each group considered. These corrections should be calculated for a composition and a temperature which represents an average of the actual values.

5. ACKNOWLEDGEMENTS

REFERENCES

1. BRISSENDEN R J & DURSTON C,
    A Method of Calculating the Spectrum in the Resonance
    Region for Dilute Fast Reactors.
    Proceedings of the Symposium on Fast and Epithermal
    Neutron Spectra in Reactors, Harwell, December 1963,
    Session 2, p. 112.

2. KIER P H & ROBBA AA,
    Rabble: a Program for Computation of Resonance Ab-
    sorption in Multiregion Reactor Cells. 1967.
    (ANL-7326).

3. TOPPEL B J, RAGO A L, & O'SHEA D M,
    MC^2: A Code to Calculate Multigroup Cross Sections. 1967.
    (ANL-7318).

4. ROWLANDS J L,
    The Effect on the Effective Cross Sections for a Resonant
    Material of Resonances in the Cross Section of a Second
    Constituent. 1963.
    (AEEW-M 398).

5. FISCHER E A,
    The Overlap Effect of Resonances of Different Fuel Isotopes
    in Doppler-Coefficient Calculations for Fast Reactors.
    Nukleonik, 8 (1966), 146.

6. NICHOLSON R B & FISCHER E A,
    The Doppler Effect in Fast Reactors.
    Advances in Nuclear Science and Technology, Vol. 4 (1968)
    p. 109.

7. HÄGGBLOM H,
    Computation of Resonance-Screened Cross Sections by the
    Dorix-Speng System. 1968.
    (AE-334).

8. GOLDSTEIN R,
    Intermediate Resonance Absorption.
    Reactor Physics in the Resonance and Thermal Regions.
    Proc. National topical meeting of the ANS, San Diego, Calif.,
    Feb. 7-9, 1966. Ed. by A J Goodjohn and G C Pomraning
    (M.I.T. Press) Cambr., Mass. 1966, Vol. 2. Resonance
    absorption, p. 37.

9. ADLER F T & ADLER D B,
    Calculation of Resonance Integrals for Fissile Materials.
    Reactor Physics in the Resonance and Thermal Regions,
    Proc. National topical meeting of the ANS, San Diego, Calif.,
    Feb. 7-9, 1966. Ed. by A J Goodjohn and GC Pomraning
    (M.I.T. Press) Cambr., Mass. 1966. Vol. 2. Resonance
    absorption. p. 47.

10. HÄGGBLOM H & TIRÉN L I,
A comparison of Theoretical and Experimental Values of the
Activation Doppler Effect in Some Fast Reactor Spectra. 1968.
(AE-331).

11. PERKIN J L et al.,
Measurements and Calculations of the Doppler Effect on the
Reactions $^{238}U(n,\gamma)$, $^{235}U(n,f)$ and $^{239}Pu(n,f)$ with Neutrons
in the Energy Range 0-25 keV.
Journ. Nucl. En. A/B, 20 (1966), 921.

12. GOLDSTEIN R,
Spectral Distribution of Neutrons in the Vicinity of a Resonance.
Nucl. Sc. Eng., 19 (1964), 359.

13. HWANG R N,
Effect of Fluctuations in Collision Density on Fast-reactor
Doppler Effect Calculations.
Proceedings of the Conference on Safety, Fuels, and Core
Design in Large Fast Power Reactors, 1965.
(ANL-7120, p. 449).

14. WALLIN M,
1968. AB Atomenergi, Sweden. (Internal report TPM-RFR-686).

15. SCHMIDT J J
Neutron Cross Sections for Fast Reactor Materials. p. 1. 1966.
(KFK-120, Part 1).

16. ASHGAR M et al.,
Low-Energy Neutron Resonance Parameters of ^{238}U.
Nucl. Phys., 85 (1966), 305.

17. DERRIEN H et al.,
Sections efficaces totale et de fission du ^{239}Pu.
Nuclear Data for Reactors. Proceedings of a Conference,
Paris, 1966. (IAEA) Vienna 1967, Vol. 2, p. 195.

18. CRAMÉR H,
Sannolikhetskalkylen och några av dess användningar. 3 uppl.
(Almqvist & Wiksell) Stockholm, 1961.

19. HÄGGBLOM H,
Calculation of Resonance Interaction Effects Using a Rational
Approximation to the Symmetric Resonance Line Shape Func-
tion. 1968.
(AE-329).

Table 1:   Resonance parameters used for ^{238}U

Statistical weight function $g = 1$ for all resonances.

E_r eV	Γ_n eV	$\Gamma_\gamma \cdot 10^3$ eV	E_r eV	Γ_n eV	$\Gamma_\gamma \cdot 10^3$ eV
4.410	$1.10 \cdot 10^{-7}$	20.00	93.20	$3.00 \cdot 10^{-3}$	23.80
6.676	$1.52 \cdot 10^{-3}$	27.00	102.58	$6.45 \cdot 10^{-2}$	25.20
10.25	$1.50 \cdot 10^{-6}$	23.80	116.80	$2.43 \cdot 10^{-2}$	22.90
11.32	$3.60 \cdot 10^{-7}$	23.80	125.00	$1.42 \cdot 10^{-5}$	23.80
19.60	$9.70 \cdot 10^{-7}$	23.80	145.80	$7.10 \cdot 10^{-4}$	23.80
20.98	$8.98 \cdot 10^{-3}$	27.60	153.00	$3.70 \cdot 10^{-5}$	23.80
36.77	$3.10 \cdot 10^{-2}$	27.60	160.00	$1.04 \cdot 10^{-5}$	23.80
45.20	$8.30 \cdot 10^{-7}$	23.80	165.40	$2.92 \cdot 10^{-3}$	18.00
63.60	$5.50 \cdot 10^{-6}$	23.80	173.00	$3.34 \cdot 10^{-5}$	23.80
66.08	$2.37 \cdot 10^{-2}$	24.90	189.96	$1.46 \cdot 10^{-1}$	23.00
80.78	$1.91 \cdot 10^{-3}$	21.17	209.46	$5.10 \cdot 10^{-2}$	22.20
83.50	$7.00 \cdot 10^{-6}$	23.80	237.40	$2.71 \cdot 10^{-2}$	20.60
89.50	$8.50 \cdot 10^{-5}$	23.80	242.90	$1.60 \cdot 10^{-4}$	23.80

Table 2:   Frequency numbers, $\overline{N^{49}}(\delta,n|E_r^{28})$, for statistical assembly 1, giving the number of ^{239}Pu resonances in the interval $(n-1)\delta$ to $n\delta$ from some ^{238}U resonance

δ eV	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
0.05		2			1				1	1							
0.10	2	2		1		2	1		1	1				3	1	2	
0.20	2	1	1		2	2	1	3	1	1		4	2	1	2	2	
0.40	4	4	4	3	6	2	2	3	4	1	6	3	2	9	4	6	4
0.80	4	10	7	11	7	8	9	4	7	7	5	13	10	6	14	8	6

Table 3: Frequency numbers, $\overline{N^{49}}$ $(\delta, n | E_r^{28})$, for statistical assemblies 1 and 2, with $\delta = 1.6$ eV

n	Assembly 1	Assembly 2
-4	13	8
-3	17	12
-2	13	6
-1	17	10
0	11	4
1	12	3
2	23	11
3	20	7
4	14	9

Table 4: Mean square deviation about the mean of the frequency numbers given in Table 3

Assembly	$\langle \overline{N^{49}} \rangle$	x^2
1	16.11	7.75
2	8.33	6.00

Table 5: Relative corrections to the resonance integral and to the effective cross sections of ^{239}Pu due to interaction with ^{238}U

T = 300 $^{\circ}$K, $\sigma^{eff}_{p,49}$ = 410.3 b, $\sigma^{eff}_{p,28}$ = 50.64 b.

E_L eV	N_{49}	N_{28}	$B^{(49)}_{is}$	$B^{(49)}_{stat}$	$B^{(49)}_{app}$	$C^{(49)}_{is}$	$C^{(49)}_{stat}$	$C^{(49)}_{app}$
4.0	10	6	-0.025	-0.063	-0.065	0.103	0.044	0.041
26.5	10	2	-0.014	-0.056	-0.057	0.094	0.036	0.035
54.0	9	2	-0.080	-0.061	-0.060	-0.008	0.017	0.017
74.5	10	4	-0.013	-0.019	-0.018	0.016	0.007	0.003
96.0	10	2	-0.101	-0.117	-0.116	0.073	0.043	0.043
122.0	10	2	-0.011	-0.008	-0.008	-0.002	0.002	0.001
143.0	5	3	-0.022	-0.020	-0.021	0.009	0.010	0.009
166.0	10	2	-0.075	-0.112	-0.117	0.077	0.026	0.020
196.0	10	2	-0.016	-0.060	-0.063	0.070	0.016	0.013
220.0	10	2	-0.013	-0.045	-0.047	0.049	0.010	0.008

Table 6: Relative corrections to the resonance integral and to the effective cross sections of ^{239}Pu due to interaction with ^{238}U

T = 1800 $^{\circ}$K, $\sigma^{eff}_{p,49}$ = 410.3 b, $\sigma^{eff}_{p,28}$ = 50.64 b.

E_L eV	N_{49}	N_{28}	$B^{(49)}_{is}$	$B^{(49)}_{stat}$	$B^{(49)}_{app}$	$C^{(49)}_{is}$	$C^{(49)}_{stat}$	$C^{(49)}_{app}$
4.0	10	6	-0.029	-0.076	-0.081	0.127	0.051	0.043
26.5	10	2	-0.015	-0.068	-0.071	0.109	0.037	0.032
54.0	9	2	-0.116	-0.085	-0.085	-0.018	0.022	0.017
74.5	10	4	-0.018	-0.029	-0.029	0.027	0.010	0.009
96.0	10	2	-0.143	-0.165	-0.169	0.090	0.046	0.038
122.0	10	2	-0.018	-0.011	-0.011	-0.008	0.001	0.001
148.0	5	3	-0.073	-0.032	-0.034	-0.037	0.012	0.010
166.0	10	2	-0.086	-0.137	-0.145	0.094	0.024	0.015
196.0	10	2	-0.035	-0.097	-0.101	0.095	0.017	0.011
220.0	10	2	-0.029	-0.079	-0.082	0.071	0.010	0.005

Table 7: Relative corrections to the resonance integral and to the effective cross sections of ^{239}Pu due to interaction with ^{238}U

$T = 300\ ^\circ K$, $\sigma_{p,49}^{eff} = 210.3$ b, $\sigma_{p,28}^{eff} = 50.64$ b

E_L eV	N_{49}	N_{28}	$B_{is}^{(49)}$	$B_{stat}^{(49)}$	$B_{app}^{(49)}$	$C_{is}^{(49)}$	$C_{stat}^{(49)}$	$C_{app}^{(49)}$
4.0	10	6	-0.027	-0.059	-0.060	0.105	0.048	0.046
26.5	10	2	-0.016	-0.058	-0.058	0.113	0.045	0.045
54.0	9	2	-0.068	-0.030	-0.076	0.052	0.022	0.030
74.5	10	4	-0.019	-0.017	-0.015	0.006	0.008	0.009
96.0	10	2	-0.105	-0.105	-0.101	0.058	0.047	0.050
122.0	10	2	-0.013	-0.007	-0.007	-0.003	0.003	0.002
148.0	5	3	-0.022	-0.016	-0.016	0.004	0.010	0.010
166.0	10	2	-0.077	-0.100	-0.100	0.069	0.033	0.033
196.0	10	2	-0.028	-0.054	-0.056	0.058	0.023	0.019
220.0	10	2	-0.016	-0.042	-0.043	0.019	0.014	0.012

Table 8: Relative corrections to the resonance integral and to the effective cross sections of ^{238}U due to interaction with ^{239}Pu

$T = 300\ ^\circ K$, $\sigma_{p,28}^{eff} = 50.64$ b, $\sigma_{p,49}^{eff} = 410.3$ b.

E_L eV	N_{28}	N_{49}	$B_{is}^{(28)}$	$B_{stat}^{(28)}$	$B_{app}^{(28)}$	$C_{is}^{(28)}$	$C_{stat}^{(28)}$	$C_{app}^{(28)}$
4.0	6	10	-0.034	-0.059	-0.064	0.119	0.069	0.063
26.5	2	10	-0.013	-0.037	-0.040	0.079	0.042	0.039
54.0	2	9	-0.032	-0.072	-0.086	0.099	0.096	0.082
74.5	4	10	-0.038	-0.075	-0.089	0.154	0.090	0.075
96.0	2	10	-0.029	-0.035	-0.041	0.058	0.045	0.038
122.0	2	10	-0.069	-0.045	-0.050	-0.001	0.023	0.018
148.0	3	5	-0.037	-0.028	-0.035	0.025	0.032	0.025
166.0	2	10	-0.015	-0.021	-0.023	0.034	0.023	0.021
196.0	2	10	-0.012	-0.026	-0.033	0.061	0.039	0.031
220.0	2	10	-0.014	-0.025	-0.034	0.059	0.041	0.031

Table 9: Relative corrections to the resonance integral and to the effective cross sections of ^{238}U due to interaction with ^{239}Pu

$T = 1800 \, ^{\circ}K, \quad \sigma_{p,28}^{eff} = 50.64 \, b, \quad \sigma_{p,49}^{eff} = 410.3 \, b$

E_L eV	N_{28}	N_{49}	$B_{is}^{(28)}$	$B_{stat}^{(28)}$	$B_{app}^{(28)}$	$C_{is}^{(28)}$	$C_{stat}^{(28)}$	$C_{app}^{(28)}$
4.0	6	10	-0.038	-0.058	-0.075	0.146	0.091	0.072
26.5	2	10	-0.014	-0.040	-0.050	0.083	0.059	0.048
54.0	2	9	-0.057	-0.064	-0.094	0.147	0.120	0.086
74.5	4	10	-0.048	-0.096	-0.112	0.176	0.089	0.073
96.0	2	10	-0.030	-0.034	-0.048	0.072	0.058	0.041
122.0	2	10	-0.107	-0.058	-0.063	-0.037	0.016	0.010
148.0	3	5	-0.114	-0.043	-0.049	-0.051	0.030	0.023
166.0	2	10	-0.013	-0.020	-0.025	0.040	0.028	0.021
196.0	2	10	-0.016	-0.024	-0.037	0.066	0.046	0.032
220.0	2	10	-0.016	-0.028	-0.039	0.065	0.044	0.032

Table 10: Relative corrections to the resonance integral and to the effective cross sections of ^{238}U due to interaction with ^{239}Pu

$T = 300 \, ^{\circ}K, \quad \sigma_{p,28}^{eff} = 50.64 \, b, \quad \sigma_{p,49}^{eff} = 210.3 \, b$

E_L eV	N_{28}	N_{49}	$B_{is}^{(28)}$	$B_{stat}^{(28)}$	$B_{app}^{(28)}$	$C_{is}^{(28)}$	$C_{stat}^{(28)}$	$C_{app}^{(28)}$
4.0	6	10	-0.052	-0.088	-0.096	0.188	0.103	0.094
26.5	2	10	-0.018	-0.061	-0.058	0.127	0.055	0.058
54.0	2	9	-0.096	-0.169	-0.149	0.287	0.116	0.136
74.5	4	10	-0.070	-0.113	-0.135	0.239	0.135	0.113
96.0	2	10	-0.044	-0.053	-0.064	0.095	0.072	0.060
122.0	2	10	-0.111	-0.076	-0.082	0.005	0.040	0.034
148.0	3	5	-0.058	-0.039	-0.049	0.027	0.044	0.035
166.0	2	10	-0.026	-0.036	-0.039	0.057	0.039	0.036
196.0	2	10	-0.021	-0.043	-0.055	0.105	0.066	0.052
220.0	2	10	-0.023	-0.043	-0.059	0.107	0.071	0.054

Table 11: The quantities h_i and h_i^x for different isotopes h_i^x calculated from Eq. (3.11a) and from Eq. (3.15)
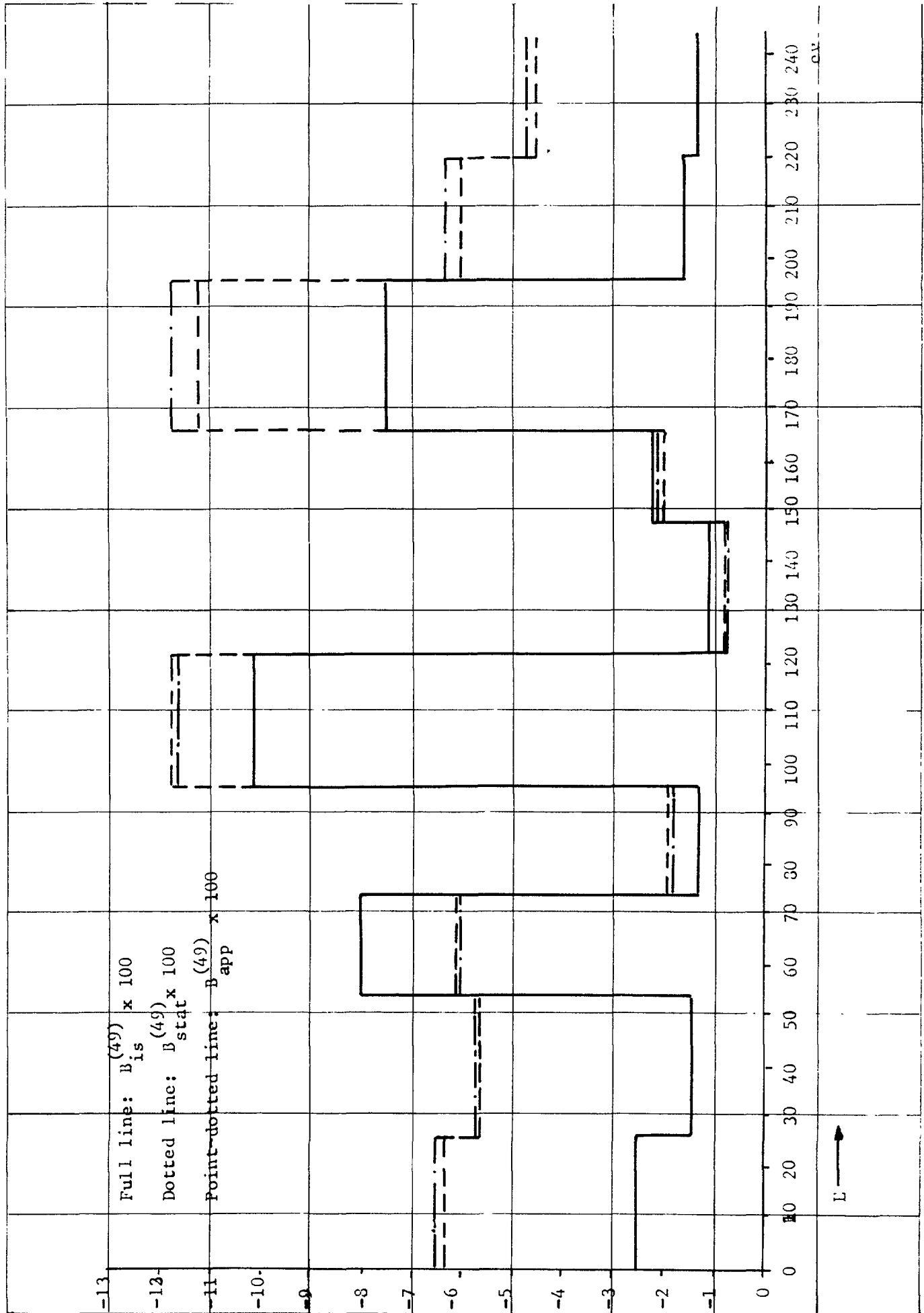
$T = 300\ ^{\circ}K$, $\sigma_{p,49}^{eff} = 410.3$ b, $\sigma_{p,28}^{eff} = 50.64$ b

E_L eV	h_{49}	h_{49}^x	$h_{49,app}^x$	h_{28}	h_{28}^x	$h_{28,app}^x$
4.0	0.129	0.053	0.051	0.107	0.058	0.054
26.5	0.079	0.031	0.031	0.093	0.050	0.046
54.0	0.172	0.041	0.044	0.081	0.046	0.040
74.5	0.165	0.050	0.056	0.027	0.015	0.012
96.0	0.032	0.023	0.023	0.162	0.009	0.008
122.0	0.069	0.013	0.011	0.010	0.003	0.003
148.0	0.062	0.021	0.020	0.031	0.016	0.013
166.0	0.046	0.009	0.007	0.140	0.075	0.070
196.0	0.066	0.015	0.012	0.077	0.047	0.038
220.0	0.067	0.013	0.011	0.056	0.035	0.028

Table 12: The quantities h_i and h_i^x for different isotopes h_i^x calculated from Eq. (3.11a) and from Eq. (3.15)

$t = 1800\ ^{\circ}K$, $\sigma_{p,49}^{eff} = 410.3$ b, $\sigma_{p,28}^{eff} = 50.64$ b

E_L ev	h_{49}	h_{49}^x	$h_{49,app}^x$	h_{28}	h_{28}^x	$h_{28,app}^x$
4.0	0.149	0.060	0.054	0.127	0.077	0.063
26.5	0.100	0.036	0.032	0.106	0.063	0.053
54.0	0.187	0.042	0.040	0.109	0.071	0.054
74.5	0.188	0.055	0.055	0.041	0.020	0.016
96.0	0.095	0.022	0.020	0.214	0.137	0.106
122.0	0.074	0.010	0.007	0.012	0.003	0.002
148.0	0.074	0.021	0.018	0.045	0.019	0.015
166.0	0.050	0.008	0.005	0.163	0.098	0.081
196.0	0.072	0.012	0.009	0.115	0.076	0.056
220.0	0.073	0.010	0.006	0.090	0.056	0.043

The percentage correction to the resonance integral
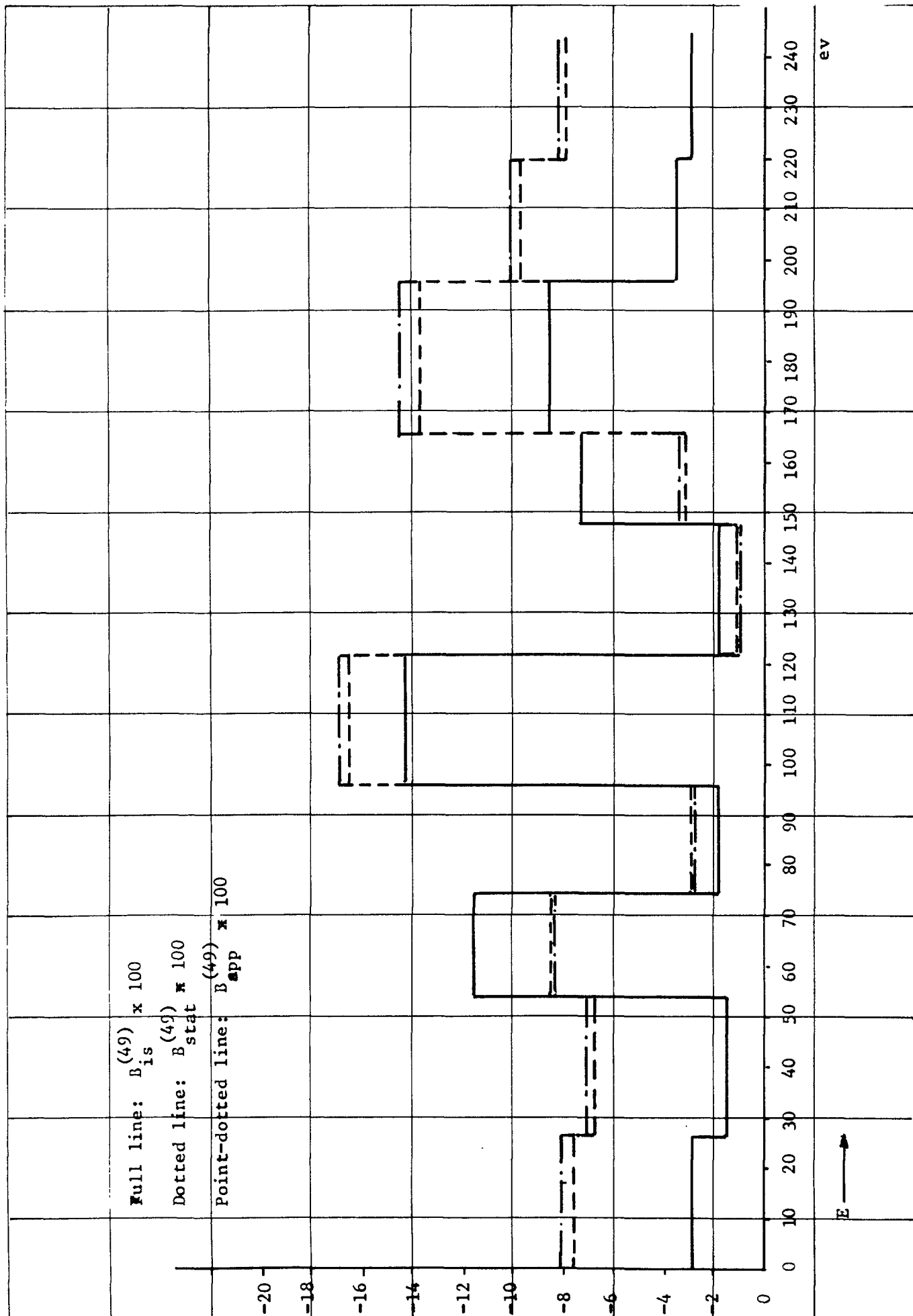of ^{239}Pu due to interaction with ^{238}U.   Fig. 1

$T = 300^\circ$K, $\sigma_{p,49}^{eff} = 410.3$ b, $\sigma_{p,28}^{eff} = 50.64$ b

Full line: $B_{is}^{(49)} \times 100$

Dotted line: $B_{stat}^{(49)} \times 100$

Point-dotted line: $B_{app}^{(49)} \times 100$

The percentage correction to the resonance integral of ^{239}Pu due to interaction with ^{238}U.   Fig. 2

$T = 1800^{\circ}K$, $\sigma_{p,49}^{eff} = 410.3$ b, $\sigma_{p,28}^{eff} = 50.64$ b

Full line: $B_{is}^{(49)} \times 100$

Dotted line: $B_{stat}^{(49)} \times 100$

Point-dotted line: $B_{pp}^{(49)} \times 100$

E

The percentage correction to the resonance integral
of ^{239}Pu due to interaction with ^{238}U.   Fig. 3

$T = 300^\circ K$, $\sigma^{eff}_{p,49} = 210.3$ b, $\sigma^{eff}_{p,28} = 50.64$ b
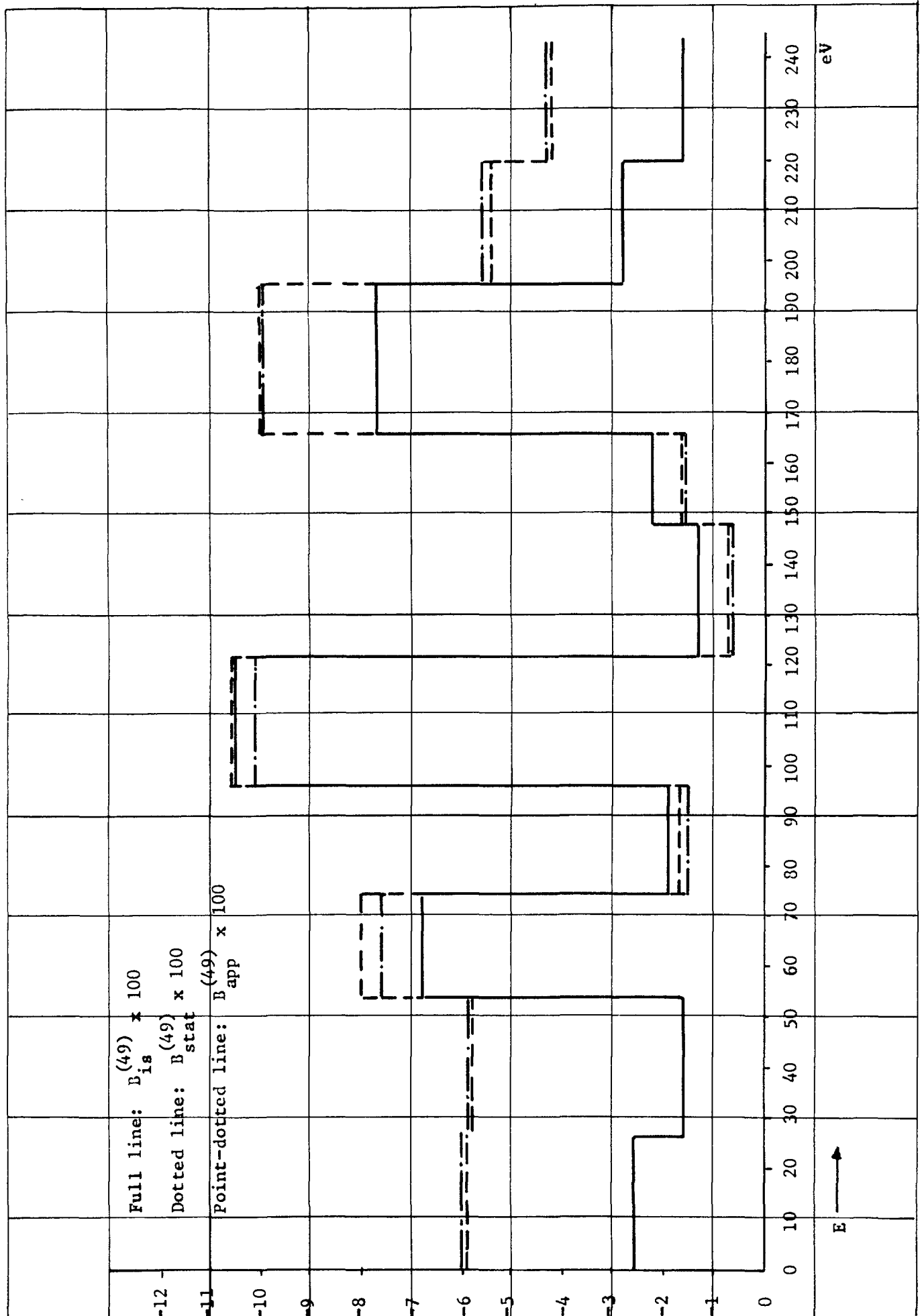
Full line: $B^{(49)}_{is} \times 100$

Dotted line: $B^{(49)}_{stat} \times 100$

Point-dotted line: $B^{(49)}_{app} \times 100$

eV

E

The percentage correction to the average cross section of ^{239}Pu due to interaction with ^{238}U. Fig. 4

$T = 300^\circ$K, $\sigma_{p,49}^{eff} = 410.3$ b, $\sigma_{p,28}^{eff} = 50.64$
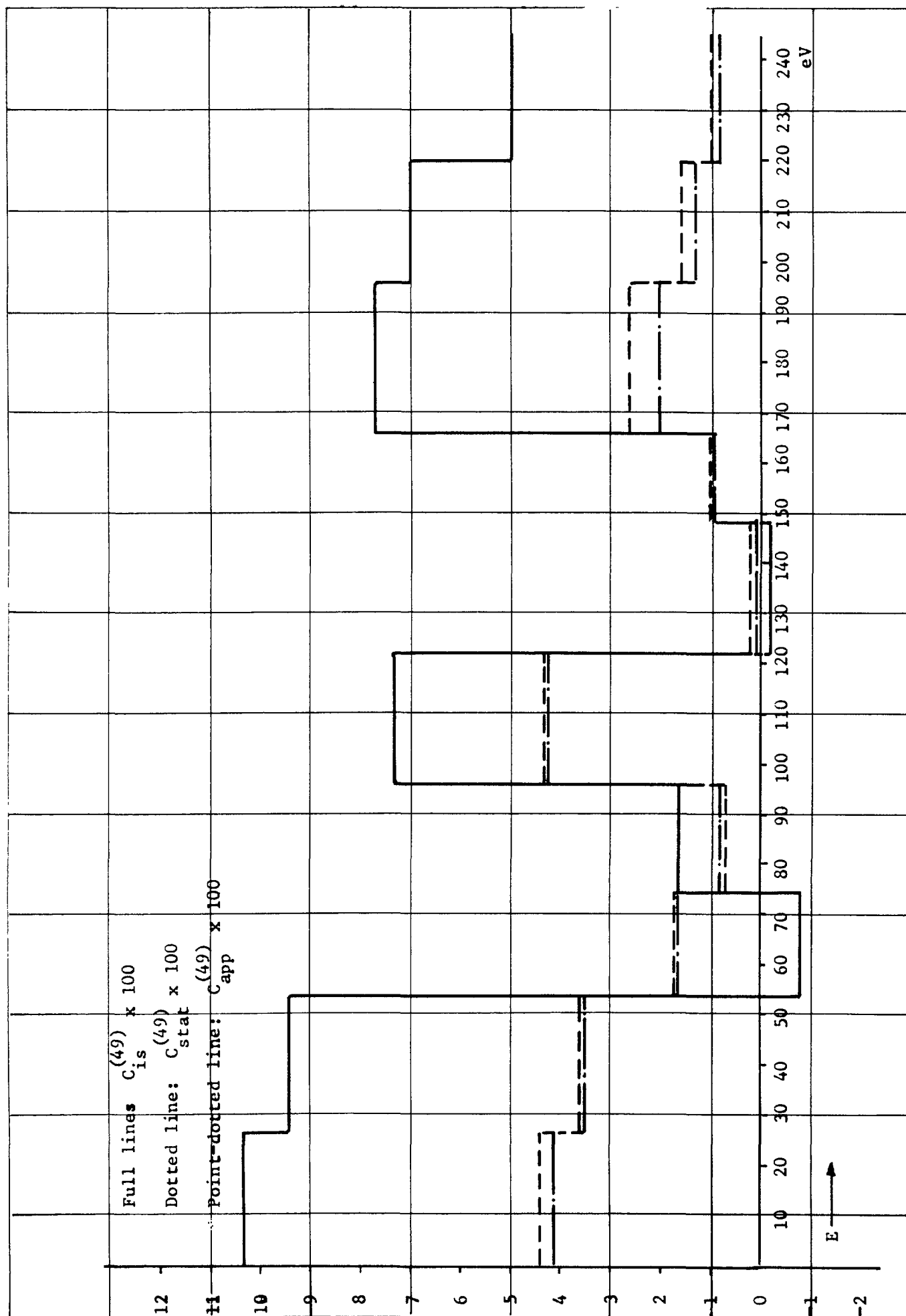
Full lines $C_{is}^{(49)} \times 100$

Dotted line: $C_{stat}^{(49)} \times 100$

Point-dotted line: $C_{app}^{(49)} \times 100$

eV

E

The percentage correction to the average cross
section of ^{239}Pu due to interaction with ^{238}U.   Fig. 5

$T = 1800^{\circ}K$, $\sigma_{p,49}^{eff} = 410.3$ b, $\sigma_{p,28}^{eff} = 50.64$ b
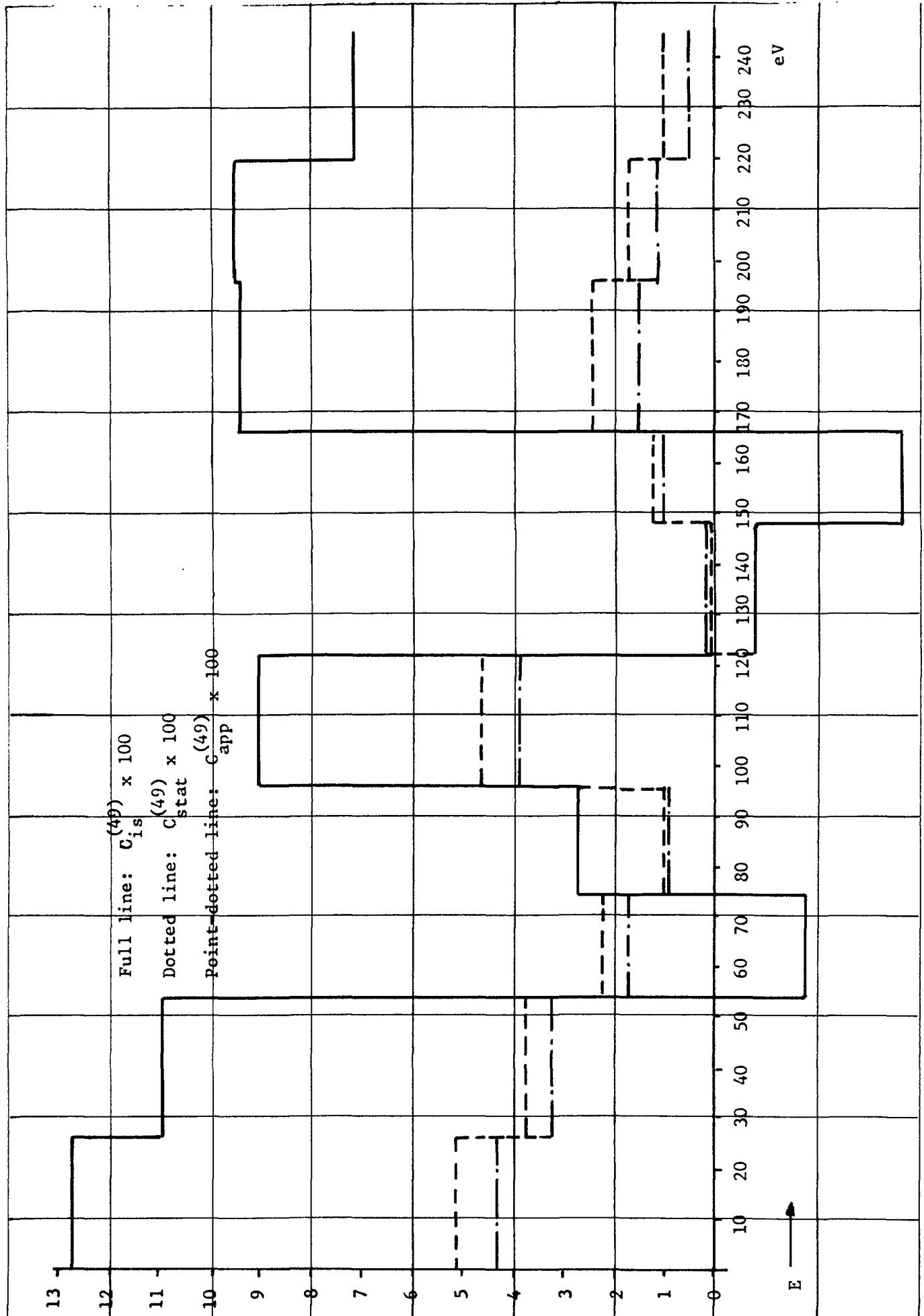
Full line: $C_{is}^{(49)} \times 100$

Dotted line: $C_{stat}^{(49)} \times 100$

Point-dotted line: $C_{app}^{(49)} \times 100$

The percentage correction to the average cross section of ^{239}Pu due to interaction with ^{238}U.   Fig. 6

$T = 300^\circ K$, $\sigma_{p,49}^{eff} = 210.3$ b, $\sigma_{p,28}^{eff} = 50.64$ b
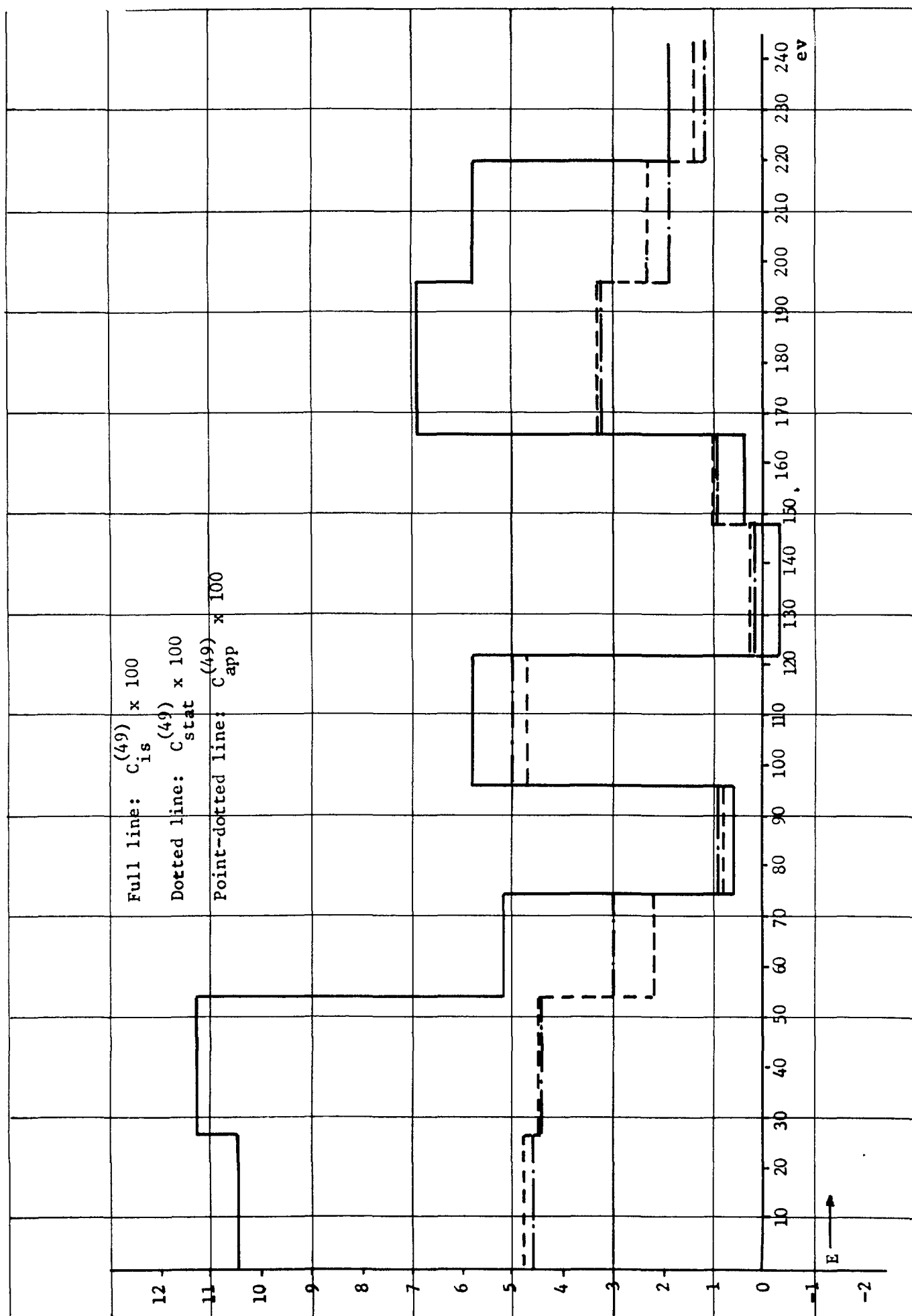
Full line: $C_{is}^{(49)} \times 100$

Dotted line: $C_{stat}^{(49)} \times 100$

Point-dotted line: $C_{app}^{(49)} \times 100$

E

ev

The percentage correction to the resonance integral
of ^{238}U due to interaction with ^{239}Pu.   Fig. 7

$T = 300^{\circ}K,\ \sigma_{p,28}^{eff} = 50.64\ b,\ \sigma_{p,49}^{eff} = 410.3\ b$
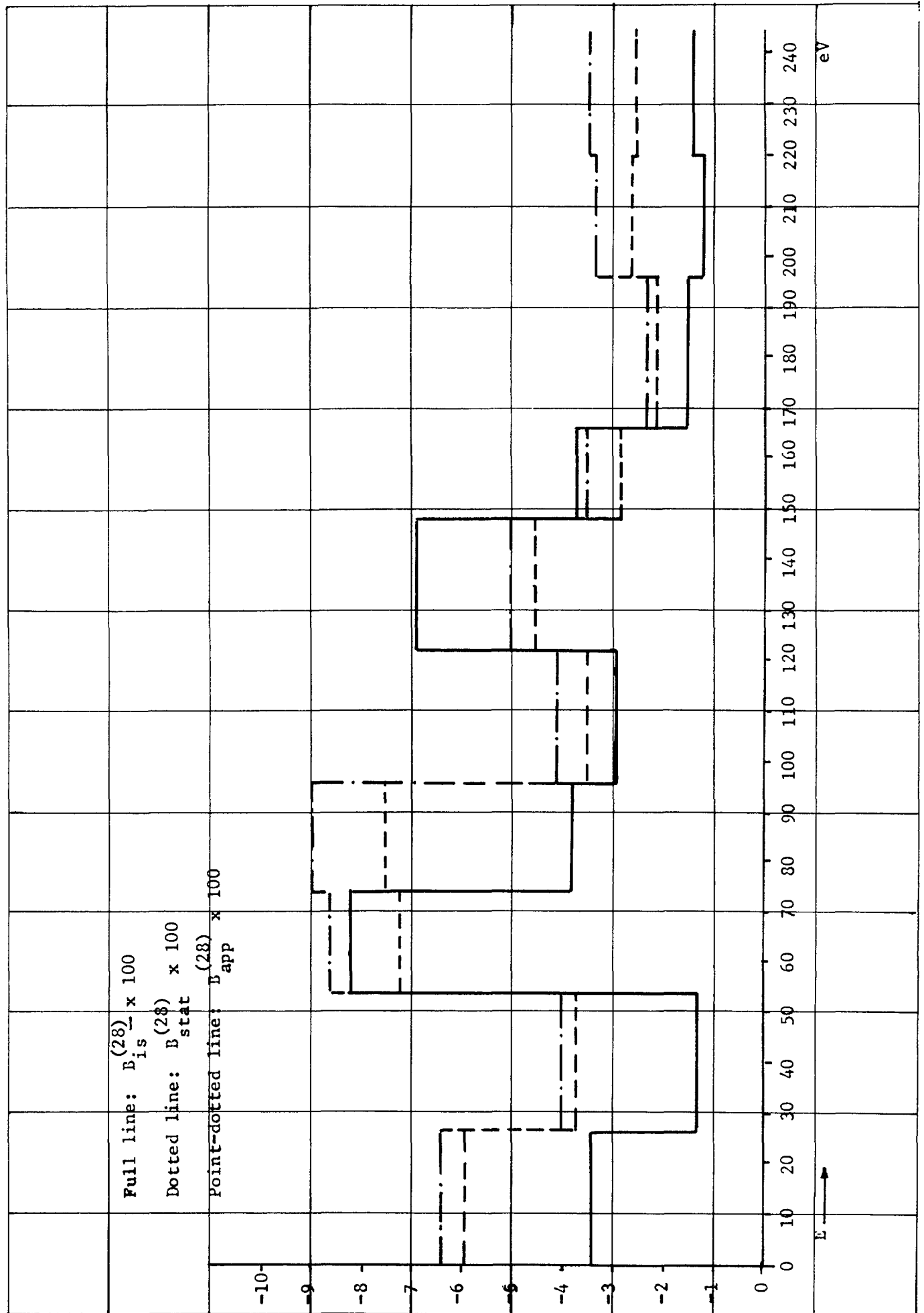
Full line: $B_{is}^{(28)} \times 100$

Dotted line: $B_{stat}^{(28)} \times 100$

Point-dotted line: $B_{app}^{(28)} \times 100$

The percentage correction to the resonance inte-
gral of ^{238}U due to interaction with ^{239}Pu.   Fig. 8

$T = 1800^{\circ}K$, $\sigma^{eff}_{p,28} = 50.64$ b, $\sigma^{eff}_{p,49} = 410.3$ b
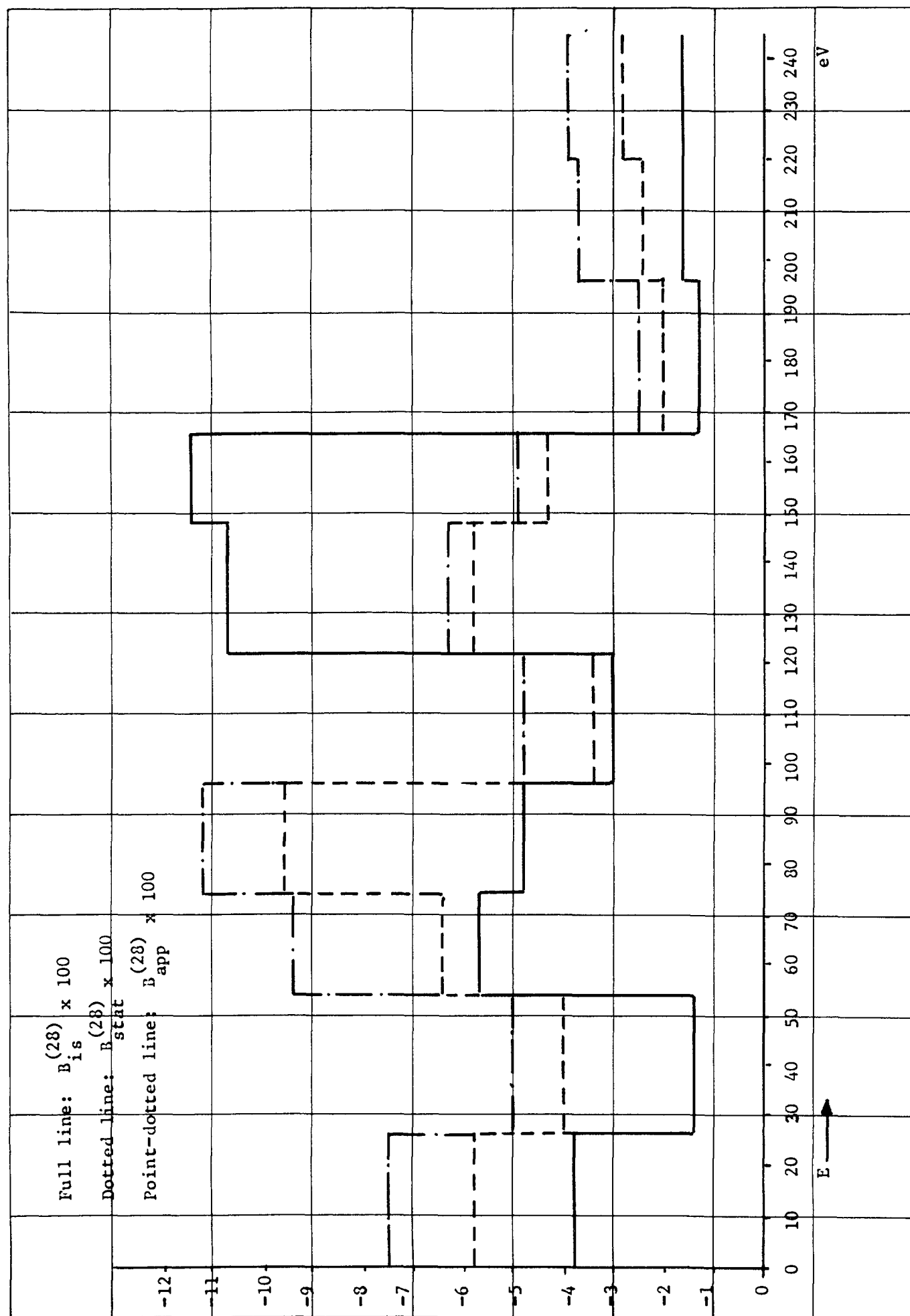
Full line: $B^{(28)}_{is} \times 100$

Dotted line: $B^{(28)}_{stat} \times 100$

Point-dotted line: $B^{(28)}_{app} \times 100$

The percentage correction to the resonance integral of ^{238}U due to interaction with ^{239}Pu.

T = 300°K, $\sigma_{p,28}^{eff}$ = 50.64 b, $\sigma_{p,49}^{eff}$ = 210.3 b

Fig. 9

Full line: $B_{is}^{(28)}$ x 100

Dotted line: $B_{stat}^{(28)}$ x 100

Point-dotted line: $B_{app}^{(28)}$ x 100
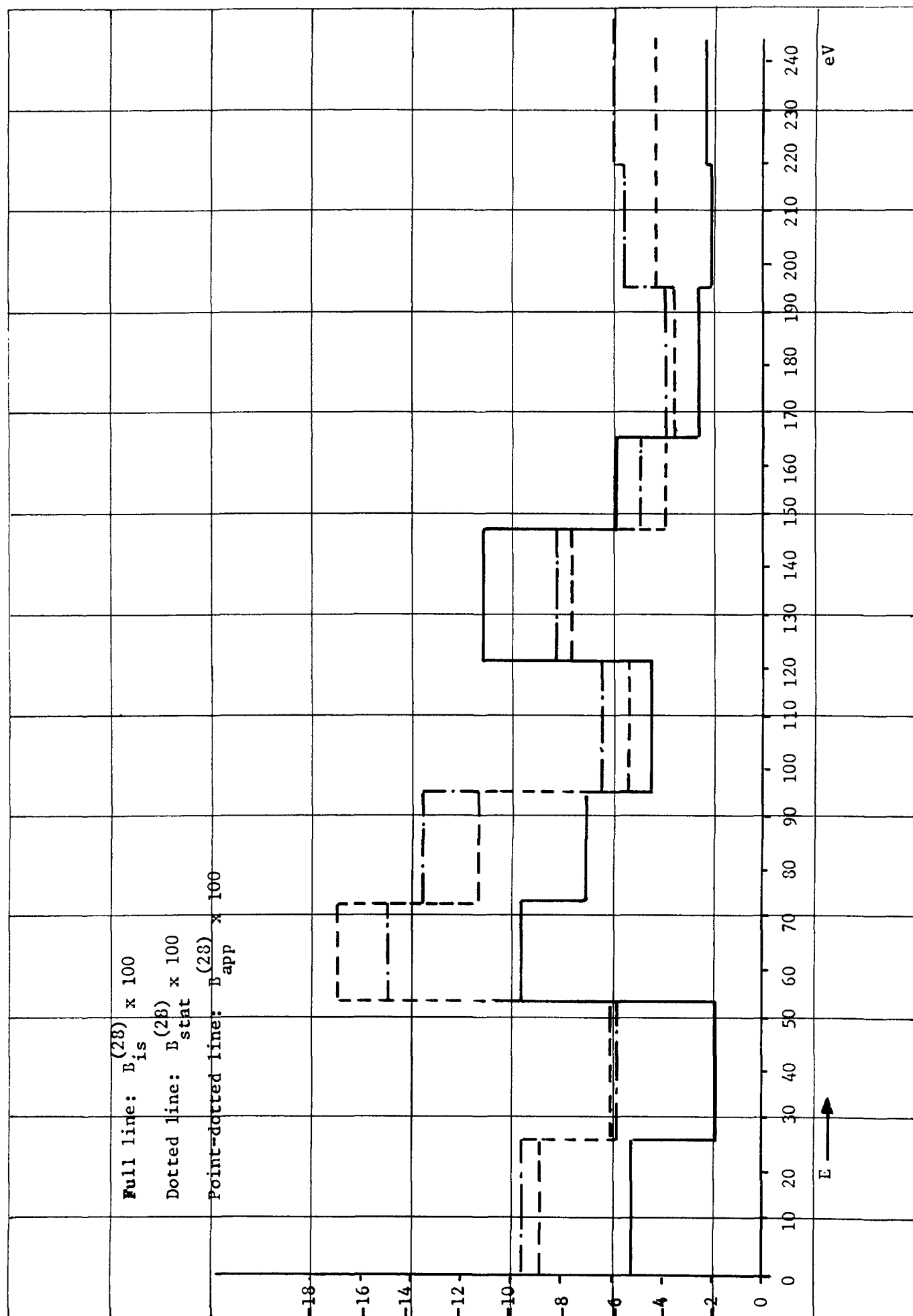
The percentage correction to the effective cross
section of ^{238}U due to interaction with ^{239}Pu.   Fig. 10

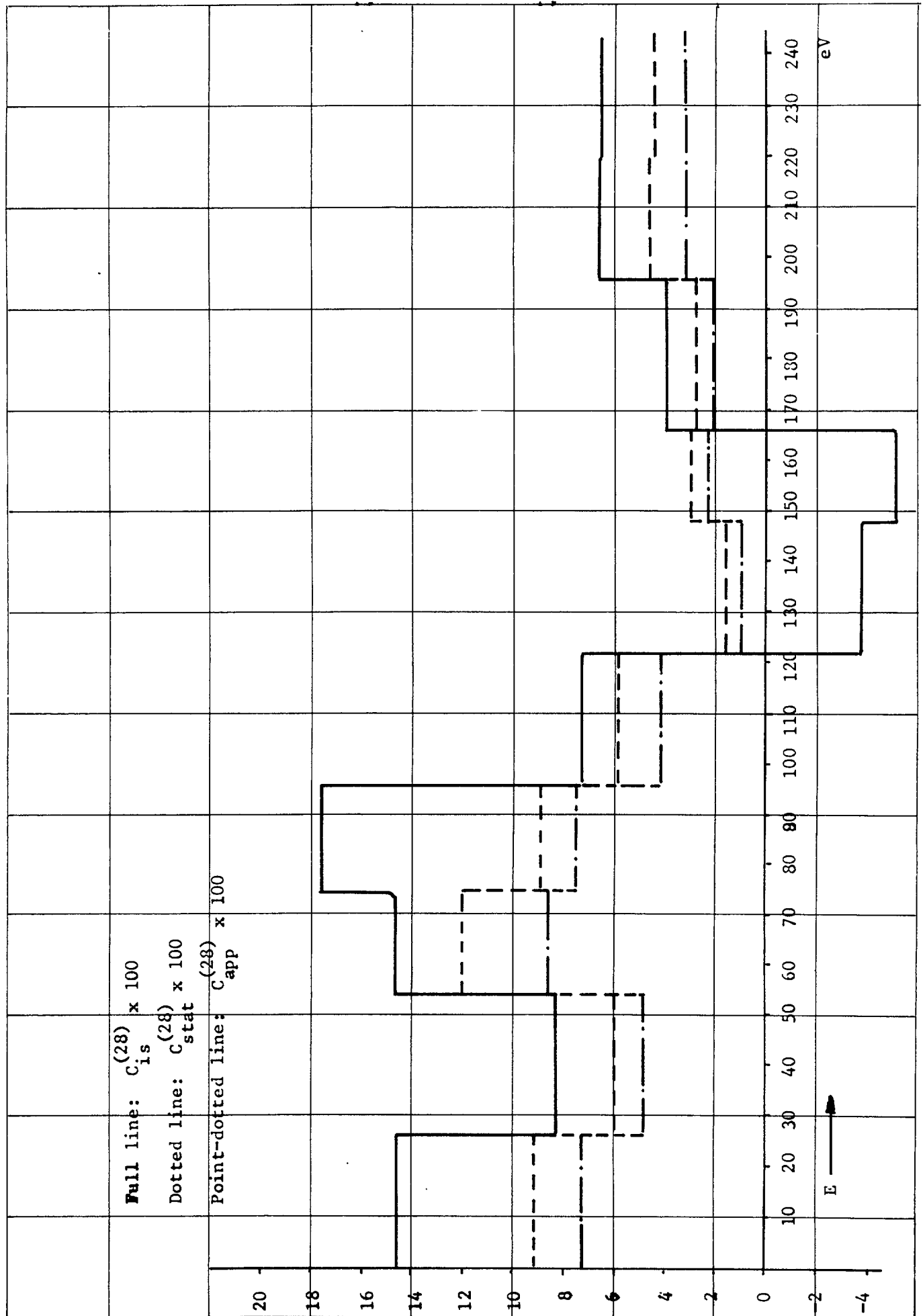$T = 300^{\circ}K$, $\sigma_{p,28}^{eff} = 50.64$ b, $\sigma_{p,49}^{eff} = 410.3$ b
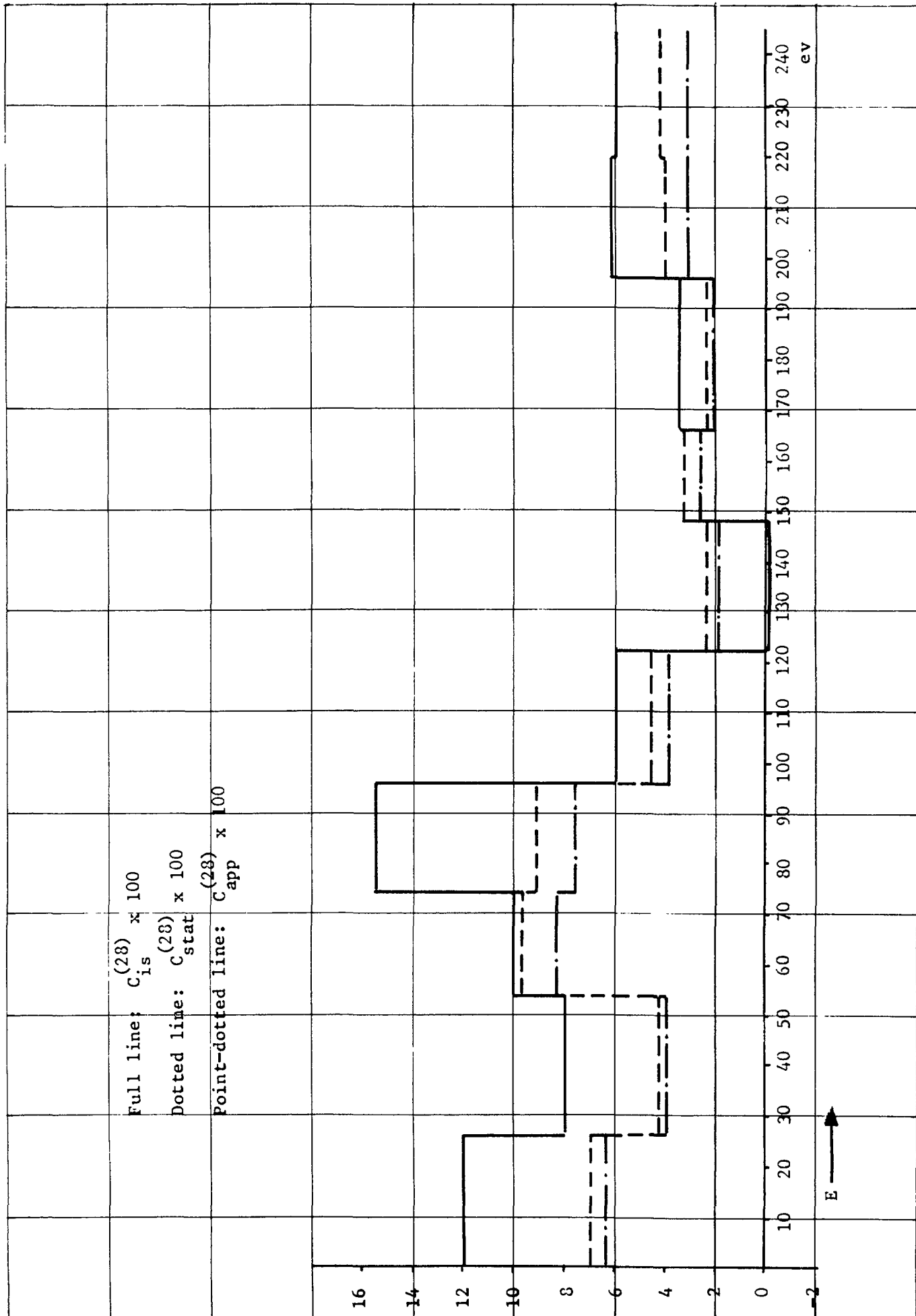
Full line: $C_{is}^{(28)} \times 100$

Dotted line: $C_{stat}^{(28)} \times 100$

Point-dotted line: $C_{app}^{(28)} \times 100$

The percentage correction to the effective cross
section of ^{238}U due to interaction with ^{239}Pu.   Fig. 11

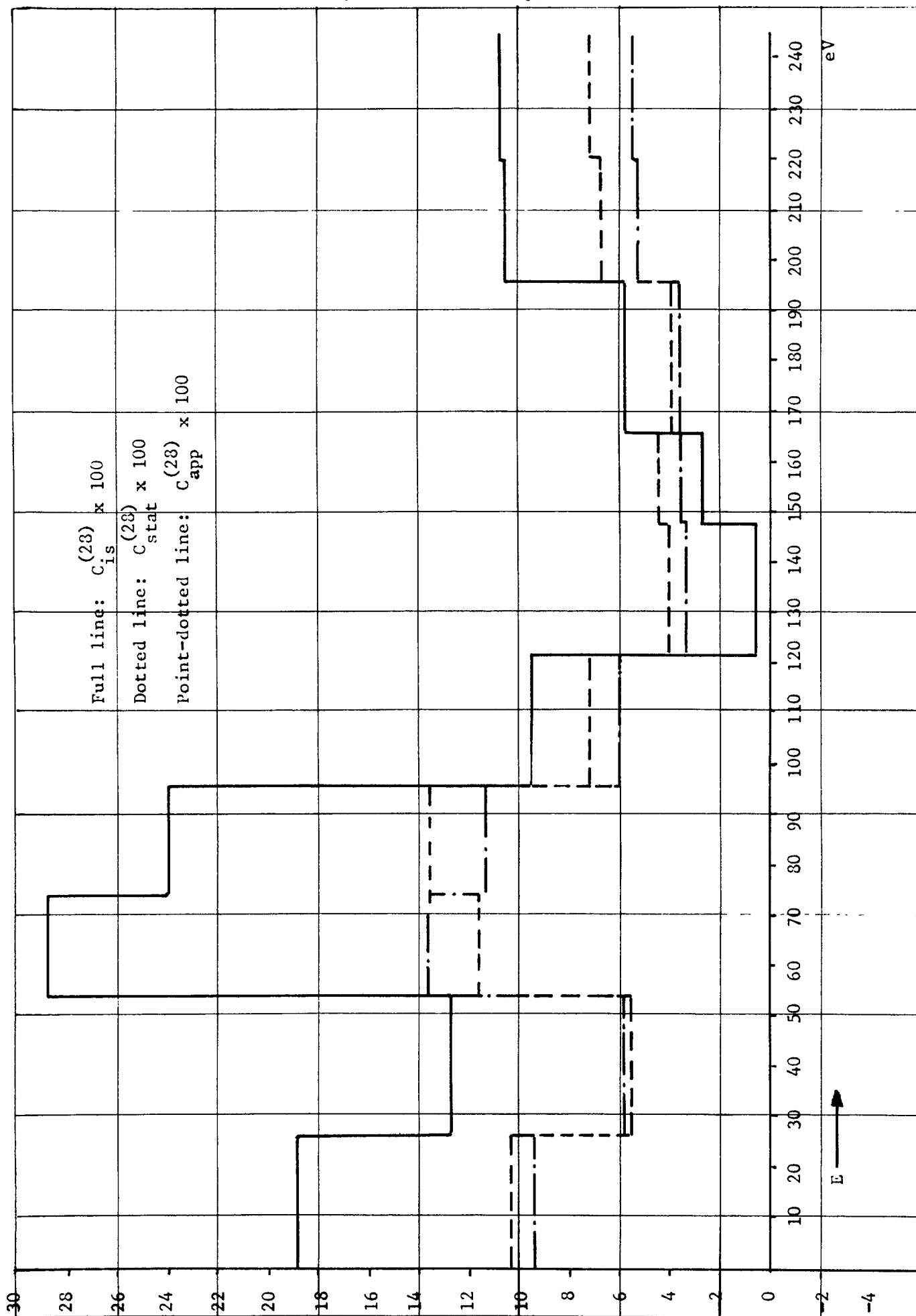$T = 1800^\circ K$, $\sigma_{p,28}^{eff} = 50.64$ b, $\sigma_{p,49}^{eff} = 410.3$ b

Full line: $C_{is}^{(28)} \times 100$

Dotted line: $C_{stat}^{(28)} \times 100$

Point-dotted line: $C_{app}^{(28)} \times 100$

The percentage correction to the effective cross
section of ^{238}U due to interaction with ^{239}Pu.   Fig. 12

$T = 300^{o}K$, $\sigma^{eff}_{p,28} = 50.64$ b, $\sigma^{eff}_{p,49} = 210.3$ b

Full line: $C^{(23)}_{is} \times 100$

Dotted line: $C^{(23)}_{stat} \times 100$

Point-dotted line: $C^{(23)}_{app} \times 100$

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